Answer to Assignment 3:

1a) To get the production function in per-worker terms, divide by N and simplify:
$\frac{Y}{N} = A\frac{K^{0.5}N^{0.5}}{N}$
$y = AK^{0.5}N^{0.5-1}$
$y = AK^{0.5}N^{-0.5}$
$y = A(\frac{K}{N})^{0.5}$
$y = Af(k) = Ak^{0.5}$

1b) From (6.10), $sy = (n+d)k$. And from above: $sy = sAk^{0.5}$. Combining we get $sAk^{0.5} = (n+d)k$. Solving for k gives:
$\frac{sA}{(n+d)}k^{0.5} = k$
$\frac{sA}{(n+d)} = k^{1-0.5}$
$\frac{sA}{(n+d)} = k^{0.5}$
$(\frac{sA}{n+d})^{\frac{1}{0.5}} = k^*.$
$(\frac{sA}{n+d})^2 = k^*.$

1c) Given $A = 1, s = 0.5, n = .10, d = .05$, then $k^* = 11.11$;
$y^* = A(k^*)^{0.5} = 1(11.11)^{0.5} = 3.33.$
$i^* = (n+d)k^* = (0.15)11.11 = 1.67.$
$c^* = y^* - i^* = 3.33 - 1.67 = 1.67.$

1d) Given $A = 1, s = 0.5, n = .01, d = .05$, then $k^* = 69.44$;
$y^* = A(k^*)^{0.5} = 1(69.44)^{0.5} = 8.33.$
$s = 0.5y = 4.17$
Or, $s = i^* = (n+d)k^* = (0.06)69.44 = 4.17.$
$c^* = (1-s)y^* = 4.17.$

1e) A higher savings rate leads to a higher long-run capital-labour ratio (a long-run level effect). This in turn, leads to more long-run output per-worker in the new steady state (also a long-run level effect). Given the assumption we are below the golden rule, consumption per-worker also increases in the new steady state (yet another long-run level effect).

To get to the new steady-state, initially c falls (as s increases for y thus far unchanged). As capital per-worker accumulates, it and output per worker are growing (short-run growth effects).

Once we arrive at the new steady state there is no growth in the per-worker variables (there is no long-run growth effect on any variable).

2a) The reason the Solow model reaches a steady state is because we assume is diminishing marginal returns to (physical) capital accumulation – each incremental investment yields less and less return. Thus the only way to grow in the long run in *per-worker* terms is by ever-increasing productivity (how efficiently outputs are created from inputs). Note: Aggregate Y, C, K all grow at the rate of population growth in the steady-state.

2b) The main departure in endogenous growth theory is relaxing the diminishing returns to capital assumption: $Y = AK$, rather than $Y = AK^{\alpha}$, where $\alpha < 1$. This new view is justified by treating capital as representing physical and human capital or 'ideas'. These author also argue for increasing returns to idea since they are 'non-rival'.

2c) A new result or insight that endogenous growth theory yields is the prediction that the savings rate will positively impact the growth of output per worker. The original Solow result about productivity improving growth still holds. Finally, depreciation has growth effects in the new growth theory, while it has only 'level' effects in the original growth theory.

2d) Copyright and patent, i.e. protect 'intellectual property rights'.

3a)

Country	% Growth GDP/worker 1950-2000	% Population Growth 1950-2000
Canada	127.7307	123.5824
Switzerland	106.4456	53.22438
UK	174.4336	18.38971
India	307.2272	175.5917
Japan	858.6412	52.72118
Trinidad	311.948	102.6394

3b) There is a weak negative relationship. (see graph from the website - beside 'Assignment 3' in the Assignment section)

3c) A short-run negative relationship (see Fig 6.5), so there is some evidence in favour of the model.

3d) There is a positive relationship. (see graph from the website - beside 'Assignment 3' in the Assignment section)

4a)
$\frac{M^d}{4} = \frac{0.02Y}{i}$
$\frac{M^d}{4} = \frac{0.02 \times 200}{0.05}$
$M^d = 320 \longrightarrow$ a nominal money demand
$\frac{M^d}{P} = 80 \longrightarrow$ a real money demand
$M^s = 320 \longrightarrow$ a money supply
V = nominal GDP / nominal money stock
= PY / M
$= (4 \times 200)/320 = 2.5$

4b)
Using the formula $\%\triangle M^d = \eta_Y \%\triangle Y + \eta_i \%\triangle i$:

Year A
$\%\triangle M^d = \eta_Y \%\triangle Y + \eta_i \%\triangle i$
$.02 = \eta_Y .08 + \eta_i .20$ (1a)
$\eta_i .20 = .02 - \eta_i Y .08$
$\eta_i = .10 - \eta_Y .4$ (1b)

Year B
$\%\triangle M^d = \eta_Y \%\triangle Y + \eta_i \%\triangle i$
$-.01 = \eta_Y(-.07) + \eta_i(-.25)$ (2a)
$\eta_i(-.25) = -.01 - \eta_Y(-.07)$
$\eta_i = .04 - \eta_Y(.28)$ (2b)

set (1b) = (2b)
$.10 - \eta_Y .4 = .04 - \eta_Y(.28)$
$.12\eta_Y = .06$
$\eta_Y = .5$

$\eta_i = .10 - (.5).4$
$\eta_i = -.1$

4c) A countercyclical variable moves in the opposite direction of aggregate economic activity. Unemployment and the unemployment rate are examples because they tend to rise as aggregate activity weakens.