Answer sheet for Assignment 4

Question 1: Money and inflation (25 Marks)

Part a) (**4 Marks**)

The income elasticity of the demand for money in this case is $\eta_Y = 0.5$ and is obtained as follows:

$$\eta_Y = [dM^d(Y)/dY](Y/M) = [d(PY^{0.5})/dY]/(Y/PY^{0.5})$$

$$= P(0.5Y^{-05})(Y/PY^{-0.5}) = 0.5$$

Part b) (**7 Marks**)

First find equilibrium in the money market, namely:

$$M^d = M^s = M$$

$$PY^{0.5} = M$$

With P=1 we get:

(1) $Y = (M)^2$

This shows the relationship between output and money supply in our simple economy. Next we use the definition of velocity, in order to get values of both M^s and Y. First:

$$V = PY/M = 2, \text{ or}$$

(2) $Y = 2M$

We now have two relationships between output and money. Setting one equal to the other we get:

$$2M = (M)^2 \text{ or}$$

(3) $M^2 - 2M = 0$

We have to solve a simple quadratic equation, which is:

$$M_1, M_2 = \{2 \pm \sqrt{4 - 0}\}/2 = (2 \pm 2)/2$$

The two roots are $M_1 = 0$ and $M_2 = 2$. We will focus on the second case, which is the more sensible one. Thus a level of $M^s = 2$ and $Y = 4$ (from either 1 or 2 above) would be consistent with velocity of 2 and a price level of 1.

Note that some students may just factor expression (3) – $M(M - 1) = 0$ – and get $M = 2$ directly as a root. This is fine as an answer.

Part c) (**7 Marks**)

We have to rely on the following equation, linking predicted or expected inflation, π^e, to money growth $\Delta M/M$ and output growth $\Delta Y/Y$:

(1) $\pi^e = \Delta M/M - \eta_y \Delta Y/Y = 0.04 - 0.5(0.05) = 0.015$

In our case, expected inflation is going to be 1.5%. Money is growing too fast compared with the increase in transactions needs: this will induce inflation in the economy.

Part d) (**7 Marks**)

Only equation (ii) is consistent with the quantity theory. The theory makes the strong assumption that the velocity of money (the rate at which it turns over in the economy) is constant. The two other equations imply that velocity will change with interest rates and real income.

Question 2: Deriving and solving the IS-LM-AD model – closed economy version (25 Marks)

Part a) (**4 Marks**)

For the IS curve, start with the desired saving-investment equilibrium condition. In particular:

(1) $S^d = Y - C^d - G = Y - 0.6(Y - T) + 50r - G$

(2) $I^D = 310 - 150r$

Setting the two equal and solving for r in terms of Y and factors that would shift the curve, we get:

(3) $r = 1.55 - 0.002Y - 0.003T + 0.005G$ (IS Curve)

Movements along the IS curve show the combinations of r and Y that are consistent with goods market equilibrium. For instance, an increase in Y implies an increase in S^d (from (1) above). The shift outward in the S^d curve will lower interest rates and in the process raising investment, bring it into equilibrium with the higher level of saving. As well, saving being equal to investment is consistent with goods market equilibrium.

The LM curve is straightforward and is simply the money demand function rewritten as r related to Y, M and π^e.

(4) $r = 0.00125Y - 0.0025(M/P) - \pi^e$ (LM curve)

The LM curve shows combinations of r and Y that are consistent with money market equilibrium, given an unchanged money supply, M, and constant inflation expectations, π^e. A rise in income will raise the demand for money and in order to

restore equilibrium with M/P unchanged, the real interest rate must rise to offset it. An increase in real balances (M/P) will shift the curve down and to the right. The key is to note that Y remains unchanged and accordingly interest rates must now be lower at every point along the curve in order to encourage people to hold the increased real balances.

Part b) (**7 Marks**)

The aggregate demand curve shows the relationship between the price level and aggregate demand. We can start by setting the IS curve equal to the LM curve so as to eliminate r. Thus:

(5) $1.55 - 0.002Y - 0.003T + 0.005G = 0.00125Y - 0.0025(M/P) - \pi^e$

Rewriting (5) as Y related to P yields:

(6) $Y = [1.55 + 0.0025(M/P) - 0.003T + 0.005G + \pi^e]/0.00325$ (AD curve)

Writing it this way has the advantage of being able to plug in M (and the other variables) to solve directly for Y.

An alternative way to write down the equation is with P on the left-hand side (as graphed in the text). This yields:

(6) $P = M/[1.3Y - (620 + 2G - 1.2T) - 400\pi^e]$

This is the aggregate demand curve, and demand (output) is inversely related to the price level. Here however the relationship is driven by the effect that P has on real balances. In particular, a decline in P will raise real balances and shift the LM curve down and out along an unchanged IS curve, resulting in both lower interest rates and higher output. The inverse relation then has everything to do with how P affects real balances and through that interest rates and demand.

Part c) (**7 Marks**)

The easiest way is to go directly to the AD curve (equation (6)) and solve for Y. Thus:

(6') $Y = [1.55 + 0.0025M/P - 0.003T + 0.005G + \pi^e]/0.00325$

$= [1.55 + 0.0025(480) - 0.003(250) + 0.005(250)]/0.00325$

$= 3.25/0.00325 = 1000$

Use this value of Y in either equation (3) or (4) to get r. Using the latter, we get:

(4') $r = 0.00125Y - 0.0025(M/P) - \pi^e$

$= 0.00125(1000) - 0.0025(480)$

$= 0.05$

To get the price level, given the nominal level of M we can use either the LM or the AD curve. The LM curve is more straightforward to use. Thus:

(4'') $r = 0.00125Y - 0.0025(M/P) - \pi^e$

$0.05 = 1.25 - 1.8/P$

Rewriting in terms of P we get:

$P = 1.8/1.2 = 1.5$

Part d) (**7 Marks**)

The key here is to assume that the price level does not adjust in the short run. Substitute into the aggregate demand relationship the new, higher money supply and solve for Y. Thus:

(6'') $Y = [2.05 + 0.0025M/P]/0.00325 = [2.05 + 0.0025(756/1.5)]/0.00325$

$= 1018.4615\ldots$

To solve the interest rate we can use either the LM or the IS curves. The latter yields:

$r = 2.05 - 0.002Y = 2.05 - 0.002(1018.4615\ldots)$

$= 0.013076\ldots$

This level of output and interest rates is not consistent with long-run equilibrium since actual output at 1018.4615… is great than its full employment level of 1000. In a situation of excess demand, prices will start to rise, causing real balances to decline. The process will continue until the LM curve has shifted back to its original position. The only way this can be accomplished is for the price level to rise until the original level of real balances is restored; i.e., by 5 per cent. The rate of inflation during the transition period goes from 0 to 5 per cent.

Question 3: Some policy implications of the IS-LM-AD model (25 Marks)

Part a) (**4 Marks**)

First we have to recalibrate the IS curve to take into account the drop in business confidence. Thus:

$r = 0.95(310)/200 - 0.002Y - 0.003T + 0.005G$

(3') $r = 1.395 - 0.002Y + 0.5 = 1.9725 - 0.002Y$

The LM curve remains the same (where we assume $\pi^e = 0$):

(4) $r = 0.00125Y - 0.0025(M/P)$

With real balances unchanged for the moment at 480, we can use (3) and (4) to solve for Y and r. Thus:

r = 0.02019…

Y = 976.1538…

Output has fallen by about 2.4 per cent below potential while the real rate of interest has dropped by close to 3 percentage points. The IS curve has shifted leftward and along the LM curve.

Part b) (**7 Marks**)

i. To calculate the amount by which the money supply must be increased to restore equilibrium we can use the aggregate demand equation, which now must be recalculated since the constant term in the IS curve has changed. Setting (3') equal to (4) so as to eliminate r, we get:

   (6'") Y = [1.9725 + 0.0025(M/P)]/0.00325

   Substituting in Y = 1000 (the full-employment level to which we wish to return) and P = 1.5 (which has not changed in the short run) we can solve for the new level of M needed to restore full employment output. The level of M is 766.5 (approximately a 6.5 per cent increase). This in turn implies that real interest rates would have to be <u>negative</u> 0.0275. Given inflation expectations of zero ($\pi^e = 0$), the nominal interest rate, and in this case the real interest rate, can only fall to zero. Monetary policy cannot get the economy back to full employment.

ii. The monetary authorities, however, can push the nominal (and accordingly the real) interest rate to zero. Putting this in the new IS curve we can solve for the resulting level of Y. Thus:

   (3') r = 1.9725 – 0.002Y

   This implies that Y is 986.25, which is still 1.375 per cent below potential although better than the 2.385 initial drop.

   The resulting money supply increase would be derived from the LM curve.

   (4) r = 0.00125Y – 0.0025(M/P)

   With r = 0 and Y = 986.25 we get M/P = 493.125 or M = 739.6875 (assuming an unchanged price level of 1.5). The money supply would increase by 2.73 per cent, short of the approximately 6.5 per cent increase initially calculated in (i) above.

iii. If the monetary authorities did nothing, the price level would fall because of the now wider output gap (deficient demand). However, the resulting rise in real money balances would still be unable to push the real interest rate beyond zero. Indeed, further drops in the price level after that would risk setting in place

deflation (although this is not modelled here), which would only raise the real rate of interest.

If inflation expectations (π^e) were equal to or above 2¾ per cent however, then monetary policy would be effective.

Part c) (**7 Marks**)

i. It is best to rewrite the AD curve with the constant term of the investment adjusted for the shock as well as with the fiscal variables clearly identified:

$Y = [1.4725 + 0.0025(M/P) - 0.003T + 0.005G + \pi^e]/0.00325$

With $\pi^e = 0$ and M/P unchanged at 480, we can re-write the new AD equation in a form suitable to our needs as:

(6') $Y = [2.6725 - 0003T + 0.005G]/0.00325$

By setting Y = 1000 and T = 250, we can solve for G that would be required to get the economy back to full employment. This yields G = 265.5.

ii. We can proceed in a similar manner and solve for size of the tax cut needed to restore full employment output. Thus T = 224.166 $\overline{6}$.

iii. Using the definition of the government's budget balance we find that the policy of increasing spending is less expensive in terms of its budget costs compared with the tax cuts, a deficit of 16.5 for spending versus a deficit of 25.833 $\overline{3}$ for the tax cut. The reason is that the spending increase has a direct effect on lowering saving versus the indirect effect of the tax cut, which works through consumption and part of which is saved. Another way to think about this is that the expenditure multiplier is greater than the tax multiplier, again due to the fact that some part of the tax cut would be saved.

Part d) (**7 Marks**)

In this case monetary policy could not move the economy back to an equilibrium position because of the zero bound on nominal interest rates, which puts a floor under the real rate. As a result, fiscal policy was the only other tool left to the authorities. The fiscal expansion moves the IS curve along an unchanged LM curve and the original real interest rate (r = 0.05) is restored.

Question 4: The open economy version of the model (25 Marks)

Part a) (**4 Marks**)

To get the new IS curve, the students should start from the open-economy equilibrium condition, which is:

$$S^d - I^d = NX$$

Using the definitions of each we get:

$0.4Y + 200r + 0.6T - G - 310 = 140 - 0.1Y - e$, or in terms of r and Y and the other variables:

(7) $r = [450 - 0.5Y - 0.6T + G - e]/200$

The LM curve is unaffected and from above it is described by:

(4) $r = 0.00125Y - 0.0025(M/P) - \pi^e$

The new AD curve is derived as usual by using (7) and (4) to eliminate r. Thus:

(8) $Y = [2.25 + 0.0025(M/P) - 0.003T + 0.005G - 0.005e + \pi^e]/0.00375$

Part b) (**7 Marks**)

With the real exchange rate at 40, and given the fiscal variables G = T = 250, we can use the new IS curve to get Y, bearing in mind that the world rate of interest is fixed at 0.05. From (7) above we get:

$r = [450 - 0.5Y - 0.6T + G - e]/200$

$0.05 = [450 - 0.5Y - 0.6(250) + 250 - 40]/200$

Which implies that Y = 1000.

From the net export equation we get:

$NX = 140 - 0.1(1000) - 40 = 0$

Trade is balanced so the economy is neither adding to nor subtracting from world saving.

The definition of the real exchange rate is:

$e = e_{nom}P/P^F$

With the domestic price level of 1.5 and the foreign price level of 0.9, the nominal exchange rate would be 24.

Part c) (**7 Marks**)

First calculate the new IS curve where the constant term has shifted down. It is:

$r = [0.95(310) + 140 - 0.5Y - 0.6T + G - e]/200$

$r = [434.5 - 0.5Y - 0.6T + G - e]/200$

Setting this equal to the LM curve we can derive a new AD curve as:

$$Y = [2.1725 + 0.0025(M/P) - 0.003T + 0.005G - 0.005e]/0.00375$$

With r = 0.05; T = G = 250 and e = 40 for the moment, the short-run equilibrium level of output would be:

$$Y = 979.33\ \overline{3}$$

The fall in demand will shift the IS curve to the left along an unchanged LM curve putting downward pressure on interest rates in the short run. From the LM curve, the short-run interest rate would be:

$$r = 0.00125Y - 0.0025(M/P) - \pi^e = 0.00125(979.33\ \overline{3}) - 0.0025(480) = 0.024166\ \overline{6}$$

The lower interest rate will cause capital to flow out of the country (in the case of perfect capital mobility, the effect is instant) until the domestic interest rate is back to the world level. The capital outflow drives down the nominal exchange rate to the point where net exports have been increased by enough to make up the short fall in demand. We can use the IS curve above to solve for e (the real exchange rate), given that output will have to return to its full-employment level while the interest rate is pegged at 0.05. Thus:

$$r = [434.5 - 0.5Y - 0.6T + G - e]/200$$

$$0.05 = [434.5 - 0.5(1000) - 0.6(250) + 250 - e]/200$$

This implies that e = 24.5 which raise net exports to 15.5, an amount sufficient to offset the fall in investment ($0.05 \times 310 = 15.5$). The nominal exchange rate can be derived from the definition of the real rate on the assumption that relative prices have not changed. Thus:

$$e = e_{nom}P/P^F$$

$$24.5 = e_{nom}(1.5/0.9)$$

$$e_{nom} = 14.7$$

This represents a fall of 38.75 per cent from its initial level of 24.

Compared to the closed economy case, the economy can return automatically to full employment since there is no zero-bound-type constraints on the exchange rate adjustment that would prevent this return from occurring.

Part d) (**7 Marks**)

The commitment to maintain the nominal exchange rate fixed means that that channel of adjustment is closed off. As the IS curve shifts leftward the nominal exchange rate will come under downward pressure. The central bank must then lower nominal M shifting the LM curve leftward until the pressure on the exchange rate is eliminated. In this fiscal policy should be used. Now the IS curve shifts back

(rightward) and because there will be upward pressure on the nominal exchange rate, the central bank intervenes, expanding the money supply in the process.

The students should illustrate their arguments with the relevant graphs.