ECON 222
Macroeconomic Theory I
Winter Term 2007/08

Midterm Exam - ANSWER KEY

PART A: Long Questions.

Question A.1: Equilibrium in the Labor Market

Part 1) West Germany.
a) Imposing the equilibrium condition in the Labor Market gets:

$$\begin{aligned} N_W^D(\omega) &= N_W^S(\omega) \\ \left(\frac{1}{\omega}\right)^2 &= \omega^2 \\ \omega^4 &= 1 \\ \omega &= 1 \end{aligned}$$

The wage rate that clears the market is $\omega_W^* = 1$. Plugging this value in either the labor demand or the labor supply gives the number of people employed in the economy, which is $N_W^* = 1$ (million):

$$\begin{aligned} N_W^* &= N_W^D(1) = \left(\frac{1}{1}\right)^2 = 1 \\ N_W^* &= N_W^S(1) = (1)^2 = 1 \end{aligned}$$

b) Since all firms are identical, you get the aggregate labor demand $N_W^D(\omega)$ simply by multiplying the firm level demand $\overline{N}_W^D(\omega)$ times the number of firms in the economy:

$$N_W^D(\omega) = \overline{N}_W^D(\omega) \times \frac{1}{2} = 2\left(\frac{1}{\omega}\right)^2 \times \frac{1}{2} = \left(\frac{1}{\omega}\right)^2$$

This shows how the aggregate labor demand used in point a) is derived from the aggregation of all the firms demanding labor in the economy.

c) Similarly to point b), you get the aggregate labor supply $N_W^S(\omega)$ simply by multiplying the individual supply $\overline{N}_W^S(\omega)$ times the number of people supplying labor in the economy:

$$N_W^S(\omega) = \overline{N}_W^S(\omega) \times 1 = \omega^2 \times 1 = \omega^2$$

Again, this shows how the aggregate labor supply used in point a) is derived from the aggregation of all the individuals supplying labor in the economy.

d) It goes without saying the $\omega_W^* = 1$ together with $N_W^* = N_W^D = N_W^S = 1$ represent the equilibrium we are looking for. The supply and demand functions did not change, hence the equilibrium is unaltered. In West Germany there are 1 million of people employed by $\frac{1}{2}$ firms, that is on average two people per firm.

Part 2) East Germany.

e) Since all firms are identical, you get the aggregate labor demand $N_E^D(\omega)$ simply by multiplying the firm level demand $\overline{N}_E^D(\omega)$ times the number of firms in the economy:

$$N_E^D(\omega) = \overline{N}_E^D(\omega) \times \frac{1}{8} = \frac{1}{2}\left(\frac{1}{\omega}\right)^2 \times \frac{1}{8} = \frac{1}{16}\left(\frac{1}{\omega}\right)^2$$

Notice that, for any wage rate ω, in the West firms demand 4 times as much labor at the firm level, and 16 times as much at the aggregate level. This is due to a "compositional" effect: in the East there are both less firms and each firm has a lower labor demand.

f) By the same arguments above:

$$N_E^S(\omega) = \overline{N}_E^S(\omega) \times 1 = \omega^2 \times 1 = \omega^2$$

This point wants to highlight that people are virtually identical in the two economies, while the conditions of production are not. The latter is what drives the different equilibria in the labor market.

g) Imposing the equilibrium condition in the Labor Market gets:

$$\begin{aligned} N_E^D(\omega) &= N_E^S(\omega) \\ \frac{1}{16}\left(\frac{1}{\omega}\right)^2 &= \omega^2 \\ \omega^4 &= \frac{1}{16} \\ \omega &= \frac{1}{2} \end{aligned}$$

The wage rate that clears the market is $\omega_E^* = \frac{1}{2}$. Plugging this value in either the labor demand or the labor supply gives the number of people employed in the economy, which is $N_E^* = \frac{1}{4}$ (million, i.e. 250,000 workers):

$$\begin{aligned} N_E^* &= N_E^D(1) = \frac{1}{16}\left(\frac{1}{1/2}\right)^2 = \frac{1}{4} \\ N_E^* &= N_E^S(1) = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \end{aligned}$$

Overall, if compared to the West, in the East there are less people employed and a lower wage rate ($N_E^* < N_W^*$ and $\omega_E^* < \omega_W^*$). The equal size of the population notwithstanding, the lower wage in the East is not enough to allow many people to have a job. On average there are two people employed per firm.

Part 3) Germany Reunified.

h) First, we need to find the aggregate labor demand $N_G^D(\omega)$ and the aggregate labor supply $N_G^S(\omega)$. In order to do so, it is enough to add up the demands and supplies of the previously separated economies. This is because firms and individuals are now free to employ and be employed anywhere in Germany.

The aggregate labor demand $N_G^D(\omega)$ is:

$$N_G^D(\omega) = N_W^D(\omega) + N_E^D(\omega) = \left(\frac{1}{\omega}\right)^2 + \frac{1}{16}\left(\frac{1}{\omega}\right)^2 = \frac{17}{16}\left(\frac{1}{\omega}\right)^2$$

The aggregate labor supply $N_G^S(\omega)$ is:

$$N_G^S(\omega) = N_W^S(\omega) + N_E^S(\omega) = \omega^2 + \omega^2 = 2\omega^2$$

The equilibrium level of employment and the wage rate that clears the labor market in the reunified Germany are obtained by imposing the equilibrium condition:

$$\begin{aligned} N_G^D(\omega) &= N_G^S(\omega) \\ \frac{17}{16}\left(\frac{1}{\omega}\right)^2 &= 2\omega^2 \\ \omega^4 &= \frac{17}{32} \\ \omega &= 0.85 \end{aligned}$$

The wage rate that clears the market is $\omega_G^* = 0.85$. Plugging this value in either the labor demand or the labor supply gives the number of people employed in Germany, which is $N_G^* = 1.46$ (millions):

$$\begin{aligned} N_G^* &= N_G^D(0.85) = \frac{17}{16}\left(\frac{1}{0.85}\right)^2 = 1.46 \\ N_G^* &= N_G^S(0.85) = 2(0.85)^2 = 1.46 \end{aligned}$$

i) The unemployment rate in the reunified Germany is zero, as it was in the West and in the East. The wage rate is flexible: after the fall of the Berlin Wall it drops in order to allow for the labor market to clear. At the equilibrium wage rate ω_G^* all the Germans that are willing to work at that wage can do so. The people that do not work, have too high a reservation wage, which leads them not to take a job.

Notice that the wage rate ω_G^* is in between the two wages found in parts 1 and 2, that is $\omega_E^* < \omega_G^* < \omega_W^*$. In terms of the labor market outcomes (assuming that the actual German economy can be represented by this simple framework), reunification could seem a good "deal" for the Easterners and not such a good one for the Westerners. But this statement is missing one important consideration: total employment goes up, and so does total production. Without additional information on the production function and on the preferences, we cannot say if total welfare increased of decreased.

What we can say is that the employment level in the reunified Germany goes up if compared to the sums of the two employment levels in the separated economies: $N_G^* = 1.46 > N_W^* + N_E^* = 1 + 0.25 = 1.25$. A larger market increases the competition among workers (employers). This drives down (up) the wage, leading to a different outcomes in both the equilibrium wage rate and the equilibrium employment level.

Part 4) The response of Germany to shocks.

j) The supply side of the economy is unaltered, while the labor demand drops by 50%:

$$\begin{aligned} N_G^D(\omega) &= \frac{1}{2}\left(\frac{1}{\omega}\right)^2 + \frac{1}{2}\frac{1}{16}\left(\frac{1}{\omega}\right)^2 = N_G^S(\omega) \\ \frac{1}{2}\frac{17}{16}\left(\frac{1}{\omega}\right)^2 &= 2\omega^2 \\ \omega^4 &= \frac{17}{64} \\ \omega &= 0.717 \end{aligned}$$

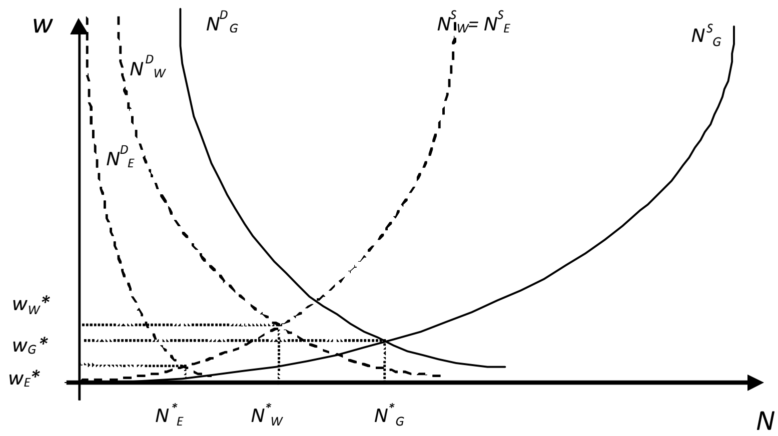


Figure 1: Equilibrium in the East, West and in the Reunified Germany

The new wage rate that clears the market is $\omega_G^{*\prime} = 0.717$. Plugging this value in either the labor demand or the labor supply gives the number of people employed in Germany, which is $N_G^{*\prime} = 1.03$ (millions). Both the wage rate and the number of employed people decrease after the shock hits the economy.

k) Had the East and the West stayed separated their equilibria would have been $\omega_W^{*\prime} = 0.84$, $N_W^{*\prime} = 0.7$, and $\omega_E^{*\prime} = 0.42$, $N_E^{*\prime} = 0.18$, respectively. This shows that a larger labor market allows for a less drastic response to negative shocks.

Note: The curves in the graph should all be concave rather than convex.

Question 2: Accounting for GDP

a) Product Approach:
Bombardier: $100 + 80 + 25 = 205$
(value of airplanes sold + inventory as if purchased by Bombardier itself)
Air Canada: $240 - 100 = 140$ (value of flight tickets - value of airplanes)
Federal Government: 100 (value of pyramids)
Total value-added: $205 + 140 + 100 = 445$

Expenditure Approach:
Bombardier: 80 (public consumption of airplanes) and 25 (inventory investment)
Air Canada: 240 (consumption of flight tickets)
Federal Government: 100
(government purchases of pyramids / purchases of services from construction workers)
Total expenditure: $C + I + G = (80 + 240) + 25 + 100 = 445$

Income Approach:
Wages: $120 + 80 + 100 = 300$ (wages paid by Bombardier, Air Canada, and the Government)
Corporate profits:
Bombardier: $100 + 80 + 25 - 120 - 35 = 50$
Air Canada: $240 - 80 - 50 - 100 - 15 = -5$
Taxes: $35 + 50 = 85$
Depreciation: 15
Total Income: $300 + 50 - 5 + 85 + 15 = 445$

b) Since the inventory stock was accumulated from 2006, the inventory investment is not made in 2007 and should not count towards 2007's GDP. Therefore using the product approach, Bombardier's value-added becomes $100 + 80 = 180$, while Air Canada and the Government's value-added remain the same. The total value-added becomes $180 + 140 + 100 = 420$.

Using the expenditure approach, everything else is the same except that Bombardier did not make any inventory investment during 2007, so the total expenditure becomes $80 + 240 + 100 = 420$.

Using the income approach, the corporate profit of Bombardier becomes $100+80-120-35 = 25$ without counting the Bombardier's inventory that was 'sold' in 2006 and not in 2007. The total income becomes $300 + 25 - 5 + 85 + 15 = 420$.

c) Using the expenditure approach: $GDP = Y = C + I + G + NX$. Now consumption C becomes $80 + 120 = 200$, and NX becomes 120. Total expenditure becomes $200 + 25 + 100 + 120 = 445$ which is the same as in part (a). The difference is that consumption C drops from 320 to 200, and NX increases from 0 to 120. As for GNP, $GNP = GDP + NFP$. The sales of flight tickets to the US is a net export (NX) transaction, not a NFP transaction (which involves Canadian workers remittance of wages from abroad or any foreign investment income). Since $NFP = 0$, $GNP = GDP$ and there are no changes for both GNP and GDP when Canada exports flight tickets to the US.

d) Uses-of-savings identity: Since national savings $S = I + CA$ and $S = S_{private} + S_{govt}$, where $S_{private}$ is Canada's private saving and S_{govt} is Canadian government saving, we can get $S_{private} = I + (-S_{govt}) + CA$, where $-S_{govt}$ is government budget deficit. From part (a) we know that $I = 25$, and from part (c) we know that $CA = NX + NFP = 120 + 0 = 120$. Also, government budget is equal to $S_{govt} = T - G = 85 - 100 = -15$, which is a deficit of \$15. So we know that private saving $S_{private}$ is used for investment of \$25, government budget deficit of \$15, and to finance net exports of \$120, which sum up to \$160.

Question 3: Firm's desired capital stocks and capital stock allocations

a) Since there is only one type of capital, the user costs of capital are the same for both firms.

$$\begin{aligned} uc &= (r+d)\times p_k \\ &= (0.05+0.1)100 \\ &= 15 \end{aligned}$$

b) First find the MPK for both firms by taking the derivative with respect to K for each firm's production function:

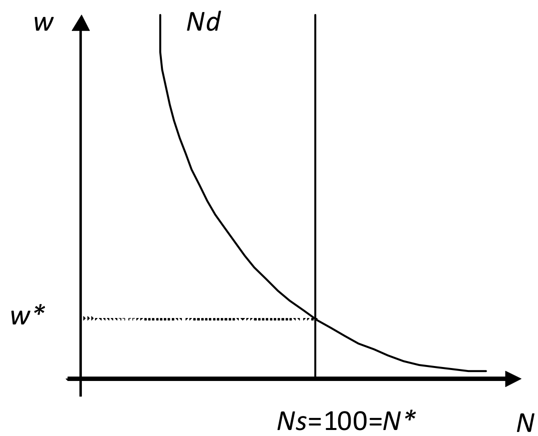
$$\begin{aligned} MPK_1 &= 0.3A_1K_1^{-0.7}N_1^{0.7} \\ MPK_2 &= 0.3A_2K_2^{-0.7} \end{aligned}$$

In this economy, labor supply is rigid (it is fixed), that is $N^S = 100$ irrespective of the going wage. Since the marginal product of labor is always positive, the first firm is going to employ all the workers in the economy.

$$MPN_1 = 0.7A_1K_1^{0.3}N_1^{-0.3} = 0.7A_1\left(\frac{K_1}{N_1}\right)^{0.3} > 0, \text{ for } 0 \le N_1 \le 100$$

For the second firm it is optimal not to employ any labor, or $N_2 = 0$. For firm 2 labor does not have any effect on production: only capital and the TFP can affect the amount of goods produced. Emplyoing any labor would not be optimal for this firm, because the wage would be paid, increasing its costs, without any effect on production.

Since firm 2 does not require any labor for production, the entire $N = 100$ work for firm 1.



Allocation of Labor

Using the information that $A_1 = 15$ and $A_2 = 150$, firms choose their desired capital stock by equating their MPK to the user cost of capital:

$$\begin{aligned} MPK_1 &= uc \\ 0.3(15)K_1^{-0.7}(100)^{0.7} &= 15 \\ K_1^* &= 17.9073 \end{aligned}$$

$$\begin{aligned} MPK_2 &= uc \\ 0.3(150)K_1^{-0.7} &= 15 \\ K_2^* &= 4.804 \end{aligned}$$

If the capital stock K_t for a firm is known at the beginning of the year, then the gross investment I_t for that firm will be given by:

$$\begin{aligned} K^* - K_t &= I_t - dK_t \\ I_t &= K^* - (1-d)K_t \end{aligned}$$

where subscript t is year 2008 for this case.

If $K^* = K_t$ then $I_t = dK_t$, which is $I^1_{2008} = 0.1 \times 17.90 = 1.79$ for the first firm, and $I^2_{2008} = 0.1 \times 4.80 = 0.48$ for the second firm.

c) For the case where $K = 20$ in the economy, because the sum of K_1^* and K_2^* is equal to $22.71 > 20$, either one or both firms cannot choose to use capital until $MPK = uc$. Capital will be allocated to both firms in a way that the rates of return on capital (MPK) for both firms are equal. Otherwise there will be incentive for owners of capital to move capital to the firm with higher MPK. Due to diminishing marginal product, more capital allocated means lower MPK, and vice-versa. These forces push the two firms' $MPKs$ to equalize. Algebraically:

$$\begin{aligned} MPK_1 &= MPK_2 \\ 0.3(15)K_1^{-0.7}(100)^{0.7} &= 0.3(150)K_2^{-0.7} \\ \frac{K_1}{K_2} &= 3.7276 \\ \frac{20-K_2}{K_2} &= 3.7276 \qquad\qquad (\Leftarrow K_1 + K_2 = 20) \\ K_2^* &= 4.23 \\ K_1^* &= 20 - 4.23 = 15.77 \end{aligned}$$

For the case where $K = 40$, the total of K_1^* and K_2^* from part (b) are smaller than 40. Since firms maximize their profits when their marginal benefits ($MPKs$) are equal to the marginal cost of capital (uc), they will not have incentives to choose more capital than their desired level, even though there is excess capital in the economy. Therefore, firm 1 chooses the desired capital stock of K_1^* and K_2^* for firm 2, both at $MPK = uc$. The remaining part of capital ($40 - K_1^* - K_2^* = 40 - 22.71 = 17.29$) will be idle, that is it will not be used in production.

PART B: Short Questions.

Question B.1:

The real interest rate that clears the good market is 8.33%. This is obtained by imposing the equilibrium in the Goods market, that is the condition that desired saving and investment are equal $I^d(r) = S^d(r)$. We already have an expression for $I^d(r)$, while we need to rely on the definition of desired savings in a closed economy in order to get $S^d(r)$:

$$
\begin{aligned}
S^d &= Y - C^d - G \\
S^d &= Y - (3000 - 2000r + 0.1Y) - G \\
S^d &= 0.9Y - (3000 - 2000r) - G \\
S^d &= 0.9(5000) - 3000 + 2000r - 1000 \\
S^d(r) &= 500 + 2000r
\end{aligned}
$$

Imposing the equilibrium condition gets:

$$
\begin{aligned}
I^d(r) &= S^d(r) \\
1000 - 4000r &= 500 + 2000r \\
r &= \frac{500}{6000} = 0.0833
\end{aligned}
$$

Question B.2:

a) The labor force consists of $LF = WAP - N = 50 - 5 = 45$ million people.

b) You get the number of employed people by exploiting the definition of employment rate, $E = WAP \times \frac{E}{WAP} = 50 \times 0.7 = 35$ million people.

c) The participation rate is the ratio of people in the labor force over the Working Age Population, that is $PR = \frac{E+U}{WAP} = \frac{LF}{WAP} = \frac{45}{50} = 90\%$.

Question B.3:

a) We know that:

$$
\begin{aligned}
Y_{05} &= A_{05} K_{05}^{0.25} N_{05}^{0.75} \\
2000 &= A_{05} 1700^{0.25} 70^{0.75} \\
A_{05} &= \frac{2000}{155.39} = 12.87
\end{aligned}
$$

and that

$$
\begin{aligned}
Y_{06} &= A_{06} K_{06}^{0.25} N_{06}^{0.75} \\
2100 &= A_{06} 1795^{0.25} 75^{0.75} \\
A_{06} &= \frac{2100}{165.89} = 12.66
\end{aligned}
$$

TFP had a negative growth rate, since it decreased by 1.64%:

$$\frac{A_{06}-A_{05}}{A_{05}}=\frac{12.66-12.87}{12.87}=-0.0164$$

b) We are looking for the level of capital K_{07} which solves the following equation:

$$\begin{aligned}
Y_{07} &= A_{07}K_{07}^{0.25}N_{07}^{0.75}=A_{06}K_{07}^{0.25}80^{0.75} \\
2200 &= 12.66K_{07}^{0.25}80^{0.75} \\
K_{07}^{0.25} &= \frac{2200}{338.63}\rightarrow K_{07}=\left[\frac{2200}{338.63}\right]^4=1781.52
\end{aligned}$$

Notice that the we need to decrease the capital stock to get the "target" production.