ECON 222 Winter 2009: Assignment 3 Solutions

March 13, 2009

Question 1 – Large Open-Economy Goods Market (25 Marks)

Japan and the US are two large open economies interacting in the world's goods market. They have the following characteristics. In Japan, suppose that full-employment output is $Y_J = 2400$, government expenditures are given by $G_J = 100$, and desired consumption is given by:

$$C_j^d = 215 + 0.75Y - 450r^w$$

Japan's desired investment can be described by:

$$I_J^d = 250 - 300r^w$$

In the U.S., desired savings is given by:

$$S_{US}^d = 425 + 400r^w$$

and desired investment by:

$$I_{US}^d = 600 - 250r^w$$

a) [10 MARKS] Find equations for Japan's desired savings, and for each country's current account balance. Then, find the interest rate r^w that clears the world market, and each country's equilibrium current account balance. Represent the resulting equilibrium graphically.

We have domestic desired savings in Japan given by:

$$\begin{aligned} S_J^d &= Y - C^d - G \\ &= 0.25Y - 215 + 450r^w - G \\ &= 600 - 315 + 450r^w \\ S_J^d &= 285 + 450r^w \end{aligned}$$

and $I_J^d = 250 - 300r^w$. Hence,

$$CA_J = S_J^d - I_J^d = 35 + 750r^w$$

and

$$CA_{US} = S_{US}^d - I_{US}^d = -175 + 650r^w$$

The equilibrium condition in the world's goods market is that $CA_J + CA_{US} = 0$. Hence, we find:

$$\begin{aligned} 35 - 175 + 750r^w + 650r^w &= 0 \\ 1400r^w &= 140 \\ r^w &= 0.10 \end{aligned}$$

which means that $CA_J = 110$ and $CA_{US} = -110$. The usual diagram follows.

b) [10 MARKS] Suppose that Japan pursues a contractionary fiscal policy. It thus decreases its expenditures to $G_J = 85$. *Ceteris paribus*, find its effect on the current account balance of both countries and on the equilibrium world interest rate, r^w. Represent the new equilibrium graphically, making sure to contrast it with the one found in a).

The new desired domestic savings in Japan is given by:

$$S_J^d = 300 + 450r^w$$

which leads to

$$CA_J = 50 + 750r^w$$

while the current account equation for the US is left unchanged:

$$CA_{US} = -175 + 650r^w$$

Imposing the equilibrium condition, we obtain:

$$\begin{aligned} CA_J + CA_{US} &= 0 \\ 300 - 175 + 1400r^{w'} &= 0 \\ 1400r^{w'} &= 125 \\ r^{w'} &\approx 0.08929 \end{aligned}$$

which leads to $CA_J \approx 116.964$ and $CA_{US} \approx -116.964$. The usual diagram follows, with Japan's desired savings shifting to the right as a result of the contractionary fiscal policy. Japan's (pre-equilibrium) current account surplus increases, which causes the equilibrium world interest rate to fall and the US to borrow more (and Japan to save more) in the new equilibrium.

c) [5 MARKS] Suppose (counter-factually) that both economies are closed. Find the equilibrium interest rates in Japan and the US using the same numbers as in a). Policy changes then lead both economies to open up to trade with to each other, leading to the situation in a). Who benefits in each economy, lenders (savers) or borrowers? Explain.

Japan as a closed economy:

$$\begin{aligned} S_J^d &= I_J^d \\ 285 + 450r_J &= 250 - 300r_J \\ 750r_J &= -35 \\ r_J &= -0.0466\bar{6} \end{aligned}$$

The US as a closed economy:

$$\begin{aligned} S_{US}^d &= I_{US}^d \\ 425 + 400r_{US} &= 600 - 250r_{US} \\ 650r_{US} &= 175 \\ r_{US} &\approx 0.2692307 \end{aligned}$$

Since $r^w < r_{US}$, borrowers in the US gain from opening up to trade with Japan while lenders lose; the opposite is true for Japan, since $r^w > r_J$.

Question 2 – Growth: Solow-Swan (35 marks)

Suppose that the United Kingdom's economy can be described in the following fashion. Its aggregate production function is:

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}$$

where Y_t is output (GDP) produced at time t, K_t is capital in use at time t, N_t is the number of workers employed at time t, and A_t is labour productivity measured at time t. (This is called a labour-augmenting technology.) In this economy, capital depreciates at rate d, and the population and labour force grow proportionately at rate $n \equiv \Delta N/N$. Moreover, the saving rate is s, and the economy is closed.

We seek to characterize growth in this economy in terms of the Solow-Swan growth model. However, in a departure from the textbook case, we want to explicitly include a measure of labour productivity growth, $g_A \equiv \Delta A/A$, in the steady-state calculations. You are therefore asked to do as follows:

a) [5 MARKS] Rewrite the production function so that you have output per effective worker (i.e. $y_t \equiv$

$Y_t/(A_tN_t)$) on the left-hand side. Find thereafter aggregate investment I_t, and investment per effective worker $\iota_t \equiv I_t/(A_tN_t)$, in this economy.

The production function can be re-written in per effective worker terms in the following way, by dividing both sides by $A_tN_t = (A_tN_t)^{\alpha}(A_tN_t)^{1-\alpha}$:

$$\begin{aligned} \frac{Y_t}{A_tN_t} &= \frac{K_t^{\alpha}}{(A_tN_t)^{\alpha}} \cdot \frac{(A_tN_t)^{1-\alpha}}{(A_tN_t)^{1-\alpha}} \\ y_t &= k_t^{\alpha} \end{aligned}$$

Since $S_t = sY_t, \forall t$, and since we are in a long run perspective ("Say's Law" is deemed to hold) and in a closed economy, $I_t = S_t, \forall t$. Hence, $I_t = sY_t = sK_t^{\alpha}(A_tN_t)^{1-\alpha}$, and $\iota_t = sy_t = sk_t^{\alpha}$.

b) [3 MARKS] In the steady state, required investment per effective worker in this economy is given by $\iota^* = (d + g_A + n)k^*$, where k^* is the steady state level of capital per effective worker (and where $k_t \equiv K_t/(A_tN_t)$). Basing yourself on the definition of the steady state and the function above, explain carefully why such a level of investment is needed in the steady state.

Such a level of investment per effective worker is required in the steady state so as to maintain the level of capital per effective worker constant at $k_t = k^*, \forall t$. This stems from the very definition of the steady state, which characterizes a balanced growth equilibrium where output and capital per effective worker are constant, while output and capital grow at a constant rate. More precisely, this very level of investment is required to compensate for depreciation (at rate d) of the capital stock per effective worker, its decrease due to population growth/growth in the labour force at rate n, and its decrease due to growth in productivity at rate g_A.

c) [12 MARKS] Now assume that $s = 0.24$, $g_A = 0.08$, $n = 0.09$, $d = 0.07$, and $\alpha = 1/4$. Using the information in a) and b), solve for the levels of capital per effective worker k^*, output per effective worker y^*, consumption per effective worker c^*, and investment per effective worker ι^*, in the steady state. Illustrate the steady-state graphically, with the help of curves representing output per effective worker, savings per effective worker, and required investment per effective worker.

From a) and b), we have the following steady state condition:

$$\begin{aligned} s\,(k^*)^{\alpha} &= (d + n + g_A)k^* \\ \iff k^* &= \left(\frac{s}{d + n + g_A}\right)^{\frac{1}{1-\alpha}} \\ \implies k^* &= \left(\frac{0.24}{0.07 + 0.09 + 0.08}\right)^{\frac{4}{3}} \end{aligned}$$

$$= \quad 1$$

We can thus find y^*:

$$y^* = (k^*)^\alpha = 1$$

consumption c^*:

$$c^* = (1-s)y^* = 0.76$$

and investment ι^*:

$$\iota^* = sy^* = 0.24$$

The usual graph follows, except that the required investment schedule differs, and so does the labelling of the axes.

d) [5 MARKS] The growth accounting equation for this economy can be written as:

$$\frac{\Delta Y}{Y} = \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta A}{A} + (1-\alpha)\frac{\Delta N}{N}$$

Given the equation above and the usual characteristics of the Solow-Swan growth model, at what rate does capital grow in the steady state? At what rate does output grow in the steady state?

We know that A grows at rate $g_A = 0.08$, and N grows at rate $n = 0.09$. Since $k_t = K_t/(A_t N_t)$ is constant for all time periods t in the steady state, it must be that the numerator grows at the same rate as the denominator, which grows at rate $g_A + n = 0.17$ (recall that the growth rate of a product is approximately the sum of the growth rates of its terms). Hence, K grows at rate $g_K = 0.17$, and by the growth accounting equation above, Y grows at rate $g_Y = 0.17$ also.

e) [10 MARKS] Find the golden rule saving rate s_g (the rate which maximizes consumption per effective worker in the steady state), ***either***:
(i) by maximizing consumption per effective worker with respect to k, finding the level of the steady state capital stock per effective worker k_g from the first order condition (FOC), and finally solving for the implied saving rate s_g;
or
(ii) by directly maximizing steady state consumption per effective worker, c^*, with respect to s, and by solving for s_g from the FOC;

As a benevolent policymaker concerned only with long run living standards, what would you recommend doing regarding savings? What effects would it have both in the short run and the long run?

(ii) We start by using the functional form for y^* found in part c), which yields the following problem:

$$\max_s c^* = (1-s)\left(\frac{s}{d+g_A+n}\right)^{\frac{\alpha}{1-\alpha}}$$

This yields the following FOC:

$$\frac{dc^*}{ds} = -\left(\frac{s}{d+g_A+n}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\alpha}{1-\alpha}\left(\frac{1-s}{d+g_A+n}\right)\left(\frac{s}{d+g_A+n}\right)^{\frac{2\alpha-1}{1-\alpha}} = 0$$

which can in turn be simplified to:

$$\begin{aligned} \iff \frac{\alpha}{1-\alpha}\left(\frac{1-s}{d+g_A+n}\right) &= \left(\frac{s}{d+g_A+n}\right) \\ \iff \frac{s}{1-s} &= \frac{\alpha}{1-\alpha} \\ \iff s_g &= \alpha = 0.25 \end{aligned}$$

(i) Using the other way, we maximize consumption w.r.t. k, where consumption is total output minus savings, subject to the constraint that savings is equal to required investment (i.e. savings in the steady state):

$$\max_k c = k^\alpha - (d+g_A+n)k$$

FOC:

$$\frac{dc}{dk} = \alpha k^{\alpha-1} - (d+g_A+n) = 0$$

This yields k_g:

$$k_g = \left(\frac{\alpha}{d+g_A+n}\right)^{\frac{1}{1-\alpha}}$$

Since we know that in the steady state, $k^* = (s/(d+g_A+n))^{1/(1-\alpha)}$, then it follows that $s_g = \alpha = 0.25$.

We are thus below the saving rate which yields the golden rule levels of capital and consumption. For a benevolent policymaker, the solution to increasing long term living standards is therefore to raise the saving rate from $s = 0.24$ to $s_g = 0.25$. The short run effect will be to slightly decrease consumption, while it will increase in the long run.

Question 3 – Asset Market (15 Marks)

Consider the following equation, which describes an economy's real money demand:

$$\frac{M^d}{P} = 250 + 0.5Y - 1000(r+\pi^e)$$

where M^d is demanded nominal money balances, P is the price level, $i = r + \pi^e$ is the nominal interest rate on non-monetary assets, r is the real interest rate on non-monetary assets, and π^e is the expected inflation rate. Note that money has a nominal rate of return of zero in this economy, that is $i^m = 0$. Furthermore, we have that $Y = 500$, $r = 0.10$, $\pi^e = 0$, and $P = 1$.

a) [7 MARKS] Assuming that the asset market is in equilibrium at $r = 0.10$, find the nominal money

balances in this economy, i.e. $M^d = M$. Proceed by finding the velocity of money in this economy, and the elasticity of money demand with respect to real output (at the equilibrium values).

Assuming that the asset market is in equilibrium, the nominal money balances in this economy (the money supply) is found by replacing the given values of Y, r, π^e, and P in the money demand equation to find $M^d = M$. This yields:

$$\begin{aligned} M &= 250 + 250 - 100 \\ &= 400 \end{aligned}$$

The velocity of money is given by the following equation:

$$MV = PY$$

which states that the quantity of money times its velocity (the speed at which it circulates, that is how frequently one unit of currency is used to purchase goods and services) must be equal to the price level times real output, or equivalently to nominal GDP. Hence, we can find that $V = PY/M = 1.25$.

Finally, the elasticity of money demand with respect to Y at the equilibrium values is given by:

$$\begin{aligned} \eta_Y &= \frac{\partial M^d}{\partial Y} \cdot \frac{Y}{M^d} \\ &= 0.5 \cdot 1.25 \\ &= 0.625 \end{aligned}$$

b) [4 MARKS] Suppose that the central bank pursues a expansionary monetary policy, causing nominal money balances to increase by 10%. Keeping output and the real interest rate constant (as the latter is determined by the goods market equilibrium), what must happen to expected inflation if the price level stays constant in the short run? Find the new value of expected inflation. What about the reverse, i.e. what must happen to the price level today if expected inflation is to remain constant? Find that new price level.

If M increases by 10%, then it means that $M = 440$. For the asset market to be in equilibrium, $M^d = M$. If the price level does not increase, then *ceteris paribus*, larger nominal money balances create expectations of deflation, which in turn induce people to hold the larger nominal balances in the short run: we find that $\pi^e = -0.04$. If expectations of inflation are to remain unchanged, then the price level must adjust immediately to keep real money balances constant, *ceteris paribus*. That yields $P = 1.1$: for expectations of future inflation to remain at $\pi^e = 0$, there needs to be an immediate increase in the price level by 10%.

c) [4 MARKS] State whether the quantity theory of money holds in this example, and for what rea-

son(s).

No, it does not. Notice that the quantity theory of money is stated thus: that real money demand M^d/P is proportional to output. That is, we would need to be able to write $M^d/P = kY$, which we are not due to the constant term in the linear function and (more crucially) due to real money demand's dependence on the real and nominal interest rates. Another way to notice this is to notice that velocity V is not constant:

$$\begin{aligned} V &= \frac{PY}{M} \\ &= \frac{PY}{P\left(250 + 0.5Y - 1000\left(r + \pi^e\right)\right)} \\ &= \frac{Y}{250 + 0.5Y - 1000\left(r + \pi^e\right)} \end{aligned}$$

which is obviously a function of Y, r, and π^e, and thus not a constant.

Question 4 – Business Cycles, and the AS-AD model (25 marks)

Read Paul Krugman's article "The Hangover Theory" (*Slate*, Dec. 4, 1998), which can be found on the course webpage. Based on the article and on the course material, answer the following questions.

a) [10 MARKS] Summarize concisely (in a few lines), what Krugman means by "the hangover theory" of recessions, and what are its implications in terms of policy. Is it closer to either of the theories of business cycles discussed in class? If so, which one?

The "hangover theory" of recessions asserts that recessions are the inevitable payback for good times. The tale is that good economic times are characterized by "over-investment" in certain sectors, which ends up triggering a slump equal in proportion to the boom that preceded it. This slump happens when investments are proven to be unsound, and investors lose out, thus causing investment to collapse, and in turn production, employment, and wages. It turns out that by removing the excess capacity off certain sectors of the economy and reallocating it to sectors in need of expansion, recessions thus act as part of the "necessary healing process". The implied policy is therefore one of *laissez-faire*, as any attempt to stimulate aggregate demand (e.g. spending on goods and services, and investment) through monetary or fiscal policy must only create "sham prosperity".

In all but name, the "hangover theory" of recessions stems from the "Austrian theory" of the business cycle, which is closer to the Classical theory of business cycles presented in class. It shares with the Classical theory of business cycles the belief that recessions are natural and transient fluctuations of output around its potential value, and that their brevity calls for no policy intervention: prices and wages being

flexible, they soon decrease after aggregate demand slumps, thus causing full-employment output to be restored.

b) [5 MARKS] What is the link between fluctuations in investment and fluctuations in output outlined in the article? Use proper terminology. Basing yourself on the article and on class material, would you say that it is a leading or a lagging variable, or neither? Explain.

Krugman writes: "Since construction projects take time to complete, however, the boom can proceed for a while before its unsoundness becomes apparent. Eventually, however, reality strikes—investors go bust and investment spending collapses." This would indicate that fixed investment (as opposed to inventory investment, as suggested by the mention "of factories that cannot find markets, of office buildings that cannot find tenants") slumps as output falls, while it was booming as output (and thus income) was previously increasing. Fixed investment therefore appears to be a pro-cyclical variable. It also seems that since Krugman refers to investment declines to "cause the whole economy to slump", it would indicate that it is either a leading or a coincidental variable. The latter interpretation is reinforced by class materials.

c) [10 MARKS] Find Krugman's explanation for why recessions occur. Based on this information and the AS-AD framework presented briefly at the end of chapter 8 (and later in chapter 9), show what Krugman describes with the support of a diagram, and what are its effects. Be sure to clarify whether Krugman describes a shock to aggregate demand or to aggregate supply. What policy response (if any) does Krugman advocate to bring the economy back to full employment? Illustrate its effects on the same diagram.

Krugman writes: "A recession happens when, for whatever reason, a large part of the private sector tries to increase its cash reserves at the same time." This would suggest that the aggregate demand for goods and services falls, which is consistent with a leftward shift of the AD curve, in the AS-AD diagram in chapter 8. This causes output to fall below its potential (full-employment) value (which is consistent with Krugman's claim that productive capacity is left "idle"), and thus calls for a policy intervention designed to make people spend. Specifically, Krugman talks about following an expansionary monetary policy ("For if the problem is that collectively people want to hold more money than there is in circulation, why not simply increase the supply of money?") or an expansionary fiscal policy (see his example of Japan's contractionary fiscal policy as something *not* to implement). This follows the Keynesian theory of intervening to restore full employment in the event of a recession, and is perfectly in line with the chapter's graphical example.