# ECON 222A
# Macroeconomic Theory I

## Productivity, Output, and Employment

## Lecture 5

# Announcements

- Homework due this Friday.
- Teaching Assistants:
  - Office hours available online
- Class representative:
  - Responsible of teacher's evaluation.

# Today's Lecture

- We turn from Measurement to Analysis
- Production Functions
- Demand for Labor
- Introduce Labor Supply

# Macroeconomic Analysis

- Goal: Build a Macroeconomic model, that is a general framework to study economic questions from an aggregate perspective.

- Ch. 3 Labor Mkt;
- Ch. 4 Goods Mkt;
- Ch. 7 Asset Mkt;
- Put it all together: Ch. 9 *IS-LM* Framework

# Production

- Output (GDP) depends on:

1. Quantity of inputs (factors of production)
    a. Labor (*N*) (workers)
    b. Capital (*K*) (factories, machines)
    c. Raw materials and energy
    d. Technology and management

2. Productivity of each input

# Productivity

- How effectively are inputs used? (capital, labor…)
- Key to a country's well-being
- The U.S. tend to be the most productive in the world
- Canada is lagging behind (productivity gap)
- Surprisingly, Italy in the late 90's had a very high productivity. But it is falling.

# Production Function

- Mathematical way of thinking about the relationship between inputs, productivity and output.
- $Y = A\,F(K,N)$

– $Y$: output in a given period of time

– $A$: measures overall productivity

– $K$: capital stock in a period

– $N$: number of workers in a period

– $F(.,.)$: function that turns inputs into output

# The Simplest Economy

- This Classroom…
- We have Capital: the laptop, the classroom, and the projector.
- We have labor: me and the computer technician.
- If we combine the inputs we get the output (from 8.30 to 10.00am): a lecture.
- We can generalize this idea to the whole economy. This gets $Y = A\,F(K,N)$

# The *F* Thing

$$Y = AF(K,N) = AK^{\alpha}N^{\beta} = AK^{\alpha}N^{1-\alpha}$$

- *F* tells us how changes in *K* and *N* change *Y*, for a given value of *A*
- Returns to scale: IRS, DRS, CRS
- We will focus on CRS, i.e. $\beta=1-\alpha$, with $0<\alpha<1$

# The *A* Thing

$$Y = AF(K,N) = AK^{\alpha}N^{\beta} = AK^{\alpha}N^{1-\alpha}$$

- That *A* thing is quite tricky: is anything else that affects *Y* (Technological change)
- If *A* goes up by 10%, *Y* goes up 10%, for given *K, N*
- In practice, we observe *Y, K,* and *N* at the firm or national level and can back out *A*

## TABLE 3.1

**The Production Function for Canada, 1981–2003**

Production function: $Y = AK^{0.3}N^{0.7}$

| Year | (1) Real GDP, Y (Billions of 1997 dollars) | (2) Capital, K (Billions of 1997 dollars) | (3) Labour, N (Millions of workers) | (4) Total Factor Productivity, A* | (5) Growth in Total Factor Productivity (% Change in A) |
|---|---|---|---|---|---|
| 1981 | 600 | 1019 | 11.3 | 13.77 | |
| 1982 | 583 | 1056 | 10.9 | 13.52 | –1.8 |
| 1983 | 599 | 1084 | 11.0 | 13.71 | 1.4 |
| 1984 | 634 | 1112 | 11.3 | 14.16 | 3.2 |
| 1985 | 664 | 1142 | 11.6 | 14.43 | 1.9 |
| 1986 | 680 | 1170 | 12.0 | 14.36 | –0.5 |
| 1987 | 709 | 1201 | 12.3 | 14.57 | 1.4 |
| 1988 | 744 | 1241 | 12.7 | 14.82 | 1.7 |
| 1989 | 764 | 1282 | 13.0 | 14.83 | 0.1 |
| 1990 | 765 | 1319 | 13.1 | 14.66 | –1.2 |
| 1991 | 749 | 1350 | 12.9 | 14.43 | –1.5 |
| 1992 | 756 | 1374 | 12.8 | 14.55 | 0.8 |
| 1993 | 774 | 1395 | 12.9 | 14.75 | 1.3 |
| 1994 | 811 | 1423 | 13.1 | 15.15 | 2.8 |
| 1995 | 833 | 1452 | 13.4 | 15.29 | 0.9 |
| 1996 | 847 | 1481 | 13.5 | 15.36 | 0.4 |
| 1997 | 883 | 1524 | 13.8 | 15.62 | 1.7 |
| 1998 | 919 | 1568 | 14.1 | 15.83 | 1.3 |
| 1999 | 970 | 1617 | 14.5 | 16.23 | 2.6 |
| 2000 | 1020 | 1669 | 14.9 | 16.62 | 2.4 |
| 2001 | 1039 | 1718 | 15.1 | 16.64 | 0.1 |
| 2002 | 1075 | 1759 | 15.4 | 16.83 | 1.2 |
| 2003 | 1096 | 1798 | 15.7 | 16.81 | –0.2 |

*Total factor productivity is calculated by the formula $A = Y/K^{0.3}N^{0.7}$.

Source: Statistics Canada, CANSIM II series v1078498, v2461119, and v3860085.

# The Shape of the Production Function

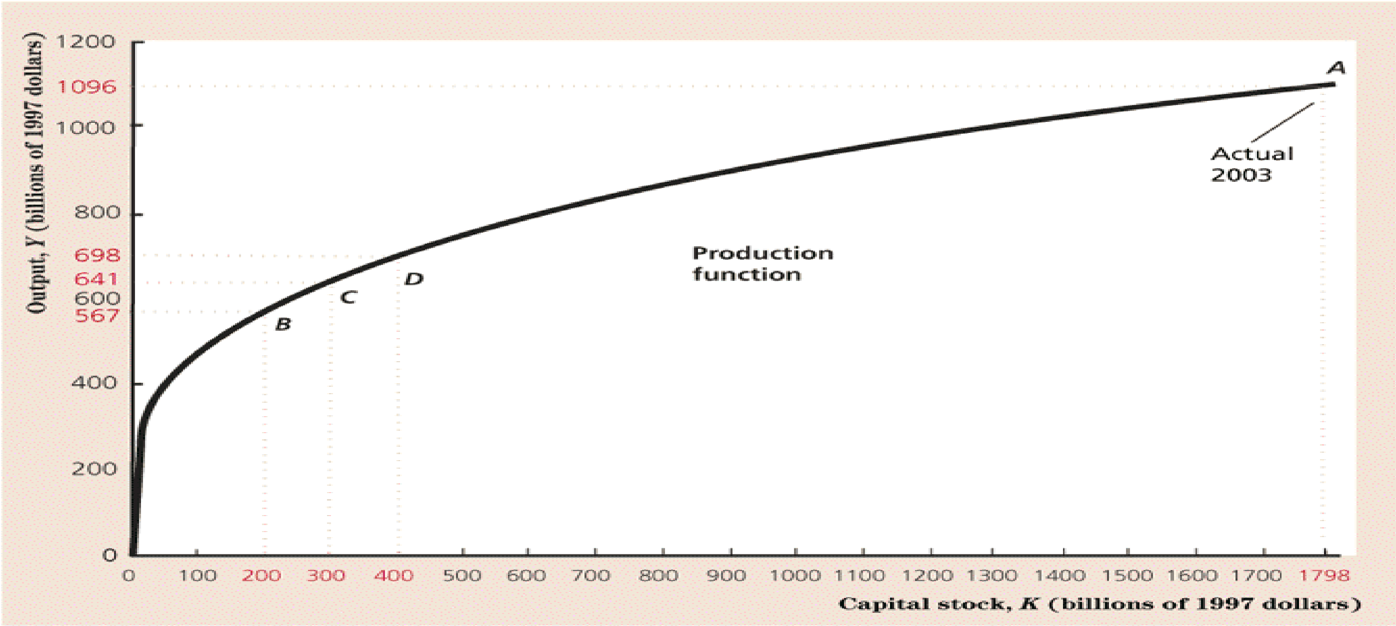
- Show it by fixing two of (*A*,*K*,*N*) and letting the remaining term vary helps us see it.
- Ex: $\alpha$=0.3, *A*=16.81, *N*=15.7 (see next fig)
- Two properties:

1. production function slopes upward from left to right, $F'>0$;
2. slope of the production function becomes flatter from left to right, $F''<0$.

## FIGURE 3.1

### THE PRODUCTION FUNCTION RELATING OUTPUT AND CAPITAL

This production function shows how much output the Canadian economy could produce for each level of Canadian capital stock, holding labour and productivity at 2003 levels. Point *A* corresponds to the actual 2003 output and capital stock. The production function has diminishing marginal productivity of capital: Raising the capital stock by $100 billion in order to move from point *B* to point *C* raises output by $74 billion, but adding another $100 billion in capital to go from point *C* to point *D* increases output by only $57 billion.

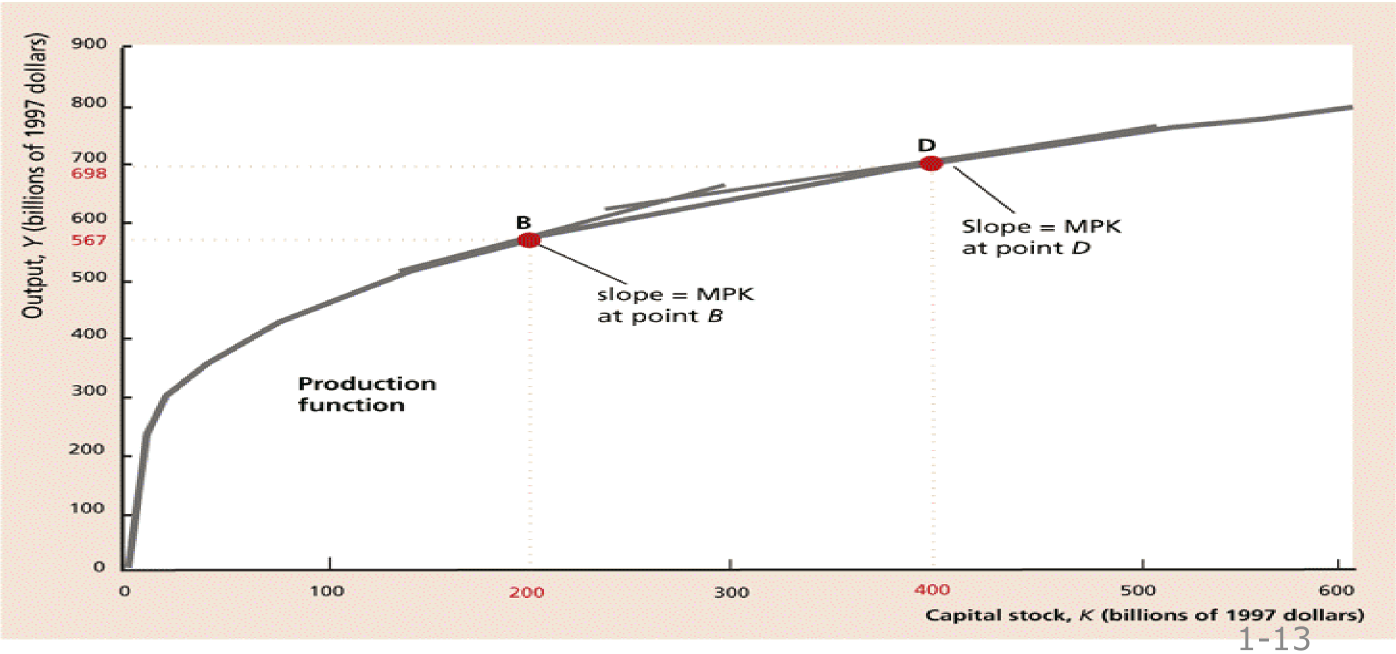





## FIGURE 3.2

### THE MARGINAL PRODUCT OF CAPITAL

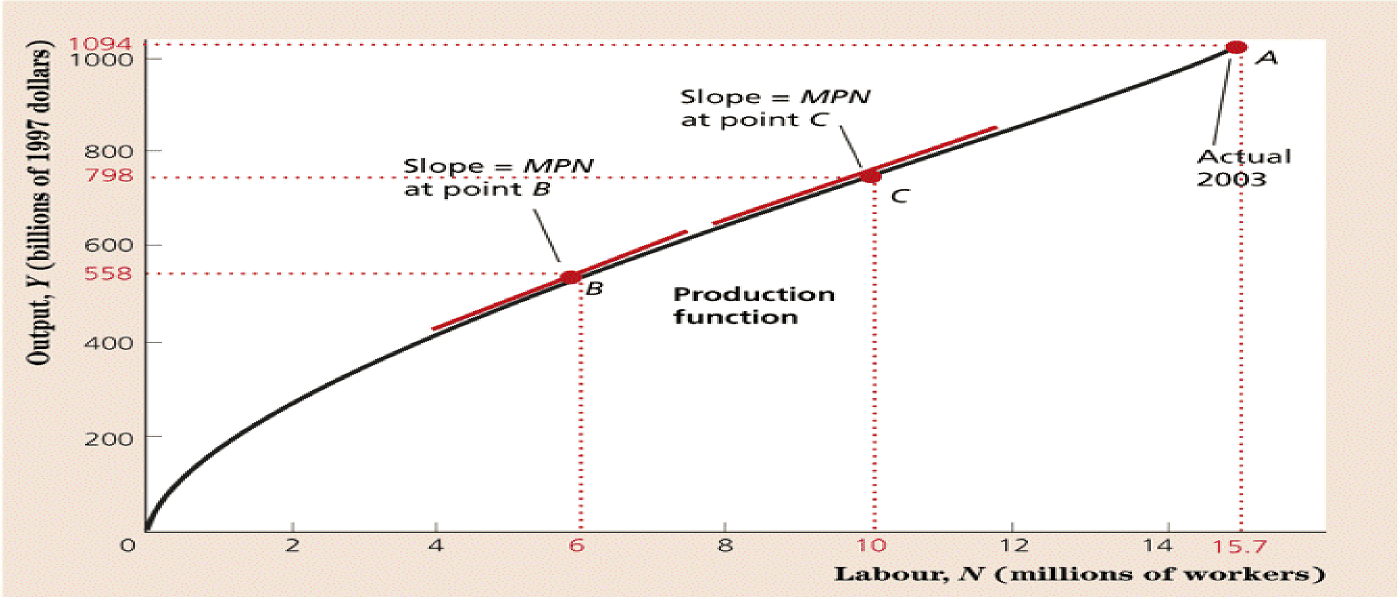
The marginal product of capital (*MPK*) at any point can be measured as the slope of the line tangent to the production function at that point. Because the slope of the line tangent to the production function at point *B* is greater than the slope of the line tangent to the production function at point *D*, we know that the *MPK* is greater at *B* than at *D*. At higher levels of capital stock, the *MPK* is lower, reflecting diminishing marginal productivity of capital.



## Figure 3.3

**The production function relating output and labour**

This production function shows how much output the Canadian economy could produce at each level of employment (labour input), holding productivity and the capital stock constant at 2003 levels. Point *A* corresponds to actual 2003 output and employment. The marginal product of labour (*MPN*) at any point is measured as the slope of the line tangent to the production function at that point. The *MPN* is lower at higher levels of employment, reflecting diminishing marginal productivity of labour.

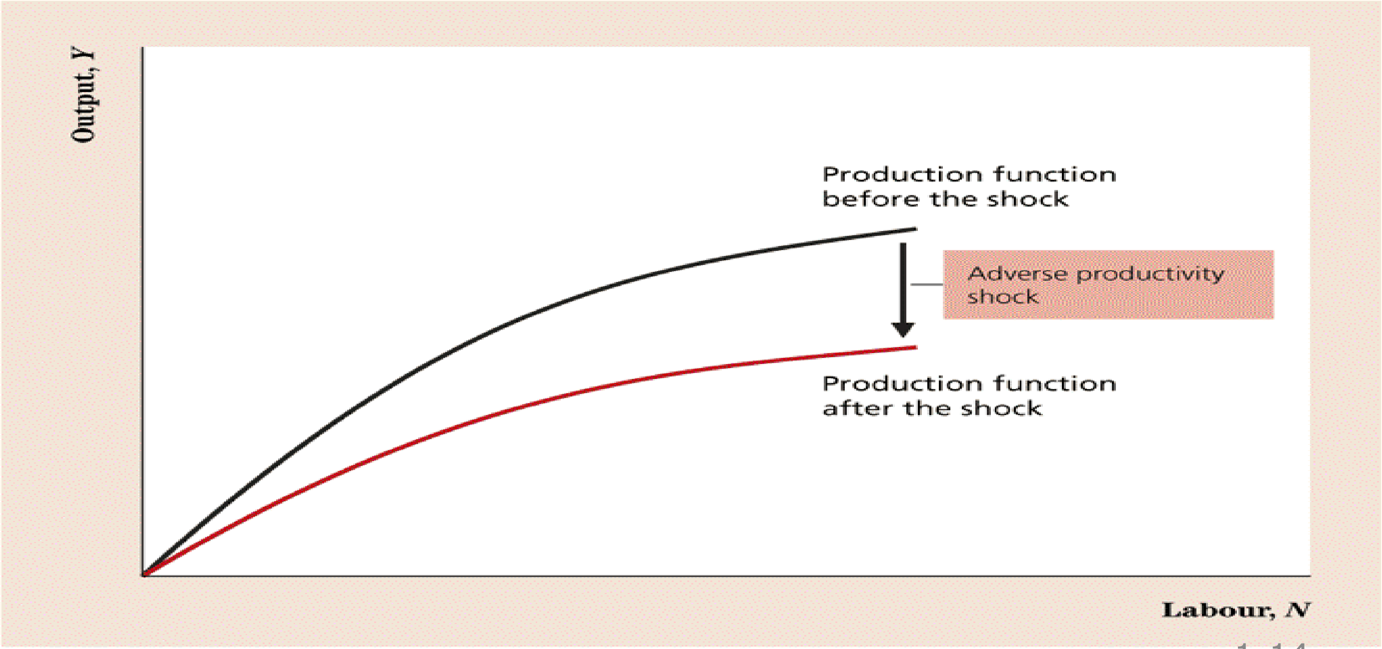





## Figure 3.4

**An adverse supply shock that lowers the *MPN***

An adverse supply shock is a downward shift of the production function. For any level of labour, the amount of output that can be produced is now less than before. The adverse shock reduces the slope of the production function at every level of employment. This corresponds to a decrease in the multiplying factor *A* in Eq. (3.2).



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# Marginal Product of Capital

- *MPK* = increase in *Y* from a 1-unit increase in *K*
- Change in variable on y-axis with a change in variable on x-axis
- Slope of the tangent
- Properties:
  - *MPK* is positive
  - *MPK* declines as *K* increases

# Marginal Product of Labor

- Same logic as before.
- *MPN* = increase in *Y* from a 1-unit increase in *N*
- Slope of the tangent
- Properties:
  - *MPN* is positive
  - *MPN* declines as *N* increases

# Diminishing Marginal Product

- What if I had 25 laptops to teach this class?
- What if McDonald's had 10,000 employees?
- How productive would some of those inputs be?

# Supply Shock

- Output changes also with changes in $A$: affects output levels for all levels of $K$ and $N$
- Thought to be one of the main causes of business cycles

- Positive supply shocks raise output that can be produced for given quantities of $K$ and $N$
- Ex: Computers, Internet

- Negative supply shocks
- Ex: Famine, wars, storm, high oil prices, viruses

# Labor Demand

- We just talked about what we can do with inputs

- Now we ask what producers would actually want to do?

- We are focusing on the short run here, fixing capital stock and varying labor

# Some assumptions

- Workers are all alike, they are homogeneous

- Firms are in competitive labor markets: take the wage ($w$) for workers as given (Firms compete for workers, they don’t collude or cooperate, and they are not big enough to affect the going wage)

- Firms profit-maximize in making their hiring decisions (behavioral assumption)

- $K$ assumed fixed (in Short Run)

# Marginal Analysis

- Profit maximization in competitive markets normally means *MB*=*MC* drives input-demand decisions
- Assume *F* looks like before (figure)
- Why?

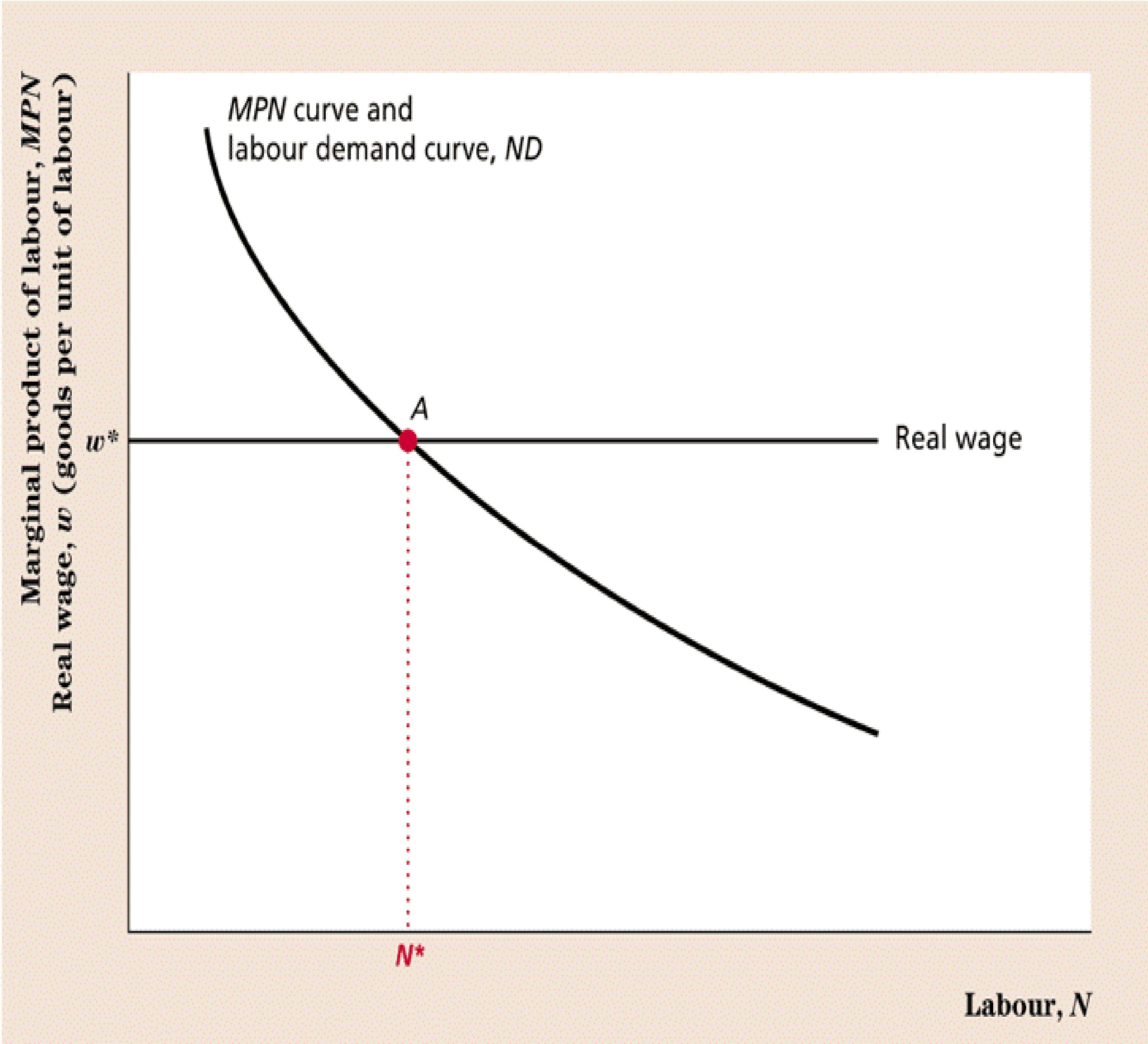
– if *MB* > *MC* -> hire more people

– if *MB* < *MC* -> fire some people

## FIGURE 3.5

### THE DETERMINATION OF LABOUR DEMAND

The amount of labour demanded is determined by locating the point on the *MPN* curve at which the *MPN* equals the real wage rate; the amount of labour corresponding to that point is the amount of labour demanded. For example, when the real wage is $w^*$, the *MPN* equals the real wage at point *A* and the quantity of labour demanded is $N^*$. The labour demand curve, *ND*, shows the amount of labour demanded at each level of the real wage. The labour demand curve is identical to the *MPN* curve.



# Real Vs. Nominal terms

- Hire so long as benefit of additional worker greater than his cost
- *MB*: Marginal Revenue Product of Labor (MRPN), or the Firm's additional revenue from hiring an additional worker

$$MRPN = P \times MPN$$

output price X additional output

- *MC*: (nominal) wage (*W*)
- Firm hires until Marginal Revenue Product of Labor equals Nominal Wage (W):

$$MRPN = W$$

# Real Vs. Nominal terms (2)

$$P \, x MPN = W$$

$$MPN = W/P$$

*(divide both sides by P)*

*Marginal Product of Labor = w* *(Real Wage)*

# Wage Changes

- If the wage changes (for example because the government changes the fiscal system), all else equal, labor demand changes

- More costly -> fire some people

- Cheaper -> hire some people

- Graph

# Real vs. Nominal Terms

- This can also be seen from the *firm's profits maximisation program :*

$$\max_{K,L} AK^{\alpha}L^{1-\alpha} - wL - rK$$

$$\Rightarrow \underbrace{w}_{MC} = \underbrace{(1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}}_{MB}$$

# Real Vs. Nominal Terms

- If *p* is the price of general output *:*

$$\max_{K,L} pAK^{\alpha}L^{1-\alpha} - WL - RK$$

$$\Rightarrow \underbrace{W}_{NMC} = \underbrace{(1-\alpha)pA\left(\frac{K}{L}\right)^{\alpha}}_{NMB}$$

- If we divide both sides by *p*, we find the equation in real terms again.

# Labor Demand

- The relationship between total labor demanded by firms and the wage level

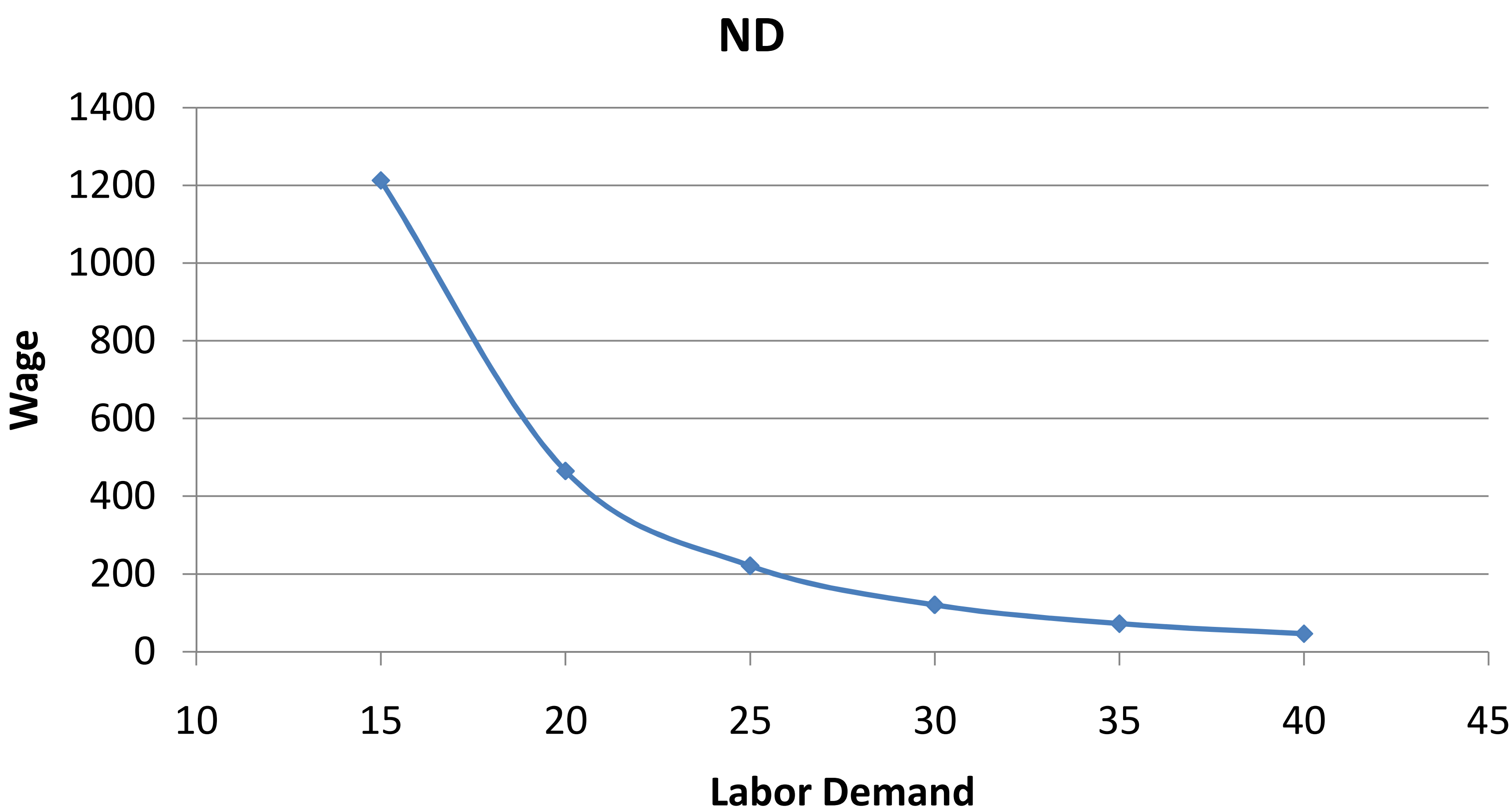
- Changes in wage move us along the *MPN* curve (does not change the curve itself)

- The curve shifts when other factors change what firms want to employ for a given wage (e.g.: shocks)

# Deriving Labor Demand (*ND*)

- One picture creates another (board)
- Labor demand is constructed by a bunch of equilibrium points as we vary the wage
- Notice *MPN*=*w* at all of these points (firms are profit maximizers!!)
- Varying *w*, traces out *MPN*; *MPN* is the *ND* curve (shows firms' best choice for any given *w*)
- *ND* Downward-sloping because of diminishing *MPN*

# Labor Demand (*ND*)



# Aggregate Labor Demand

- So far we have thought of just one firm's decision
- Studying macro though is about all of the firms
- Aggregate labor demand = Sum of all the individual firms' demand
- So the analysis is basically the same

# All else equal…

- We often say this quick
- It’s a “thought experiment”
- When we talk about labor demand shifts we assume everything in the universe is fixed
- Also, the capital stock has been the same the whole time!

# Aggregate ND shifts

- Increase in productivity -> ND curve shifts right
  - MPN rises with technology, so for a given wage, hiring more people is worth it
- Decrease in capital stock -> ND curve shifts left
  - MPN rises with capital stock, worth it to hire more people to make use of the capital

# Labor Supply

- Remember that we are studying the labor Mkt, so we need both sides: demand and supply
- With labor markets, it's about individuals deciding how much to work for a given wage, given alternatives
- Aggregate supply of labor: sum of labor supplied by everyone

# Alternatives

- Working has a cost: time and effort that are no longer available for other activities

- Leisure: all the off-the-job activities like eating, working around the house, family, friends, etc.

# Income-Leisure Tradeoff

- Utility (happiness) – depends on G&S you consume, and the amount of time you relax.

- Unfortunately, have to work to make money to buy G&S.

- Same idea: *MB* = *MC* to determine utility maximizing work/leisure levels

# Real Wages and LS

- Real wage – amount of income in real terms a worker receives in exchange for giving up leisure

- Increase in real wage has two effects
  - substitution effect
  - income effect

# Consumption Leisure Trade Off

- Optimization problem slightly different from the firm's problem:

$$\max_{c,l} u(c,(h-l))$$

$$\text{s.t. } wl \geq c$$

- Consumers are *constrained* by their income.

# Summary

I. The Production Function:

1. $Y = AF(K, N)$

2. Marginal Product of Capital/Labor

   - Diminishing Marginal Productivity of Capital/Labor

II. Labor Demand:

1. $MRPN = W$ or $MPN = w$

2. Labor Demand Curve

3. Aggregate Labor Demand

# Ex.2 page 101

- An economy has the Production function:

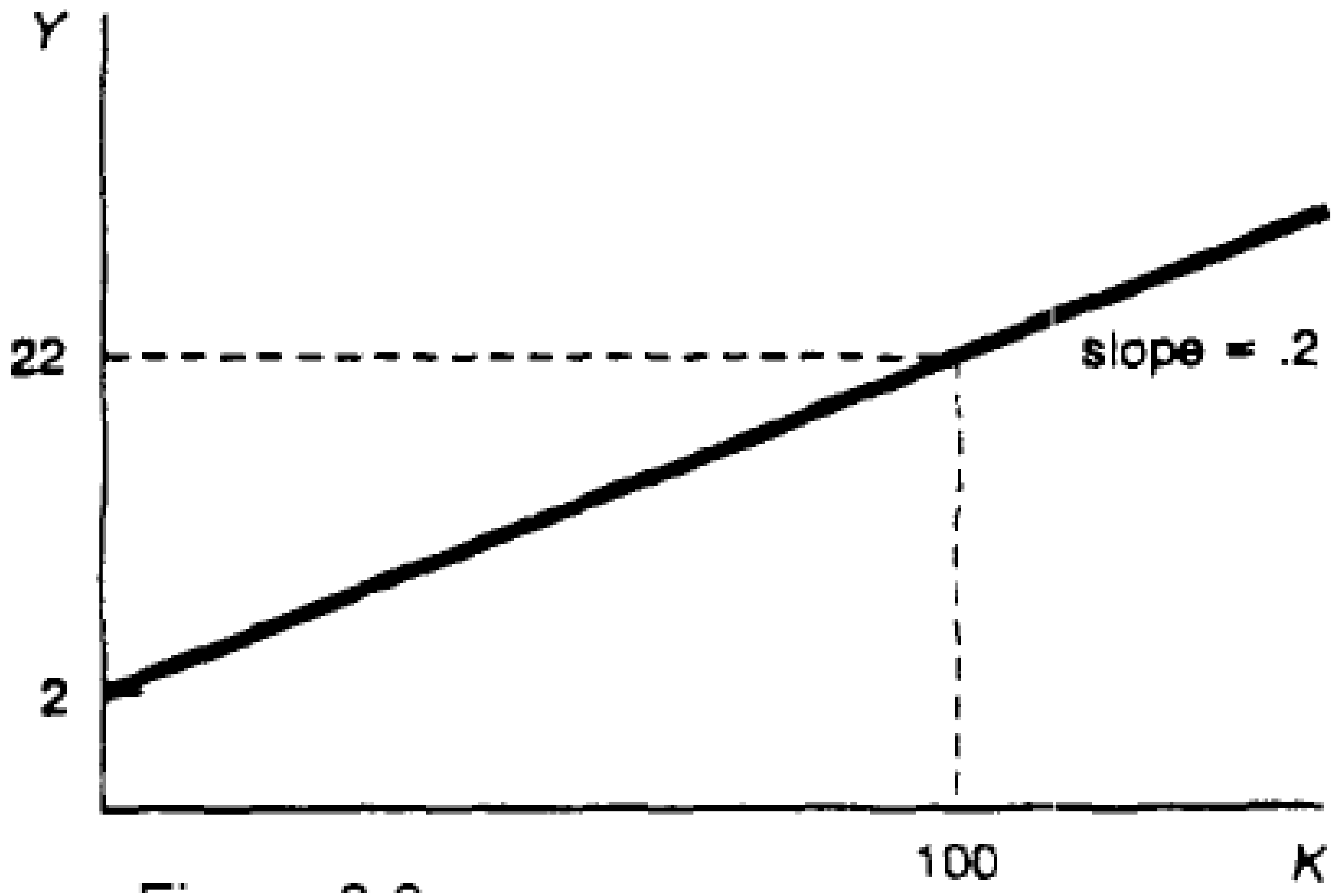
$$Y = AF(K,N) = 0.2\left(K + \sqrt{N}\right)$$

In the current period $K$=$N$=100.

a. Graph the relationship between $Y$ and $K$, holding labor constant. What is the *MPK*?

b. Graph the relationship between $Y$ and $N$, holding labor constant. What is the *MPN* when $N$ raises from 100 to 110? And from 110 to 120?

a. The *MPK* is 0.2, because for each additional unit of capital, output increases by 0.2 units. The slope of the production function line is 0.2. There is no diminishing marginal productivity of capital in this case, because the *MPK* is the same regardless of the level of *K*. The production function is a straight line.


Y
22
2
slope = .2
100
K

b. When $N$ is 100, output is $Y = 0.2(100 + 10) = 22$. When N is 110, Y is 22.0976. So the MPN for raising N from 100 to 110 is

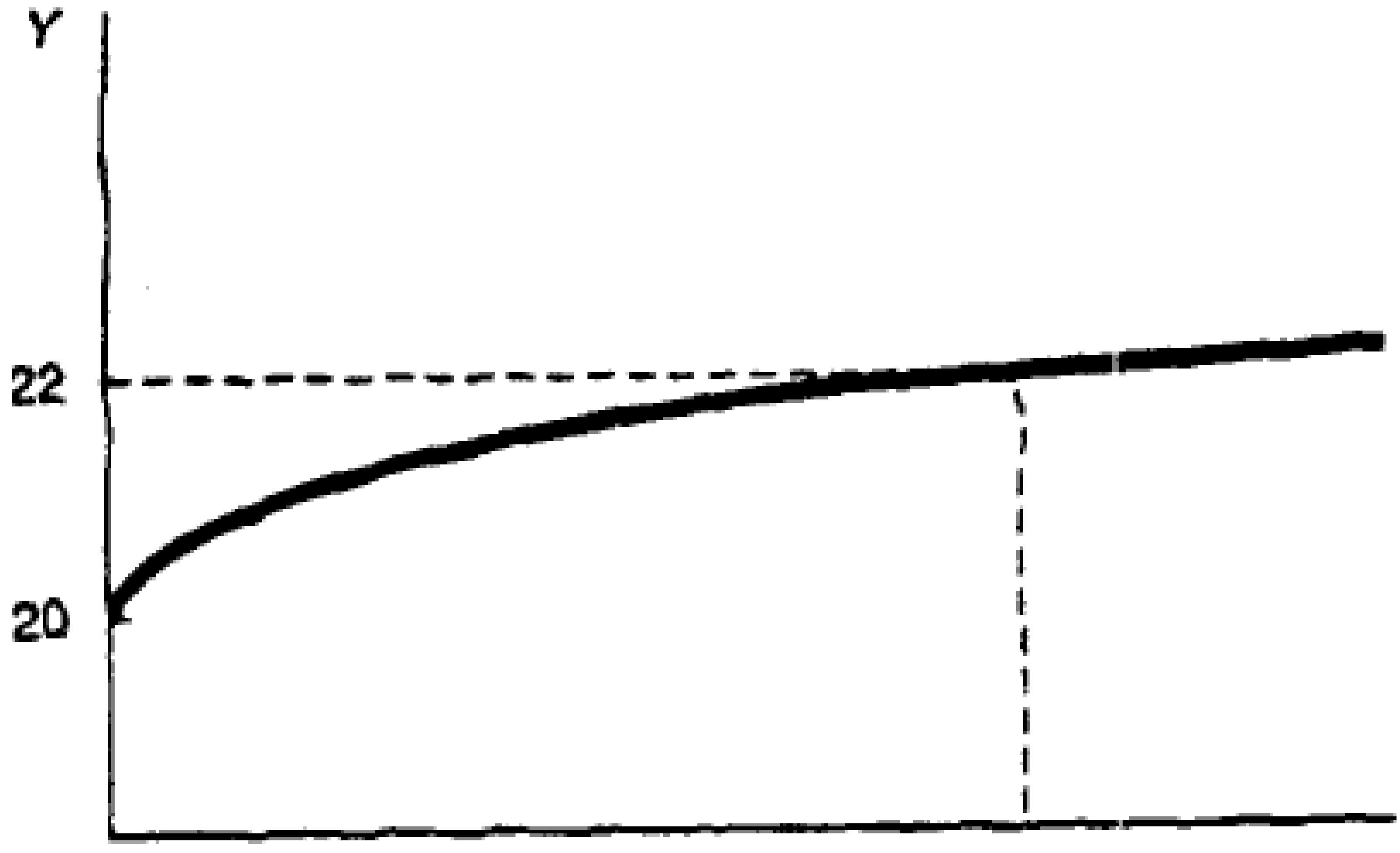
(22.0976 – 22) / 10 = 0.00976.

When N is 120, Y is 22.1909. So the MPN for raising N from 110 to 120 is

(22.1909 – 22.0976) / 10 = 0.00933.

This shows diminishing marginal productivity of labor because the MPN is falling as N increases.

This is shown as a decline in the slope of the production function as N increases.


Y
22
20