ECON 222 – Answers for Assignment 2

Question 1: (Consumption and saving)

First part:

For the diagram, the two extreme points are $c_{(cf = 0)} = 220.238$ and $c^f{}_{(c = 0)} = 231.250$

The inter-temporal budget constraint is:

(1) $c + c^f/(1 + r) = y + y^f/(1 + r)$

From the point of tangency we have $1.22(c^f/c) = 1.05$, which implies that:

(2) $c^f = (1.05/1.22)c$.

Using the inter-temporal budget constraint (1) as well as (2) we have:

$c + c/1.22 = 220.238$ and $c = 121.032$ and $c^f = 104.166$

Using the values obtained for c and c^f as input into (1) we have:

$121.032 + 104.21/(1.05) = 220.238$

The point to note is that in deciding how much to consume in period 1, the agent looks at the present value of lifetime resources or permanent income.

Second part:

Defining the rebate as "b", which is the face value of the bond, and future taxes as t^f, the new inter-temporal budget constraint is:

(1) $c + c^f/(1 + r) = (y + b) + (y^f - t^f)/(1 + r)$

The government will have to raise taxes in the future in an amount sufficient to repay the bond taken out in the present period. As they will owe $(1 + r)b$, the value of extra future taxes is $t^f = b(1 + r)$. Rational consumers know this and realize that their lifetime resources are unchanged.

Students should talk about conditions under which pure RE doesn't hold.

Third part:

The new extreme points are now:

$c_{(cf = 0)} = 215.909$ and $c^f{}_{(c = 0)} = 237.500$

To show where the curves cross, start with the following:

(1) $c^f = (y - c)(1 + r) + y^f$

The new relationship with the higher interest rate r' is:

(2) $c^f = (y - c)(1 + r') + y^f$

Since both y and y^f are unchanged and only the interest rate is, then the point where they cross is:

(3) $(y - c)(1 + r) = (y - c)(1 + r')$

This can only hold if c = y. Given that then (1) implies that $c^f = y^f$.

If students provide an intuitive proof that is fine.

To get the two new consumption bundles, proceed as before to get:

c = 118.653 and c^f = 106.982

These two values satisfy the inter-temporal budget constraint, since 118.653 + 106.983/(1.10) = 215.10 (which is close enough).

Saving in the case where r = 5% was y – c = 125 – 121.032 = 3.968, while now they are 6.347, which implies that the substitution is dominating.

Fourth part:

When demonstrating graphically the income and substitution effects, the students should first locate their indifferences curves above the no-saving/borrowing point. If the indifference curves were located below this point, then the substitution effect would unambiguously dominate, as the household would be a debtor.

Question 2: (Investment and goods market equilibrium)

First part:

The production that they will derive will look like $Y = AK^{ß}N^{(1-ß)}$, which with the values for A, N and ß would be $Y = 156.55K^{ß}$. The $MPK^f = 46.97K^{-0.3}$. This later relationships yields the numbers shown in the table. Depending on how the students round up the values for MPKf, their calculated implied interest rates could be different. What is important is that they are consistent.

Value of MPK, r for given K		
Capital stock	MPK^f	Implied interest rate
900	0.40	10.20
950	0.38	8.94
1000	0.37	8.31
1050	0.36	7.68
1100	0.35	7.05
1150	0.34	6.42
1200	0.33	5.79
1250	0.32	5.16
1300	031	4.53
1350	0.30	3.90
1400	0.295	3.59

The pattern the students should notice is the positive relation between the MPK^f and r and since MPK^f is a demand for capital equation (as MPKf declines, the demand for K is in effect increasing), lower interest rates are needed to increase the demand for capital.

Second part:

The students should the equation $MPK^f = 46.97K^{*-0.7}$. By subbing in K* =1100, they will get MPKf = 0.35. Based on the tax adjusted user cost, they can solve for r, as (1 – tax)(0.35) – 0.15 (depreciation), which is 7.05%. The students can get this number from the above table as well.

To get the effect of a 10% productivity improvement, the student must first multiply A by 1.1 (24.2), calculate the new production function $Y = 172.21K^{*0.3}$ and then the new $MPK^f = 51.66K^{*-0.7}$, which with K* = 1100, yields the new $MPK^f = 0.384$. The new interest rate that clears the market is the 0.384*.63 – 0.15 = 9.19%.

For the effective tax rate change (37 to 35%), go back to the original marginal product of capital with K* = 1100, which is 0.35. Then the effect of the lower tax is 0.35(1 – 0.35) – 0.15 = 7.75.

Question 3: (Investment and goods market equilibrium, continued)

First part:

The students should use the MPK^f equation to back out the desired amount of K*. In particular, the equation $MPK^f = 46.97K^{*-0.7}$ can be written as:

$K = (MPK^f/46.97)\exp(1/(-0.7))$

Since MPK^f = uc/(1 – tax) = (.05 + .15)/(1 – 0.37) = 0.317, we have

K* = (0.317/46.97)exp(-1.4285…) = 1270.907

The gross investment relationship is:

$I_t = K_t^* - K_t + dK_t$, = 1270.907 -1100 + 0.15(1100) = 335.907 (the desired amount of gross investment).

Second part:

With a saving function described as S = 256 + 150r, the amount of desired saving at a 5% interest rate is 263.5, which is less than the desired amount of investment (335.907, from the first part). What these facts tell us is that the market is in disequilibrium, with excess demand. A rise in interest rates to 6%, would increase saving to 265, and, through the user cost, lower K* to 1200 (see table above). Based on the desired amount of investment equation, the new investment rate would be 265. This would clear the market, with a new K* of 1200 (greater than the initial K* of 1100).

Third part:

The students should use the new K* and the original production function to derive a new level of $Y = 156.55K^{*0.3} = (156.55)(1200^{0.3}) = 1313.43$. Then from the national accounts identity, we get 1313.43 = C + 265 (I) + 200 (G). This implies consumption of 848.43 and private saving of Y – C – G = 265.

Question 4: (Saving in an open economy)

First part:

First, use the saving function (S = 256 + 150r) to determine the level of saving at 8%, which is 268. Next, use first the equation $MPK^f = 46.97K^{*-0.7}$ to solve for the new capital stock, given that the user cost with 8% interest rates is now higher at 0.365 . The result is K8 = 1031.83. This gives investment of $I_t = K_t^* - K_t + dK_t = 1031.83 – 1200 + 180 = 11.83$. In this case, there is an excess of saving (268 – 11.83 = 256.17). and a current account surplus.

Proceeding as above, but with interest rates now at 5%, S = 7.5 and I = 239.47. Now there is a current account deficit.

Their figure should illustrate this by showing a fixed world interest rate line.

Second part:

With the 10% improvement in productivity, the production function now becomes

$Y = 172.21K^{*0.3}$ implying $MPK^f = 51.663K^{*-0.7}$. With interest rates at 5%, the user cost is (.05 + .15)/(1 - .37) = 0.317. Setting this equal to MPK^f we can solve for K* as 1443.02. This implies that I = 1443.02 – 1200 + 180 = 423.02. Graphically, this is a rightward shift in the investment curve and a worsening current account position.

Third part:

Students should use Figure 5.11 in Chapter and discuss the various factors that might cause the current account to worsen or not. The key is to focus on consumption and ask whether or not it will change with the tax cut. If it does, saving shifts leftward and the current account tends fall. If consumers are Richardian, then consumers save the amount of their cut to pay future taxes. They should as well discuss some of the empirical evidence on this relationship.