QUEEN'S UNIVERSITY

FACULTY OF ARTS AND SCIENCE

DEPARTMENT OF ECONOMICS

Economics 222 A & B
Macroeconomic Theory I

Suggested Answers to the Midterm Questions

Part A – Short Questions (True/False/Uncertain)

*Do **three (3)** of the **six (6)** questions. Each question is worth **10 marks**, for a total of **30 marks**.* State whether you think the statement is true/false/uncertain, and explain why, using the appropriate diagrams if applicable. ***No marks*** will be given for answers lacking an explanation.

A1

The savings-investment identity in a closed economy, $S = I$, describes how an increase in government spending, G, must crowd out investment one-for-one.

FALSE. This is an *accounting identity, not a behavioural relationship.* It holds regardless of the assumptions of the model being analyzed, and thus tells nothing about how private savings, output, and investment react to an increase in G. As long as one can show that C or Y is affected by the increase in G (which the former is in the model presented in class, through the increase in r that results from the increase in G), then one has effectively shown that the statement does not hold.

A2

The government finances a transfer to households in $t = 1$ by debt emission. The debt must be reimbursed in $t = 2$, and to this end the government levies a lump-sum tax on the households in $t = 2$. As a result, the households' consumption bundles are left unchanged.

UNCERTAIN. This depends whether one assumes Ricardian equivalence to hold. If it does, then the statement is true: households foresee the increase in future taxes, and thus save the entirety of the transfer, thus leading them to consume exactly the same amount. If Ricardian equivalence does not hold, however, then the statement is false. For instance, credit-constrained households would spend at least a fraction of the transfer in $t = 1$, thus changing their optimal consumption bundle. This goes also for myopic households.

A3

Two large economies (the US and China) interact in the world's economy. Assume that Home initially has a current account surplus. An increase in Home's government deficit due to an increase in G will cause Foreign to save more.

TRUE. An increase in the US's government deficit due to an increase in G causes domestic desired savings to decrease (S^d_{US} shifts left), and Home's desired international lending to fall, *ceteris*

paribus. This causes an imbalance, such that at the previous world interest rate, China's desired international borrowing exceeds the US's desired international lending, also *ceteris paribus*. The world interest rate must increase in order for China's desired international borrowing to fall (and the US's desired international lending to increase), and an equilibrium being reached, therefore implying that China must save more than before. Note however that in China's $S^d - I^d$ diagram, the S^d_{CN} and I^d_{CN} curves do not move.

A4

The introduction of a tax on labour income will necessarily reduce the quantity of aggregate labour supplied in equilibrium.

FALSE. It may, but it certainly does not *necessarily* result in a decrease in the quantity of aggregate labour supply. The former is true only if the substitution effect (work less since after-tax gain lower, and hence opportunity cost of leisure has fallen) outweighs the income effect (work more because income is lower – ceteris paribus – after the introduction of a tax). It may well be that the income effect prevails for enough people, causing aggregate labour supply to increase in equilibrium.

A5

Classical theory provides a good framework for considering the aggregate fluctuations of unemployment in the short run.

FALSE. Classical theory states that all markets clear, even in the short run, and thus do not remain out of equilibrium. This means that there can be no unemployment (except as a strictly transitory phenomenon) even in the short run. This therefore does not allow for the study of fluctuations in *market disequilibria*, such as fluctuations in unemployment (in the short or long run). Keynesian theory, which states that wages can be sticky in the short run, and thus explains why there may be persisting disequilibria, is better suited to explain fluctuations in unemployment.

A6

Knowing where the no-lending/no-borrowing point is on a consumer's inter-temporal budget constraint allows economists to derive the pure substitution effect of a change in the real interest rate.

TRUE. A change in the real interest rate involves pivoting the budget line through the endowment point to find the total effect on the consumption bundle. The substitution effect is obtained by drawing a hypothetical budget line at the new interest rate, but that goes through

the original consumption bundle such that it can still be bought. This allows to remove any income effects caused by a change in relative prices for current and future consumption.

Part B – Long Questions

*Do **two (2)** of the **four (4)** questions. Each question is worth **35 marks**, for a total of **70 marks**.*

B1 – Employment, Labour, and Production

An economy is characterized by the following equations, describing respectively the production technology and the supply of labour in this economy.

$$Y = 2K^{\frac{1}{2}}N^{\frac{1}{2}}$$

$$N^S = (1-\tau)w$$

where τ is the marginal tax rate on labour income. The capital stock is given by $\bar{K} = 125$. Using this information, answer the following questions.

a) [5 MARKS] *Show algebraically* or *argue rigourously* that this production technology exhibits constant returns to scale (CRS).

Algebraically: suppose that we multiply all inputs by a factor $\theta > 1$. We therefore have:

$$\begin{aligned} 2\left(\theta K\right)^{\frac{1}{2}}\left(\theta N\right)^{\frac{1}{2}} &= \theta 2K^{\frac{1}{2}}N^{\frac{1}{2}} \\ &= \theta Y \end{aligned}$$

QED.

This could also be argued rigourously by saying that what characterizes this class of CRS production functions is that the exponents sum to 1 (verified here), which means that, for instance, doubling all inputs will double output.

b) [10 MARKS] Assume initially that $\tau = 0$. Find the labour market equilibrium wage rate, w^*, and level of employment, N^*.

The marginal product of labour is given by:

$$MPN = \frac{\partial Y}{\partial N} = \sqrt{\frac{125}{N}}$$

In equilibrium, by the profit maximization motive of the firm, $MPN = w$. Hence, we have:

$$w = \sqrt{\frac{125}{N^D}}$$

$$\iff N^D = \frac{125}{w^2}$$

an equation for aggregate labour demand.

The equilibrium condition on the labour market is that $N^D = N^S = N^*$. Hence, using the equation for labour demand found above, along with the equation for labour supply with $\tau = 0$ given in the question, we find:

$$\frac{125}{w^2} = w$$

$$\iff w^* = 5$$

to which corresponds $N^* = 5$.

c) [10 MARKS] Suppose now that $\tau = 0.25$. Find the new labour market equilibrium wage rate and level of employment. Illustrate this new equilibrium in a diagram, and contrast it with the equilibrium in c) by drawing the appropriate curves.

In a way similar to the one above we find that:

$$\frac{125}{w^2} = 0.75w$$

$$\iff w^{**} \approx 5.503$$

and $N^{**} \approx 4.127$. This implies a leftward shift in the labour supply (N^S) curve, thus raising the equilibrium wage rate and lowering the equilibrium amount of labour hired.

d) [10 MARKS] Assume again that $\tau = 0$. Suppose that the government imposes a minimum wage of $\underline{w} = 5.25$. Find the new labour market level of employment. What is the unemployment rate after the minimum wage has been imposed? Illustrate the situation graphically. Would you expect the effect to be the same if $\tau = 0.25$? Explain.

The demand for labour (equivalent to *employment*) implied by a minimum wage rate of $\underline{\mathrm{w}} = 5.25$ is $N^D \approx 4.535$ while the quantity of labour supplied (equivalent to the size of the *labour force*) at that wage is $N^S = 5.25$. This leads us to conclude that the unemployment rate

is given by:

$$\begin{aligned} u &= \tfrac{LF-E}{LF} \cdot 100 \\ &= \tfrac{5.25-4.535}{5.25} \cdot 100 \\ &\approx 13.616\% \end{aligned}$$

Graphically, this means that there is a floor price above the equilibrium wage rate, causing therefore a surplus of labour on the market.

If, on the other hand, $\tau = 0.25$, then this minimum wage would not have any effect since it would be below the equilibrium wage rate in that aggregate labour market with a positive tax on labour income.

B2 – Savings, Investment, and the Goods Market Equilibrium

Suppose that the production side of an economy is characterized by the following equations:

$$MPK^f = 150 - 0.2K^f \tag{1}$$

$$\bar{Y} = 1600 \tag{2}$$

$$K = 700 \tag{3}$$

$$d = .20 \tag{4}$$

$$p_K = 1 \tag{5}$$

where MPK^f describes the relationship between the future marginal product of capital and the future capital stock K^f, $\bar{Y}$ represents full-employment output, K is the initial capital stock, d is the depreciation rate, and p_K is the price of capital.

a) [10 MARKS] We first seek to characterize the firms' investment decisions. To this extent, find the following: (i) an equation relating K^f to the interest rate, r; (ii) use this equation to find the firms' desired (gross) investment, I^d, as a function of r.

The firms' choice of future capital stock is given by the following equation:

$$MPK^f = p_K(d + r)$$

$$\iff 150 - 0.2K^f = r + 0.20$$

$$\iff K^f = 749 - 5r$$

Hence, desired gross investment is given by:

$$I^d = K^f - (1-d)\bar{K}$$

$$\iff I^d = 189 - 5r$$

Suppose now that aggregate desired consumption in this economy is given by:

$$C^d = 160 + 0.6Y - 150r$$

Meanwhile, government expenditures are given by $G = 300$. No taxes are levied, hence $T = 0$.
b) [5 MARKS] Find an equation for desired national saving, S^d, as a function of the interest rate r.

We find that desired national savings is equal to:

$$\begin{aligned} S^d &\equiv S_{pvt} + S_{govt} \\ &= \left(\bar{Y} - C^d\right) + (-G) \\ &= 180 + 150r \end{aligned}$$

c) [10 MARKS] Find the goods market equilibrium, that is, the market clearing interest rate r^*, and equilibrium levels of investment and saving.

This is done by making use of the equilibrium condition of the goods market, namely:

$$I^d = S^d$$

$$\iff 189 - 5r = 180 + 150r$$

$$\iff r^* \approx 0.05806$$

and $S = I \approx 188.71$.

d) [10 MARKS] Now assume that this is a small open economy, and that $NFP = 0$. The world's interest rate is given by $r^w = 0.02$. Holding national income at its full-employment level, compute the desired saving S^d and desired investment I^d, as well as the current account balance, CA. Is this economy a net lender or borrower on the world market? Represent the situation

graphically.

For a small open economy, the goods market equilibrium is given by $S^d = I^d + CA$, and r^w is taken is as given. We therefore have:

$$180 + 150(0.02) = 189 - 5(0.02) + CA$$

$$\iff CA = NX = -5.9$$

This economy is therefore a *net borrower* on the world market, which can be illustrated by the usual graph.

B3 – National Accounts

The table below summarizes an economy, which consists of three sectors, potato farmers, McCain's (a producer of potato chips) and a government sector. Farmers have a current crop (valued at 200) as well as some left over potatoes from the previous year's crop held as inventories (valued at 20). Farmers sell most of their crop to McCain's (170) and the rest, including the previous year's inventories, to the public. Farmers pay wages (100) and they receive a subsidy from the government (20).

McCain's potato chips are popular and are sold in both Canada (350) and the United States (150). To produce them, McCain's buys potatoes from farmers and pays wages (200) to its employees. As well the firm pays taxes to the government equal to 20% of its profits.

The government sector produces a service, which is valued at cost (200, the amount paid to civil servants to deliver the service). In addition, the government also pays subsidies to farmers. The only source of revenue that the government has is corporate taxes, which it levies on McCain's profits at a 20% rate.

Potato Farmers	
Crop in current year	200
Inventories from previous year	20
Amount sold to public (from crop and inventories)	50
Amount sold to McCain's	170
Wages paid to employees	100
Subsidies to farmers	20

McCain's	
Amount of potatoes purchased from farmers	170
Value of potato chips sold in Canada	350
Value of potato chips sold in US	150
Wages paid to employees	200
Tax rate on corporate profits	20%

Government	
Wages paid to civil servants to perform services	200
Subsidies paid to farmers	20
Revenues from corporate profits	20% of profits

In answering the following questions, be sure to give the reasons behind each calculation. ***No marks*** will be given for answers lacking an explanation.

a) [15 MARKS] Use the example to show that the three approaches to national accounting are equivalent. In the income approach, what is the value of net domestic product at factor costs? How does one go from that concept to GDP?

Product approach:
Farmers produce 200, McCain's sells 350 (to Canadians) + 150 (to US) but subtracts 170 purchased from farmers and government produces 200 for a total of **730**.

Expenditure approach:
Consumption is 50 (sold by farmers to Canadians) + 350 (McCain's sales) – 20 (the decline in inventories, which is investment, and offsets purchase made by farmer's clients) + 200 (government) + 150 (exports by McCain's) = **730**.

Income approach:
Wages are 100 (farmers) + 200 (McCain's) + 200 (government) = 500.

Profits for farmers are 200 (revenues from current production – the crop) + 20 (transfers from

government) – 20 (decrease in inventories) – 100 (wages) = 120 (NB the revenues from inventories are not counted as they were taken into account in previous period).

Profits for McCain's are 350 (sales to Canadians) + 150 (receipts from exports) – 200 (wages paid) – 170 (cost of inputs from farmers) = 130.
Net national income at factor costs (NNI) = 500 + 120 + 130 = **750**
$GDP = NNI$ plus indirect taxes less subsidies (indirect taxes are 0 while subsidies are 20) = 750 – 20 = **730**.

The three approaches are the same.

b) [10 MARKS] Show the relation for and calculate the fiscal balance of the government. Show the relation for and calculate private saving. Using your results, calculate national saving. Is saving equal to investment, and if not, why not?

The government's budget balance (or its saving) is G_{gvt} = T – Subsidies – Spending = 26 (.20 time McCain's profits of 130) – 20 – 200 = –194 (they are dissaving).

Private saving S_{pvt} = GDP – T + Transfers (i.e., subsidies) – domestic consumption = $730 - 26 + 20 - 50 - 350 = 324$.

National saving (S) = $S_{pvt} + S_{gvt} = -194 + 324 = 130$

Because this is an open economy $S = I + NX = -20 + 150 = 130$.

c) [10 MARKS] Show the relation for and calculate domestic absorption. Is Canada experiencing a capital inflow or an outflow? In all cases, state why when giving your answer.

Domestic absorption is $C + I + G$ and is equal to $50 + 350(C) - 20(I) + 200(G) = 620$.

Subtracting absorption from output yields $730 - 620 = 150$ (the value of exports to the US). In this case Canada has a (net) capital outflow of 150.

B4 – Consumption and Saving

A consumer divides his/her life into two periods, working and retirement. The consumer has a current income (y) of 250 and an expected future income (y^f) of 200. The agent knows the rate of interest ($r = 5\%$). The agent is a consumption smoother and he/she needs to determine how much to consume in the current period (c) and how much to consume in the future (c^f).

a) [5 MARKS] Use the information given to calculate the present value of the agent's lifetime resources. As well, calculate the maximum amount that the agent could consume in the future. Use that information to plot the agent's budget constraint line.

The $PVLR = y + y^f/(1+r) = 250 + 200/1.05 = 440.476\ldots$ (if 440.5 is used that is fine). The maximum value of $c^f = y(1+r) + y^f = 250(1.05) + 200 = 462.5$.

These are the extreme values on the budget line and that line should be plotted.

b) [10 MARKS] The agent faces a set of downward sloping indifference curves with each having a slope given by $-(10/9)(c^f/c)$. Use this information, and the agent's inter-temporal budget constraint, to calculate the optimal levels of current and future consumption. Show your results graphically.

Given a slope of $-(10/9)(c^f/c)$ must equal the slope of the budget line $-(1+r)$, we have $(10/9)(c^f/c) = (1.05)$ (the minus signs cancel) $\Longrightarrow c^f = (1.05) \cdot .9 \cdot c$. Using this in the inter-temporal budget constraint we get:

$$c + .9c = y + y^f/(1.05) = 440.476...$$

$$\Longrightarrow c = (1/1.9)(440.476\ldots) = 231.82957\ldots$$

which from c^f:

$$c^f = (1.05)(0.9)c = 219.078947\ldots$$

Current saving is:

$$y - c = 250\text{–}231.82957\ldots = 18.17043\ldots$$

The students should locate these points (fairly closely) on their graph with the budget constraint line.

c) [10 MARKS] Suppose now that the agent is made aware of a gift of 50 to be received in the second period. Calculate the effect that this has on both current and future consumption as well as current saving.

Going back to our present consumption relationship we have:

$c = (1/1.9)([y+(y^f+50)/(1.05)] = (1/1.9)(250+250/1.05) = (1/1.9)(488.0952\ldots) = 256.8922\ldots$

As before:

$$cf = (1.05).9c = 242.7631\ldots$$

With current income unchanged at 250, current saving falls from $18.17043\ldots$ to $(250-256.8922\ldots) = -6.8922\ldots$

d) [10 MARKS] Starting from the original income position (no gift of 50), assuming that interest rates rise to 10%. What is the effect on both current and future consumption as well as on current saving? What can we say about the strength of the income and substitution effects?

Representing the new interest rate as r', start as before with:

$$c = (1/1.9)[y + y^f/(1 + r')]$$

Then:

$$c = (1/1.9)[250 + 200/(1.1)] = (1/1.9)[431.8181\ldots] = 227.2727\ldots$$

Proceeding as before:

$$c^f = (1.1)(0.9)c = 224.999\ldots.$$

Before the rise in the interest rate saving was 18.17043 and now saving is:

$$(y - c) = 250 - 227.2727\ldots = 22.727272\ldots$$

Accordingly saving has risen and the substitution dominates the income effect.