ECON 222A
Macroeconomic Theory I

Long-Run Economic Growth

Lecture 12

Today’s Lecture

- Final on April 16th
- Second Problem set is ready to collect
- PS3 is due on the 18th
- Today’s office hours are postponed to Friday

Today's Lecture

- Conclude the Neoclassical Growth Model
- Discuss the Golden Rule
- Introduce Endogenous Growth Theory

Neoclassical Growth Model: Assumptions

- Workforce is a fixed proportion of population (ignore movements in/out of *LF*)
- Population (and Workforce) grow at constant rate n

$$N_{t+1} = N_t(1 + n)$$

- Closed economy (NX=0 and S^d=I^d)

- There is no government (G=0) which simplifies Income-Expenditure Identity to Y=C+I, or

$$C_t = Y_t - I_t$$

Neoclassical Growth Model

- Trick: Write everything in per-worker terms (gets rid of N in the production function)
- $y_t = Y_t/N_t$ = output per worker in year t
- $c_t = C_t/N_t$ = consumption per worker in year t
- $k_t = K_t/N_t$ = capital stock per worker in year t
- k_t is also called the capital-labor ratio

Neoclassical Growth Model

- Getting the per-worker production function, $f(.)$; start from the production function

$$Y_t=A_tF(K_t,N_t)$$

- Set A=1(for now) and divide the production function by N_t:

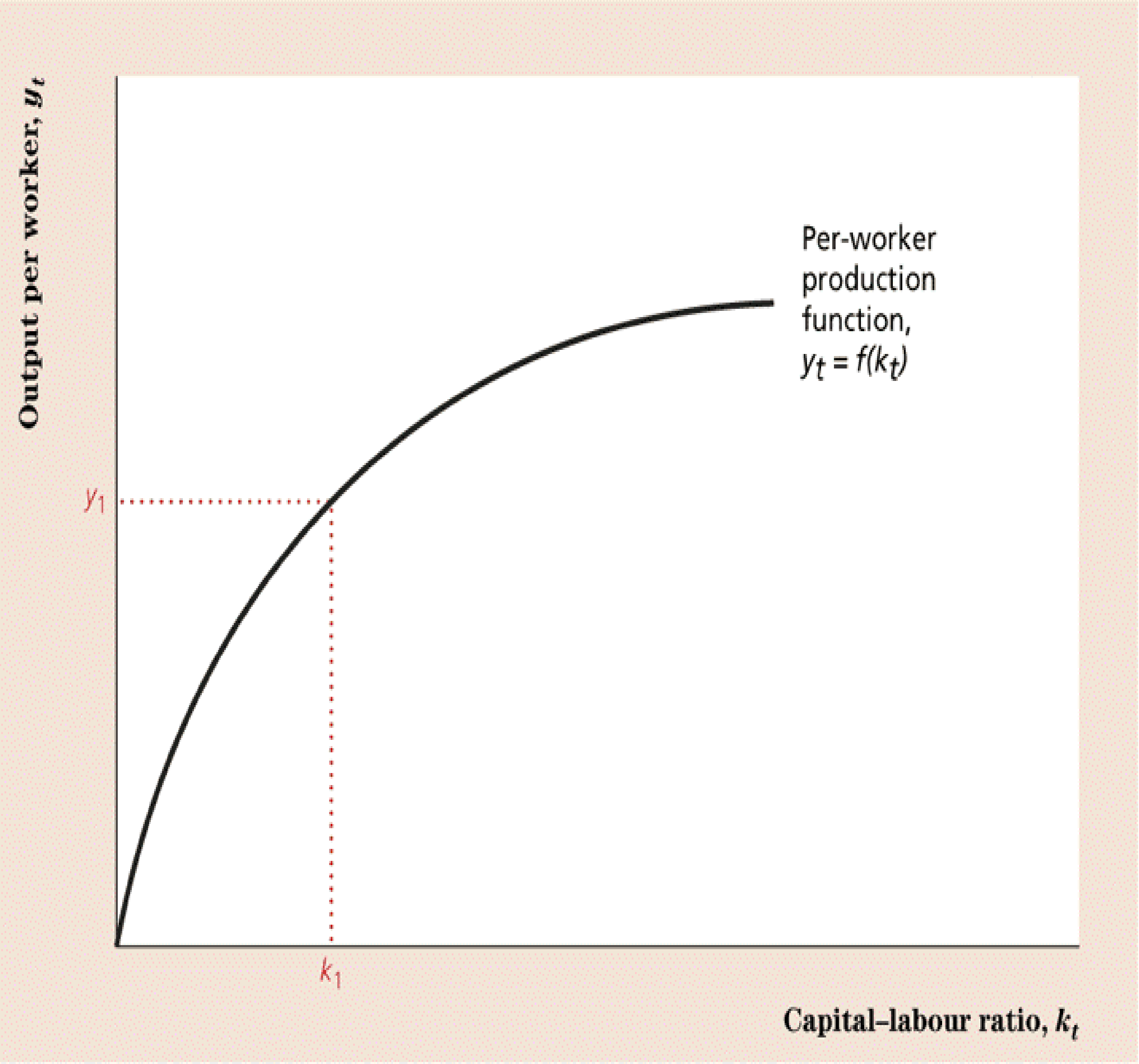
$$y_t=(K_t)^{\alpha}(N_t)^{-\alpha}=(k_t)^{\alpha}$$

- Same properties as the aggregate production function but different units. It slopes up, and shows diminishing *MPk*

FIGURE 6.1

THE PER-WORKER PRODUCTION FUNCTION

The per-worker production function, $y_t = f(k_t)$, relates the amount of output produced per worker, y_t, to the capital–labour ratio, k_t. For example, when the capital–labour ratio is k_1, output per worker is y_1. The per-worker production function slopes upward from left to right because an increase in the capital–labour ratio raises the amount of output produced per worker. The bowed shape of the production function reflects the diminishing marginal productivity of capital.



Equilibrium Concept: Steady State

- Equilibrium in this model is called the steady state: over time we have constant **<u>per worker</u>** values of y_t, c_t, and k_t
- In the absence of productivity growth the economy reaches a steady state in the long run.
- Since y_t, c_t, and k_t are constant in a steady-state, Y_t, C_t, and K_t all grow at rate n, the rate of growth of the workforce.

- Consider savings and investment decisions

Equilibrium Concept: Steady State

- Savings are simple since we suppose that people save a constant fraction of their income

$$S_t = sY_t$$

S_t : aggregate national savings in year t

s: savings rate, between (0,1)

- Relaxing this doesn't change the results, if given the choice in the model that's what people would do

Equilibrium Concept: Steady State

- Investment
- Equation for evolution of *K* stock:

$$K_{t+1} = (1\text{-}d)K_t + I_t$$

- where d= depreciation rate
- *K* stock in next period = undepreciated capital + investment

- Closed economy: $S^d=I^d$, so substitute S_t for I_t

Step by step…Until

- $K_{t+1} = (1\text{-}d)K_t + I_t$
- $K_{t+1} = (1\text{-}d)K_t + sY_t$
- $K_{t+1} / N_{t+1} = [(1\text{-}d)K_t]/N_{t+1} + sY_t/N_{t+1}$
- $K_{t+1} / N_{t+1} = [(1\text{-}d)K_t]/[(1+n)N_t] + sY_t/[(1+n)N_t]$
- $K_{t+1} / N_{t+1} = [(1\text{-}d)/(1+n)][K_t/N_t] + sY_t/[N_t(1+n)]$
- $k_{t+1} = k_t(1\text{-}d)/(1+n) + sf(k_t)/(1+n)$

Step by step…Until

- $k_{t+1} = k_t = k^*$
- $k^* = k^*(1\text{-}d)/(1+n) + sf(k^*)/(1+n)$
- $k^* - k^*(1\text{-}d)/(1+n) = sf(k^*)/(1+n)$
- $k^*\,[1+n-(1\text{-}d)]/(1+n) = sf(k^*)/(1+n)$
- $k^*\,(1+n\text{-}1+d)/(1+n) = sf(k^*)/(1+n)$
- $k^*\,(n+d)/(1+n) = sf(k^*)/(1+n)$
- $k^*\,(n+d) = sf(k^*)$

Step by step…Until

$$(n + d)k^* = sf(k^*)$$

- This equation tells us what is the steady state level of capital per worker k^*.
- S.s. investment per worker: Invest enough to cover depreciation and equip new workers, but k capital per-worker constant.

$$i^* = (n + d)k^*$$

- Given k^*, solve for s.s. output per worker, y^*

$$y^* = f(k^*)$$

- S.s. consumption per worker c^*, (given y^* and k^*):

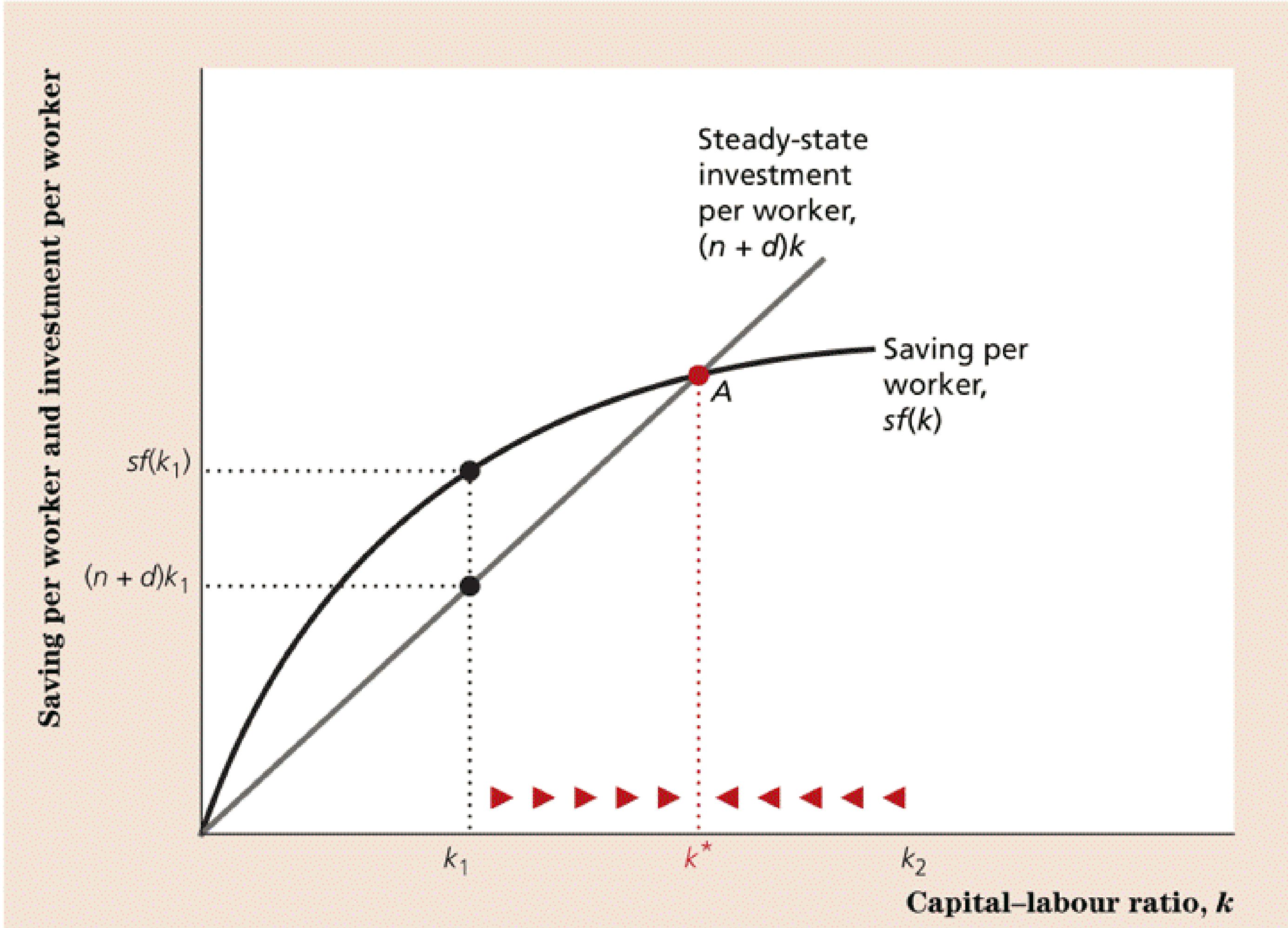
$$y^* = c^* + i^*$$

$$c^* = y^* - i^* = (1-s)f(k^*)$$

FIGURE 6.3

DETERMINING THE CAPITAL–LABOUR RATIO IN THE STEADY STATE

The steady-state capital–labour ratio, k^*, is determined by the condition that saving per worker, $sf(k)$, equals steady-state investment per worker, $(n + d)k$. The steady-state capital–labour ratio k^* corresponds to point A, where the saving curve and the steady-state investment line cross. From any starting point, eventually the capital–labour ratio reaches k^*. If the capital–labour ratio happens to be below k^*, say, at k_1, saving per worker, $sf(k_1)$, exceeds the investment per worker, $(n + d)k_1$, needed to maintain the capital–labour ratio at k_1. As this extra saving is converted into capital, the capital–labour ratio will rise, as indicated by the arrows. Similarly, if the capital–labour ratio is greater than k^*, say, at k_2, saving is too low to maintain the capital–labour ratio, and it will fall over time.



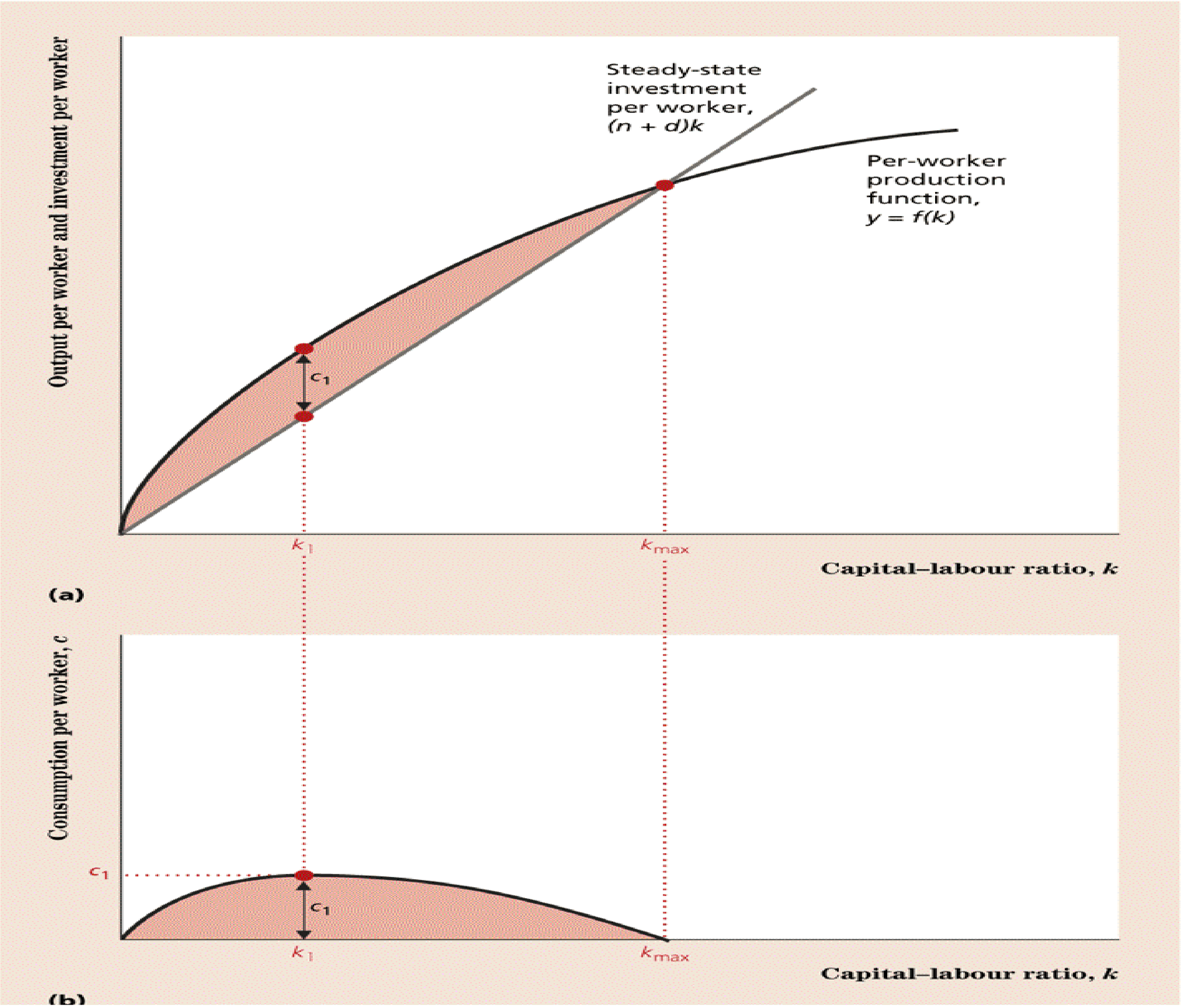
Golden Rule

- Natural question to ask:
  – how much k^* should we have?
- Suppose my policy is to make k^* as big as possible, that is $s=1$
  – not a smart one because of diminishing *MPK* and I am not consuming anything
- Focus on the Golden Rule k-level
  – is such that in steady state the consumption per worker is maximized

FIGURE 6.2

THE RELATIONSHIP OF CONSUMPTION PER WORKER TO THE CAPITAL–LABOUR RATIO IN THE STEADY STATE

(a) For each value of the capital–labour ratio, k, steady-state output per worker, y, is given by the per-worker production function, $f(k)$. Steady-state investment per worker, $(n + d)k$, is a straight line with slope $n + d$. Steady-state consumption per worker, c, is the difference between output per worker and investment per worker (the shaded area). For example, if the capital–labour ratio is k_1, steady-state consumption per worker is c_1.
(b) For each value of the steady-state capital–labour ratio, k, steady-state consumption per worker, c, is derived in (a) as the difference between output per worker and investment per worker. Thus, the shaded area in (b) corresponds to the shaded area in (a). Note that starting from a low value of the capital–labour ratio, an increase in the capital–labour ratio raises steady-state consumption per worker. However, starting from a capital–labour ratio greater than k_1, an increase in the capital–labour ratio actually lowers consumption per worker. When the capital–labour ratio equals k_{max}, all output is devoted to investment, and steady-state consumption per worker is zero.



In any economy in the world today, could a higher capital stock lead to less consumption in the long run? A recent study of seven advanced industrial countries concluded that the answer is "no." Even for high-saving Japan, further increases in capital per worker would lead to higher steady-state consumption per worker.[22]

22. See Andrew B. Abel, N. Gregory Mankiw, Lawrence H. Summers, and Richard J. Zeckhauser, "Assessing Dynamic Efficiency: Theory and Evidence," *Review of Economic Studies*, January 1989, pp. 1–20.

Golden Rule

- Level of k^* that maximizes s.s. consumption per worker: explicit solution obtained by differentiating $c^* = f(k^*) - (n + d)k^*$ wrt k^*

- An increase in k^* at low values of k^* increases s.s. c^*

- An increase in k^* at high values of k^* decreases s.s. c^*
- Consider $f(k^*) = (n + d)k^*$ then $c^* = 0$, too costly to keep investing to keep k from falling

Dynamics

- Fig 6.3 shows k dynamics to $k*$
- $sf(k)$ scaled-down version of per-worker production function

- Model implies that if $k \neq k^*$ then k will move to k^*

i) $k < k^*$, then $sf(k) > (n + d)k$ and k rises until k^*

ii) $k > k^*$, then $sf(k) < (n + d)k$ and k falls until k^*

Levels

- K_t, C_t and Y_t grow at rate n

- Since $k=K_t/N_t$, and in s.s. $k_t - k_{t-1} = 0$
- k doesn't grow so numerator and denominator grow at same rate.
- Denominator grows at n by assumption, so too must numerator K.
- c and y don't grow in s.s.

Summarizing

- With no productivity growth, economy reaches a steady state, with constant *k* capital-labor ratio, *y* output per worker and *c* consumption per worker

- Model highlights diminishing returns to physical K accumulation

- Changes in productivity are needed to keep economy growing, eventually raising inputs doesn’t pay off because of diminishing returns

Determinants of Long-Run Living Standards

- What affects individuals' consumption?

- Look at three ways this might change

– savings

– population growth

– technological change

Savings Rate

- What happens when *s* rises?

– could happen because of government policy

- Allows for more investment
- Results in higher output, consumption, and capital per worker in the LR
- So policy: make *s* high…

– not so fast…

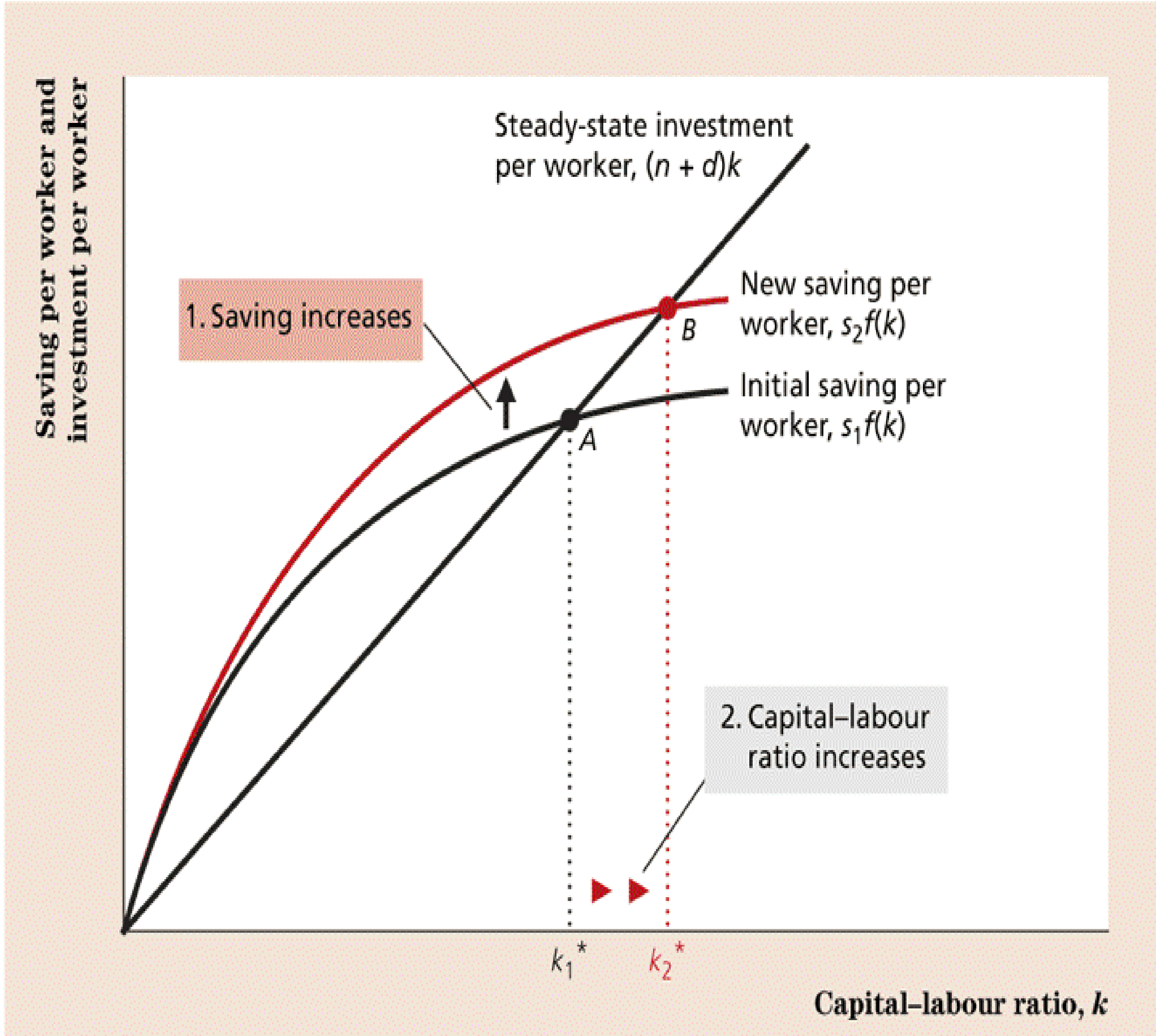
Long Run Vs. Short Run

- Policy of make *s* high ignores an important issue
- LR gain, with SR pain
- To get higher long-run consumption, individuals‘ consumption today has to fall
- Trade off: need to lower current consumption to increase savings
- Golden Rule yields the maximum SS level

Figure 6.4

The effect of an increased saving rate on the steady-state capital–labour ratio

An increase in the saving rate from s_1 to s_2 raises the saving curve from $s_1 f(k)$ to $s_2 f(k)$. The point where saving per worker equals steady-state investment per worker moves from point *A* to point *B*, and the corresponding capital–labour ratio rises from k_1^* to k_2^*. Thus, a higher saving rate raises the steady-state capital–labour ratio.



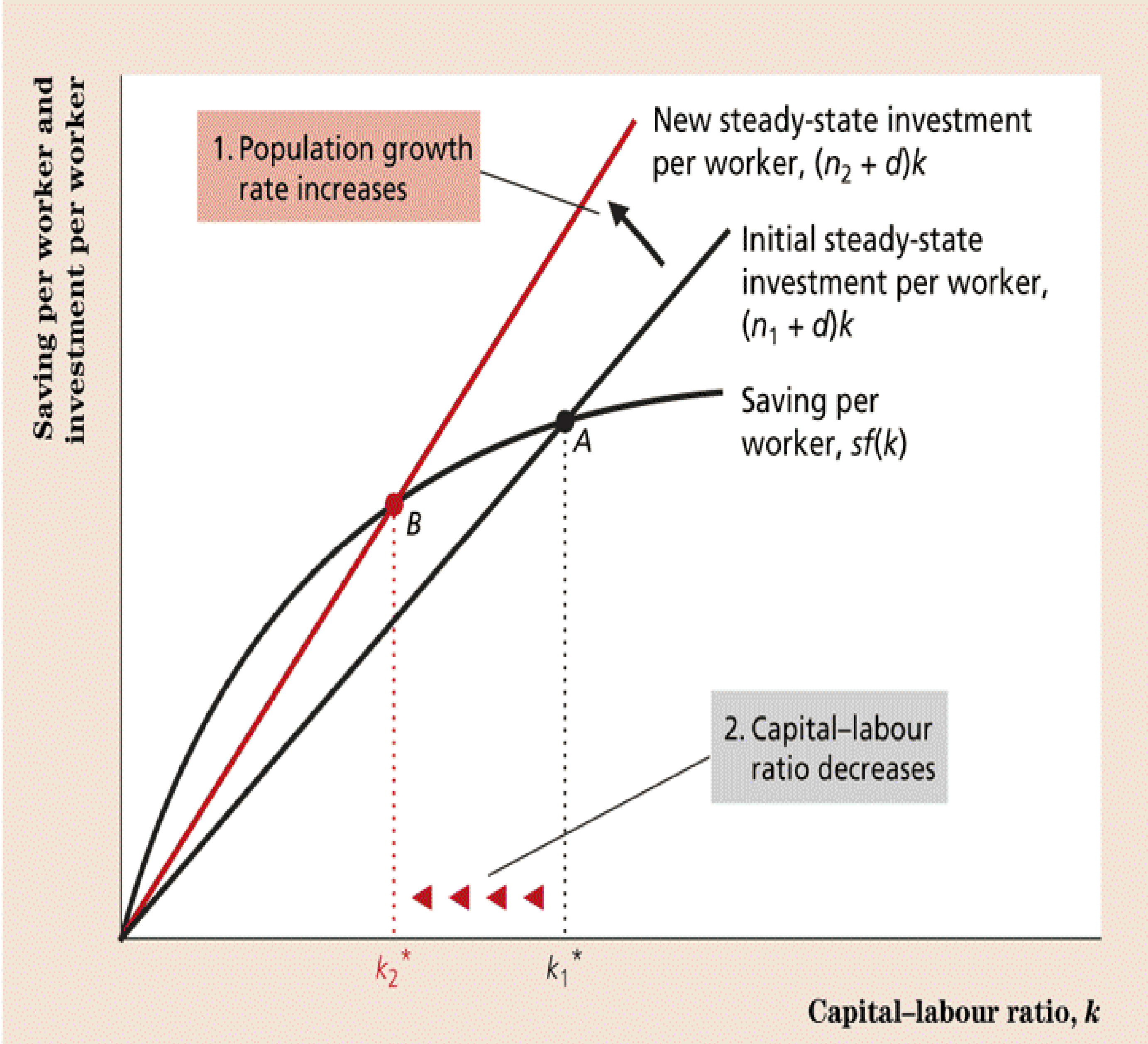
Population Growth

- What happens when n rises, that is when the population expands more rapidly?
- More workers come in, and they have to be equipped with k too, so k thins out.
- Implication: a rise in n yields lower living standards, c falls.
- Solow model assumes LF is fixed proportion working age population: not true if change in population growth is large

FIGURE 6.5

THE EFFECT OF A HIGHER POPULATION GROWTH RATE ON THE STEADY-STATE CAPITAL–LABOUR RATIO

An increase in the population growth rate from n_1 to n_2 increases steady-state investment per worker from $(n_1 + d)k$ to $(n_2 + d)k$. The steady-state investment line pivots up and to the left as its slope rises from $n_1 + d$ to $n_2 + d$. The point where saving per worker equals steady-state investment per worker shifts from point A to point B, and the corresponding capital–labour ratio falls from k_1^* to k_2^*. A higher population growth rate therefore causes the steady-state capital–labour ratio to fall.



Productivity Growth

- Most important factor determining growth
- Is there a role for Government policy? Yes, improve infrastructure, education, R&D

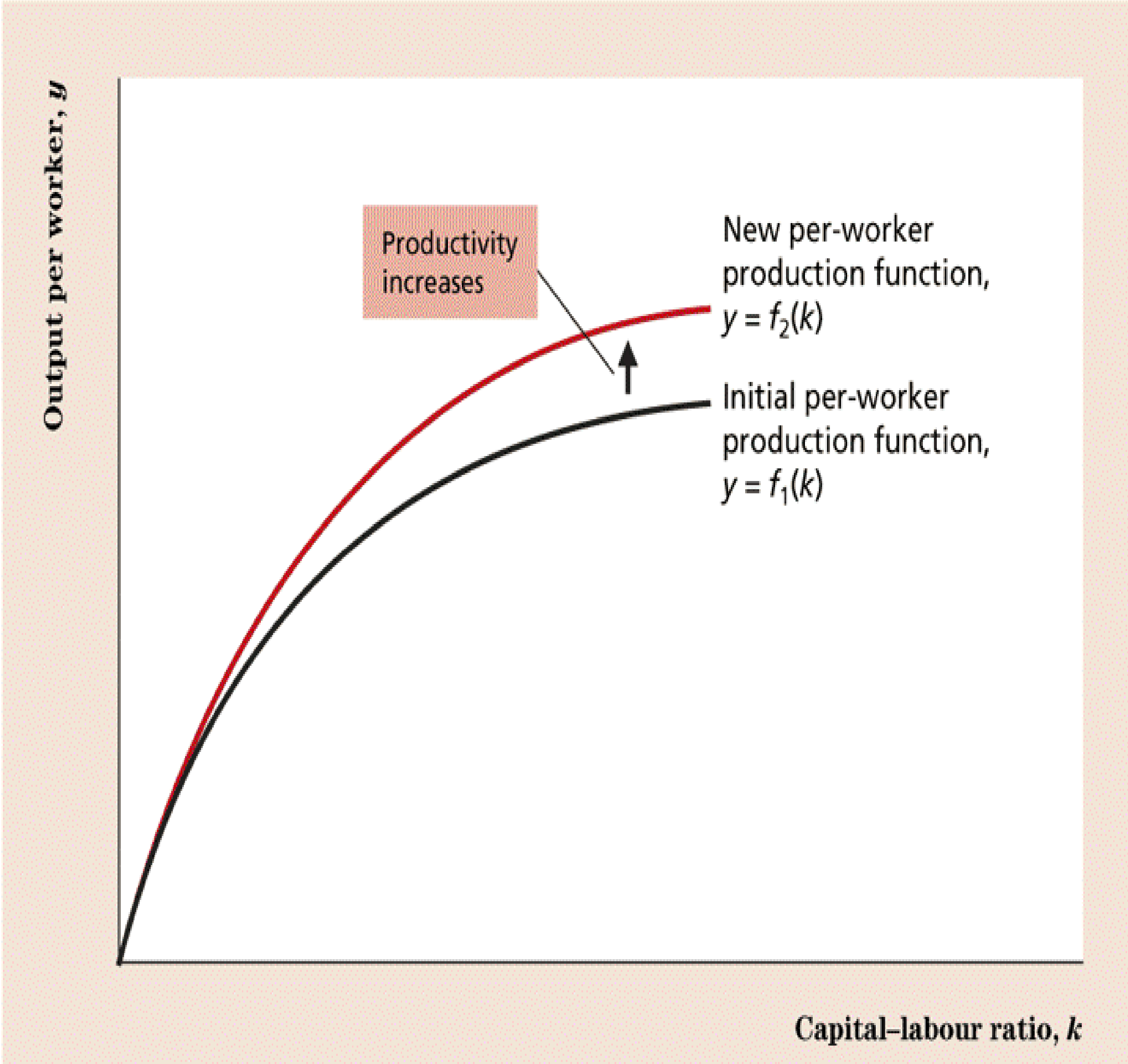
- If A increases, then s.s. y, k, c because:

1) More output produced for given k
2) Higher supply of savings as s is a fixed fraction of y

FIGURE 6.6

AN IMPROVEMENT IN PRODUCTIVITY

An improvement in productivity shifts the per-worker production function upward from the initial production function $y = f_1(k)$ to the new production function $y = f_2(k)$. After the productivity improvement, more output per worker y can be produced at any capital–labour ratio k.



Productivity Growth

- Consider a one time increase in *A*
- Move from one steady state to a higher one:
  - once new steady state is reached, *c*, *y* are constant again
  - *C*, *Y* grow at rate n in steady state again

- Rate of Productivity growth is very important in determining Long-Run Growth

  i) *s* can only increase by so much (to 100% which would mean c=0)

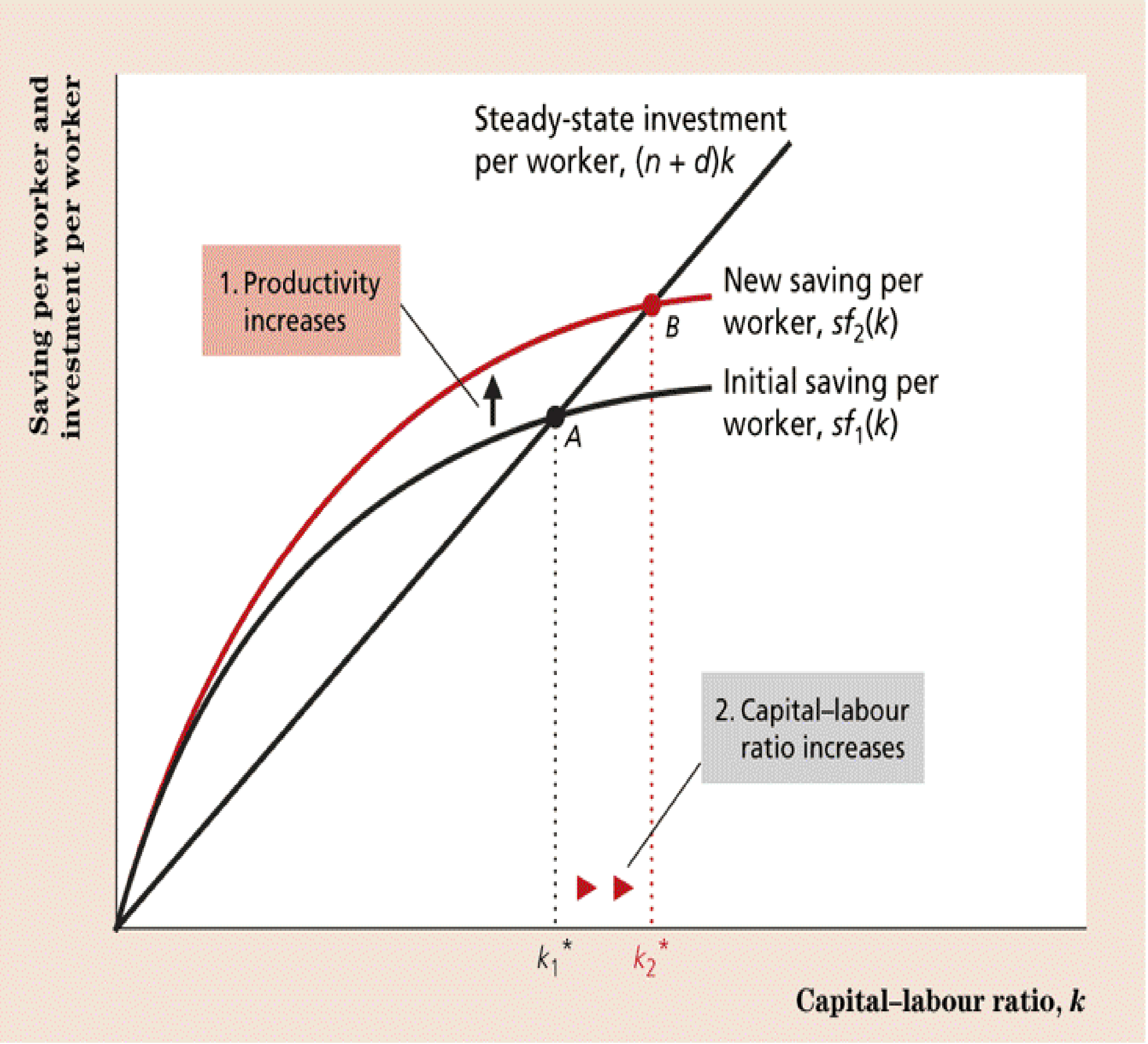
  ii) *n* can only drop so much

- Sustained growth in output, consumption per-worker requires on-going productivity improvements

FIGURE 6.7

THE EFFECT OF A PRODUCTIVITY IMPROVEMENT ON THE STEADY-STATE CAPITAL–LABOUR RATIO

A productivity improvement shifts the production function upward from $f_1(k)$ to $f_2(k)$, raising output per worker for any capital–labour ratio. Because saving is proportional to output, saving per worker also rises, from $sf_1(k)$ to $sf_2(k)$. The point where saving per worker equals steady-state investment per worker shifts from point A to point B, and the corresponding steady-state capital–labour ratio rises from k_1^* to k_2^*. Thus, a productivity improvement raises the steady-state capital–labour ratio.



Productivity Growth

- Result: s.s. consumption and output per worker rises
- Two effects at play:

– direct effect: $f(k)$ better makes y go up

– indirect effect: supply of savings rises, so there is more investment

- But there is no SR loss here, only LR gain

Do Economies Converge?

1) Unconditional (Absolute) convergence

- Naïve Idea: poor countries will eventually catch up to rich
- Rationale: poor countries have lower k so their MPK is greater than rich countries'

  $k_{poor} < k_{rich}$ hence $r_{poor} > r_{rich}$ (diminishing MPK)

- Even if $s_{poor} < s_{rich}$, foreign investment should flow into poor Countries (due to larger MPK)
- But we don't observe poor Countries catching up, or funds flowing into poor countries.

Do Economies Converge?

2) Conditional convergence

- Idea: only countries with similar fundamentals (n, s, f) will converge

- Some empirical support for this
- Can include human and physical capital, (education investment): in Canadian provinces, once education converged, income followed soon after

Do Economies Converge?

3) No convergence

- Poor countries won't catch up
- If there are permanent differences in productivity, technologies f differ, or poor can't access same technology (lack of info, skills, patents).
- No fall in Global income inequality

Endogenous Growth Theory

- Inherent problem with the Solow-Swan model
  - Technological improvement is the key driver of growth
  - But this is completely defined outside of the model
  - We can't explain where it comes from (Manna from Heaven).
- Troublesome
  - Exogenous: assumed, outside the model
  - Endogenous: solved from within the model

Endogenous Growth Theory

- Endogenous growth theory: developed to explain productivity growth, and hence output growth, within the model
- Gives rise to additional importance to a country's saving and investment
- Key difference: human capital and/or R&D

Model Setup

- Aggregate production: $Y = AK$

– no diminishing returns to K (always A)

– Intuition: as K rises, so does human capital (R&D, training), so MPK stays constant

- Consider the case with no population growth
- National saving: $S = sAK$
- Investment: $I_t = (K_t - K_{t-1}) + dK_t$

Model Setup

- Closed economy equilibrium condition:

$$\text{Aggregate } I = \text{Aggregate } S$$

$$\text{Net investment} + \text{depreciation} = sY$$

$$\Delta K + dK = sAK$$

- Divide by K:

$$\Delta K/K = sA - d = \Delta Y/Y$$

because Y is proportional to K

- Changes in productivity and savings rate affects Long Run growth rate in this Endogenous Growth Model