APPENDIX 4.A

A FORMAL MODEL OF CONSUMPTION AND SAVING

This appendix analyzes more formally the decision about how much to consume and how much to save. We focus on the decisions of a consumer named Prudence. To help keep the analysis manageable, we make three simplifying assumptions:

1. The time horizon over which Prudence makes plans consists of only two periods: the present, or current, period and the future period. The current period might represent Prudence's working years and the future period might represent her retirement years, for example.
2. Prudence takes her current income, future income, and wealth as given.
3. Prudence faces a given real interest rate and can choose how much to borrow or save at that rate.

HOW MUCH CAN THE CONSUMER AFFORD? THE BUDGET CONSTRAINT

To analyze Prudence's decision about how much to consume and save, we first examine the choices available to her. To have some specific numbers to analyze, let us suppose that Prudence receives a fixed after-tax income, measured in real terms,[18] of $14 000 in the current period and expects to receive a real income of $11 000 in the future period. In addition, she begins the current period with real wealth of $6000 in a savings account, and she can borrow or lend at a real interest rate of 10% per period.

Next, we list the symbols used to represent Prudence's situation:

y = Prudence's current real income (14 000);
y^f = Prudence's future real income (11 000);[19]
a = Prudence's real wealth (assets) at the beginning of the current period (6000);
r = real interest rate (10%);
c = Prudence's current real consumption (not yet determined);
c^f = Prudence's future real consumption (not yet determined).

In general, any amount of current consumption, c, that Prudence chooses will determine the amount of future consumption, c^f, that she will be able to afford. To work out this relationship, note that the funds that Prudence has on hand in the current period are her current income, y, and her initial wealth, a. If her current consumption is c, then at the end of the current period she has $y + a - c$ left.

Prudence can put these leftover current resources, $y + a - c$, in the bank to earn interest. If the real interest rate that she can earn on her deposit is r, the real

18. The units in which Prudence's income is measured are base-year dollars.
19. We do not include in future income y^f the interest that Prudence earns on her saving. Future income, y^f, includes only labour income or transfers received, such as Canada Pension Plan payments.

value of her bank account (principal plus interest) in the future period will be $(y + a - c)(1 + r)$. In addition to the real value of her bank account in the future period, Prudence receives income of y^f, so her total resources in the future period equal $(y + a - c)(1 + r) + y^f$. Because the future period is the last period of Prudence's life, she spends all of her remaining resources on consumption.[20] Thus, Prudence's future consumption, c^f, is

$$c^f = (y + a - c)(1 + r) + y^f \qquad (4.A.1)$$

Equation (4.A.1) is called the *budget constraint*. It shows for any level of current consumption, c, how much future consumption, c^f, Prudence can afford, based on her current and future income and initial wealth.[21] The budget constraint in Eq. (4.A.1) is represented graphically by the *budget line*, which shows the combinations of current and future consumption that Prudence can afford, based on her current and future income, her initial level of wealth, and the real interest rate. Figure 4.A.1 depicts Prudence's budget line, with current consumption, c, on the horizontal axis and future consumption, c^f, on the vertical axis.

The budget line slopes downward, reflecting the trade-off between current and future consumption. If Prudence increases her current consumption by one unit, her saving falls by one unit. Because saving earns interest at rate r, a one-unit decline in saving today implies that Prudence's future resources—and thus her

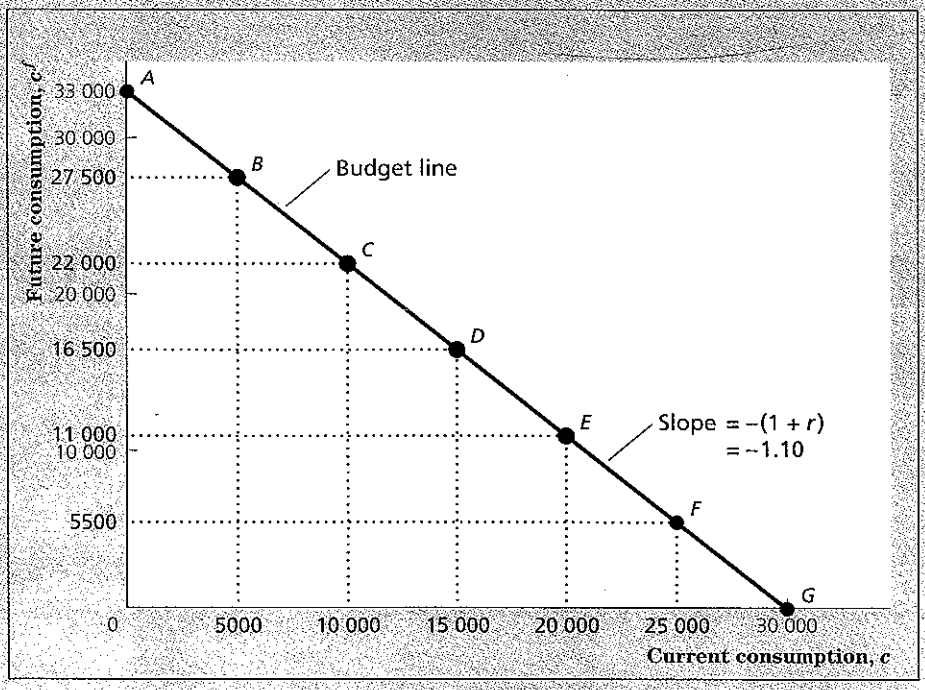


FIGURE 4.A.1

THE BUDGET LINE

The budget line shows the combinations of current and future consumption, c and c^f, available to Prudence. The slope of the budget line is $-(1 + r) = -1.10$. The horizontal intercept is at $c = PVLR = 30\,000$. You can verify that the combinations of current and future consumption at each of the lettered points (as well as any point on the budget line) satisfy $c + c^f/(1 + r) = PVLR = 30\,000$.

20. Here we assume that Prudence does not wish to leave a bequest to anyone. Later, we will briefly examine the effect of bequests on saving decisions.

21. In our derivation of Eq. (4.A.1), we assumed that Prudence's current consumption was less than her total [illegible] resources so that she had some resources left to deposit in the bank. However, the budget constraint, [illegible] that she must

future consumption—will be lower by $1 + r$ units. Because a one-unit increase in current consumption lowers future consumption by $1 + r$ units, the slope of the budget line is $-(1 + r)$. In our numerical example, the real interest rate is 10%, so the slope of the budget line in Figure 4.A.1 is −1.10.

PRESENT VALUES

We can conveniently represent Prudence's budget constraint by using the concept of *present value*. The present value measures the value of payments to be made in the future in terms of today's dollars or goods. To illustrate this concept, suppose that you must make a payment of $13 200 one year from now. How much money would you have to put aside today so that you could make that future payment? The answer to this question is the present value of $13 200.

The present value of a future payment depends on the interest rate. If the current nominal interest rate, i, is 10% per year, the present value of $13 200 to be paid one year from now is $12 000. The reason is that $12 000 deposited in the bank today at a 10% interest rate will earn $1200 (10% of $12 000) of interest in one year, which, when added to the initial $12 000, gives the $13 200. Therefore, at an interest rate of 10%, having $13 200 one year from now is economically equivalent to having $12 000 today.

More generally, if the nominal interest rate is i per year, each dollar in the bank today is worth $1 + i$ dollars one year from now. To have $13 200 one year from now requires $13 200/(1 + i) in the bank today; thus, the present value of $13 200 to be paid one year from now is $13 200/(1 + i). As we have already shown, if i = 10% per year, the present value of $13 200 one year from now is $13 200/1.10 = $12 000. If i = 20% per year, the present value of $13 200 one year in the future is $13 200/1.20 = $11 000. Hence an increase in the interest rate reduces the present value of a future payment.

If future payments are measured in nominal terms, as in the preceding example, the appropriate interest rate for calculating present values is the nominal interest rate, i. If future payments are measured in real terms, present values are calculated in exactly the same way, except that we use the real interest rate, r, rather than the nominal interest rate, i. In analyzing Prudence's consumption–saving decision, we are measuring everything in real terms, so we use the real interest rate, r, to calculate the present values of Prudence's future income and consumption.

PRESENT VALUE AND THE BUDGET CONSTRAINT

We define the *present value of lifetime resources* (*PVLR*) as the present value of the income that a consumer expects to receive in current and future periods plus initial wealth. In the two-period case, the present value of lifetime resources is

$$PVLR = y + y'/(1+r) + a \qquad (4.A.2)$$

which is the sum of current income, y,[22] the present value of future income, $y'/(1 + r)$, and current wealth, a. In our example, Prudence has $PVLR$ = 14 000 + 11 000/1.10 + 6000 = 30 000.

22. Note that the present value of current income is just current income.

Next, we divide both sides of Eq. (4.A.1) by $(1 + r)$ and then add c to both sides to get

$$c + c^f/(1 + r) = y + y^f/(1 + r) + a$$
$$PVLC = PVLR \qquad (4.A.3)$$

The left side of Eq. (4.A.3) is the present value of lifetime consumption, $c + c^f/(1 + r)$, which we denote *PVLC*. The budget constraint in Eq. (4.A.3) states that the *present value of lifetime consumption (PVLC)* equals the present value of lifetime resources *PVLR*.

In terms of Figure 4.A.1, and indeed for any graph of the budget line, *PVLR* equals the value of current consumption, c, at the horizontal intercept of the budget line because the horizontal intercept is the point on the budget line at which future consumption, c^f, equals zero. Setting future consumption, c^f, to zero in Eq. (4.A.3) yields current consumption, c, on the left side of the equation, which must equal *PVLR* on the right side. Thus, $c = PVLR$ at the horizontal intercept of the budget line.

WHAT DOES THE CONSUMER WANT? CONSUMER PREFERENCES

The budget constraint, represented graphically as the budget line, shows the combinations of current and future consumption *available* to Prudence. To determine which of the many possible consumption combinations Prudence will choose, we need to know something about Prudence's preferences for current versus future consumption.

Economists use the term *utility* to describe the satisfaction or well being of an individual. Preferences about current versus future consumption are summarized by how much utility a consumer obtains from each combination of current and future consumption. We can graphically represent Prudence's preferences for current versus future consumption through *indifference curves*, which represent all combinations of current and future consumption that yield the same level of utility. Because Prudence is equally happy with all consumption combinations on an indifference curve, she does not care (that is, she is indifferent to) which combination she actually gets. Figure 4.A.2 shows two of Prudence's indifference curves. Because the consumption combinations corresponding to points X, Y, and Z all are on the same indifference curve, IC^1, Prudence would obtain the same level of utility at X, Y, and Z.

Indifference curves have three important properties, each of which has an economic interpretation and each of which appears in Figure 4.A.2:

1. *Indifference curves slope downward from left to right.* To understand why, let us suppose that Prudence has selected the consumption combination at point Y, where $c = 15\,000$ and $c^f = 15\,000$.[23] Now, suppose that Prudence must reduce her current consumption to $c = 13\,000$. Clearly, if she reduces current consumption while maintaining future consumption at 15 000, she will suffer a

23. Point Y lies below Prudence's budget line, shown in Figure 4.A.1, which means that not only could Prudence afford this consumption combination but she would also have resources left over at the end of the future period. Unless she wants to leave a bequest, she would not actually choose such a combination for the resources shown in Figure 4.A.1.

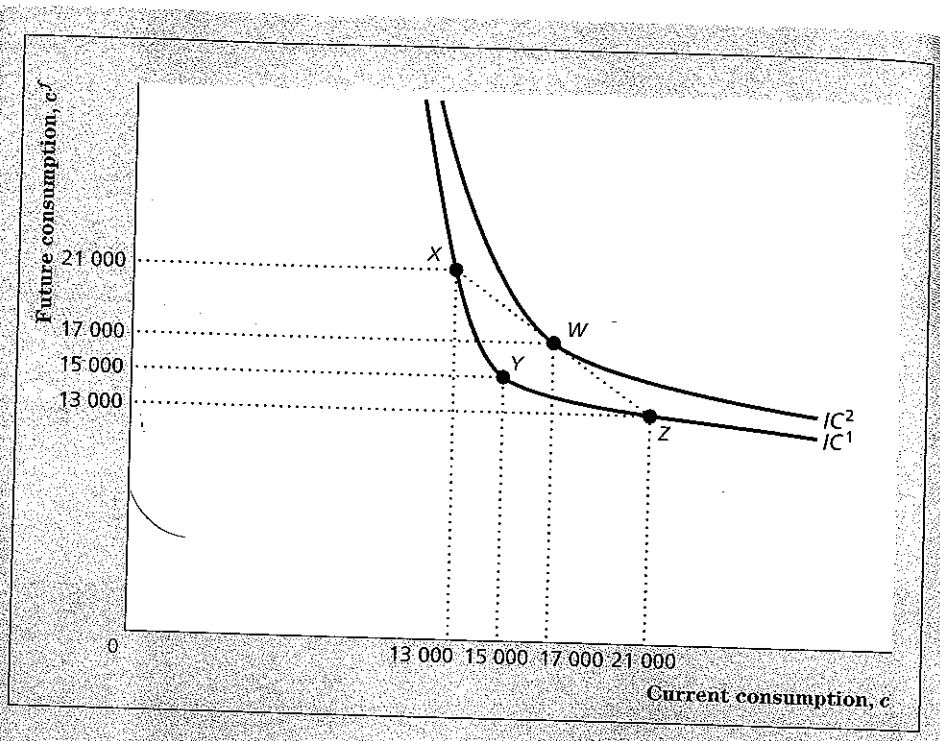


FIGURE 4.A.2

INDIFFERENCE CURVES

All points on an indifference curve represent consumption combinations that yield the same level of utility. Indifference curves slope downward because a consumer can be compensated for a reduction in current consumption by an appropriate increase in future consumption. All points on IC^2 represent consumption combinations that are preferred to all consumption combinations represented by points on IC^1. Indifference curves are bowed toward the origin to reflect the consumption-smoothing motive. Prudence prefers the consumption combination at point *W*, which is an average of the combinations at point *X* and *Z*, because *W* represents a smoother pattern of consumption. Thus the indifference curve containing *W* (IC^2) lies above and to the right of the indifference curve containing *X*, *Y*, and *Z* (IC^1).

reduction in utility. However, Prudence can be compensated for this reduction in current consumption by additional future consumption. Suppose that if she increases her future consumption to $c' = 21\,000$ when her current consumption falls to $c = 13\,000$, so that she moves to point *X*, her level of utility remains unchanged. In such a case, she is indifferent to the consumption combinations at *X* and *Y*, and points *X* and *Y* must lie on the same indifference curve. In general, any change in the level of current consumption must be accompanied by a change in the *opposite* direction in the level of future consumption so as to keep Prudence's level of utility unchanged. Thus, indifference curves, which represent consumption combinations with equal levels of utility, must slope downward from left to right.

2. *Indifference curves that are farther up and to the right represent higher levels of utility.* Consider for example point *W*, which lies above and to the right of point *Y* in Figure 4.A.2. Both current consumption and future consumption are higher at *W* than at *Y*. Because Prudence obtains utility from both current and future consumption, *W* offers a higher level of utility than does *Y*; that is, Prudence prefers *W* to *Y*. In fact, as all points on the indifference curve IC^1 yield the same level of utility as *Y*, Prudence prefers *W* to all points on the indifference curve IC^1. Furthermore, as all points on indifference curve IC^2 yield the same level of utility as *W*, Prudence prefers all points on IC^2 to all points on IC^1. In general, for any two indifference curves, consumers prefer consumption combinations on an indifference curve that is above and to the right of the other indifference curve.

3. *Indifference curves are bowed toward the origin.* This characteristic shape of indifference curves captures the consumption-smoothing motive, discussed in Chapter 4. Under the consumption-smoothing motive, consumers prefer a relatively smooth pattern of consumption over time to having large

amounts of consumption in one period and small amounts in another period. We can illustrate the link between the shape of indifference curves and the consumption-smoothing motive by considering the following three consumption combinations in Figure 4.A.2: point X (c = 13 000; c' = 21 000), point W (c = 17 000; c' = 17 000), and point Z (c = 21 000; c' = 13 000). Note that W corresponds to complete consumption smoothing, with equal consumption occurring in both periods. In contrast, X and Z represent consumption combinations with large changes in consumption between the first period and the second period. In addition, note that W represents a consumption combination that is the average of the consumption combinations at X and Z. Current consumption at W, 17 000, is the average of current consumption at X and Z, 13 000 and 21 000, respectively; similarly, future consumption at W, also 17 000, is the average of future consumption at X and Z, 21 000 and 13 000 respectively.

Even though point W essentially is an average of points X and Z, and Prudence is indifferent between X and Z, she prefers W to X and Z because W represents much "smoother" (more even) consumption. Graphically, her preference for W over X and Z is indicated by W's position above and to the right of indifference curve IC^1 (which runs through X and Z). Note that W lies on a straight line drawn between X and Z. The only way that W can lie above and to the right of IC^1 is if IC^1 bows toward the origin, as depicted in Figure 4.A.2. Thus, the bowed shape of the indifference curve reflects the consumption-smoothing motive.

THE OPTIMAL LEVEL OF CONSUMPTION

Combining Prudence's budget line (which describes her available consumption combinations) and her indifference curves (which describe her preferences for current versus future consumption), we can find the levels of current consumption and saving that make her happiest. This best available, or *optimal*, level of current consumption and saving is represented graphically by the point at which Prudence's budget line is tangent to an indifference curve, shown as point D in Figure 4.A.3.

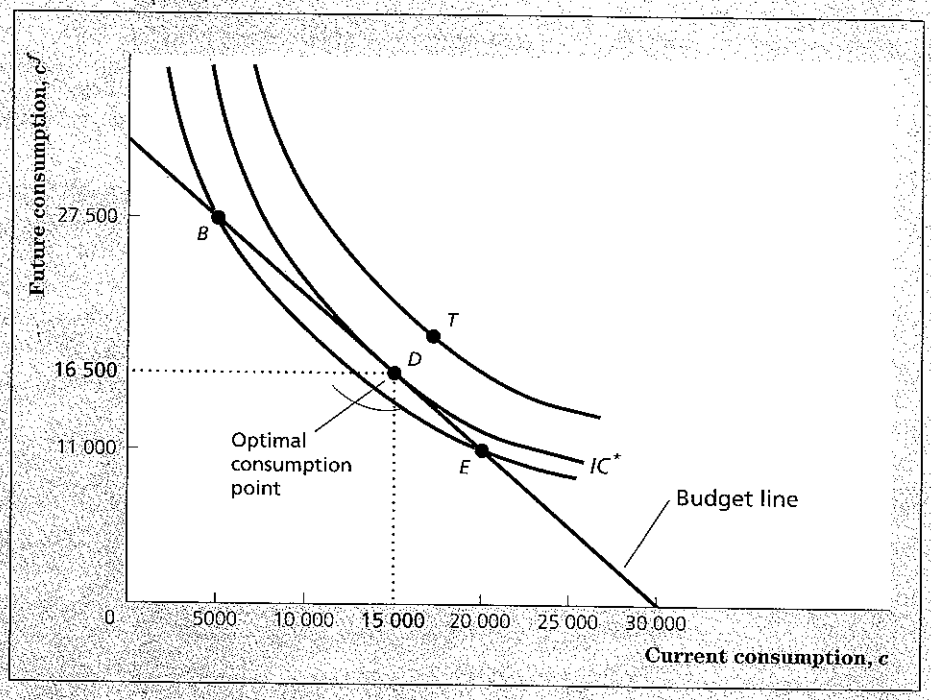
To see why Prudence achieves her highest possible level of satisfaction, or utility, at point D, first note that D lies on indifference curve IC^*, which means that all consumption combinations on IC^* yield the same level of utility as D. All points on Prudence's budget line other than point D—points such as B and E, for example—lie on indifference curves that are below and to the left of IC^*. Thus, the consumption combinations represented by all of these other points yield a lower level of utility than the consumption combination at D. Prudence would prefer the consumption combination represented by a point such as T in Figure 4.A.3 to the consumption combination represented by D, because T lies on an indifference curve above and to the right of IC^*; because T also lies above the budget line, however, Prudence cannot afford the consumption combination represented by that point. With her budget constraint, Prudence cannot do any better than D.

We conclude that Prudence's utility-maximizing consumption and saving choice is represented by point D, where her budget line is tangent to an indifference curve. Here, her optimal level of current consumption is 15 000, and her optimal level of future consumption is 16 500. Prudence's choice of current

FIGURE 4.A.3

THE OPTIMAL CONSUMPTION COMBINATION

The optimal (highest utility) combination of current and future consumption is represented by the point of tangency between the budget line and an indifference curve (point *D*). All other points on the budget line, such as *B* and *E*, lie on indifference curves below and to the left of indifference curve *IC** and thus yield lower utility than the consumption combination at *D*, which lies on *IC**. Prudence would prefer the consumption combination at point *T* to the one at *D*, but as *T* lies above the budget line she cannot afford the consumption combination that *T* represents.



consumption automatically determines her current saving, s, which equals her current income, minus her optimal current consumption:

$$s = y - c = 14\,000 - 15\,000 = -1000.$$

Thus, Prudence chooses to dissave (decrease her initial assets) by 1000.

THE EFFECTS OF CHANGES IN INCOME AND WEALTH ON CONSUMPTION AND SAVING

The formal model developed in this appendix provides a helpful insight: *The effect on consumption of a change in current income, expected future income, or wealth depends only on how that change affects the consumer's present value of lifetime resources, or* PVLR.

AN INCREASE IN CURRENT INCOME

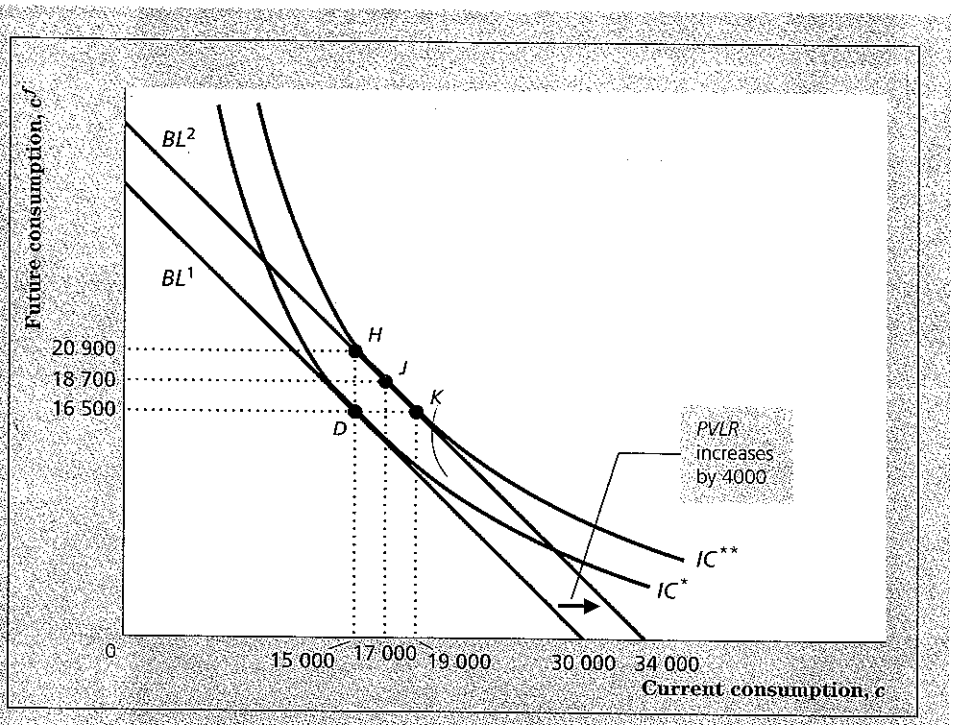
Suppose that Prudence receives a bonus at work of 4000, which raises her current real income from 14 000 to 18 000. Her initial assets (6000), future income (11 000), and the real interest rate (10%) remain unchanged; hence the increase of 4000 in current income implies an equal increase in Prudence's present value of lifetime resources, or *PVLR*. If she has not yet committed herself to her original consumption–saving plan, how might Prudence revise that plan in light of her increased current income?

We use the graph in Figure 4.A.4 to answer this question. In Figure 4.A.4, BL^1 is Prudence's original budget line, and point *D*, where c = 15 000 and c^f = 16 500, represents Prudence's original, pre-bonus consumption plan. Prudence's

FIGURE 4.A.4

AN INCREASE IN INCOME OR WEALTH

An increase in current income, future income, and/or initial wealth that raises Prudence's *PVLR* by 4000 causes the budget line to make a parallel shift to the right by 4000, from BL^1 to BL^2. If Prudence's original consumption plan was to consume at point *D*, she could move to point *H* by spending all the increase on future consumption and none on current consumption; or she could move to point *K* by spending all the increase on current consumption and none on future consumption. However, if Prudence has a consumption-smoothing motive she will move to point *J*, which has both higher current consumption and higher future consumption than *D*. Point *J* is optimal because it lies where the new budget line BL^2 is tangent to an indifference curve, *IC***.



bonus will allow her to consume more, both now and in the future, so the increase in her income causes her budget line to shift. To see exactly how it shifts, note that the increase of 4000 in Prudence's current income implies that her *PVLR* also increases by 4000. Because the horizontal intercept of the budget line occurs at $c = PVLR$, the bonus shifts the horizontal intercept to the right by 4000. The slope of the budget line, $-(1 + r) = -1.10$, remains unchanged because the real interest rate r is unchanged. Thus, the increase in current income of 4000 causes a parallel shift of the budget line to the right by 4000, from BL^1 to BL^2.

That shift demonstrates graphically that after receiving her bonus, Prudence can enjoy greater current and future consumption. One strategy for Prudence, represented by point *K* on the new budget line BL^2, is to use the entire bonus to increase her current consumption by 4000 while leaving her future consumption unchanged. Another strategy, represented by point *H* on BL^2, is to save all of her bonus while keeping her current consumption unchanged, and then use both the bonus and the interest of 400 earned on the bonus to increase her future consumption by 4400.

If Prudence operates under a consumption-smoothing motive, she will use her bonus to increase *both* her current consumption and (by saving part of her bonus) her future consumption, thereby choosing a point on BL^2 between point *K* (consume the entire bonus) and point *H* (save the entire bonus). If her indifference curves are as shown in Figure 4.A.4, she will move to *J*, where her new budget line, BL^2, is tangent to the indifference curve *IC***. At *J*, current consumption, *c*, is 17 000, future consumption, c^f, is 18 700, and saving, *s*, is 18 000 − 17 000 = 1000. Both current and future consumption are higher at *J* than at *D* (where c = 15 000 and c^f = 16 500). Prudence's current saving of 1000 at *J* is higher than her saving was at *D* (where she dissaved by 1000) because the increase in her current

consumption of 2000 is less than the increase in her current income of 4000. This example illustrates that an increase in current income raises both current consumption and current saving.

AN INCREASE IN FUTURE INCOME

Suppose that Prudence does not receive her bonus of 4000 in the current period, so that her current income, y, remains at its initial level of 14 000. Instead, because of an improved company pension plan, she learns that her future income will increase by 4400, so y^f rises from 11 000 to 15 400. How will this good news affect Prudence's current consumption and saving?

At a real interest rate of 10%, the improvement in the pension plan increases the present value of Prudence's future income by 4400/1.10, or 4000. So, as in the case of the current-period bonus just discussed, the improved pension plan raises Prudence's *PVLR* by 4000 and causes a parallel shift of the budget line to the right by that amount. The effects on current and future consumption are therefore exactly the same as they were for the increase of 4000 in current income (and Figure 4.A.4 applies equally well here).

Although increases in current income and expected future income that are equal in present value will have the same effects on current and planned future consumption, the effects of these changes on current saving are different. Previously, we showed that an increase in current income raises current saving. In contrast, because the increase in future income raises current consumption (by 2000 in this example) but does not affect current income, it causes saving to fall (by 2000, from –1000 to –3000). Prudence knows that she will be receiving more income in the future, so she has less need to save today.

AN INCREASE IN WEALTH

Changes in wealth also affect consumption and saving. As in the cases of current and future income, the effect of a change in wealth on consumption depends only on how much the *PVLR* changes. For example, if Prudence finds a passbook savings account in her attic worth 4000, her *PVLR* increases by 4000. To illustrate this situation, we use Figure 4.A.4 again. Prudence's increase in wealth raises her *PVLR* by 4000 and, thus, shifts the budget line to the right by 4000, from BL^1 to BL^2. As before, her optimal consumption choice goes from point D (before she finds the passbook) to point J (after her increase in wealth). Because the increase in wealth raises current consumption (from 15 000 at D to 17 000 at J) but leaves current income (14 000) unchanged, it results in a decline in current saving (from –1000 at D to –3000 at J). Being wealthier, Prudence does not have to save as much of her current income (actually, she is increasing her dissaving) to provide for the future.

The preceding analyses show that changes in current income, future income, and initial wealth all lead to parallel shifts of the budget line by the amount that they change the *PVLR*. Economists use the term *income effect* to describe the impact of any change that causes a parallel shift of the budget line.

THE PERMANENT INCOME THEORY

In terms of our model, a temporary increase in income represents a rise in current income, y, with future income, y^f, held constant. A permanent increase in income raises *both* current income, y, *and* future income, y^f. Therefore, a permanent