

ECON 222
Macroeconomic Theory I
Winter Term 2010/11

Assignment 1 - ANSWER KEY

Question 1: Some Canadian Macroeconomic Data (30 Marks)

- a) The three time series are plotted in figure (1). There are some interesting patterns:
- 1) The number of employed people showed a substantial increase over these 20 years;
 - 2) In the early 90's the number of employed had a short run behaviour different from the long run one: it fell for a couple of years but then bounced back and started rising at a rate similar to the one showed in the 80's.
 - 3) The number of unemployed people did not show any long run trend: it kept oscillating around the same value, approximately equal to 1.5 million.
 - 4) The Labor Force, being the sum of employed and unemployed people, inherits the behaviour of the these two. In the long run we observe an increasing trend, with the rate of growth (the slope) decreasing in the early 90's and increasing again in the mid 90's.

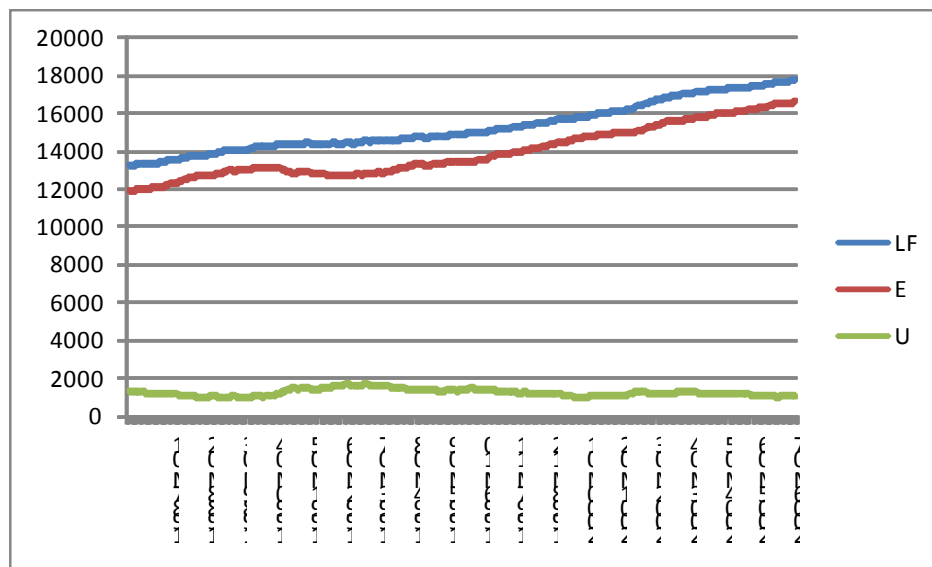


Figure 1 - Employed, Unemployed, and People in the Labor Force, 1986-2006.

- b) The Canadian unemployment rate is plotted in figure (2). In the 80's the unemployment rate had a sharp decrease, it plateaued at the 8.00% level for two years and it increased abruptly and quite steadily for five years. From 1993 to 2006 the unemployment rate decreased by six percentage points. The series is quite volatile, that is there are several sudden changes from one month to the next. From what we discovered in figure (1), it seems safe to conclude that the decrease in the unemployment rate in the last 15 years was mainly driven by more people finding jobs than people leaving the labor force.

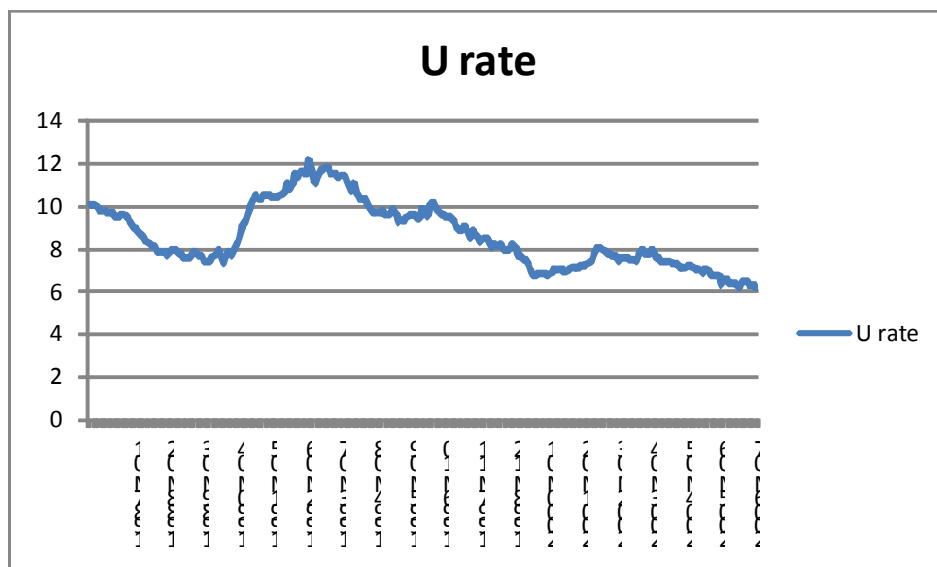


Figure 2 - Unemployment Rate, 1986-2006.

c) The average unemployment rates for the years 1986, 1996, 2006 are reported in table 1.

d) The time series for the share of immigrants in the population is plotted in figure (3). This series seems not to be seasonally adjusted: we detect a 'sawtooth' behaviour, with frequent and repeated ups and downs. This might be explained by seasonal workers migrating into the Country for a short period of time, for example with a fixed term work permit. The medium run trends show a 17 percentage points increase in the 80's and early 90's, a 12 percentage points decrease in the remaining part of the 90's, and a 8 percentage points increase in the more recent past. The Immigrants' share behaves exactly as the series of the number of immigrants, since the data for the population are a smooth line.

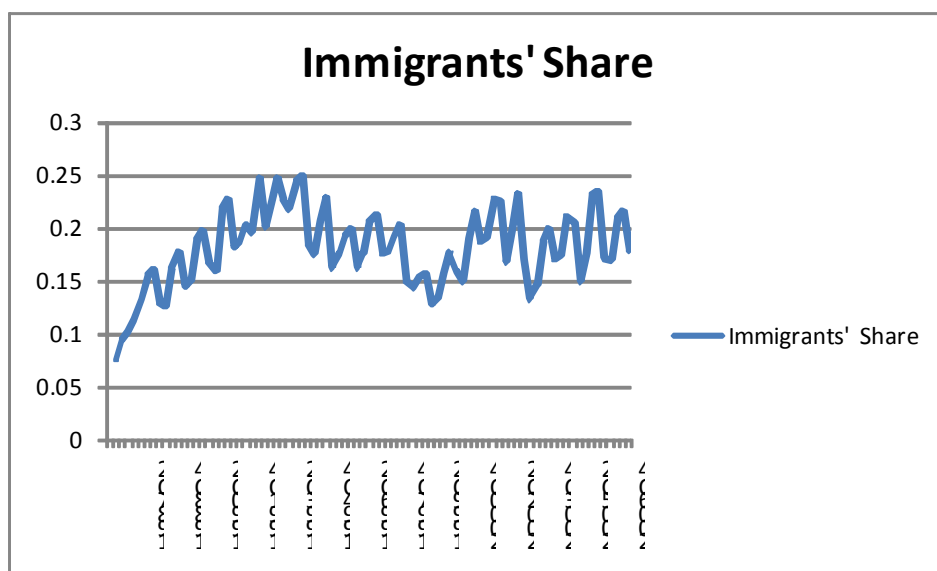


Figure 3 - Share of Immigrants, 1986-2006.

e) See table 2 below. Both the population and the immigrants growth rates seem to have slowed down over time. This finding is a clear one for the population growth rate, while it is more debatable for the growth rate of immigrants, given the high volatility of the series. Taking growth rates for different time periods would lead to very different results.

f) See table 3 below. The participation rate has been relatively stable for the first ten years of the sample, while it has increased by more than 3 percentage points in the second half of the sample. This could be explained by an increased availability of jobs, as seen in figure (1), that might have lead some 'discouraged workers' to enter the labor force or people close to retirement to defer the end of their working life.

Question 2: Employment, Output, Productivity and Inflation (20 Marks)

Part 1)

a. The average labor productivity of labour decreased by 1.56%

$$\begin{aligned}\frac{8000}{700} &= \frac{80}{7} \\ \frac{9000}{800} &= \frac{90}{8} \\ \frac{\frac{90}{8}}{\frac{80}{7}} - 1 &= -0.0156\end{aligned}$$

b. The inflation rate between 2009 and 2010 was 12.5%:

$$\frac{9.00}{8.00} - 1 = 0.125$$

c. The unemployment rate in 2009 was $\frac{70}{770} = 9.09\%$. In 2010 it was $\frac{100}{900} = 11.1\%$.

Part 2)

a. The real quantity for each category is obtained by dividing the nominal quantity by the Price index and multiplying by 100. They are reported in table 4.

b. The nominal *GDP* is equal to 691.32 (we rely on the expenditure approach to computing GDP, that is we add up all nominal quantities, remembering to put a minus in front of the imports). The same procedure is followed to get the real *GDP*, the only difference being that we add up the real quantities, obtaining 557.45.

c. The implicit price deflator is given by the ratio (Nominal *GDP*)/(Real *GDP*) multiplied by 100, which is equal to 124.

Question 3: More on Employment, Output, Productivity and Inflation (20 Marks)

Part 1)

The inflation rates were equal to 6.11%, 3.06%, 2.90%, and 2.75%. Hence, in 2007 the inflation rate was the highest.

Part 2)

(a) Use $CA = NX + NFP$ to get $NFP = 25$.

(b) Use $\frac{T}{Y} = 0.4$ and $Y = 2120$ to get $T = 848$. Then use $S_{govt} = T - G - TR - INT$ to get $G = 353$. Finally, use $Y = C + I + G + NX$ to get $C = 1299$. (Alternatively, use $S = I + CA$ and $S = Y + NFP - C - G$ to get $C = 1299$.)

Question 4: The Aggregate Production Function (30 Marks)

Part 1)

The expressions for the marginal productivities are as follows:

$$MPK = \alpha K^{\alpha-1} N^{\beta} L^{\gamma}$$

$$MPN = \beta K^{\alpha} N^{\beta-1} L^{\gamma}$$

$$MPL = \gamma K^{\alpha} N^{\beta} L^{\gamma-1}$$

Part 2)

a) As we can see from figure (4), the marginal productivity of labor is decreasing, but the rate at which it decreases diminishes with N . That is, the MPN is a convex function.

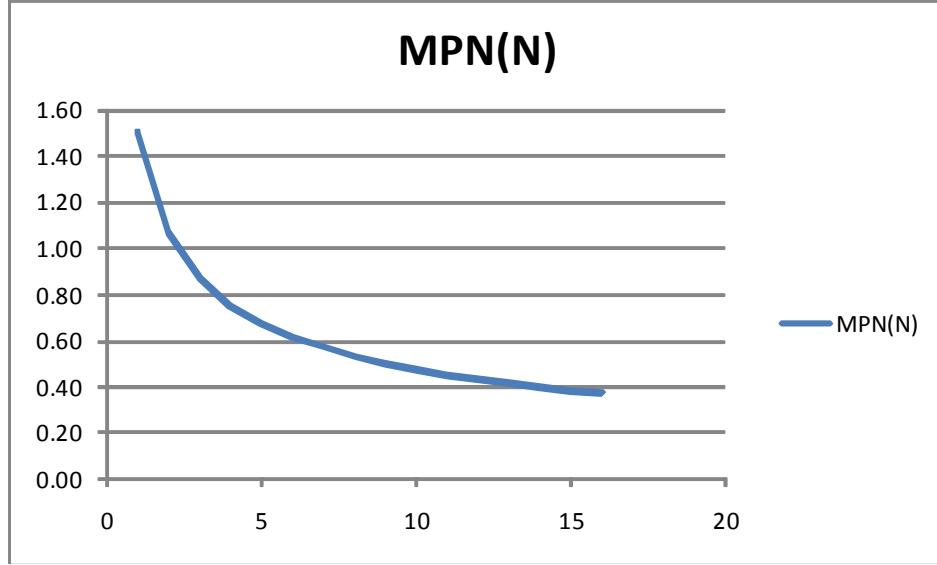


Figure 4 - Marginal Product of Labor

b) When the wage paid to labour is $p_N = \frac{1}{2}$ the level of employment in the economy is $N = 9$. To get this result we need to impose that the labor market is in equilibrium, that is that the price of labor is equal to its marginal product:

$$\begin{aligned} p_N &= MPN(N) \rightarrow \frac{1}{2} = \beta K^{\alpha} N^{\beta-1} L^{\gamma} = \frac{1}{2} \sqrt{9} N^{-\frac{1}{2}} \sqrt{1} \\ 1 &= 3 N^{-\frac{1}{2}} \rightarrow N^{\frac{1}{2}} = 3 \rightarrow N = 9 \end{aligned}$$

c) We need to see the behaviour of the derivative of the MPN :

$$\frac{\partial MPN(K, N, L)}{\partial N} = \beta(\beta - 1) K^{\alpha} N^{\beta-2} L^{\gamma} = \beta(\beta - 1) \frac{K^{\alpha} L^{\gamma}}{N^{2-\beta}}$$

This quantity is negative, since $\beta > 0$, $(\beta - 1) < 0$, and $\frac{K^{\alpha} L^{\gamma}}{N^{2-\beta}} > 0$. Again, we get decreasing marginal product of labor.

d) Table 4 report the values requested. Notice that each input is essential to production, that is if we do not use any labor we don't get any output no matter what the capital stock is. The same comment applies for capital (and land). Production is increasing monotonically in the quantity of inputs. But, since in our example the curvature is more pronounced in K (that is $\alpha > \beta$), if we compare the change in output

deriving from a change in one of the inputs (keeping the other fixed) this is higher for capital. In other words, if we compare the marginal product of capital and the marginal product of labor, for the same pairs of values of K and N (with inverted roles in the two marginal products), the MPK is always higher than the MPN .

Part 3)

a) The expressions for the factors' incomes are as follows:

$$p_K K = MPK \cdot K = \alpha K^{\alpha-1} N^{\beta} L^{\gamma} \cdot K = \alpha K^{\alpha} N^{\beta} L^{\gamma}$$

$$p_N N = MPN \cdot N = \beta K^{\alpha} N^{\beta-1} L^{\gamma} \cdot N = \beta K^{\alpha} N^{\beta} L^{\gamma}$$

$$p_L L = MPL \cdot L = \gamma K^{\alpha} N^{\beta} L^{\gamma-1} \cdot L = \gamma K^{\alpha} N^{\beta} L^{\gamma}$$

b) If they are between 0 and 1, α , β , and γ always represent the income share of the factor they are the exponent of:

$$\text{For capital } \frac{p_K K}{Y} = \frac{\alpha K^{\alpha} N^{\beta} L^{\gamma}}{Y} = \frac{\alpha K^{\alpha} N^{\beta} L^{\gamma}}{K^{\alpha} N^{\beta} L^{\gamma}} = \alpha$$

$$\text{For labor } \frac{p_N N}{Y} = \frac{\beta K^{\alpha} N^{\beta} L^{\gamma}}{Y} = \frac{\beta K^{\alpha} N^{\beta} L^{\gamma}}{K^{\alpha} N^{\beta} L^{\gamma}} = \beta$$

$$\text{For land } \frac{p_L L}{Y} = \frac{\gamma K^{\alpha} N^{\beta} L^{\gamma}}{Y} = \frac{\gamma K^{\alpha} N^{\beta} L^{\gamma}}{K^{\alpha} N^{\beta} L^{\gamma}} = \gamma$$

<i>Year</i>	<i>Unemployment Rate (%)</i>
<i>1986</i>	9.69
<i>1996</i>	9.65
<i>2006</i>	6.30

Table 1: Question 1, Part1

<i>Year</i>	<i>Population</i>	<i>Pop. Growth rate</i>	<i>Immigrants</i>	<i>Imm. Growth rate</i>
<i>1986</i>	26,068,353	—	24,836	—
<i>1996</i>	29,570,577	13.43%	32,720	31.75%
<i>2006</i>	32,603,606	10.26%	38,008	16.16%

Table 2: Question 1, Part2

<i>Year</i>	<i>Participation Rate (%)</i>
<i>1986</i>	50.91
<i>1996</i>	50.24
<i>2006</i>	53.96

Table 3: Question 1, Part2

	<i>Real Value</i>
<i>Consumption</i>	338.2
<i>Fixed investment</i>	100.9
<i>Government purchases</i>	134.6
<i>Exports</i>	178.8
<i>Imports</i>	192.1
<i>Change in inventories</i>	-3.0

Table 4: Question 2, Part2

	<i>K=0</i>	<i>K=1</i>	<i>K=2</i>	<i>K=4</i>	<i>K=16</i>
<i>N=0</i>					
<i>N=1</i>					
<i>N=2</i>					
<i>N=4</i>					
<i>N=16</i>					

Table 5: Question 4, Part2