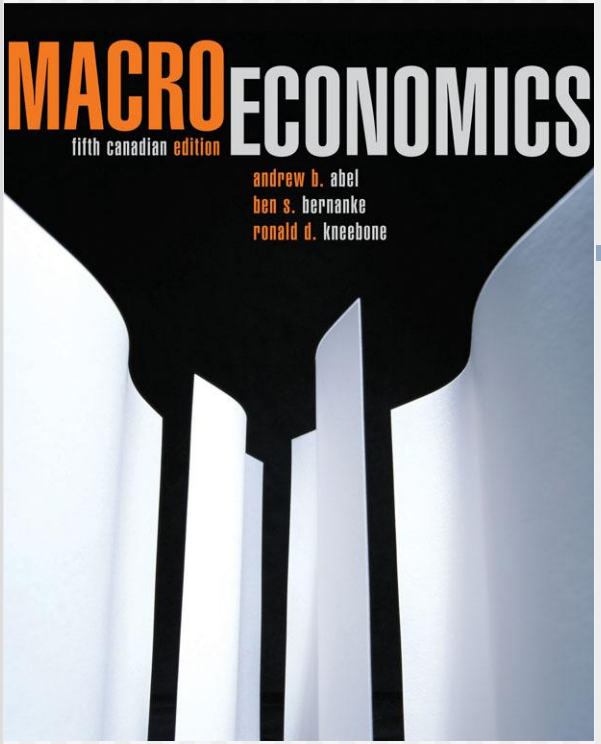


Chapter 3

Productivity Output and the Demand and Supply of Labour

A bit of a review

- What have we covered so far:
  - A broad brush picture of what some key variables in the macro economy look like.
  - A discussion of what lies behind the concepts in the national accounts and what they mean.
  - How to convert these measures in something we call “real”. This is important.
  - Measures of inflation and real interest rates – also important.

Where are we headed to now?

- The first two chapters have been an introduction.
- We now turn from *Description* and *Measurement* to *Analysis*
- In this lecture we will discuss:
  - Production Functions
  - Demand for Labour
  - Introduce Labour Supply

Macroeconomic Analysis

- Goal: Build a Macroeconomic model, that is a general framework to study economic questions already raised from an aggregate perspective.
  - Chap. 3 Labour Market;
  - Chap. 4 Goods Market;
  - Chap. 7 Asset Market.
- Then we will put it all together in Chap. 9 IS-LM Framework – we want to see how they interact as this will tell us how the macro-economy works.
- We will be assuming that the economy is at full employment – but we will be touching on unemployment as well.

The Production Function

- How do we describe production in the economy?
- Factors of production are inputs to the production process. We will be concerned with:
  - capital (factories, machines) and labour (workers);
  - and how effectively they are used – *e.g.*, productivity.

The Production Function (continued)

- Here we are going to measure the physical capacity of the economy to produce goods and services.
- A production function is a mathematical expression relating the amount of output produced to quantities of capital and labour utilized.

The Production Function (continued)

In its general form, it looks like this:

$$Y = AF(K,N)$$

Y is real output produced

A is a number measuring overall productivity

K is the quantity of capital used (capital stock)

N is the number of workers employed

F is a function relating Y to K and N; it tells us how changes in *K* and *N* change *Y*, for a given value of *A*

The production function (continued)

$$Y = AF(K,N) = AK^{\alpha}N^{\beta} = AK^{\alpha}N^{1-\alpha}$$

- This is perhaps the most well-known and well-used (Cobb-Douglas).
- The exponents (α and β) represent shares in income – in fact knowing income shares allows us to estimate these parameters.
- Because their values are less than unity, we get diminishing returns to factors.
- The "A" represents total factor productivity.

The production function (continued)

- In the real world, there are returns to scale (RS): Increasing (IRS), decreasing (DRS), constant (CRS).
- We will focus on CRS, *i.e.* $\beta = 1 - \alpha$, with $0 < \alpha < 1$.
- With CRS we say that the function is homogeneous of degree one. This means that if we double all inputs (K and N) we double the output.

The production function (continued)

- A formal demonstration of constant returns to scale. Given a Cobb-Douglas production function multiply each input by λ, a scaler.

$$Y = A(\lambda K)^{\alpha}(\lambda N)^{1-\alpha}$$

- The scaler is just a number which scales up or down the input. We can factor out this scaler:

$$Y = \lambda A(K)^{\alpha}(N)^{1-\alpha} \text{ (since } \alpha \text{ and } 1-\alpha \text{ sum to one)}$$

- Thus output rises by the same amount as the increase in inputs.

The Production Function (continued)

- Total factor productivity (productivity) is a measure of overall effectiveness with which capital and labour are used.
  - An improvement in production technology or just using existing factors more efficiently, allows capital and labour to be utilized more effectively.
  - We measure this using the production function as a residual; it is what we can't explain that well.

TABLE 3.1

The Production Function for Canada, 1981–2006

Production function: $Y = AK^{0.3}N^{0.7}$

Year	(1) Real GDP, Y (Billions of 2002 dollars)	(2) Capital, K (Billions of 2002 dollars)	(3) Labour, N (Millions of workers)	(4) Total Factor Productivity, A^*	(5) Growth in Total Factor Productivity (% change in A)
1981	647	827	11.3	15.80	
1982	629	844	10.9	15.60	–1.3
1983	646	830	11.0	16.03	2.7
1984	683	834	11.3	16.64	3.8
1985	716	850	11.6	16.99	2.1
1986	733	863	12.0	16.96	–0.2
1987	765	863	12.3	17.33	2.2
1988	803	883	12.7	17.70	2.1
1989	824	906	13.0	17.74	0.3
1990	825	927	13.1	17.57	–1.0
1991	808	897	12.9	17.59	0.1
1992	815	894	12.7	17.88	1.6
1993	834	900	12.8	18.20	1.8
1994	874	927	13.1	18.63	2.4
1995	899	936	13.3	18.87	1.2
1996	913	952	13.4	18.95	0.4
1997	952	984	13.7	19.27	1.7
1998	991	1037	14.0	19.41	0.7
1999	1046	1057	14.4	20.01	3.1
2000	1101	1064	14.8	20.66	3.2
2001	1120	1087	14.9	20.71	0.3
2002	1153	1099	15.3	20.89	0.9
2003	1175	1070	15.7	21.11	1.0
2004	1211	1089	15.9	21.38	1.3
2005	1248	1111	16.2	21.69	1.4
2006	1282	1138	16.5	21.84	0.7

* Total factor productivity is calculated by the formula $A = Y/(K^{0.3}N^{0.7})$

Source: Statistics Canada, CANSIM II series v1997756, v1070274, v2461119, and v3860085.

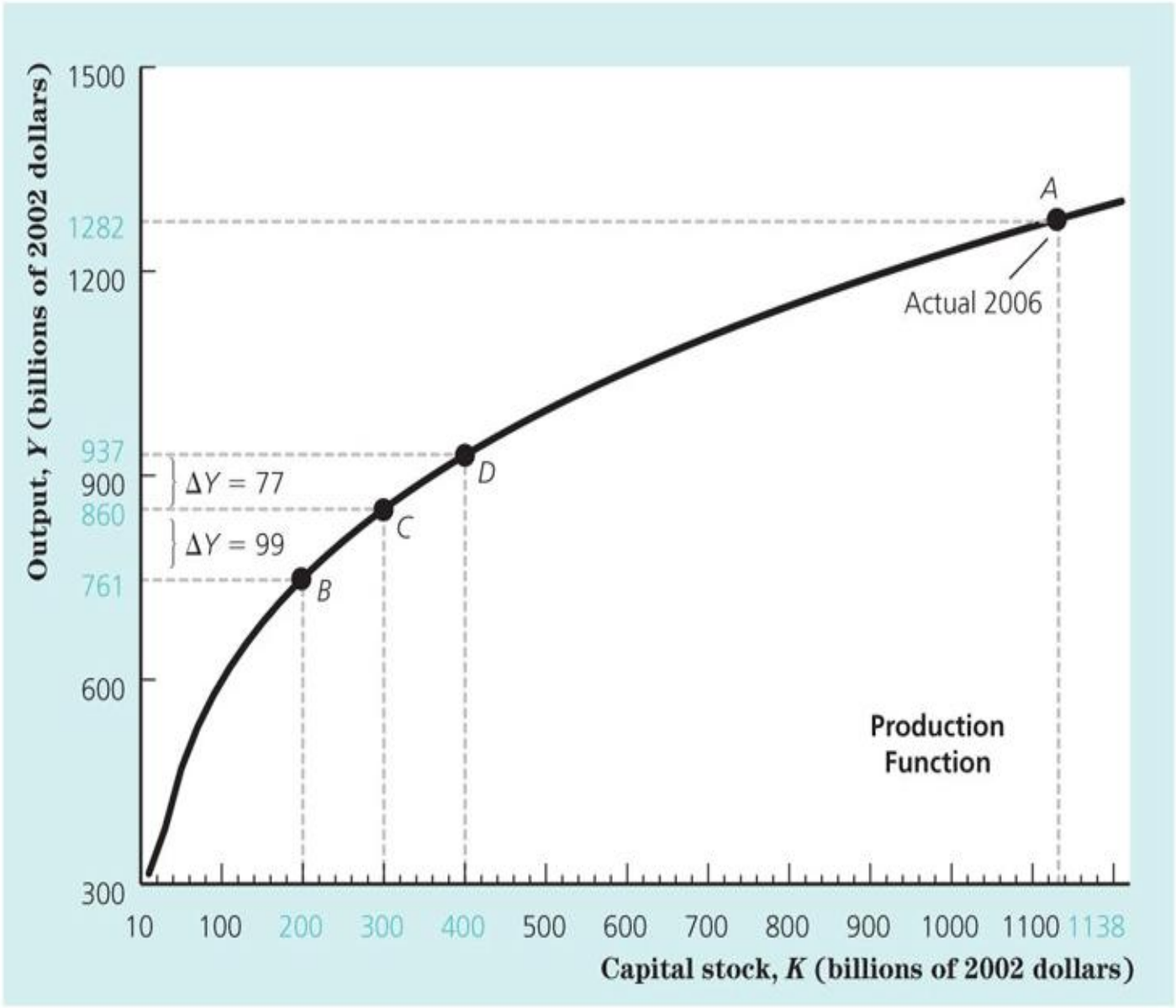
The Production Function (continued)

- Properties of production functions when one factor is changed holding the others constant:
  - they slope upward from left to right;
  - the slope becomes flatter from left to right (fig 3.1).

FIGURE 3.1

THE PRODUCTION FUNCTION RELATING OUTPUT AND CAPITAL

This production function shows how much output the Canadian economy could produce for each level of Canadian capital stock, holding labour and productivity at 2006 levels. Point *A* corresponds to the actual 2006 output and capital stock. The production function has diminishing marginal productivity of capital: Raising the capital stock by $100 billion in order to move from point *B* to point *C* raises output by $99 billion, but adding another $100 billion in capital to go from point *C* to point *D* increases output by only $77 billion.



The Marginal Product of Capital

- The marginal product of capital (MPK) is the increase in output produced resulting from a one-unit increase in the capital stock (other factors held constant).
- It can be considered as a demand function since it measures the benefits of an additional unit of capital.

The Marginal Product of Capital (continued)

- The *MPK* equals the slope of the line tangent to the production function at a given point.

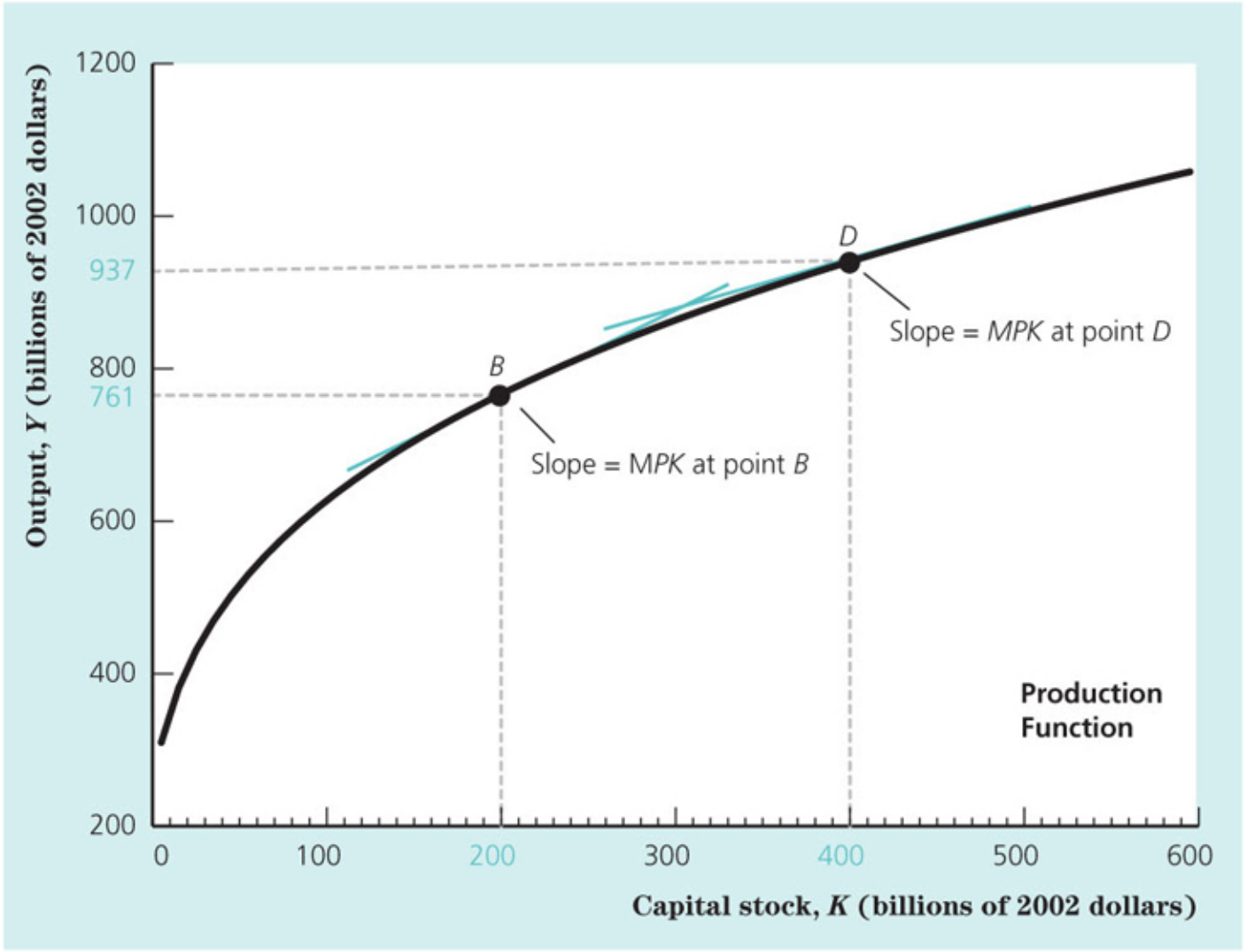
$$MPK = \frac{\Delta Y}{\Delta K}$$

- It is the return or reward to capital.

FIGURE 3.2

THE MARGINAL PRODUCT OF CAPITAL

The marginal product of capital (*MPK*) at any point can be measured as the slope of the line tangent to the production function at that point. Because the slope of the line tangent to the production function at point *B* is greater than the slope of the line tangent to the production function at point *D*, we know that the *MPK* is greater at *B* than at *D*. At higher levels of capital stock, the *MPK* is lower, reflecting diminishing marginal productivity of capital.



[4] For definitions and a discussion of slopes of lines and curves, see the Appendix, Section A.2.

[5] We often refer to the slope of the line tangent to the production function at a given point as simply the slope of the production function at that point, for short.

The Marginal Product of Capital (continued)

- For those taking calculus, it is the first derivative of Y with respect to (*wrt)* to K.

$$MPK = \Delta Y/\Delta K = dY/dK = \alpha A K^{(\alpha-1)} N^{(1-\alpha)} > 0$$

- The properties of the production function:
  - the *MPK* is positive;
  - the *MPK* declines as the capital stock increases, holding the other factors constant.
- Diminishing marginal productivity is the tendency for the marginal product of capital to decline as the amount of capital increases.

The Marginal Product of Labour

- The marginal product of labour (*MPN*) is the increase in output produced by each additional unit of labour (other factors held constant). The concept is the same as *MPK*.
- The marginal productivity of labour is diminishing for similar reasons as with capital.
- It is also a first derivative, this time of Y *wrt* N.

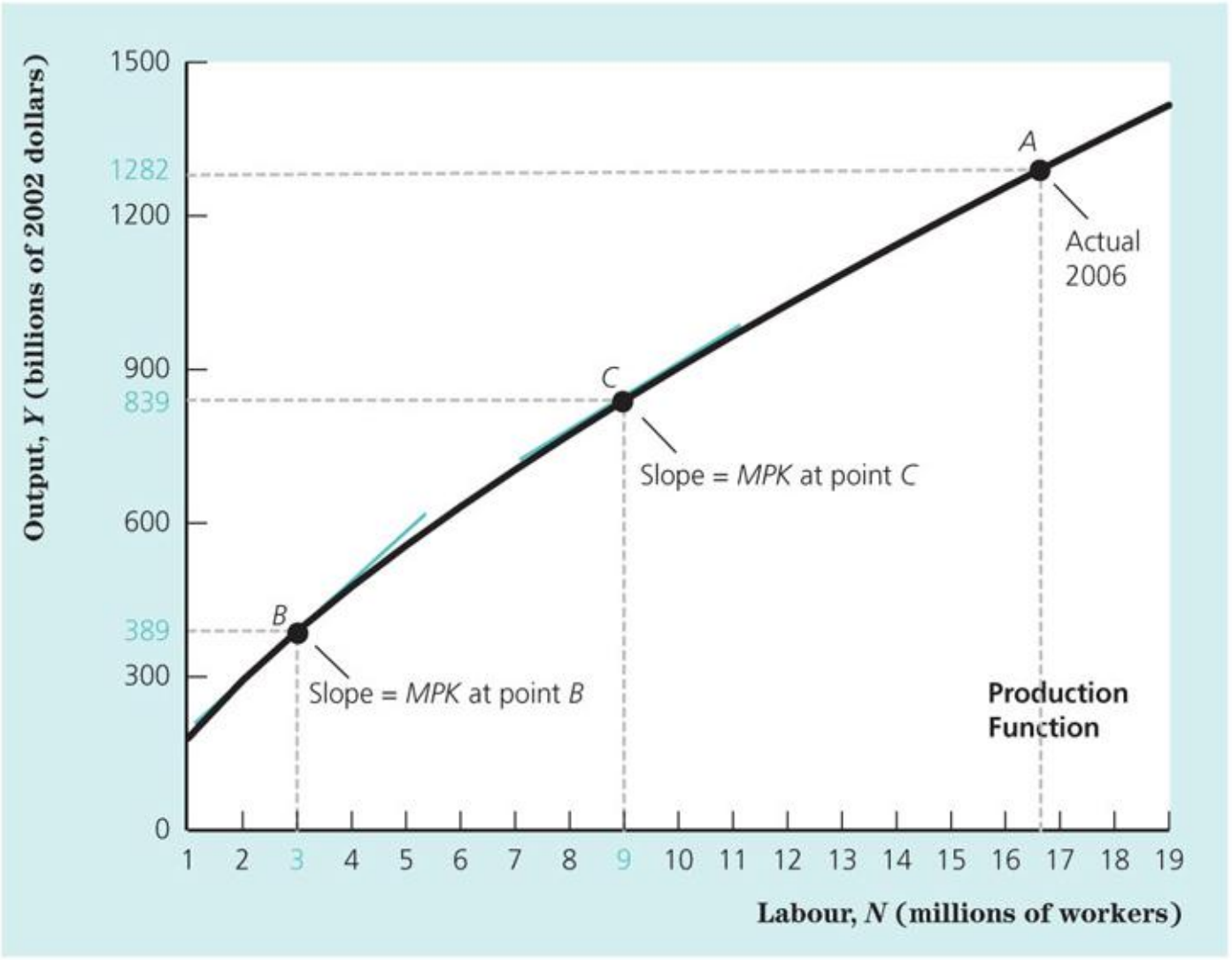
$$\text{MPN} = \Delta Y/\Delta N = dY/dN = (1-\alpha)\text{AK}^{\alpha}\text{N}^{(-\alpha)} > 0$$

- The properties of the MPN are the same as those of the MPK.

FIGURE 3.3

THE PRODUCTION FUNCTION RELATING OUTPUT AND LABOUR

This production function shows how much output the Canadian economy could produce at each level of employment (labour input), holding productivity and the capital stock constant at 2006 levels. Point *A* corresponds to actual 2006 output and employment. The marginal product of labour (*MPN*) at any point is measured as the slope of the line tangent to the production function at that point. The *MPN* is lower at higher levels of employment, reflecting diminishing marginal productivity of labour.



[6] Because N is raised to the power of 0.7 but K is raised to the power of 0.3, the production function relating output and labour is not as sharply bowed as the production function relating output and capital.

The Production Function Supply Shocks

- A supply shock (productivity shock) is a change in an economy's production function.
- It usually is represented as a change in "A".
- A positive (beneficial) shock (new technology, innovation) raises the amount of output which can be produced with each capital-labour combination.

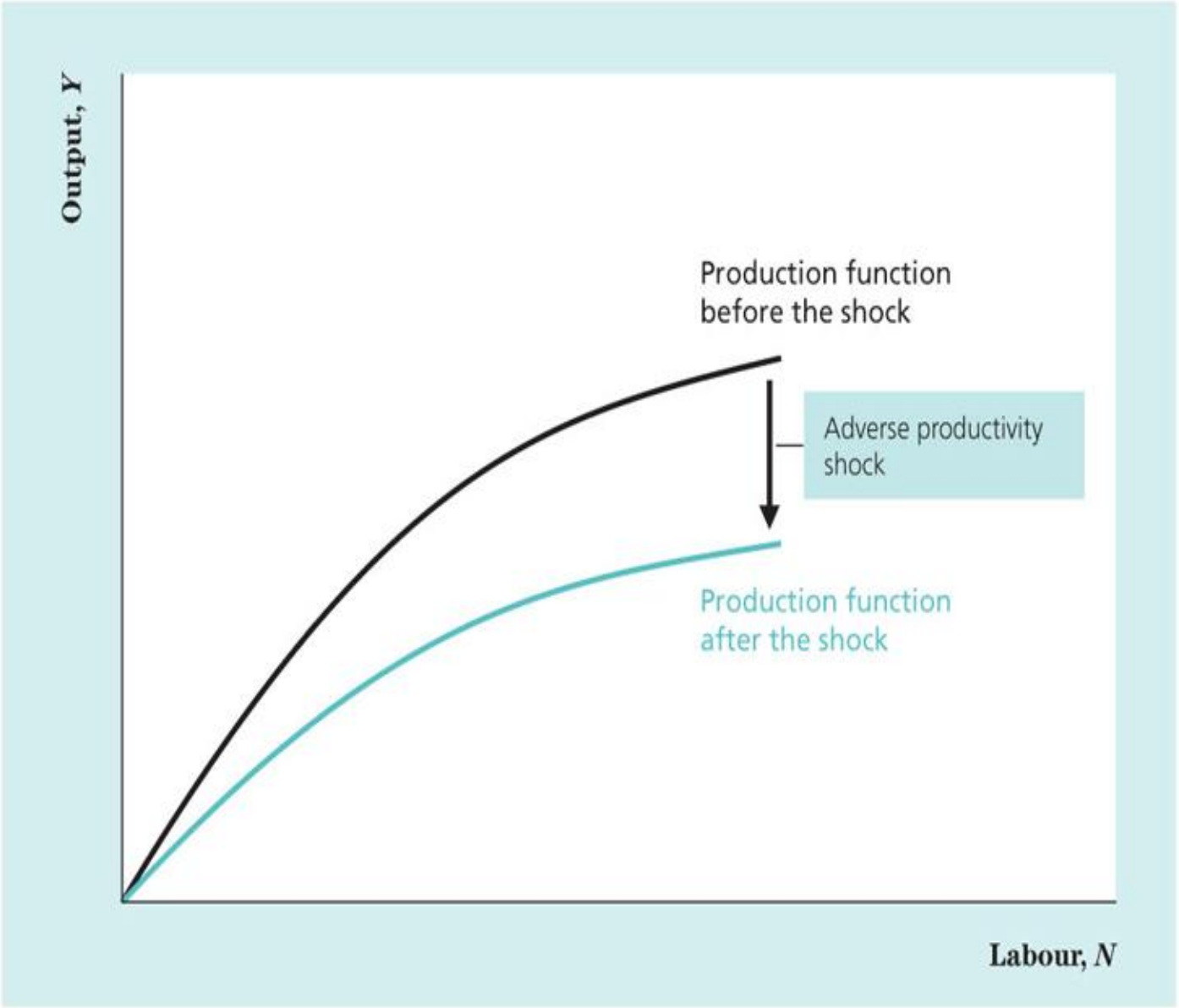
The Production Function Supply Shocks (continued)

- A negative (adverse) shock (drought, oil price hike) lowers the amount of output which can be produced with each capital-labour combination.
  - Positive shocks shift the production function upward.
  - Negative shocks shift the production function downward.
- Sometimes we don't really understand the source of the shock.

FIGURE 3.4

AN ADVERSE SUPPLY SHOCK THAT LOWERS THE *MPN*

An adverse supply shock is a downward pivot of the production function. For any level of labour, the amount of output that can be produced is now less than before. The adverse shock reduces the slope of the production function at every level of employment. This corresponds to a decrease in the multiplying factor *A* in Eq. (3.2).



What determines the demand for inputs?

- The two most important inputs are capital and labour.
- The capital stock is slow to change because it is built up over years and is long lived.
- We will treat the capital stock as fixed for now and this allows us to focus on labour.

The Demand for Labour

- Assuming that the capital stock is fixed, let's also assume the amount of labour is variable in the short run.
- Firms may employ and lay off workers without much notice.
- These assumptions may not seem reasonable – real world – but we can test them later.

The Demand for Labour (continued)

- Also let's assume (heroically) that:
  - Workers are all alike.
  - The wage is determined in a competitive market.
  - A firm employs workers to earn the highest possible level of profit (up to MPN equals wage, in real terms – the real wage is the cost of using labour).

The *MPN* and the Labour Demand

- The *MPN* measures the benefit of employing an additional worker in terms of the extra output produced – remember the figure on the production function.
- This is the demand curve for labour.
- It is a first derivative of the production *wrt* to N.
- The demand for labour is embedded in the production function.

The *MPN* and the Labour Demand (continued)

- The marginal revenue product of labour *(MRPN)* measures the benefit to the firm of employing an additional worker in terms of the extra revenue produced.

$$MRPN = P \times MPN$$

- P is the price of output.

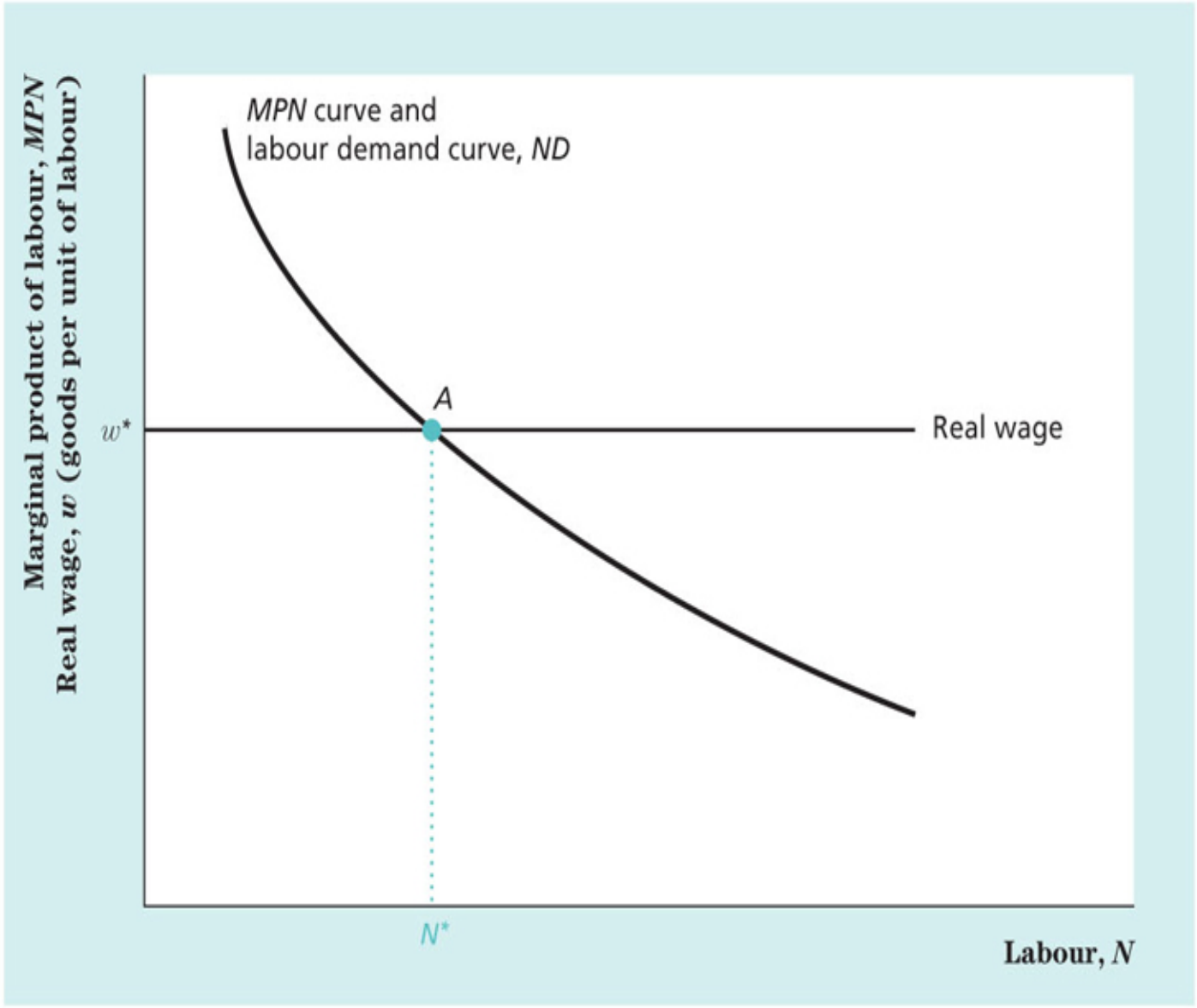
The *MPN* and the Labour Demand (continued)

- To an employer the benefit is *MRPN* and the cost is the nominal wage (*W*).
- In real terms the benefit is *MPN* and the cost is the real wage (*w*) - the nominal wage (*W*) divided by the price of output (*P*).
- The *w* line is horizontal, the wage is constant in a competitive labour market.
- The profit-maximizing amount of labour input is the point where the *MPN* curve and *w* line intersect, *MPN*=*w*.

FIGURE 3.5

THE DETERMINATION OF LABOUR DEMAND

The amount of labour demanded is determined by locating the point on the *MPN* curve at which the *MPN* equals the real wage rate; the amount of labour corresponding to that point is the amount of labour demanded. For example, when the real wage is w^*, the *MPN* equals the real wage at point *A* and the quantity of labour demanded is N^*. The labour demand curve, *ND*, shows the amount of labour demanded at each level of the real wage. The labour demand curve is identical to the *MPN* curve.



The Labour Demand Curve Shifters

- Economists are always interested in the differences between movements along a curve and shifts in a curve.
  - Changes in the real wage are represented as movements along the labour demand curve.
  - If the real wage falls (rises) the the amount demanded will rise (fall).
  - This will be a *movement* along the curve.

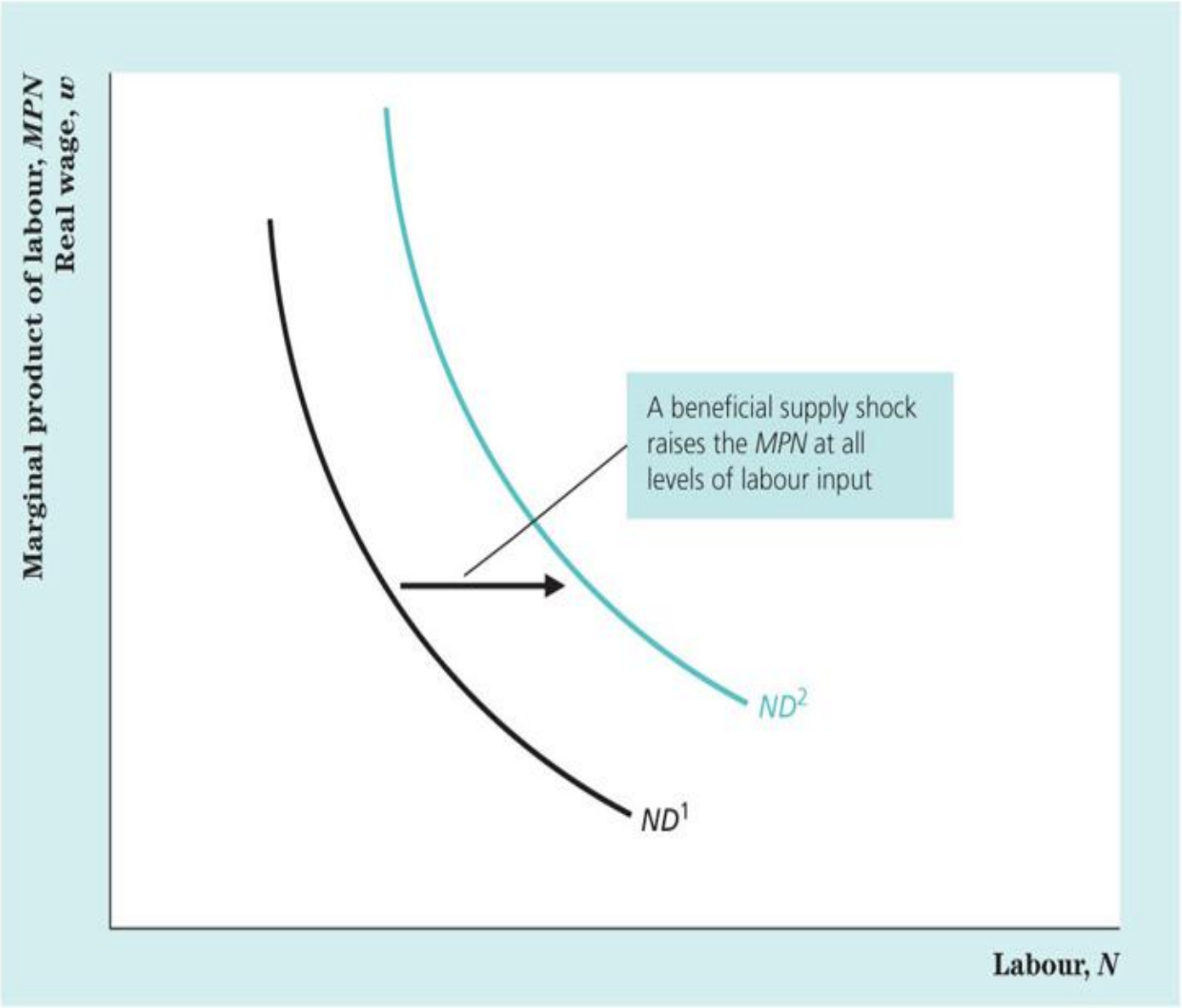
The Labour Demand Curve Shifters (continued)

- A positive supply (productivity) shock increases the *MPN* and increases the quantity of labour demanded at each real wage level.
- Higher capital stock increases the *MPN* and increases the quantity of labour demanded at each real wage level.
- All of this is represented by a *shift* in the labour demand curve.

FIGURE 3.6

THE EFFECT OF A BENEFICIAL SUPPLY SHOCK ON LABOUR DEMAND

A beneficial supply shock that raises the *MPN* at every level of labour shifts the *MPN* curve upward and to the right. Because the labour demand curve is identical to the *MPN* curve, the labour demand curve shifts upward and to the right from ND^1 to ND^2. For any real wage, firms demand more labour after a beneficial supply shock.



Aggregate Labour Demand

- Aggregate labour demand is the sum of the labour demands of all the firms in the economy.
- The aggregate labour demand curve looks the same and behaves the same as a labour demand curve for an individual firm.

The Supply of Labour

- Demand is determined by firms but supply by individuals.
- Aggregate supply of labour is the sum of labour supplied by everyone in the economy.
- Each person must decide how much time to work for income (the principal benefit) versus how much time to allocate for leisure (off-work activities, the cost of working).

The Supply of Labour (continued)

- When talking about individuals making choices the concept of utility is helpful.
- The utility (happiness) from income for one more hour at work is compared to the cost (lost utility) of one less hour of leisure – there is a trade-off.
- Utility is maximized when these values are the same.

Real Wages and Labour Supply

- The real wage is real income received in exchange for giving up leisure.
- There are two effects at work here.
- The substitution effect of a higher real wage is the tendency of workers to supply more labour and reduce leisure hours in response to a higher real wage.

Real Wages and Labour Supply (continued)

- The income effect of a higher real wage is the tendency of workers to supply less labour and increase leisure hours as they enjoy higher incomes.
- The two effects work in opposite directions.
  - The substitution effect of a higher real wage leads to an increase in the quantity of labour supplied.
  - The income effect leads to a decrease in the quantity of labour supplied.

Real Wages and Labour Supply (continued)

- We can imagine a pure substitution and a pure income effect.
- Pure substitution effect:
  - A temporary increase in the wage rate (which doesn't affect wealth) like an overtime bonus.
- A pure income effect:
  - A one time increase in wealth (which raises long-term income).
- An important point:
  - The *longer* an increase in the real wage is expected to last, the *larger* is the income effect.

Real Wages and Labour Supply (continued)

- Because of conflicting effects there is some ambiguity about how labour supply will respond to a real wage change – it becomes an empirical question.
- The empirical evidence suggests that:
  - when wage increases are perceived as temporary, the substitution effect dominates; but
  - when permanent, the income effect dominates.

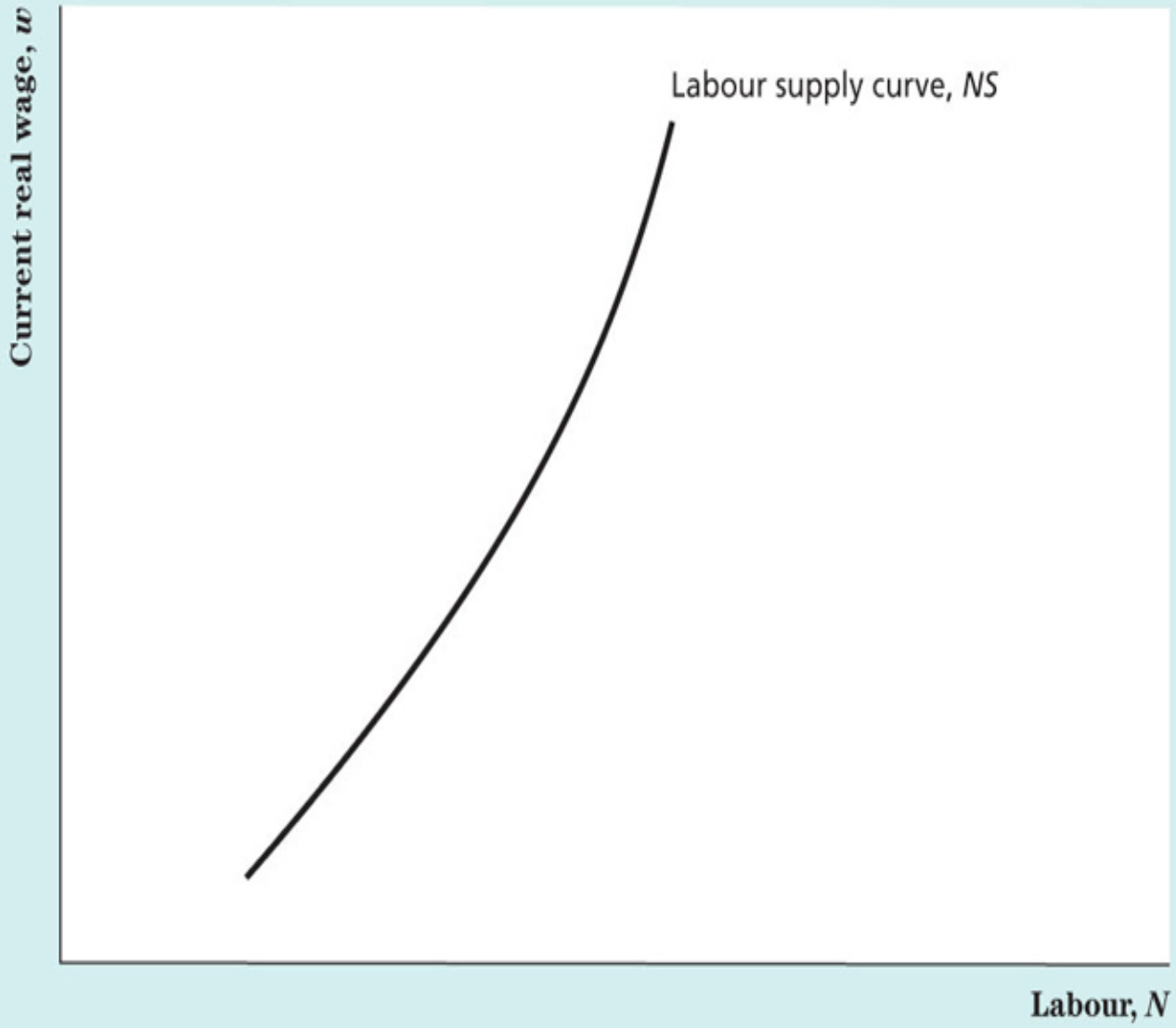
The Labour Supply Curve

- The labour supply curve is the curve which relates the amount of labour supplied to the current real wage (other factors held constant, including the real wage expected in the future).
- The labour supply curve is upward sloping. An increase in the current real wage leads to an increase in labour supplied.
- With the exception of the real wage, any factor which changes the amount of labour supply will shift the labour supply curve.

FIGURE 3.7

THE LABOUR SUPPLY CURVE OF AN INDIVIDUAL WORKER

The horizontal axis shows the amount of labour that a worker will supply for any given current real wage on the vertical axis. The labour supply curve slopes upward, indicating that—with other factors including the expected future real wage held constant—an increase in the current real wage raises the amount of labour supplied.



Aggregate Labour Supply

- Aggregate labour supply is the total amount of labour supplied in the economy.
- More things are going on at the aggregate level than at the level of the individual.

Aggregate Labour Supply (continued)

- *Moving along the curve*.
- An increase in the current economy-wide real wage raises the aggregate quantity of labour supplied:
  - people already working supply more hours (more overtime, second job);
  - some people are induced to join the labour force (reservation wage).
- We conclude that the aggregate labour supply curve slopes upward.

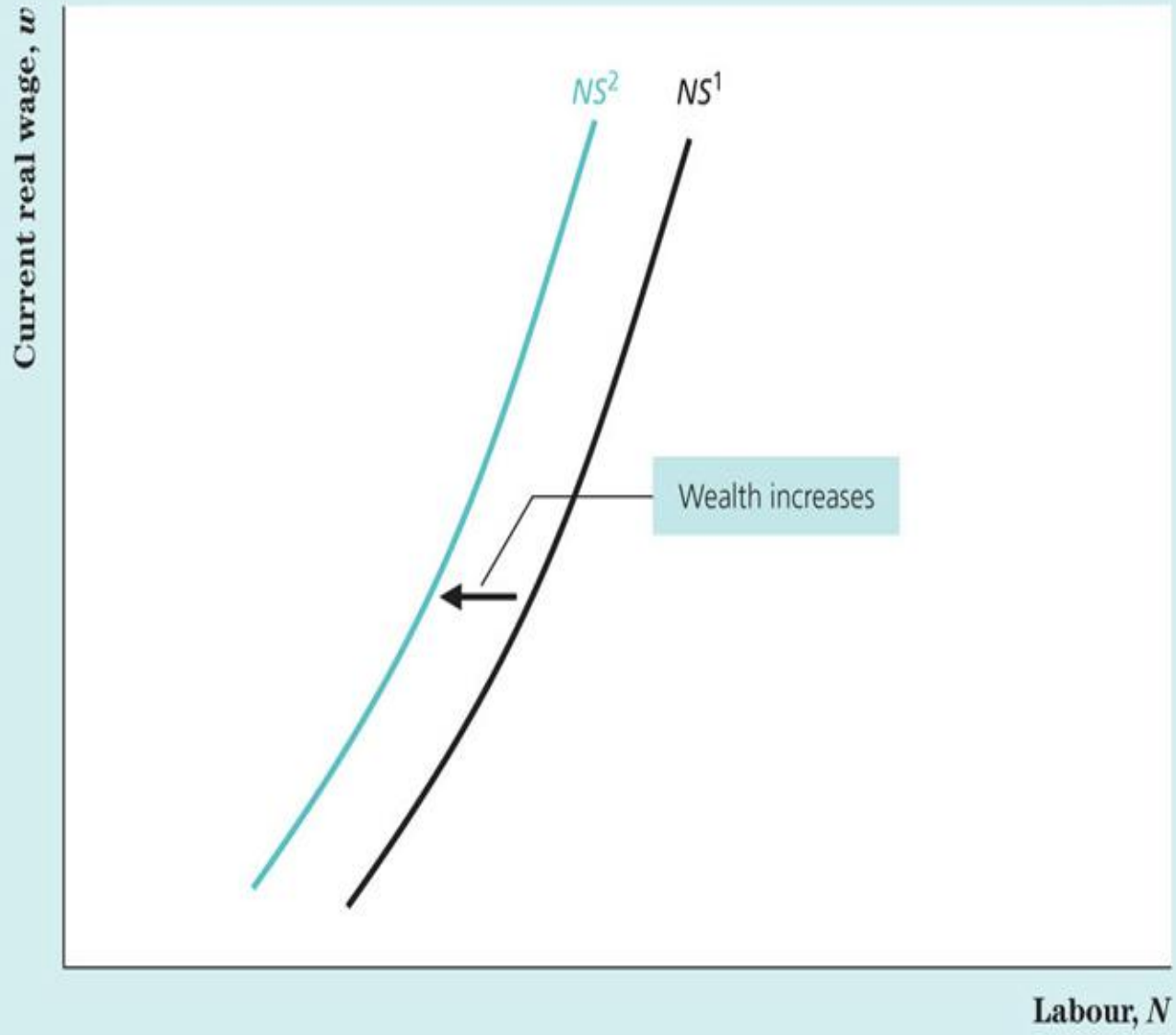
Aggregate Labour Supply (continued)

- *Shifts in the curve:*
  - An increase in wealth or future income will shift the curve leftward.
  - An increase in the working age population or a change in the participation rate will shift the curve rightward.
    - The rise in the participation of women is an interesting example – social attitudes changed – as is the elimination of mandatory retirement.
  - The recent reductions in wealth (due to the recession) will cause people to postpone retirement or re-enter the workplace.

FIGURE 3.8

THE EFFECT ON LABOUR SUPPLY OF AN INCREASE IN WEALTH

An increase in wealth reduces the amount of labour supplied at any real wage. Therefore, an increase in wealth causes the labour supply curve to shift to the left. Similarly, an increase in the expected future real wage, which has the effect of making the worker wealthier, reduces the amount of labour supplied at any given current real wage and shifts the labour supply curve to the left.



What do the Data Show on Hours Worked

- Over time, as Canada has become wealthier (a permanent rise in real wages), hours worked and the length of time in work (both measures of labour supply) have declined.
- This pattern shows up in cross section analysis where individual country experiences are compared at a point in time.

FIGURE 3.9

AVERAGE WEEKLY HOURS, CANADIAN MANUFACTURING

Reflecting the income effect of wages on labour supply, the steady increase in the real wage in Canada during the 20th century tended to reduce the average weekly hours of manufacturing workers. Weekly hours fluctuated sharply during the Great Depression and World War II, then declined further in the postwar period.

Source: For 1901–1927, average weekly hours of machinists in Halifax, Montreal, Toronto, Winnipeg, and Vancouver: Adapted from *Wages and Hours of Labour in Canada, 1901–1920,* Report No. 1, Department of Labour, Canada, Table II(d) and *Wages and Hours of Labour in Canada, 1920–1927,* Report No. 11, Department of Labour, Canada, Table I(b). For 1926–1955, average weekly hours of non-agricultural workers: *Historical Statistics of Canada* (1st ed., 1965), Series D408. For 1945–1970, average weekly hours in manufacturing: *Historical Statistics of Canada,* Series E131. For 1961–2006, Statistics Canada, CANSIM series v718395 and v15901097.

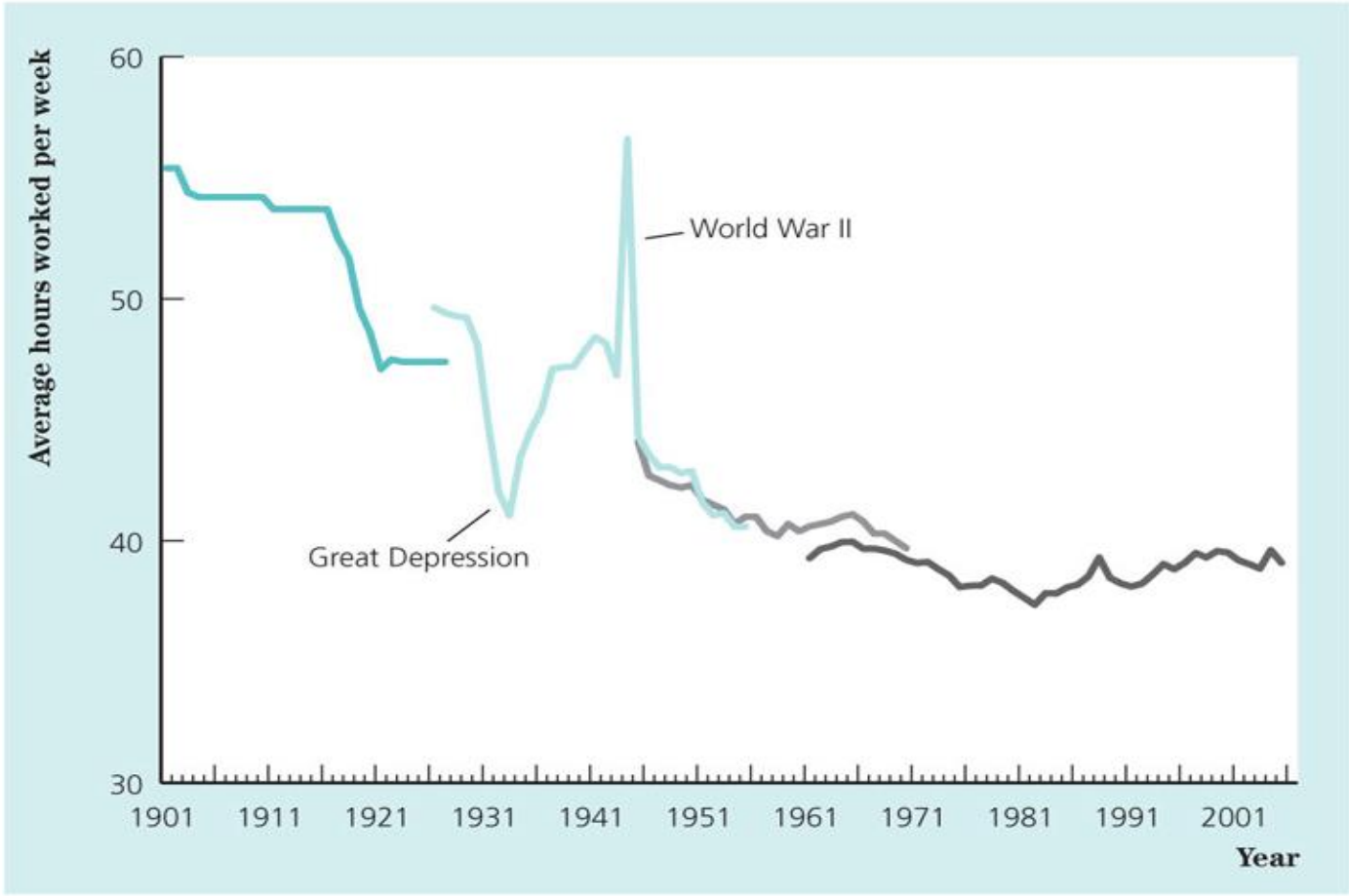
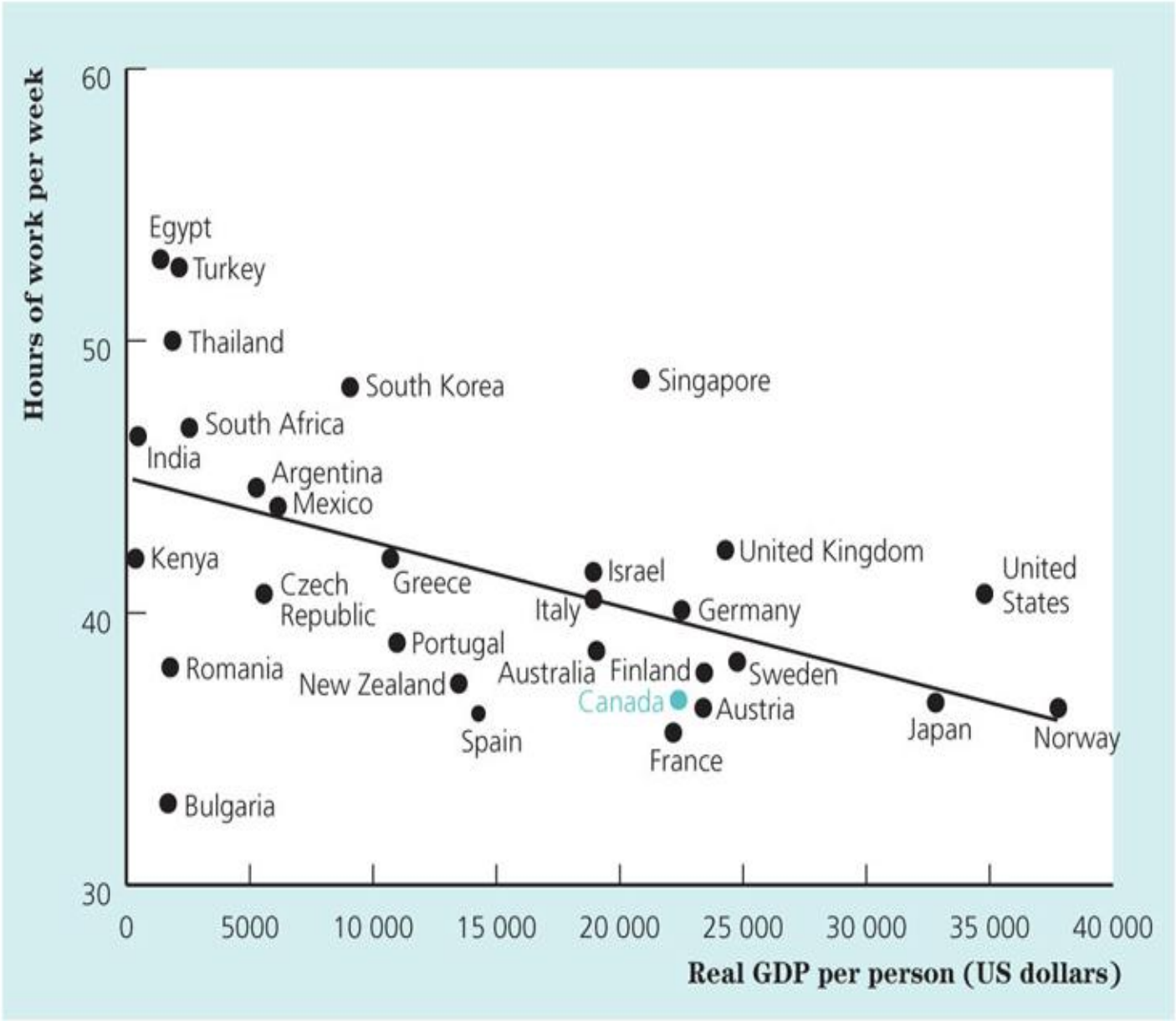


FIGURE 3.10

THE WORKWEEK AND REAL GDP PER PERSON IN 30 COUNTRIES

The point corresponding to each country shows the country's real GDP per person in 2001 on the horizontal axis and the average number of hours worked per week in manufacturing on the vertical axis. A trend line has been drawn through these data. Because of the income effect on labour supply, richer countries tend to have short workweeks.

Source: International Labour Office Bureau of Statistics (*http://laborsta.ilo.org*).



Factors that Shift the Aggregate Labour Supply Curve – A Summary

All else equal an increase in	Causes the Labour Supply Curve to Shift	Reason
Wealth	Left	Increases in wealth increase the amount of leisure workers can afford
Expected future real wage	Left	Increases in expected future real wage increases amount of leisure workers can afford.
Working age population	Right	Increased number of potential workers increases amount of labour supplied.
Participation rate	Right	Increased number of people wanting to work increases amount of labour supplied.