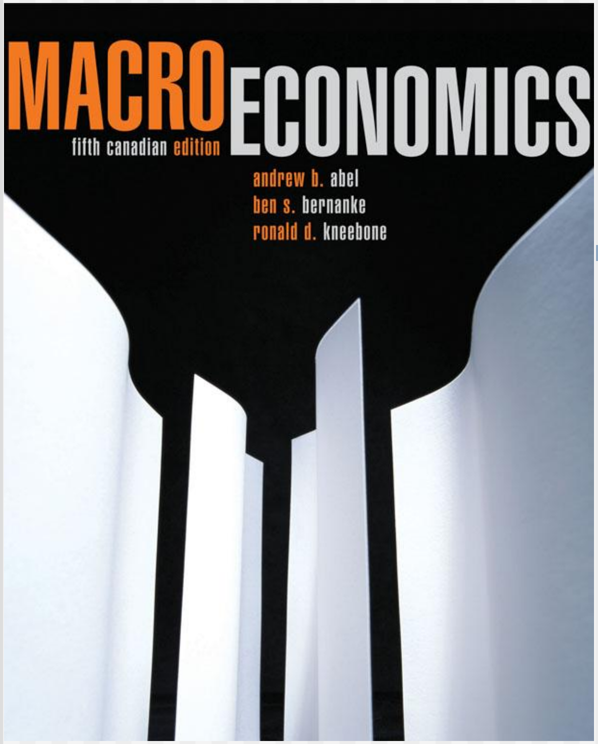


Chapter 6

Long-Run Economic Growth

Table 6.1

Economic Growth in Eight Major Countries, 1870–1998

	Levels of Real GDP per Capita				Annual growth rate
Country	**1870**	**1913**	**1950**	**1998**	**1870–1998**
Australia	3 645	5 715	7 493	20 390	1.4%
Canada	1 695	4 447	7 437	20 559	2.0
France	1 876	3 485	5 270	19 558	1.8
Germany	1 821	3 648	3 881	17 799	1.8
Japan	737	1 385	1 926	20 084	2.6
Sweden	1 664	3 096	6 738	18 685	1.9
United Kingdom	3 191	4 921	6 907	18 714	1.4
United States	2 445	5 301	9 561	27 331	1.9

Note: Figures are in US dollars at 1990 prices, adjusted for differences in the purchasing power of the various national currencies.

Source: Data from Angus Maddison, *The World of Economy: A Millennial Perspective*, Paris: OECD, 2001.

The Sources of Economic Growth

- The relationship between output and inputs is described by the production function:

$$\boldsymbol{Y = AF(K, N)}$$

- For Y to grow, either quantities of K or N must grow or productivity (A) must improve, or both.

The Growth Accounting Equation

- The growth accounting equation:

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha_K \frac{\Delta K}{K} + \alpha_N \frac{\Delta N}{N}$$

$\Delta Y/Y$ is the rate of output growth;
$\Delta K/K$ is the rate of capital growth;
$\Delta N/N$ is the rate of labour growth;
$\Delta A/A$ is the rate of productivity growth.

The Growth Accounting Equation (continued)

α_K = elasticity of output with respect to capital (about 0.3 in Canada);

α_N = elasticity of output with respect to labour (about 0.7 in Canada).

- The elasticity of output with respect to capital/labour is the percentage increase in output resulting from a one per cent increase in the amount of capital stock/labour.

The Growth Accounting Equation (continued)

- There is another way to derive the equation using logs. The production function can be written as:

$$ln(Y) = ln(A) + \alpha_K ln(K) + \alpha_L ln(N)$$

- The term "*ln*" means the natural log of the variable in question. Since the first derivative of the log of a variable is approximately equal to the proportional change then:

$$dln(Y) = dln(A) + \alpha_K dln(K) + \alpha_L dln(N)$$

- This is approximately equal to growth accounting equation in slide 4.

Growth Accounting

- Growth accounting measures empirically the relative importance of capital stock, labour and productivity for economic growth.
- The impact of changes in capital and labour is estimated from historical data.
- The impact of changes in total factor productivity is treated as a residual, that is, not otherwise explained.

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha_K \frac{\Delta K}{K} - \alpha_N \frac{\Delta N}{N}$$

TABLE 6.2

The Steps of Growth Accounting: A Numerical Example

Step 1. Obtain measures of output growth, capital growth, and labour growth over the period to be studied.

Example:

$$\text{Output growth} = \frac{\Delta Y}{Y} = 40\%;$$

$$\text{Capital growth} = \frac{\Delta K}{K} = 20\%;$$

$$\text{Labour growth} = \frac{\Delta N}{N} = 30\%.$$

Step 2. Using historical data, obtain estimates of the elasticities of output with respect to capital and labour, a_K and a_N.

Example: $a_K = 0.3$ and $a_N = 0.7$.

Step 3. Find the contributions to growth of capital and labour.

Example:

$$\text{Contribution to output growth of growth in capital} = a_K \frac{\Delta K}{K} = (0.3)(20\%) = 6\%;$$

$$\text{Contribution to output growth of growth in labour} = a_N \frac{\Delta N}{N} = (0.7)(30\%) = 21\%.$$

Step 4. Find productivity growth as the residual (the part of output growth not explained by capital or labour).

Example:

$$\text{Productivity growth} = \frac{\Delta A}{A} = \frac{\Delta Y}{Y} - a_K \frac{\Delta K}{K} - a_N \frac{\Delta N}{N}$$

$$= 40\% - 6\% - 21\% = 13\%.$$

Growth Accounting and the Productivity Slowdown

- Rapid output growth during 1962-1973 has slowed in 1974-2006.
- Much of the decline in output growth can be accounted for by a decline in productivity growth.
- The slowdown in productivity starting in 1974 was widespread, suggesting a global phenomenon.

Table 6.3

Sources of Economic Growth in Canada (Percent per Year)

	(1) 1891–1910	(2) 1910–1926	(3) 1926–1956
Source of Growth			
Labour growth	1.8	1.0	0.6
Capital growth	0.8	0.3	0.6
Total input growth	2.6	1.3	1.2
Productivity growth	0.8	1.2	2.7
Total output growth	**3.4**	**2.5**	**3.9**

	(4) 1962–1973	(5) 1974–1986	(6) 1987–2006
Source of Growth			
Labour growth	2.2	1.5	1.1
Capital growth	1.3	1.0	0.4
Total input growth	3.5	2.5	1.5
Productivity growth	1.8	0.6	1.3
Total output growth	**5.3**	**3.1**	**2.8**

Source: Adapted from the following: 1891–1956: N. Harvey Lithwick, *Economic Growth in Canada: A Quantitative Analysis*, 2nd ed., Toronto: University of Toronto Press, 1970; 1962–2006: Statistics Canada, CANSIM II series v1078498, v246119, v3860085, and Labour Force Historical Review, 2004. The Lithwick findings do not incorporate the recent Urquhart revisions to pre-1926 GDP.

The Post-1973 Slowdown in Productivity Growth

- Explanations of the reduced growth in productivity are:
  - Output measurement problem:
    - *Quality of output and inputs*
    - *Shifts to lower productivity sectors*
    - *Measurement problems have always been there*
  - Technological depletion and slow commercial adaptation:
    - *The easy stuff has been used up*
    - *Firms slow to take up new technologies*

The Post-1973 Slowdown in Productivity Growth (cont’d)

- The dramatic rise in oil prices:
  - *Old capital was energy inefficient*
  - *Timing and the fact that the slowdown was international in scope make this an attractive story*
  - *But price of capital did not fall and energy was not that important for several sectors*
  - *As well, productivity should have picked up when oil prices fell in the 1980s – it didn’t*
- The beginning of new industrial revolution:
  - *The beginning of the computer age*
  - *Takes time to adopt new technologies*
  - *Have seen some pick up in productivity*
  - *The industrial revolution was like this*

Growth Dynamics: The Neoclassical Growth Model

- Accounting approach is just that – it is not an explanation of growth.
- The neoclassical growth model:
  - *clarifies how capital accumulation and economic growth are interrelated;*
  - *explains the factors affecting a nation's long-run standard of living;*
  - *is suggestive of how a nation's rate of economic growth evolves over time; and*
  - *can say something about convergence – do poor countries/regions catch up?*

Assumptions of The Model of Economic Growth

- Assume that:
  - *population (N_t) is growing;*
  - *at any point in time the share of the population of working age is fixed;*
  - *both the population and workforce grow at a fixed rate n;*
  - *the economy is closed and there are no government purchases.*

Setup of the Model of Economic Growth

- Part of the output produced each year is invested in new capital or in replacing worn-out capital (I_t).
- The part of output not invested is consumed (C_t).

$$C_t = Y_t - I_t$$

The per-Worker Production Function

- The production function in per worker terms is:

$$y_t = A_t f(k_t) \qquad (6.5)$$

$y_t = Y_t / N_t$ *is output per worker in year t*

$k_t = K_t / N_t$ *is capital stock per worker in year t*

A_t = *the level of total factor productivity in year t*

- When the production is written like (6.5) it is often called the intensive form.

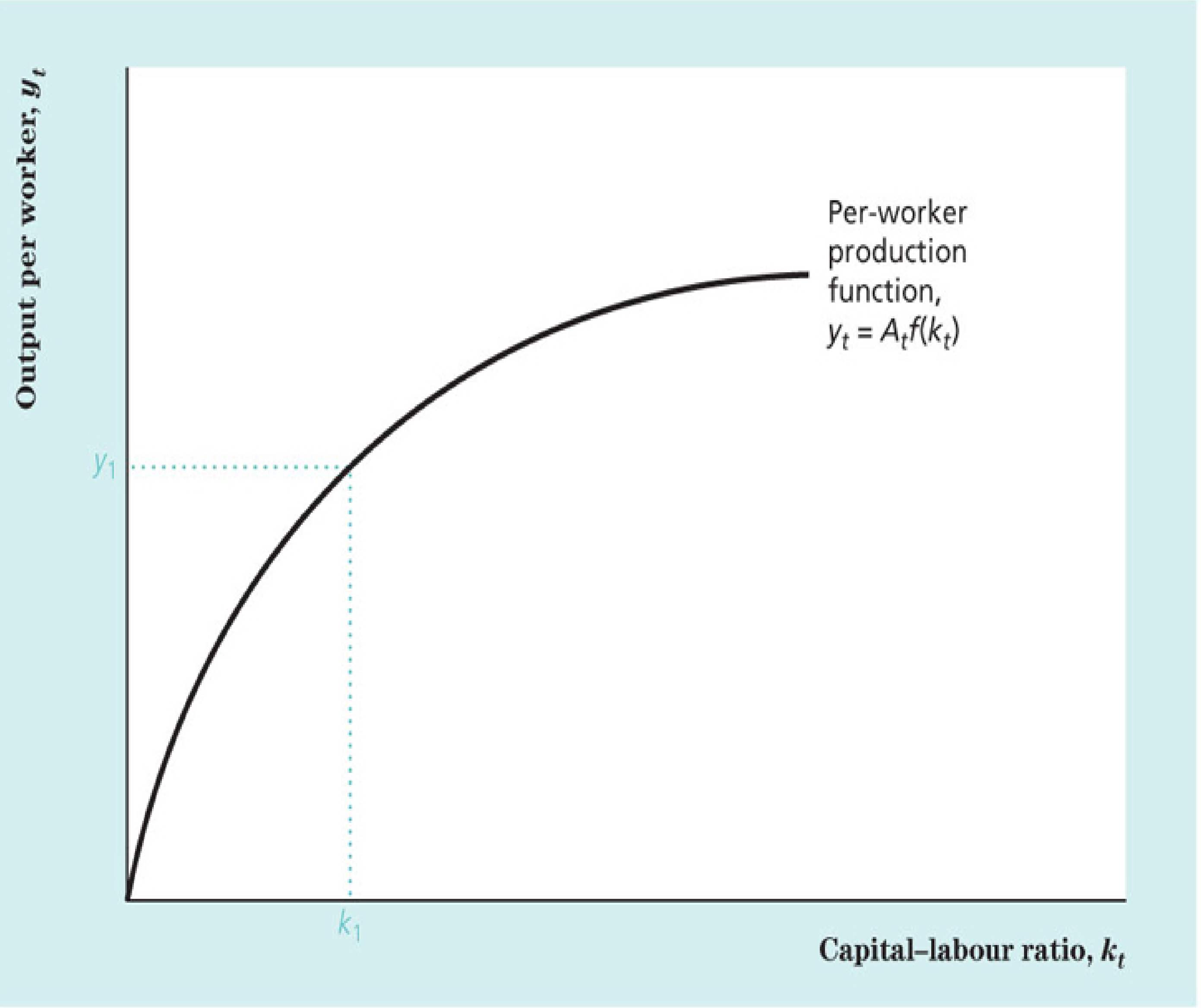
Graph of the per-Worker Production Function

- The production function slopes upward. As we move rightward, K is rising faster than N so that k increases.
- With more capital, each worker can produce more output.
- The slope gets flatter at higher levels of capital per worker. This reflects diminishing *MPK*.

Figure 6.1

The per-worker production function

The per-worker production function, $y_t = A_t f(k_t)$, relates the amount of output produced per worker y_t to the capital–labour ratio k_t and the level of productivity A_t. For example, for a given level of productivity A_t, when the capital–labour ratio is k_1 output per worker is y_1. The per-worker production function slopes upward from left to right because an increase in the capital–labour ratio raises the amount of output produced per worker. The bowed shape of the production function reflects the diminishing marginal productivity of capital.



Steady States

- In a growth model, equilibrium is defined by something called the steady state.
- A steady state is a situation in which the economy's output per worker (y_t), consumption per worker (c_t), and capital stock per worker (k_t) are constant; they do not change over time.
- Remember that these variables are all ratios to N_t so that for example both Y and N are growing.

Steady States (continued)

- *In the absence of productivity growth* the economy reaches a steady state in the long run.
- Since y_t, c_t and k_t are constant in a steady-state, Y_t, C_t and K_t all grow at rate n, the rate of growth of the workforce.
- As noted above, this is the definition of the steady state.

Characteristics of a Steady State

- The gross investment in year *t* is:

$$I_t = (n + d)K_t \qquad (6.6)$$

- K_t grows by nK_t in a steady state, which ensures that K/N is constant.
- K_t depreciates by dK_t where d is the capital depreciation rate.
- Is this equation consistent with what we have already studied?

Characteristics of a Steady State (continued)

- Eq (6.6) can be shown to be consistent with what we have already studied. Start by differentiating (K/N) and setting that derivative to zero (*i.e.*, fulfilling the condition that K/N does not change). Using "*Δ*" to represent changes:

$$\Delta(K/N) = [N\Delta K - K\Delta N]/N^2 = \Delta K/N - (\Delta N/N)(K/N)$$

$$= \Delta K - nK = 0$$

(note $\Delta N/N = n$, the growth rate of the labour force and we have multiplied the expression by N)

- Using the gross investment identity ($I = K^* - K + dK$) and remembering that $\Delta K = K^* - K = nK$ *in the steady state* we get:

$$I = (n + d)K$$

Characteristics of a Steady State (continued)

- Consumption is total output less the amount used for investment.

$$C_t = Y_t - (n+d)K_t \qquad (6.7)$$

- Put Eq. (6.7) in per-worker terms.
- Replace y_t with $A_t f(k_t)$ (Eq. (6.5)).

$$c = Af(k) - (n+d)k \qquad (6.8)$$

Steady-State Consumption per Worker

- An increase in the steady-state capital-labour ratio has two opposing effects on consumption per worker:
  1) *it raises the amount of output a worker can produce, $Af(k)$; and*
  2) *it increases the amount of output per worker that must be devoted to investment, $(n + d)k$.*

Steady-State Consumption per Worker (continued)

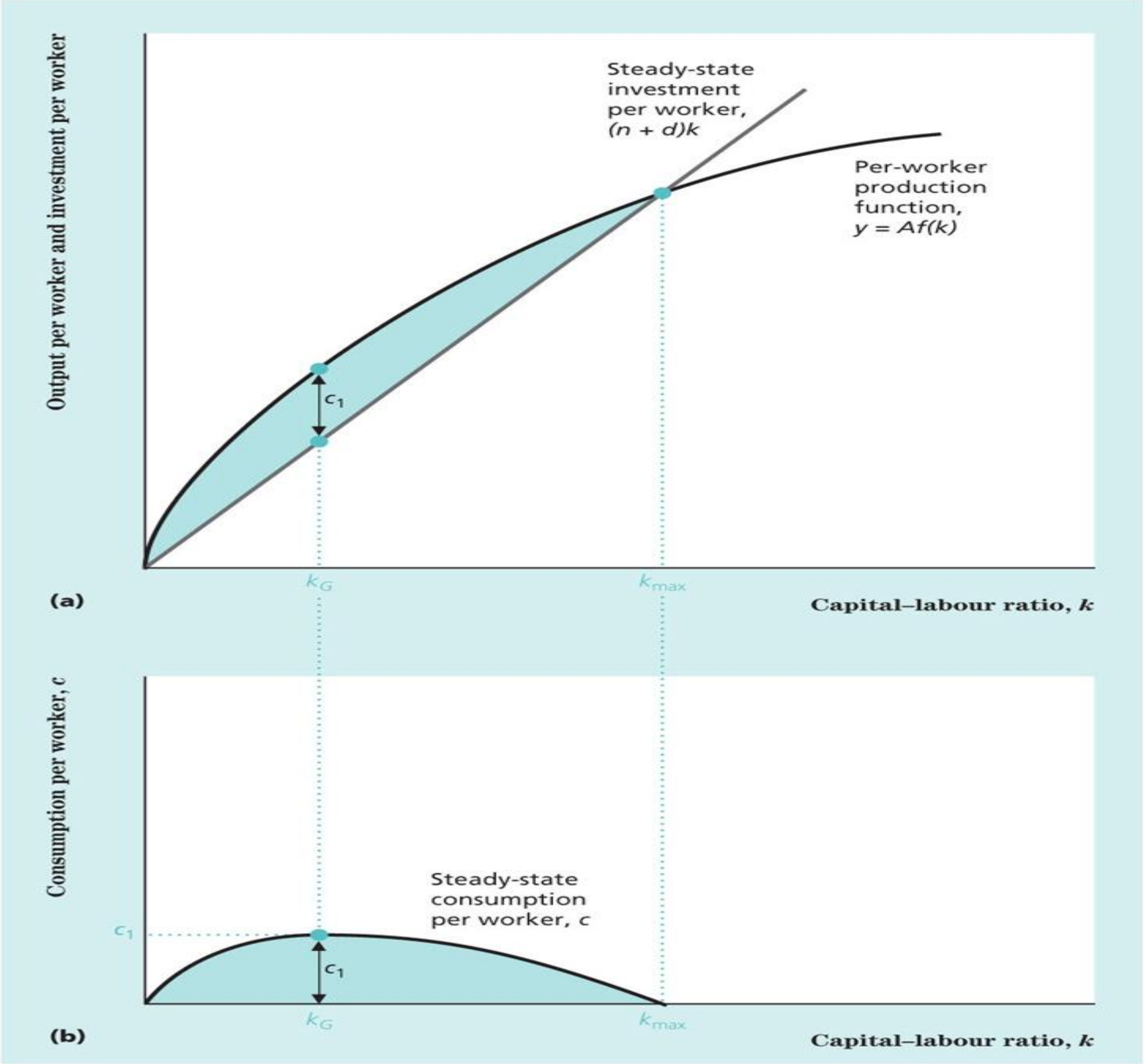
- The Golden Rule level of the capital stock maximizes consumption per worker in the steady state.
- At that point the slope of the production function (it's derivative *wrt* to k) equals $(n + d)$, the slope of the investment line.
- From this we can show that $r = n$, sometimes referred to as the biological interest rate. The key here is to use the definition of the user cost of capital assuming no taxes and a price of capital equal to one.

FIGURE 6.2

THE RELATIONSHIP OF CONSUMPTION PER WORKER TO THE CAPITAL–LABOUR RATIO IN THE STEADY STATE

(a) For each value of the capital–labour ratio, k, steady-state output per worker, y, is given by the per-worker production function, $Af(k)$. Steady-state investment per worker, $(n + d)k$, is a straight line with slope $n + d$. Steady-state consumption per worker, c, is the difference between output per worker and investment per worker (the shaded area). For example, if the capital–labour ratio is k_G, steady-state consumption per worker is c_1.

(b) For each value of the steady-state capital–labour ratio, k, steady-state consumption per worker, c, is derived in (a) as the difference between output per worker and investment per worker. Thus, the shaded area in (b) corresponds to the shaded area in (a). Note that starting from a low value of the capital–labour ratio, an increase in the capital–labour ratio raises steady-state consumption per worker. However, starting from a capital–labour ratio greater than k_G, an increase in the capital–labour ratio actually lowers consumption per worker. When the capital–labour ratio equals k_{max}, all output is devoted to investment, and steady-state consumption per worker is zero. The steady state at which consumption per worker is maximized, k_G in this diagram, is known as the Golden Rule steady state.



Steady-State Consumption per Worker (continued)

- The model shows that economic policy focused solely on increasing capital per worker may do little to increase consumption possibilities of the country citizens, if we are close to the Golden Rule level of k.
- Empirical evidence is that, given existing starting conditions, a higher capital stock would not lead to less consumption in the long run; *i.e.*, economies are far away from k_G.
- We will assume that an increase in the steady-state capital-labour ratio raises steady-state consumption per worker.

Reaching the Steady State

- We haven’t described how an economy would *reach* a steady state.
  - *Why will the described economy reach a steady state?*
  - *Which steady state will the economy reach?*

Reaching the Steady State (continued)

- The piece of information we need is saving.
- Assume that saving in this economy is proportional to current income:
- $$S_t = sY_t \qquad (6.9)$$

  "s" is a number between 0 and 1. It represents the faction of current income saved.

Reaching the Steady State (continued)

- National saving (in this case, private saving as there is no government in the model) has to equal investment.
- Here we we set our simple saving function equal to investment:

$$sY_t = (n + d)K_t \qquad (6.10)$$

Reaching the Steady State (continued)

- Put Eq. (6.10) in per-worker terms.
- Replace Y_t with $A_t f(k_t)$ (Eq. (6.5))

$$sAf(k) = (n + d)k \qquad (6.11)$$

- Subscript t is dropped because the variables are constant in the steady state.

Steady-State Capital-Output Ratio

- Equation 6.11 says that the steady state capital-labour ratio must ensure that saving per worker and investment per worker are equal.
- k^* is the value of k at which the saving curve and the steady-state investment line cross.
- k^* is the only possible steady-state capital-output ratio for this economy.

The Steady-State Consumption per Worker

- Steady-state output per worker is:

$$y^* = Af(k^*)$$

- Then, steady-state consumption per worker is:

$$c^* = Af(k^*) - (n + d)k^*$$

- While steady-state investment per worker is:

$$i^* = (n + d)k^*$$

FIGURE 6.3

DETERMINING THE CAPITAL–LABOUR RATIO IN THE STEADY STATE

The steady-state capital–labour ratio, k^*, is determined by the condition that saving per worker, $sAf(k)$, equals steady-state investment per worker, $(n + d)k$. The steady-state capital–labour ratio k^* corresponds to point X, where the saving curve and the steady-state investment line cross. From any starting point, eventually the capital–labour ratio reaches k^*. If the capital–labour ratio happens to be below k^*, say, at k_1, saving per worker, $sAf(k_1)$, exceeds the investment per worker, $(n + d)k_1$, needed to maintain the capital–labour ratio at k_1. As this extra saving is converted into capital, the capital–labour ratio will rise, as indicated by the arrows. Similarly, if the capital–labour ratio is greater than k^*, say, at k_2, saving per worker, $sAf(k_2)$, is too low relative to the investment per worker, $(n + d)k_2$, needed to maintain the capital–labour ratio at k_2. As a result, the capital–labour ratio will fall until it adjusts to k^*.

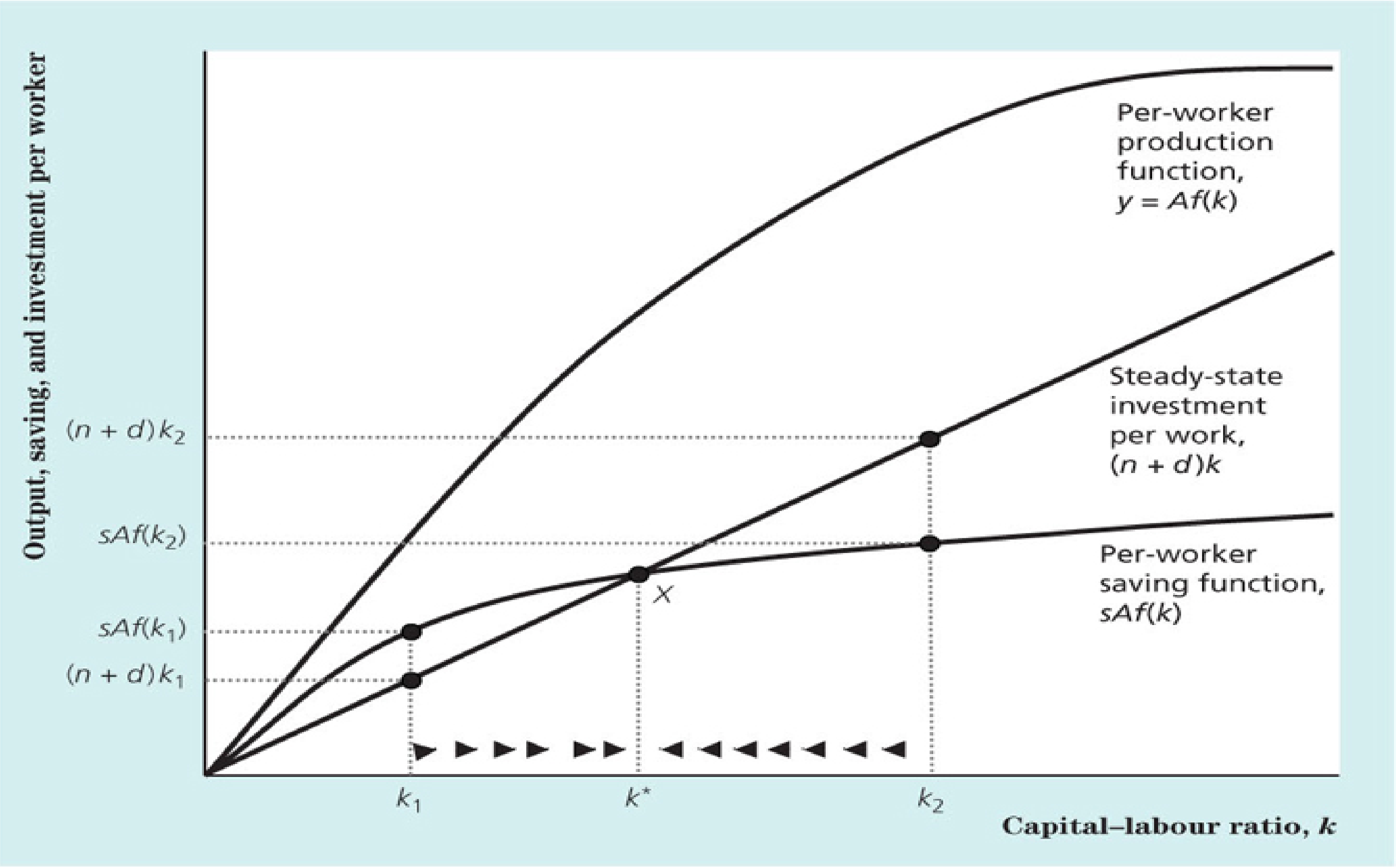
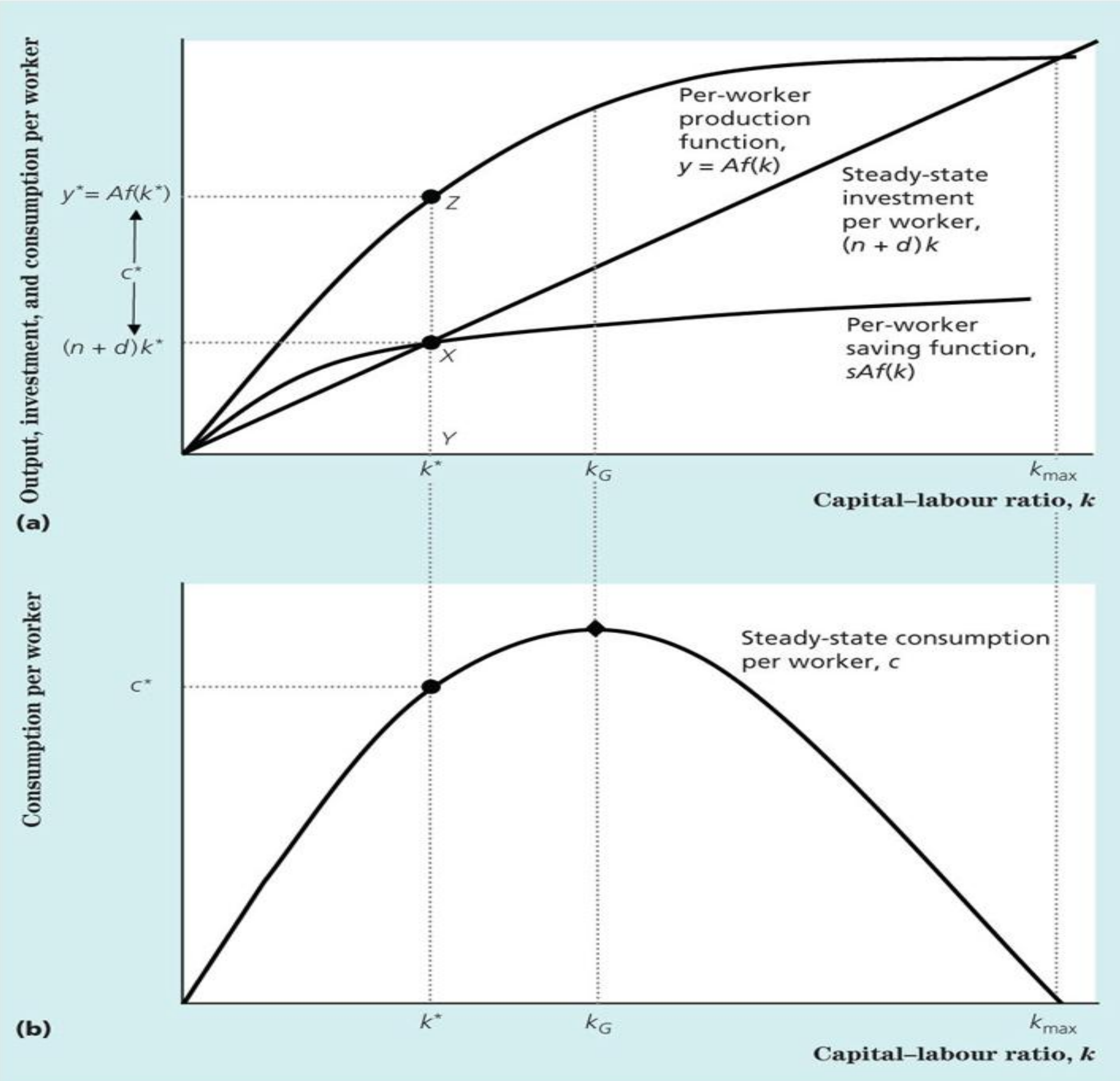


FIGURE 6.4

IDENTIFYING THE STEADY STATE AND THE GOLDEN RULE STEADY STATE

Panel (a) shows a per-worker production function, a per-worker saving function, and a steady-state per-worker investment line. The position of these curves reflects assumptions about the size of the productivity parameter, A, the saving rate, s, the rate of depreciation, d, and the rate of population growth, n. Given those values, the steady state occurs at capital–labour ratio k^*. Panel (b) shows the amounts of consumption per worker that can be realized for various savings rates, s, given values of the productivity parameter, A, the rate of depreciation, d, and the rate of population growth, n. The Golden Rule steady state is identified as capital–labour ratio k_G while the steady state that results with savings rate s is identified as k^*

Note that the scale of the vertical axis in panel (b) is exaggerated in order to more clearly show values of consumption per worker, c.



The Model Implications

- The economy's capital-labour ratio has a tendency to go to k^*. It will remain there forever, unless something changes.
- In this steady state the capital-labour ratio, output per worker, investment per worker, and consumption per worker all remain constant over time.
- The model determines an equilibrium but not growth – that is given by assumption.

The Model Implications (continued)

- If the level of saving were greater than the amount of investment needed to keep k constant, then that extra saving gets converted into capital and k rises.
- If saving were less than the amount needed to keep k constant, the reverse would happen – k would fall.
- Note that there is no reason to suppose that the steady state is at a point of maximum consumption – the "Golden Rule".

The Determinants of Long-Run Living Standard

- Long-run well-being is measured here by the steady-state level of consumption per worker.
- Its determinants are:
  1) *the saving rate (s);*
  2) *the population growth rate (n);*
  3) *the rate of productivity growth (how fast A grows).*

Long-Run Living Standard and the Saving Rate

- A higher saving rate implies a higher living standard. The increased saving rate raises output at every level of capital per worker.
- A steady-state with higher output and consumption per worker is attained in the long run.

The Saving Rate (continued)

- An increase in the saving rate has a cost – a fall in current period consumption.
- As before in the decision to consume, there is a trade-off between current current and future consumption.
- Beyond a certain point the cost of lost consumption today will outweigh the future benefits.

The Saving Rate (continued)

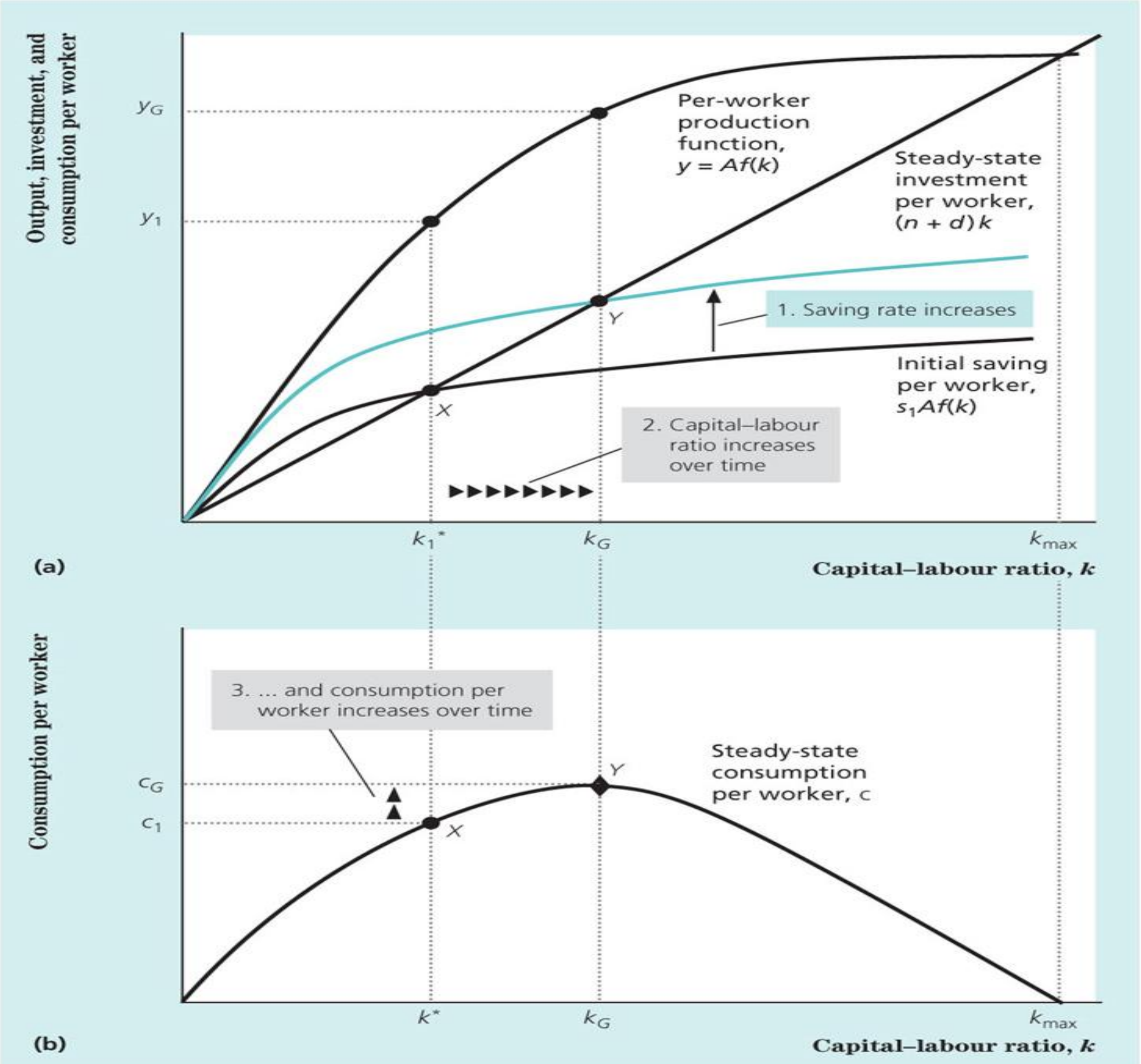
- It is also the case that a policy that increases saving will generate a temporary spurt in the growth rate.
- Since $y = Y/N$ and N is growing at a constant rate (n), then as we move to a new and higher k^* output must grow faster than n at least temporarily.

FIGURE 6.5

THE EFFECT OF AN INCREASED SAVING RATE ON THE STEADY-STATE CAPITAL–LABOUR RATIO

An increase in the saving rate causes the per-worker saving curve to pivot upward. The point where saving per worker equals steady-state investment per worker moves from point *X* to point *Y* in panel (a). Over time, the steady-state capital–labour ratio rises from $k_1{}^*$ to k_G. In panel (b) we see that the rise in the capital–labour ratio causes consumption per worker to increase over time from c_1 to c_G. In the short term, however, consumption per worker falls because at the initial capital–labour ratio, increases in saving and investment leave less output available for current consumption. Thus, the long-term gain in living standards comes at the cost of short-term pain.

Note that the scale of the vertical axis in panel (b) is exaggerated in order to more clearly show values of consumption per worker, *c*.



Long-Run Living Standard and Population Growth

- Increased population growth tends to lower living standards.
- When the workforce is growing rapidly, a larger part of current output must be devoted to just providing capital for the new workers to use.
- Absent here is any effect increased population may have on output – increased immigration of highly skilled workers would improve growth.

Population Growth (continued)

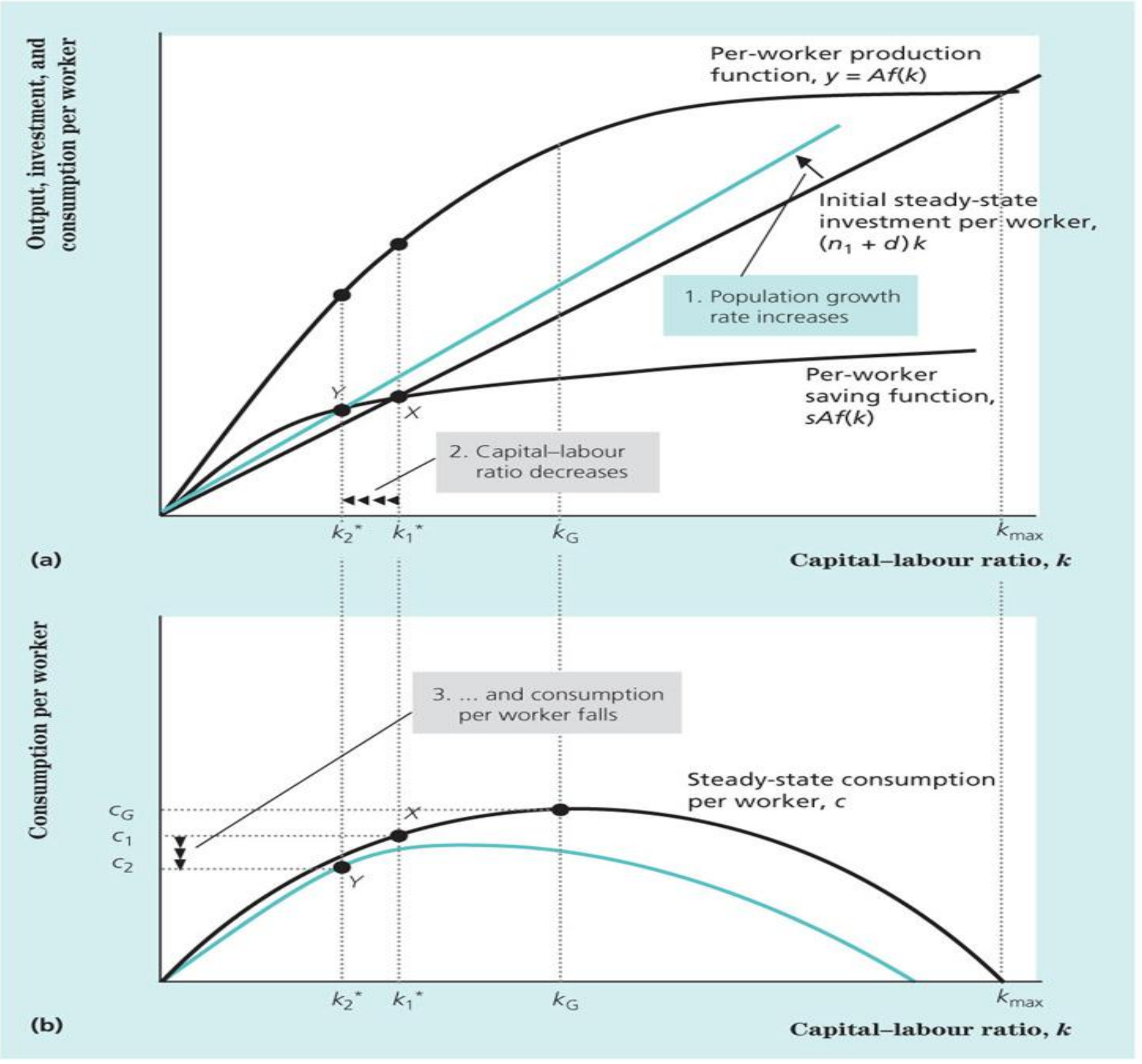
- However, a reduction in population growth means:
  - *lower population and lower total productive capacity;*
  - *lower ratio of working-age people to the population and perhaps an unsustainable pension system.*
- In some countries, low population growth can be raised by encouraging immigration and higher female participation.

FIGURE 6.6

THE EFFECT OF A HIGHER POPULATION GROWTH RATE ON THE STEADY-STATE CAPITAL–LABOUR RATIO

An increase in the population growth rate causes the steady-state investment per worker line to pivot upwards. The point where saving per worker equals steady-state investment per worker shifts from point *X* to point *Y* in panel (a). As a consequence, the steady-state capital–labour ratio will over time fall from $k_1{}^*$ to $k_2{}^*$. The increase in the population growth rate reduces the difference between the production function and the investment line and so reduces consumption per worker at every capital–labour ratio. The steady-state consumption per-worker curve is now shown by the teal line in panel (b). The fall in the steady-state capital–labour ratio causes consumption per worker to fall from c_1 to c_2 as shown in panel (b). A higher rate of population growth therefore results in a fall in living standards.

Note that the scale of the vertical axis in panel (b) is exaggerated in order to more clearly show values of consumption per worker, *c*.



Long-Run Living Standard and Productivity Growth

- The model accounts for the sustained growth by incorporating productivity growth.
- Increased productivity will improve living standards:
  - *it raises y at every k;*
  - *then saving per worker increases;*
  - *and higher k^* is attained.*
- This is important. Without productivity improvements, living standards would remain unchanged.

Productivity Growth (continued)

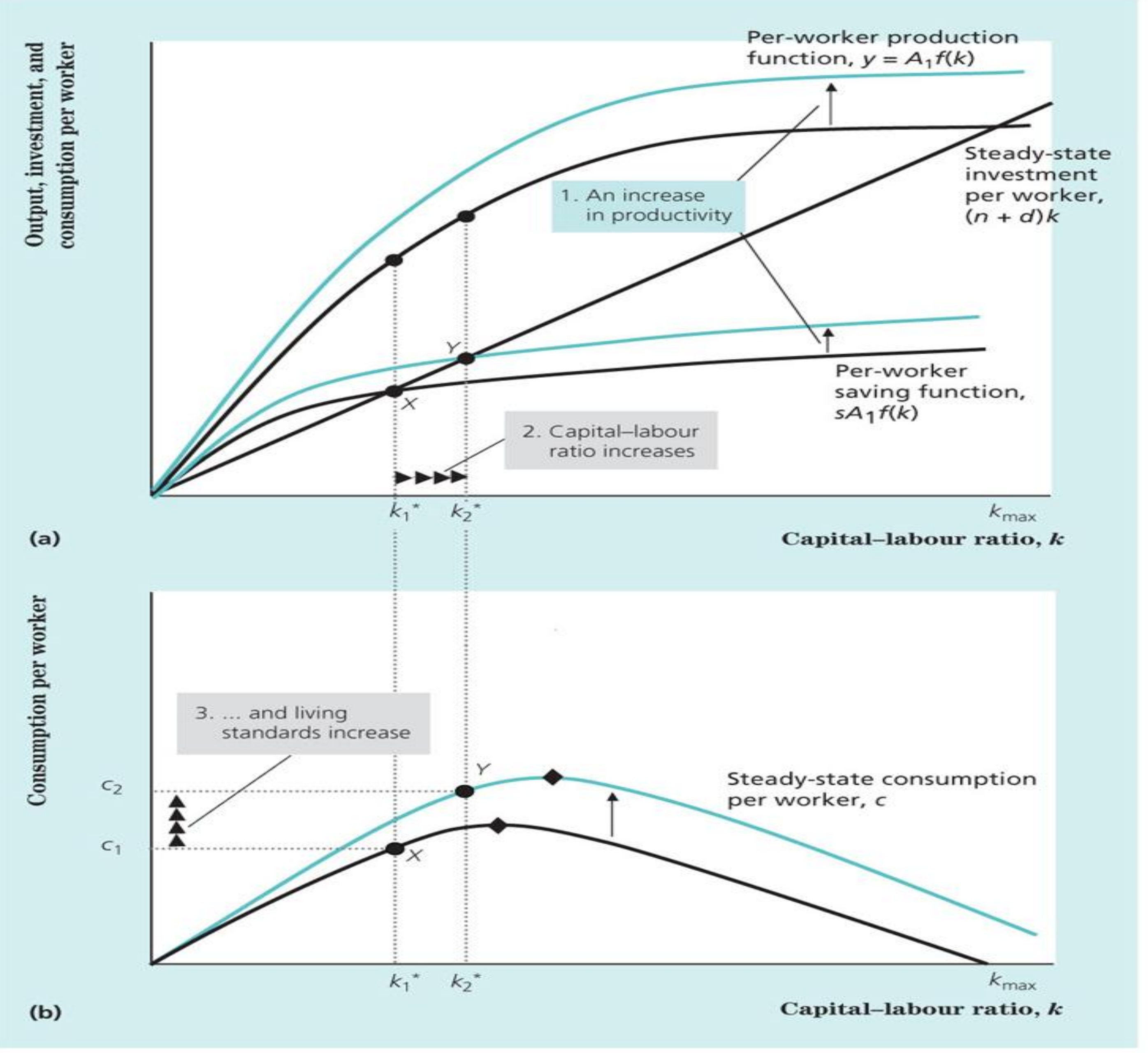
- A one-time productivity improvement shifts the economy only from one steady state to a higher one.
- Only continuing increases in productivity can perpetually improve living standards.
- Remember again, productivity growth is exogenous in this model.

Figure 6.7

The effect of a productivity improvement on the steady-state capital–labour ratio

In the growth model, a productivity improvement is represented by an increase in parameter A. As a result, a productivity improvement causes two curves in Figure 6.7 (a) to pivot upward; the per-worker production function $Af(k)$ and the per-worker saving function $sAf(k)$. The point where saving per worker equals steady-state investment per worker shifts from point X to point Y and the corresponding steady-state capital–labour ratio rises from k_1^* to k_2^*. Because it enables capital and labour to be used more productively, a productivity improvement also means that fewer resources need to be devoted to maintaining any capital–labour ratio. As a result, the amount of consumption per worker available at any steady state also increases. This is shown in Figure 6.7 (b) by the upward pivot in the steady-state per-worker consumption curve c. As the steady-state capital–labour ratio increases, so too does consumption per worker, as shown by the arrows. Thus, a productivity improvement raises the capital–labour ratio and increases living standards.

Note that the scale of the vertical axis in panel (b) is exaggerated in order to more clearly show values of consumption per worker, c.



Do Economies Converge?

- Unconditional convergence is a situation when the poor countries eventually catch up to the rich countries so that in the long run, living standards around the world become more or less the same.
- Certain conditions apply. For example, if the only difference is K/N but all else is the same (s, n, A) then the model predicts that living standards will converge.
- Some evidence suggests yes – poorer countries have tended to grow faster than richer ones.
- Trade and capital flows may be routes that facilitate unconditional convergence (Fischer's results).

Do Economies Converge? (continued)

- Conditional convergence is a situation when living standards will converge only within a group of countries with similar characteristics.
- OECD and developing economies could be considered two such groups.
- This result occurs if there are differences in s, n and A.

Do Economies Converge? (continued)

- Most findings find support for the idea of conditional convergence.
- Studies show that low saving (including human capital) in developing countries are important in explaining growth differences.
- Capital flows are again important.
- Other studies highlight importance of competition as well as the functioning of labour markets and macroeconomic policy.

Implication of the Neoclassical Growth Theory

- The neoclassical model highlights the role and importance of productivity
- However, it assumes, rather than explains, productivity – the crucial determinant of living standards (see first part of chapter).
- In other words, the model does not explain growth in output *per capita* which is what is of great interest.

Endogenous Growth Theory

- Endogenous growth theory tries to explain productivity growth within the model (endogenously).
- An implication of endogenous growth theory is that a country's growth rate depends on its rate of saving and investment, not only on exogenous productivity growth.

Setup of the Endogenous Growth Model

- Assume that the number of workers remains constant.
- This implies that the growth rate of output per worker is simply equal to the growth rate of output.
- The aggregate production function is:

$$Y = AK \qquad (6.11)$$

Where A is a positive constant.

Setup of the Endogenous Growth Model (continued)

- The marginal product of capital (*MPK*) is equal to *A* and does not depend on the capital stock (*K*).
- The *MPK* is not diminishing, it is constant. This is a major departure from the previous growth model.

Constant MPK and Human Capital

- One explanation of constant *MPK* is human capital – the knowledge, skills, and training of individuals.
- As an economy's physical capital increases its human capital stock tends to increase in the same proportion.
- Workers get better at using capital – there is *learning-by-doing* – an idea due to Arrow.

Constant MPK and Research and Development

- Another explanation of constant *MPK* is research and development (R&D) activities.
- The resulting productivity gains offset any tendency for the *MPK* to decrease.
- As the economy grows, firms have an incentive to invest in R&D.

The Model of Endogenous Growth

- Assume that national saving, S, is a constant fraction s of aggregate output, AK, so that $S = sAK$.
- In a closed economy $I=S$.
- As we know, total gross investment equals net investment plus depreciation

$$I = \Delta K + dK$$

The Model of Endogenous Growth (continued)

- Therefore: $$\Delta K + dK = sAK \quad (6.13)$$

  or $$\frac{\Delta K}{K} = sA - d \quad (6.14)$$

  and $$\frac{\Delta Y}{Y} = sA - d \quad (6.15)$$

- Since the growth rate of output is proportional to the growth rate of capital stock.

Implication of The Model of Endogenous Growth

- The endogenous growth model places greater emphasis on saving, human capital formation and R&D as sources of long-run growth.
- Higher saving and capital formation generate investment in human capital and R&D raising A.
- Remember, in the neoclassical (or Solow-Swan) growth model, the saving rate affects only the level of output, not growth.
- One problem is that it can be difficult to distinguish this model from conditional convergence.

Economic Growth and the Environment

- So far we have assumed that there are no natural limits to growth, like declining non-renewable resources or the environment.
- The empirical facts:
  - *Levels of many pollutants rise and then fall as economy grows.*
  - *The costs of controlling pollution are rising but remain relatively constant as a fraction of GDP.*
  - *Pollution emissions per unit of GDP have been falling since the late 1940s.*

Economic Growth and the Environment (continued)

- During the rapid initial economic growth phase the impact of output growth overwhelms the improvements in pollution-abatement technology.
- Near the steady state economic growth slows down and technological progress in pollution control overwhelms the impact of economic growth.
- These results are often due to policy choices.

Government Policies and Long-Run Living Standards

- Government policies that are useful in raising a country’s long-run standard of living are:
  - *polices to raise the saving rate;*
  - *policies to raise the rate of productivity.*

Policies to Affect the Saving Rate

- By taxing consumption a government can exempt from taxation the income that is saved.
- A government can increase the amount that it saves by reducing its deficit.

Affecting productivity Growth: Improving Infrastructure

- Some research finds a link between productivity and the quality of nation's infrastructure.
- Other research finds that public investments cannot explain cross-country differences.
- Higher growth in productivity may lead to more infrastructure, and not vice versa; that is, richer countries may want better infrastructure, like roads, schools and hospitals.

Affecting productivity Growth: Building Human Capital

- Recent research finds a strong connection between productivity growth and human capital.
- Governments affect human capital through education policies, training programs, health programs, etc.
- Productivity growth may increase if barriers to entrepreneurial activity are removed and competition increased.

Affecting productivity Growth: Research and Development

- Direct government support of basic research is a good investment for raising productivity.
- Some economists believe that even commercially-oriented research deserves government aid.
- Here public-private partnerships may help.

Affecting productivity Growth: Industrial Policy

- Industrial policy is a growth strategy in which the government attempts to influence the country’s pattern of industrial development.
- The arguments for the industrial policy are borrowing constraints, spillovers, and nationalism. The danger is favouritism.
- “Government’s are not very good at finding winners, but losers are good at finding governments.” *Sylvia Ostry*

Affecting productivity Growth: Market Policy

- Market policy is government restriction on free markets.
- Economists favour respect for property rights and a reliance on free markets to allocate resources efficiently.
- The reasons for government to interfere are market failures and efficiency vs. equity trade-off.

Solving the Model for Key Variables

- We can use the neo-classical growth model to solve for various key variables.
- To determine the steady-state capital-labour ratio (k^*), start with the equilibrium condition that $S=I$ in per capita terms. Thus:

$$(n+d)k^* = sAk^{*\alpha}$$

- This implies that k* is:

$$k^* = [sA/(n+d)]^{1/(1-\alpha)}$$

Solving the Model for Key Variables (continued)

- Suppose we want to know the Golden Rule level of the capital-labour ratio, k_G.
- From Figure 6.2, when the marginal productivity of k equal $n + d$, we know that consumption is maximised and the capital-labour ratio = k_G.

Solving the Model for Key Variables (continued)

- Assuming that the production function, in intensive form, is Cobb-Douglas ($y = Ak^{\alpha}$), then the marginal product of k is $\alpha Ak^{\alpha-1}$. Substituting in k_G into this relationship and setting it equal to $n + d$, we get:

$$\alpha A k_G^{\alpha-1} = n + d$$

- It then follows that:

$$k_G = [(\alpha A)/(n+d)]^{1/(1-\alpha)}$$

- We can now solve for y, investment and c.

Solving the Model for Key Variables (continued)

- If we wanted to know what saving rate (s) would get us to k_G, (whose value we now know) we go back to our old friend saving = investment and assume that we were at the point k_G.

$$sAk_G^{\alpha} = (n + d)k_G$$

- Then s is given by:

$$s = [(n + d)/A]k_G^{1-\alpha}$$

  - The strategy is to go to the point k_G and ask the question: what must have been s to get us to this point.
  - Once we know k_G we can solve for y and investment, which of course gives us c.

Solving the Model for Key Variables (continued)

- Using the saving = investment identity we can solve for the effects of other changes.
- Suppose we want to know what is the effect of higher labour for growth (n'). From the S/I identity we have:

$$sAk^{\alpha} = (n' + d)k$$

- Which yields a new *k* equal to:

$$k_{n'} = [sA/(n' + d)]^{1/(1-\alpha)}$$

- We can use $k_{n'}$ (the capital/labour ratio resulting from n') to calculate the new y, investment and c.

Solving the Model for Key Variables (continued)

- A productivity improvement (call it A') can be handled in a similar fashion; i.e., through the saving-investment identity.

$$sA'k^{\alpha} = (n + d)k$$

- Then as before, we solve for a new k,

$$k_{A'} = [sA'/(n + d)]^{1/(1-\alpha)}$$

- Once we have $k_{A'}$ we proceed as before and get the other variables of interest.
- Remember, when re-calculating y, adjust it for the now larger productivity, A'.

Solving the Model for Key Variables in Discrete Time

- Start with the following

  $K_{t+1} - K_t = I_t - dK_t$, which is the capital accumulation identity and can be written as:

  $K_{t+1} = (1 - d)K_t + I_t$
- The labour force (or population) evolves as:

  $N_{t+1} = (1 + n)N_t$, where n is the growth rate of labour.
- Divide both sides by N_{t+1} bearing in mind the growth of labour equation and that $S_t = I_t$:
- $K_{t+1}/N_{t+1} - K_t/(1+n)N_t = sAK_t^{\alpha}/(1+n)N_t - dK_t/(1+n)N_t$

Solving the Model for Key Variables in Discrete Time (con't)

- Remembering that $k = K/N$, the equation on the previous slide can be written as:

$$k_{t+1} = (1-d)/(1+n)k_t + [s/(1+n)]Ak_t^{\alpha}$$

- This expression shows what is called the law of motion of capital. Simply stated, it shows how the capital stock per worker (k_{t+1}) evolves over time.
- As can be seen, k_{t+1} depends on the amount of capital already in place – k_t, times $(1-d)/(1+n)$ – as well as the amount of new capital being added – $[s(1+n)]Ak_t^{\alpha}$, divided by $(1+n)$.
- Next we show that the implied steady state capital labour ratio is the same as derived in slide 70.

Solving the Model for Key Variables in Discrete Time (con't)

- Multiplying both sides by 1+n we get:

$$(1+n)\, k_{t+1} - (1-d)\, k_t = sAk_t^{\alpha}$$

- In the steady $k_{t+1} = k_t = k$ the equation becomes:

$$(n+d)k = sAk^{\alpha}$$

- The steady state level of capital per worker (k^*) is then:

$$k^* = [sA/(n+d)]^{1/(1-\alpha)}$$

- This is identical to the expression in slide 70.

