ECON 222
Macroeconomic Theory I
Winter Term 2011
Answers to assignment 2

Question 1 (**25 points**): Labour markets

(a) Given the standard production function, $Y = A[K^{\alpha} + N^{1-\alpha}]$, the students should first derive the marginal product of labour and set that equal to the wage rate (w). Thus: $dY/dN = (1-\alpha)A[N^{-\alpha}]=w$. Then re-arranging to get an expression in terms of just N we get the demand for labour (N^d) as:

$$N^d = [(1-\alpha)A/w]^{1/\alpha}$$

(b) Assuming that A = 4 and $\alpha = 0.5$, and that $N^d = N^s = w$, we get:

$$N^d = [(1-\alpha)A/w]^{1/\alpha} = 0.25w^2, \text{ which can be solved for w as}$$

$$0.25w^2 = [(1-\alpha)A/w]^{1/\alpha} = [2/w]^2 \Rightarrow w^4 = 4/0.25 \Rightarrow w = 2$$

Labour demand is $N^d = [(1-\alpha)A/w]^{1/\alpha} = (2/2)^2 = 1$

(c) The new wage is now 2(1.10) or 2.2. The amount of labour demanded will be:

$N^d = [(1-\alpha)A/w]^{1/\alpha} = 0.826$ (approximately), which is total employment. Households will be willing to supply $N^s = 0.25w^2 = 0.25(2.2)^2 = 1.21$. Total unemployment equal those will to work (1.21) less those employed (1.21–0.826 = 0.384). The unemployment rate would be U/(U + E) or 32%.

(d) The graph should show a downward sloping demand curve for labour and an upward sloping in w intersecting the same demand curve at w (=2) and a lower level of N (1). The minimum wage is above this level and the graph should identify both N^d and N^s.

Question 2 (**25 points**): Consumption

(a) The students can start with the identity showing the relationship between future consumption (c^f) and present consumption (c), current and future income (y and y^f). Thus:

(1) $c^f = (1 + r)(y - c) + y^f$

This expression shows how much one could consume in the future given how much one saved (y – c), the return on those saving (1 + r) and the amount of future income available. This expression can be re-arranged into the inter-temporal budget constraint putting consumption and income on different sides of the equation. Thus:

(2) $c + c^f/(1 + r) = y + y^f/(1 + r)$

In this form it states that the present value of lifetime consumption [PVLC = $c + c^f/(1 + r)$] has to be identically equal to the present value of lifetime resources [LVLR = $y + y^f/(1 + r)$].

To graph the budget constraint the students need to use equation (1). The explanation is that it shows all the combinations of c^f and c that are available to the consumer.

(b) Start by equating the slope of the utility curve to the slope of the budget constraint. Thus:

(3) $(11/9)c^f/c = (1+r) \Rightarrow c^f = (9/11)c(1+r)$

Then substitute (3) into (2) to get:

(4) $c(1+9/11) = [y + y^f/(1 + r)] \Rightarrow c = 0.55[100 + 120/1.20] = 110$

From (3), $c^f = (9/11)(110)(1.20) = 108$

Using the values of c and c^f in (2) we can verify that the inter-temporal budget constraint is satisfied. Thus:

$110 + 108/1.20 = 200 = (100 + 120/1.20)$

(c) The consumers were borrowers since c(=108) is greater than y(=100). If the interest rate falls to 10% then, using (4) again we get:

$c(1+9/11) = [y + y^f/(1 + r)] = 0.55(100 + 120/1.10) = 209.0909$

Since consumption rises with the fall in the interest rate we can say the substitution and income effects go in the same direction, which is to reduce present consumption (raise saving). This also follows directly from fact that they were borrowers in the initial period.

(d) This question asks to demonstrate the Ricardian Equivalence (RE) hypothesis. They can start with the inter-temporal budget constraint but with the lump sum transfer (Tr) thrown in. They should also account for the fact that the transfer will be clawed back in the future. Thus:

$c(1+9/11) = [y +Tr + [y^f - Tr(1+r)]/(1 + r)]$

As can be seen, PVLR is not affected so consumption remains unchanged. The policy could work to the extent that households are not as forward looking as suggested by RE.

Question 3 (**25 points**): Investment in a closed economy

(a) The students should start with the production ($Y = AK^{\alpha}N^{(1-\alpha)}$) and set the marginal product of capital equal to the user cost. Thus

(1) $MPK = \alpha AN^{(1-\alpha)}/ K^{(1-\alpha)} = uc$ and solving for K we get:

(2) $K = [\alpha AN^{(1-\alpha)}/uc]^{1/(1-\alpha)}$

(b) Noting that $uc = \{(r + d)/(1 - \tau)\}P_k = 0.5$ we can solve for the level of K as:

(3) $K = [(0.25)(2)(16^{0.75})/0.50]^{1/0.75} = 16$

(c) We can start with the demand for capital, equation (2). Thus:

$K = [\alpha AN^{(1-\alpha)}/uc]^{1/(1-\alpha)}$

With everything unchanged except A = 1.75, we have:

$K = [(0.25)(\mathbf{1.75})(16^{0.75})/0.50]^{1/0.75} = 13.3905$ (approximately)

They students can show the graph (Figure 4.5 in the text) with a downward sloping MPK line intersecting an unchanged uc line. The shock shifts the curve to the left and lowers the equilibrium level of capital.

(d) The easiest way is to start with equation (1) and find the new uc that would be consistent with the now lower A (=1.75) but the original K (=16). Thus:

$uc = \alpha AN^{(1-\alpha)}/ K^{(1-\alpha)} = (0.25)(1.75)[(16/16)]^{0.25} = 0.4375$

From the definition of $uc = \{(r + d)/(1 - \tau)\}P_k$ we get $P_k = 2.1875$

(e) Their explanation should follow the text (page 131) and they should start with the net investment relationship and re-write in terms of gross investment (I) as:

$I_t = K_{t+1} - (1 - d)K_t$

The key is to equate K_{t+1} with the desired stock of capital. They should then note that they can derive a relationship between K^* and the interest rate (via the cost of capital).

Question 4 (**25 points**): Saving and investment in a closed and open economy.

(a) Start by deriving desired saving, which is:

$S^d = Y - C^d - G$

From the C^d function we get:

$S^d = Y - 40 - 0.4Y + 150r - G = 100 - 40 - 40 + 150r - 15 = 5 + 150r$

Then setting $S^d = I^d$ we get:

$5 + 150r = 27.5 - 300r \Rightarrow r = 5.0\%$

This implies that $I^d = S^d = 12.5$ and $C^d = 72.5$, then

$Y = C^d + I^d + G = 72.5 + 12.5 + 15 = 100$

$S_{PVT} = Y - T - C = 100 - 15 - 72.5 = 12.5$. Since government saving is zero private and national saving are the same.

(b) If government spending rises to 20, desired saving would be:

$S^d = Y - 40 - 0.4Y + 150r - G = 100 - 40 - 40 + 150r - \mathbf{20} = 0 + 150r$

Setting $S^d = I^d$ we get:

$150r = 27.5 - 300r \Rightarrow r = 6.111...\%$. With Y unchanged, the interest rate has to rise to accommodate the increase in G. Desired investment now falls to 9.1666..., a drop of 3.333..., which is less than the increase of 5 in G. Some but not all investment is crowded out by the rise in G. The other effect is to cause C^d to decline to 70.8333... and **private** saving to rise.

$S_{PVT} = Y - T - C = 100 - 15 - 70.8333... = 14.1666...$

Now private and national saving are not identical. In particular, the rise in private saving has to be enough to offset the decline in government saving.

$S_{GVT} = T - G = -5$

Note that

$S_{PVT} + S_{GVT} = S^d = I^d = 9.1666...$

(c) Students should note that now the conditions for goods market equilibrium has changed In particular, the saving/investment identity now becomes: $S^d = I^d + CA = I^d + NX$

With the world rate of interest (r^W) given, the model now solves for net exports. The other way of saying this is national desired saving is used to finance desired investment and the current account balance (either accumulating foreign assets or foreign borrowing. The diagram should be that shown in Figure 5.2 or 5.3.

(d) Start with the equilibrium condition above:

$S^d = I^d + NX$

$5 + 150r = 27.5 – 300r + NX$

With $r = 0.05$, then,

$NX = 2.25$

Net exports are in surplus because the world rate of interest is above what it was in a closed economy. As a result, S^d is greater than I^d, with the difference being used to accumulate financial assets abroad.

Desired investment and consumption are:

$I^d = 27.5 – 300r = 11$

$C^d = 40 – 150r + 0.4Y = 71.75$

$Y = C^d + I^d + G + NX = 71.75 + 11 + 15 + 2.25 = 100$

Absorption, is defined as $C^d + I^d + G = 71.75 + 11 + 15 = 97.75$ß

(e) The productivity shock has increased desired investment, which is shown as an increase in the constant term from 27.5 to 30. The new investment function is:

$I^d = 30 – 300r$

Using the equilibrium condition ($S^d = I^d + NX$) we get:

$5 + 150r = 30 – 300r + NX$, which implies

$NX = – 0.25$

The increased in desired investment has to be financed by drawing on foreign sources of saving.