Answers for ECON222 exercise 2 Winter 2010

Question 1: Productivity, Output and Employment (20 Marks)

Part a): (6 Marks)

Start by taking the derivative of the production wrt labour, which is then set equal to the real wage (w). Thus:

(1) $\partial Y/\partial N = (1 - \alpha)AK^{\alpha}N^{-\alpha} = w$

Then solve this for N, which is the demand for labour, N^d. Thus:

(2) $N^d = [(1 - \alpha)AK^{\alpha}/w]^{1/\alpha}$

Inserting values for the parameter α and the other variables, and with w = 9.25, we get:

(3) $N^d = [(1 - \alpha)AK^{\alpha}/w]^{1/\alpha} = \{[0.75\times5\times(625)^{\alpha}]/9.25\}^4 \approx 16.882$

The students should show a graph of a downward sloping demand curve for labour as the quantity of labour demand rises (noting the role of diminishing marginal product) and the curve will cut the flat line where the real wage rate is constant.

Part b): (9 Marks)

Start with the labour demand function in (2):

(2”) $N^d = [(1 - \alpha)AK^{\alpha}/w]^{1/\alpha}$

In equilibrium, labour demand, N^d, has to be equal to supply, $N^s = 1.5w$. Thus:

(4) $N^d = [(1 - \alpha)AK^{\alpha}/w]^{1/\alpha} = 1.5w$

We can now rewrite (4) in terms of w, which, after some manipulation yields:

(4’) $w = \{[(1 - \alpha)AK^{\alpha}]^{1/\alpha}/1.5\}^{1/(1+1/\alpha)} = \{(0.75\times5\times625^4)/1.5\}^{0.2} \approx 9.620$

(It looks horrible but if the students take their time they will get it.)

With this as the wage rate, labour demand from (2”):

(2”) $N^d = [(1 - \alpha)AK^{\alpha}/w]^{1/\alpha} = [(0.75\times5\times625^{0.25})/9.620]^4 \approx 14.430$

Compared with the case were the supply curve was flat, the introduction of an upward sloping labour supply implies that workers require a higher wage to supply the original amount. The result is that the wage rate rises but now firms demand fewer workers. In the final equilibrium, fewer workers are employed (14.43 versus 16) but their wage rate is higher (9.62 versus 9.25).

Part c): (5 Marks)

The question calls on the students to use the following form of Okun's Law:

(5) $\Delta Y/Y$ = potential growth – $2\Delta u$, where u is the unemployment rate.

Given the information (potential growth is now 3% and the unemployment rate has rise by 2 percentage points) we could expect growth to fall from 3.5% to -1%. Using the same relationship, growth prior to the recession was 3.5% (it goes to its potential rate since unemployment had not been changing).

Question 2: Determining consumption (30 Marks)

Part a): (5 Marks)

First derive the equation for the budget constraint, which, in this case, is:

$c^f = (y - c)(1 + r) + y^f$

The intuitive explanation is that the relationship shows the various combinations of c^f and c that are available to individuals (given y, y^f and r) and how decisions about present consumption affect what is available in the future. It is negatively sloped because any increase (decrease) in consumption today means less (more) resources are available for future consumption. The rate at which these resources fall (rise) is given by 1 + r, which is applied to saving.

To plot the relationship, calculate the maximum values of c and c^f, which are determined as follows:

c^f (max) = $(1 + r)y + y^f = 10(1.10) + 11 = 22$
c (max) = $y + y^f/(1 + r) = 10 + 11/(1.10) = 20$

A straight line connecting the two points shows all the combinations of c^f and c available to the consumer when they are deciding how much to consume today.

Part b): (10 Marks)

Start first with the budget constraint above and then divide both sides by 1 + r. The resulting expression shows that the present value of lifetime consumption (PVLC) must equal the present value of lifetime resources (PVLR).

(1) $c + c^f/(1 + r) = y + y^f/(1 + r)$

We know that the slope of the utility curve (between c^f and c) is $-(13/12)(c^f/c)$ and at its optimum level it equals the slope of the budget constraint, $-(1 + r)$. Thus:

(2) $(13/12)(c^f/c) = (1 + r)$

We can use equation (2) to get an expression for c^f in terms of c. Thus:

(3) $c^f = (12/13)(1 + r)c$

Putting this expression into the inter-temporal budget constraint (equation (1)) we have:

(4) $c + \{(12/13)(1 + r)c\}/(1 + r) = y + y^f/(1 + r)$

On the left hand side, the interest rate terms cancel out and we get:

$c(1 + 12/13)) = y + y^f/(1 + r)$, which becomes:

(5) $c = 0.52(y + y^f/(1 + r)) = 0.52(20) = 10.4$

Given that c = 10.4 we can use either (1) or (3) to get c^f. Using the latter:

(6) $c^f = (12/13)(1 + r)c = 10.56$

Substituting the values of c and c^f from (5) and (6) into (1) it can be verified that the choices are consistent with the inter-temporal budget constraint.

(7) $10.4 + 10.56/(1.10) = 10 + 11/(1.10)$
$20 = 20$

Locate the values of c and c^f on the graph and then draw an indifference curve tangent to that point.

Part c): (5 Marks)

Once we have the results from b) this part is straightforward. Start with (5) above

$c = 0.52(y + y^f/(1 + r)) = 0.52(10 + 11/(1.20)) = 9.9666…$

Then from (6) above we have:

$c^f = (12/13)(1 + r)c = (12/13)(1.20)(9.9666\ldots) = 11.04$

Using the values for c and c^f, it can be shown that the choices are consistent with the inter-temporal budget constraint. In this case each side equals 19.1666….

Compared with b) present consumption has fallen from 10.4 to 9.9666…. Given income unchanged at y = 10, saving rises from −0.4 to +0.0333…. The substitution effect dominates. This is not surprising since the starting point was one in which there was dissaving.

Part d); (5 Marks)

To derive the new budget constraint, first calculate the maximum consumption points, which are:

$c^f\,(\text{max}) = (1 + r)y + y^f = 10(1.20) + 11 = 23$
$c\,(\text{max}) = y + y^f/(1 + r) = 10 + 11/(1.20) = 19.1666\ldots$

When plotted the two lines will cross at the point where present consumption is equal to present income and where future consumption is equal to future income. This point is referred to as the no-borrowing, no-lending point. In the area below this point, consumers are dis-saving and accordingly the substitution effect dominates. The point is also useful in determining the relative strengths of the substitution and income effects.

Part e): (5 Marks)

The new inter-temporal budget constraint becomes:

$c + c^f/(1 + r) = y + y^f/(1 + r) + a$ (where a is assets)

Since the slope of the budget line is unchanged at −(1 + r), the budget line is shifted outward by an amount equal to the rise in wealth.

To find c and c^f, proceed as in part b) with the slope of the indifference curve set equal to the slope of the budget constraint:

$c^f = (12/13)(1 + r)c$

This is substituted into the inter-temporal budget constraint above:

$c + (12/13)(1 + r)c/(1 + r) = y + y^f/(1 + r) + a$

$c(1 + 12/13)) = y + y^f/(1 + r) + a$

$c = 0.52(10 + 11/1.10 + 5) = 13$ and

$c^f = (12/13)(1 + r)c = (12/13)(1.10)13 = 13.2$

Consumption is now higher in each period because of the wealth effect. As well saving falls from –0.4 to –3.0.

Question 3: Determining goods market equilibrium (20 Marks)

Part a): (5 Marks)

The key point here is that the MPK is equal to the user cost of capital, which is defined as:

(1) $uc = ((r + d)/(1 - \tau))P_k = ((0.05 + 0.08)/(1 - 0.50))150 = 39$

In equilibrium, MPK = uc (the students should show a downward sloping MPK that crosses the flat uc line.) which yields the following:

$70 - 2K_{t+1} = 39$, which implies $K_{t+1} = 15.5$

If the price of capital rises by 25%, then the new user cost will be:

$uc = ((0.05 + 0.08)/(1 - 0.50))187.5 = 48.75$

The new capital stock will be found by solving:

$70 - 2K_{t+1} = 48.75$, which implies $K_{t+1} = 10.625$

In the productivity case, the new intercept is now 77 and with an unchanged user cost the new capital stock will rise to:

$77 - 2K_{t+1} = 39$, which implies $K_{t+1} = 19$

Part b): (5 Marks)

Start from the capital accumulation equation:

(1) $K_{t+1} = K_t + I_t - dK_t$, which can be re-written in terms of I as:

(2) $I_t = K_{t+1} - (1 - d)K_t$

Then re-write the MPK relation (from above) in terms of K_{t+1} as:

(3) $K_{t+1} = 35 - 0.5uc = 35 - 0.5((r + d)/(1 - \tau))P_k$

$= 35 - 0.5(d/(1 - \tau))P_k - \{0.5P_k/(1 - \tau)\}r$
$= 35 - 12 - 150r$
$= 23 - 150r$

This final expression is substituted into the rearranged capital accumulation identity (2), and assuming that $K_t = 10$ and $d = 0.08$, yields:

(4) $I^d_t = 13.8 - 150r$

Part c): (3 Marks)

We can start by looking at the definition of desired national saving, S^d from manipulating the national accounts identity. Thus:

(1) $S^d = Y - C^d - G$

Next substitute in the expression for desired consumption to get:

(2) $S^d = Y - 14 + 100r - 0.5Y - G$

Then substituting in the values for Y (= 32) and G (= 6), we get:

(3) $S^d = -4 + 100r$

Part d): (7 Marks)

Goods market equilibrium requires that S^d is equal to I^d. We have a desired investment equation from Part b). Setting the two equal yields:

$-4 + 100r = 13.8 - 150r$

Solving for r, we get:

$250r = 17.8$
$r = 7.12\%$

Saving would be 3.12 ($-4 + 100\times0.0712$) while investment would also be 3.12 ($13.8 - 150\times0.0712$).

Question 4: Saving and Investment in an open economy (20 Marks)

Part a): (5 Marks)

Start with the goods market equilibrium conditions when the economy is open to trade. This condition is:

(1) $Y = C^d + I^d + G + NX$

Desired domestic saving is the same as before:

(2) $S^d = Y - C^d - G$

Substituting (1) into (2) we get:

(3) $S^d = I^d + NX$

This says that total national desired saving are used to finance desired investment and the current account imbalance (assuming that NFP are zero). Whether NX is positive will depend here on the level of the world interest rate, This should be illustrated in a graph.

Part b): (5 Marks)

Start with the investment and saving functions derived in Question 2, Part b and c. These are:

$I^d = 13.8 - 150r$

$S^d = -4 + 100r$

With a world interest rate of 8% then I^d would be 1.8 and S^d would be 4. With national saving greater than desired investment the country would be running a current account surplus equal to 2.2 – it would be a net supplier of saving to the rest of the world.

Part c): (5 Marks)

In this question only the investment function is affected; national saving would stay constant at 4, given a world interest rate of 8%.

The key is that both the slope and the constant term will be affected. Start again with the investment identity:

(1) $I_t = K_{t+1} - (1 - d)K_t = 35 - 0.92K_t - 0.5uc$
(2) $I_t = 25.8 - 0.5(d/(1 - \tau))P_k - \{0.5P_k/(1 - \tau)\}r$

(3) $I_t = 17.8 - 100r$

This has the advantage of isolating the various influences on the real rate of interest. With a lower price of capital ($P_k = 100$) we get,

(4) $I^d = 17.8 - 100r$

With a world interest rate of 8% the new investment function implies a new, higher level of desired investment of 9.8. The shift in the investment is enough to move the current account from a surplus of 2.2 to a deficit of 1.8 (NX = −5.8). The key for any new graph is that the constant term and the slope have changed. With the lower interest rate coefficient the I^d curve flattens out.

Part d): (8 Marks)

Start with the expanded version of the investment function above:

(1) $I^d = 19.8 - \{0.5P_k/(1 - \tau)\}r$

With the tax rate zero the interest rate term in the investment function becomes:

(2) $I^d = 19.8 - \{0.5P_k/(1 - \tau)\}r = 19.8 - 75r$

With r = 8% then I^d is 13.8.

Some of this increase in investment will be financed by an increase in national saving following the reduction of government spending. The new saving function is:

(3) $S^d = 2 + 100r - G = -1 + 100r$

The level of national saving is now equal to 7. The current account balance is:

(4) NX = 7 − 13.8 = − 6.8

Without the reduction in spending the current account imbalance would have been the difference between the original level of national saving and the new level of private saving; i.e. NX = 4 − 13.8 = − 9.8.

In the graph, the students should show that the I^d curve has changed its slope with the S^d curve has shifted (only the constant term has been affected).