ANSWER SHEET

Question A.1

Consumption:

In the inter-temporal model, consumption is based on the present value of lifetime resources (PVLR). A change in current income will have a smaller effect on PVLR than a change in permanent income and accordingly the former has a smaller effect on current consumption. The model assumes that individuals can borrow at the going rate of interest. If there is a borrowing constraint and the individual needs to borrow to smooth income then more will be consumed out of current income than the theory suggests. If the individual does not need to borrow then the constraint does not matter (it is non-binding). This is sometimes pointed out as a reason why current consumption appears to be excessively volatile compared with the predictions of the theory.

Investment:

Given that $MPK^f = 9.7 - K^d$ must equal the user cost of capital then:

$$9.7 - K^d = uc = [(r+d)/(1-\tau)]$$

Then

$$K^d = 9.7 - [(r+d)/(1-\tau)]P_k$$

Since investment is given by:

$$I = K^d - K + dK = K^d - (1 - d)K,$$

Then using the depreciation rate of 0.10 and an initial capital stock of 7 we get:

$$I = 3.4 - [(r+d)/(1-\tau)]P_k$$

Using $r = 0.07$; $d = 0.10$; $\tau = 0.32$; and $P_k = 1.2$ we get $I = 3.1$

Question A.2

Starting from a position of a US surplus and an Asian deficit, an increase in the US fiscal deficit will shift the US saving curve up and to the left (a decrease in national saving), resulting in an increase in the world interest rate and a deterioration of the US external balance. Some of this effect could be muted to the extent that some form of Ricardian Equivalence holds. With a smaller amount of US saving going to the rest of the world (Asia), the world interest rate will have to rise. In response, Asia will now save more and invest less (but their S and I curves do not move) and their current account balance will improve.

Starting from the same position, now assume that Asia increases it's saving. Their S function shifts to the right causing an excess of world saving, forcing down world interest rates. Asia now has a surplus and the US a deficit on current account but crucially the world real interest rate is lower.

Question A.3

1) Economies can grow, and fairly rapidly, by increasing inputs of both capital and labour. However, because of diminishing marginal product, it is difficult to sustain increases in living standards. Indeed, once economies converge to a steady state output per capita will stop growing. As well, there are limits as to how much population growth can slow or saving can rise. Once in a steady state, assuming saving is high enough to achieve maximum consumption, only improving TFP has the possibility of raising living standards.

2) Poorer economies will converge to richer ones, provided they differ only in terms of their initial capital-labour ratios. In other words, they must have similar population growth rates, saving rates and access to the same technologies (similar production functions) to experience unconditional convergence. Note that there is evidence that countries (OECD ones) with similar characteristics did experience unconditional convergence. Openness to trade is a factor supporting unconditional convergence.

3) If countries differ in these key characteristics then they will tend to converge to those with similar characteristics – those with low saving, poorer technologies, etc. Some empirical evidence should be cited.

Question A.4

1) Use the equilibrium relationship between money demand and supply and non-monetary asset demand and supply; that is $(M^d - M) + (NM^d - NM) = 0$. Note first that when Md = M, then the market for non-monetary assets is also in equilibrium. An excess supply of money will cause asset holders to start buying NM assets thereby bidding up their prices and lower rates of return on those assets. An excess demand for money works in the opposite direction.

2) Velocity is nominal GDP divided by the nominal money supply. It tells us by the number of times the money stock turns over in the economy during a particular period (say one year). If it rises, each unit of the currency is being used more intensively; i.e., a given stock of money is supporting more transactions. In general, if money responds to changes in interest rates or if the demand for money is less than unit elastic (if they say that the coefficient on money is less than one, that would be fine), then velocity would not be a constant. Broader definitions of money tend to be less sensitive to interest rates (because they include more interest bearing assets, which negate the non-interest bearing effects).

3) The meaning of “money being neutral” is that increases in it cannot have an effect on any real variables in the economy (output, real interest rates, etc.) in long-run equilibrium. The velocity of money can change and you still get

neutrality. This is shown in our models by a money demand function written as $M^d = P\ F(Y,r)$.

Question B.1, Part (a)

The production function is:

$$Y = AK^{\alpha}N^{(1-\alpha)}$$

Dividing both sides by N, we get:

$$Y/N = AK^{\alpha}(N^{(1-\alpha)})N^{-1} = Ak^{\alpha}$$

In the steady state y and k are constants. Differentiating the definition of $k=(K/N)$, we get:

$$\Delta(K/N) = (\Delta KN - \Delta NK)/N^2 = \Delta K/N - (K/N)(\Delta N/N) = 0$$

Multiply the final expression by N and setting $\Delta N/N = n$, we get:

$\Delta K = nK$. Using this in the definition of investment, I, we get:

$I = \Delta K + dK = (n + d)K$ or in per worker terms:

$I/N = (n + d)k$

Investment per worker has to grow by enough to cover depreciation as well as to supply K to the labour force, which is growing at a rate n.

Question B.1, Part (b.1)

Start with the relationship for consumption per worker (c):

$c = y - I/N = Ak^{\alpha} - (n + d)k$, taking the derivative wrt to k yields:

$$\Delta c/\Delta k = \alpha Ak^{(\alpha-1)} - (n + d) = 0$$

The level of capital be worker that maximises c (k_G) is:

$$k_G = [(n + d)/\alpha A]^{1/(\alpha-1)} = [\alpha A/(n + d)]^{1/(1-\alpha)}$$

Given $\alpha = 0.25$; $A = 8$; $n = 0.03$; and $d = 0.22$ then:

$$k_G = 16$$

Given k_G, the saving rate that would maximise consumption is:

$$sAk_G^{\alpha} = (n + d)k_G \text{ or,}$$

$$s = [(n + d)/A]\ k_G^{(1-\alpha)} = [(0.03 + 0.22)/8]16^{0.75} = 0.25$$

The maximum level of consumption would be:

$$c = Ak_G^{\alpha} - (n + d)k_G = 12$$

The diagram should show that in the steady state, with k at k_G, consumption is maximised. As well, the saving function should be cutting the investment requirement line from above.

Question B.1, Part (b.2)

If the price of capital is unity and there are no taxes, then the user cost of capital (uc) is:

$$uc = d + r$$

From the maximum consumption relationship above, we have that the marginal product of capital at k_G is:

$$\alpha Ak^{(\alpha - 1)} = n + d$$

Since the user cost in equilibrium must equal the marginal product of capital, the

$$d + r = n + d \text{ or}$$

$$r = n$$

Question B.1, Part (b.3)

To find the steady-state level of output, first use the saving = investment relationship to get the steady-state level of k:

$$sAk^{\alpha} = (n + d)k \text{ or}$$

$$k = [sA/(n + d)]^{1/(1 - \alpha)} = [(0.20)8/(0.03 + 0.22)]^{1/0.75}$$

$$k = 11.8825$$

Next use this level of the capital in the production function to get y, thus:

$$y = Ak^{\alpha} = 8(11.825^{0.25}) = 14.853$$

With a constant saving rate of 0.20, consumption is:

$$c = (1 - s)Ak^{\alpha} = 11.88$$

To get to a position of maximum consumption, s would have to rise by 5 percentage points.

The usual diagram applies in B.1 part (B.1) but it is important to note that we get to the steady state because of diminishing marginal product of capital (or capital per worker) and that the equilibrium is to the left of the k_G point.

Question B.2, Part (a)

$C^d = 300 + 0.7(Y – T) – 75r$

$I^d = 185 – 75r$

$M^d/P = 0.6Y – 100(r + \pi^e)$

From the above, start by defining desired saving and remembering T = 150 and G = 100, then:

$S^d = Y – C^d – G = Y – 300 – 0.7(Y – T(=150)) + 75r – G(= 100)$

$S^d = – 295 + 0.3Y + 75r$

Setting S^d = Id, we get: $– 295 + 0.3Y + 75r = 185 – 5r$, or

$r = 3.2 – 0.002Y$ (IS)

Solving the money demand equation in terms of r, yields:

$100r = 0.6Y – (M/P) – 100\pi e$

$r = 0.006Y – 0.001(M/P) – \pi^e$ (LM)

Setting the IS and LM curves equal yields:

$Y = 400 + 1.25(M/P) + 125\pi^e$ (AD)

The AD curve slopes downward in terms of Y and P, because a fall in the price level shifts the LM curve down and to the right, lowering interest rates and increasing output, holding other variables constant.

Question B.2, Part (b)

If r = 0.05, then use the IS curve to solve for Y as:

$0.05 = 3.2 – 0.002Y$

$Y = 1575$

Given the interest rate and output, and assuming $\pi^e = 0$, we can use the LM curve to get the level of M:

$0.05 = 0.006Y – 0.001M$ (remembering P=1)

$M = 940$

From the consumption and investment functions we have:

$C^d = 300 + 0.7(Y – T) – 75r = 300 + 0.7(1575 – 100) – 0.002(0.05)$

$\mathbf{C^d = 1293.75}$

$I^d = 185 – 75r = 185 – 75(0.05)$

$\mathbf{I^d = 181.25}$

Question B.2: Part (c)

In the short, the rise in π^e, with the money supply constant, shifts the LM curve to the right and causes the real interest rate to fall. With the IS curve unaffected, the new, short-run equilibrium is found where these two curves intersect. Thus we can use the AD curve to get the new level of output:

$\mathbf{Y = 400 + 1.25(M/P) + 125\pi^e = 400 + 1.25(940) +125(0.02)}$

$\mathbf{Y = 1577.5}$

Now that Y is known, we can use the IS relation to get the short-run level of r:

$\mathbf{r = 3.2 – 0.002Y = 3.2 – 0.002(1577.5)}$

$\mathbf{r = 0.045}$

Since the nominal money supply unchanged, and given that π^e is transitory, the old equilibrium is restored. The price level will not be affected in long-run equilibrium.

Show the usual graph, LM shifting down and then back.

Question B.2: Part (d)

Lowering taxes by 10 will affect the IS curve. Starting from the relationship for private saving, we have:

$S^d = Y – C^d – G = Y – 300 – 0.7(Y – T(=\mathbf{140})) + 75r – G(= 100)$

S^d = – 302 + 0.3Y + 75r which, given the investment equation above (Id = 185 – 75r), yields the following IS curve:

$\mathbf{r = 3.24666 – 0.002Y}$

Given an unchanged LM curve, the new AD relationship is:

$\mathbf{Y = 405.8333 + 1.25(M/P) = 405.8333 = 1.25(940)}$

$\mathbf{Y = 1580.8333}$

Given this value of Y and the new IS curve, we get:

$r = 3.24666 – 0.002Y = 3.24666 – 0.002(1580.8333)$

$r = 0.085$

These levels of Y(=1580.83) and r(=0.085) are the new short-run equilibrium levels. Show the usual graph with only the IS curve shifted.

In the long run, the economy returns back to its equilibrium level because the LM curve will shift (from short-run equilibrium, $Y > Y^{FE}$, so the price level will start to rise). With taxes now lower, the government sector is making a larger claim on resources and this has affected the IS curve. We can use it to solve for the new interest rate:

$r = 3.24666 – 0.002Y = 3.24666 – 0.002(1575)$

$r = 0.0966$

Using the new interest rate in the consumption function we get:

$C^d = 300 + 0.7(Y – T) – 75r = 300 + 0.7(1575 – 140) – 75(0.09666)$

$C^d = 1297.25$

Using the new interest rate in the investment function we get:

$I^d = 185 – 75r = 185 – 75(0.09666)$

$I^d = 177.75$

Both consumption and investment have fallen compared with the original levels (1293.75 and 181.25, respectively). Regarding investment, there has been some crowding out (a fall of 3.5) but it is not complete as consumption made of the rest of the difference.

Show IS curve shifting up and then LM shifting back as price level rises to a new equilibrium with a higher interest rate.