ECON 222
Macroeconomic Theory I
Winter Term 2011
Answers to assignment 4

Question 1: Money and inflation (20 Marks)

(a) By neutrality economists mean that, over the long term, changes in the money supply have no effect on real variables in the economy like Y and r. The equation shown in the question would have this property. It could be written as: $M^d = M = P(36 + 0.5Y)$, where the un-superscripted M is the money supply. The equation implies that, with <u>unchanged</u> real income, a rise in M will result in a proportional rise in prices. (**5 Marks**)

(b) If the economy is growing at 3 per cent, velocity is 1 and the inflation target is 2 per cent, then the central bank must maintain a growth rate of money of 3.5 per cent. To see this, differentiate both sides of the M/P equation:
$d(M/P) = dM/P - (dP/P)(M/P) = [dM/M - dP/P](M/P)$
Setting this equal to the derivative of the right hand side:
$[dM/M - dP/P](M/P) = 0.5dY$
Re-arranging terms we get:
$dM/M = dP/P + 0.5dY(P/M) = dP/P + 0.5(dY/Y)[(PY)/M]$ then with velocity $(PY/M) = 1$, then $dM/M = 2.0 + 0.5(3.0)(1) = 3.5\%$.

(c) If the price level is 1.0 then $M^d = 36 + 0.5Y$. Taking the first derivative yields: $\Delta M/\Delta Y = 0.5$. The elasticity of M wrt Y is $(\Delta M/M)/(\Delta Y/Y)$, which can be written as $(\Delta M/\Delta Y)(Y/M)$. Using this we get: $(\Delta M/M)/(\Delta Y/Y) = 0.5(Y/M)$.
The income elasticity would be constant as long as the velocity of money (Y/M) is constant, which could change because of financial innovation. (**5 Marks**)

(d) The money demand function is now written as $M^d/P = 36 + 0.5Y - 100i$. With $Y = 100$ and $M/P = 80$, then $80 = 36 + 0.5(100) - 100i$ which implies that $i = (86 - 80)/100 = 0.06$. If inflation is maintained at 2 per cent, then $\pi^e = 0.02$. The real rate of interest is $r = i - \pi^e = 0.06 - 0.02 = 0.04$. (**5 Marks**)

Question 2: Deriving and solving the IS-LM-AD model – closed economy version (25 Marks)

(a) To get the IS curve, we need to first find desired saving (S^d), which is:

$$S^d = Y - C^d - G = Y - (25 + 0.5(Y - T) - 100r) - G$$

$$= -25 - G + 0.5T + 0.5Y + 100r$$

And then set S^d equal to I^d:

$$30 - 200r = -25 - G + 0.5T + 0.5Y + 100r$$

Solving for r as a function of Y and the exogenous variables that will shift the curve yields:

$r = (55 - 0.5Y + G - 0.5T)/300$ (**IS curve**)

The IS curve shows the combinations of Y and r that give goods market equilibrium. The curve is downward sloping in Y: an increase in Y will raise saving and a lower interest rate is required to induce a corresponding increase in investment. The curve will shift out (in) if G (T) were to rise. Shifts in the constant term (325/300) reflect changed in consumer sentiment/productivity and or business sentiment, among other things.

The LM curve can be derived by re-writing the demand for money function in terms of r and Y and noting that $M^d = M$ (the money supply). Thus:

$r = (12 + 0.7Y - 300\pi^e - M/P)/300$ (**LM curve**)

The curve shows the combinations of Y and r that give asset market equilibrium. The LM curve is upward sloping in Y. An increase in Y will raise M^d but with the money supply unchanged, r needs to rise to restore equilibrium. Increases in M/P and π^e will shift the curve outward. (**8 Marks**)

(b) To solve for both Y and r, start by setting the LM curve equal to the IS curve. This yields:

$12 + 0.7Y - 300\pi^e - M/P = 55 - 0.5Y + G - 0.5T$

Collecting up like terms we get:

$1.2Y = 55 - 12 + G - 0.5T + M/P + 300\pi^e$

Given G=T=10, $\pi^e = 0$ and M/P = 90/1.5, we get:

$1.2Y = 108 \Rightarrow Y = 90.$

We can use either the IS or LM curve to solve for r. Using the former we get:

$r = (55 - 0.5Y + G - 0.5T)/300 = 15/300 = 0.05$

The graph should show an upward sloping LM curve intersecting the downward sloping IS curve at an interest rate of 5 per cent and Y = 90. (**8 Marks**)

(c) Start with the equality of the IS and LM curve as above:

$12 + 0.7Y - 300\pi^e - M/P = 55 - 0.5Y + G - 0.5T$, this can be written as:

$Y = (43 + G - 0.5T + M/P + 300\pi^e)/1.2$ (**AD curve**)

This shows that the AD curve is downwards sloping in P. Given, G=T=10, $\pi^e = 0$, M = 90 and now P =1.0 we get:

$Y = (138)/1.2 = 115$

A fall in the price level raises real output from 90 to 115. The mechanism is as follows, the drop in P shifts to the right the LM curve along an unchanged IS curve. This lowers interest rates, which stimulates both investment and consumption, raising output. The students should illustrate this in a graph showing in the upper panel the shift in the LM curve and in the lower panel the movement along the AD curve. (**9 Marks**)

Question 3: Some policy implications of the IS-LM-AD model (30 Marks)

(a) The now more pessimistic consumers are represented by the now lower constant term in the consumption equation. The new desired national saving function becomes:

$S^d = Y - C^d - G = Y - (25 + 0.5(Y - T) - 100r) - G$

$= -\mathbf{20} - G + 0.5T + 0.5Y + 100r$

Setting the desired saving function equal to the investment function yields:

$30 - 200r = -\mathbf{20} - G + 0.5T + 0.5Y + 100r$

$r = (\mathbf{50} - 0.5Y + G - 0.5T)/300$

Setting the new IS curve equal to the LM curve yields:

$12 + 0.7Y - 300\pi^e - M/P = \mathbf{50} - 0.5Y + G - 0.5T$

$Y = (\mathbf{50} - 12 + G - 0.5T + M/P + 300\pi^e)/1.2$

with the given values of G=T=10, M/P = (90/1.5) and $\pi^e = 0$, we get:

$Y = [\mathbf{50} - 12 + 10 - 5 + 90/1.5]/1.2 = 85.8\ \bar{3}\ \bar{3}$

Using the IS curve we can solve for r now that we have the new level of Y:

$r = (\mathbf{50} - 0.5(85.8\ \bar{3}\ \bar{3}) + 10 - 5)/300 = 4.02\ \bar{7}\ \bar{7}$

Output has fallen by 4.8% (approximately) and the real rate of interest is down by a little less than 100 basis points. Students should show a graph illustrating the leftward shift in the IS curve. (**8 Marks**)

(b) The most direct way to solve this problem is to start with the AD curve above in part (a). Thus:

$Y = [\mathbf{50} - 12 + G - 0.5T + M/P + 300\pi^e]/1.2$

This time we are given M/P = (90/1.5) and $\pi^e = 0$. We also know the equilibrium level of output (Y = 90), which is going to be the goal of fiscal policy. Then use the equation to solve for in turn T (given G = 10) and G (given T = 10). Thus:

$90 = [\mathbf{50} - 12 + 10 - 0.5T + 60]/1.2 \Rightarrow T = 0$ (taxes fall by 10).

As for government spending

$90 = [\mathbf{50} - 12 + G - 0.5(10) + 60]/1.2 \Rightarrow G = 15$ (spending increases by 5).

In each case, the effect on the interest rate is the same. To see this, start with the IS curve that has the lower level of consumption. Thus:

$r = (\mathbf{50} - 0.5Y + G - 0.5T)/300$

First solve for r assuming output is back to equilibrium ($Y = 90$), $G = 10$ and $T = 0$. Thus:

$r = (\mathbf{50} - 0.5(90) + 10)/300 = 0.05$

Now solve the same equation except that now $T = 10$ and $G = 15$, in addition to $Y = 90$. Thus:

$r = (\mathbf{50} - 0.5(90) + 15 - 0.5(10))/300 = 0.05$

The effect on interest rates is the same because each fiscal instrument had the same effect on the IS curve. (**10 Marks**)

(c) As Minister of Finance worried about the budget balance you would prefer spending increases to tax cuts. The effect of the spending increase would result in a budget deficit of 5 versus a deficit of 10 for a tax cut. Achieving the same effect through a tax cut requires a larger effort since part of the tax cut would be saved. (**4 Marks**)

(d) Left on its own, the economy would return to equilibrium. The fall in output below its equilibrium level would put downward pressure on the price level, which, with an unchanged nominal money supply (M), would shift the LM curve out and to the right. The shift occurs because the real money supply (M/P) is increased. The process would continue until the new LM curve intersected with the old IS curve at full employment output but with a lower level of interest rates. In fact the new level of interest rates would be lower than the initial short run levels. The students should show this graphically.

If monetary policy were used, the only difference is that the price level would be unchanged. The real money supply would still increase by enough to shift the LM curve back to equilibrium and interest rates would still be lower than the initial short-run equilibrium. (**8 Marks**)

Question 4: Solving the open economy version of the model and its policy implications (25 Marks)

(a) The LM curve remains unchanged and is:

$r = (12 + 0.7Y - 300\pi^e - M/P)/300$ (**LM curve**)

To derive the IS curve we start with the goods market equilibrium condition for an open economy:

$S^d - I^d = NX$

$S^d = Y - C^d - G = Y - (25 + 0.5(Y - T) - 100r) - G$

$\quad = -25 - G + 0.5T + 0.5Y + 100r$, while

$I^d = 30 - 200r$

Thus:

$-25 - G + 0.5T + 0.5Y + 100r - 30 + 200r = 46 - 0.3Y - 2e$

Collecting like terms yields:

$r = (101 - 0.8Y + G - 0.5T - 2e)/300$ (**IS curve**)

Compared with the closed economy case, the IS curve now is more steeply sloped wrt to Y, while there is an additional shift variable, in the form of the real exchange rate. Using the two relationships to eliminate r we get:

$(12 + 0.7Y - 300\pi^e - M/P)/300 = (101 - 0.8Y + G - 0.5T - 2e)/300$

Collecting like terms and remembering that $e = e_{nom}P/P^F$, where P^F is the foreign price level, we get:

$Y = (89 + M/P + 300\pi^e + G - 0.5T - 2(e_{nom}P/P^F))/1.5$ (**AD curve**)

Note that the price level has two separate effects, one working through the LM curve and the other through its effect on the real exchange rate and through that on the IS curve. Since the effects reinforce each other the final effect of a change on prices on Y is larger than in the case of a closed economy – the slope of the curve is flatter. (**5 Marks**)

(b) Given that we know $Y = 100$, we can use the LM curve to solve for r. Remembering that $M/P = 90/1.5 = 60$ and $\pi^e = 0$, then:

$r = (12 + 0.7Y - 300\pi^e - M/P)/300$

$\quad = 22/300 = 0.07\overline{3}$

We can use the AD curve to solve for e:

$Y = (89 + M/P + 300\pi^e + G - 0.5T - 2e)/1.5 \Rightarrow e = 2$

Given $Y = 100$, $r = 0.07\overline{3}$ and $e = 2$, then:

$C^d = 25 + 0.5(Y - T) - 100r = 25 + 0.5(100) - 0.5(10) - 100(0.07\overline{3}) = 62.67$

$I^d = 30 - 200r = 30 - 200(0.07\overline{3}) = 15.33$

$NX = 46 - 0.3Y - 2e = 46 - 0.3(100) - 4 = 12$

With G = 10 then the components sum to 100.

The definition of the real exchange rate is:

$e = e_{nom}P/P^F$ and with $P = 1.5$ and $P^F = 0.9$ then $e_{nom} = 1.2$ (**6 Marks**)

(c) Government spending will fall from 10 to 9. In the short run, the IS curve will shift leftward as a result of the fiscal contraction and this will put temporary downward pressure on domestic interest rates. We can solve the model for this short-run equilibrium by using the AD relationship and assuming e does not change. This gives:

$Y = (89 + M/P + 300\pi^e + G - 0.5T - 2(e_{nom}P/P^F))/1.5$ (**AD curve**)

$Y = (89 + 60 + 9 - 5 - 4)/1.5 = 99.\overline{3}$

From either the LM or IS curve this will put downward pressure on the domestic interest rate. Using the LM curve, we get:

$r = (12 + 0.7Y - 300\pi^e - M/P)/300$ (**LM curve**)

$r = (12 + 0.7(99.\overline{3}) - 60)/300 = 0.071\overline{7}$

The downward pressure on the domestic interest rate will generate capital outflows, pushing down the nominal exchange rate, which will increase net exports (NX). The rise in NX will start pushing the IS curve back towards equilibrium. This process will continue until the domestic interest rate is once again equal to the world interest rate ($r = 0.07\overline{3}$). The process can only stop when NX has risen by enough to offset the fiscal contraction (which removes the downward pressure on the exchange rate). We can use the IS curve to determine by how much e and e_{nom} (given unchanged P and P^F) have to fall:

$r = (101 - 0.8Y + G - 0.5T - 2e)/300$ (**IS curve**)

$0.07\overline{3} = (101 - 0.8(100) + 9 - 5 - 2e)/300 \Rightarrow e = 1.5$

We could have used the net export equation and solved for e, assuming that NX had to be larger by 1, the amount by which G has declined.

If $e = 1.5$, then given $P = 1.5$ and $P^F = 0.9$, $e_{nom} = 0.9$ (compared with an original values of 1.2).

The reduction in G, in the long run, does not result in lower Y but rather a change in its composition away from government spending and towards net exports. Investment is unaffected as it is not possible to change the interest rate.
(**6 Marks**)

(d) We first have to determine the short-run effect of a change in the nominal exchange rate from a peg of 1.2 to 0.9. For simplicity we can assume that during this period, P does not change and that the new fixed exchange rate is maintained. Therefore the new, lower *real* exchange rate is also unchanged at e=0.9(1.5/0.9)=1.5. Using the AD curve, we proceed in two steps, finding a temporary initial effect and then the short-run effect. Thus:

$Y = (89 + M/P + 300\pi^e + G - 0.5T - 2(e_{nom}P/P^F))/1.5$

$= (89 + 60 + 10 - 5 - 2(1.5))/1.5 = 100.\bar{6}\bar{6}$

The higher Y will put upward pressure on the domestic interest rate, which can be calculated using the LM curve. Thus:

$r = (12 + 0.7Y - 300\pi^e - M/P)/300 = (12 + 0.7(100.\bar{6}\bar{6}) - 60)/300 = 0.074\bar{8}\bar{8}$

Since the potentially higher interest rates will put upward pressure on the exchange rate, the central bank must now enter the market, expanding the money supply to offset the effect. The Bank increases M (holding P unchanged) by enough to reduce the upward pressure on interest rates. The LM curve will now intersect the IS curve which has shifted out at the world rate of interest ($0.07\bar{3}\bar{3}$). The short-run equilibrium is given by the IS curve, with the world rate of interest and the new, lower value of e (due to the new peg):

$r = (101 - 0.8Y + G - 0.5T - 2e)/300$ (**IS curve**)

$0.07\bar{3}\bar{3} = (101 - 0.8Y + 10 - 0.5(10) - 3)/300 \Rightarrow Y = 101.25$

This is larger than the initial effect ($100.\bar{6}\bar{6}$). This addition to Y is due to the monetary expansion necessary to maintain the nominal exchange rate peg. From the LM curve, the nominal money supply has increased by:

$r = (12 + 0.7Y - 300\pi^e - M/P)/300$ (**LM curve**)

$0.07\bar{3}\bar{3} = (12 + 0.7(101.25) - M/1.5)/300 \Rightarrow M = 91.3125$

Notice that part of the short-run increase in Y is due to net exports and the rest is due to consumption (because of the higher level of income). Investment has not changed.

The economy is now operating above its full-employment level and this will put upward pressure on domestic prices. This in turn will appreciate the real exchange rate until the expansionary effect of the lower nominal (and real) exchange rate has been offset. The process will shift both the LM curve (due to an increasing P) and the IS curve (as a result of a rising real exchange rate, e).

In the end, equilibrium is restored with the original level of Y(=100), r($=0.07\bar{3}\bar{3}$) and importantly the original *real* exchange rate, e(=2). With the nominal peg of e_{nom} = 0.9, the new price level have risen from 1.5 to 2.0. (**8 Marks**)