ECON 222
Macroeconomic Theory I
Fall Term 2010

Assignment 3 SOLUTION
Due: Drop Box 2nd Floor Dunning Hall by ***noon November 12th 2010***
No late submissions will be accepted
No group submissions will be accepted
No identical copies will be accepted

Remarks: Write clearly and concisely. Devote some time to give the graphs, plots and tables a format easy to understand. Also, the way you present your answers matters for the final grade. Even if a question is mainly analytical, ***briefly*** *explain what you are doing, stressing the economic meaning of the various steps. Being able to convey your thoughts effectively is an asset also in real life.*

Question 1: Savings and Investment in a large open economy (25 Marks)

Imagine a world with two large economies. In the first economy, Derekland, savings, investement and consumption decisions are governed by the following two equations :

$$S_d(r) = 450 + 1000r$$
$$I_d(r) = 550 - 1500r$$

In the other economy, Pabstania, the same decisions are governed by :

$$S_p = 900 + 1000r$$
$$I_p = 950 - 1400r$$

Furthermore, both governments spend an equal amount of $G_d = G_p = 400$ and production is slightly higher in Derekland with $Y_d = 2000$ while $Y_p = 1950$ in Pabstania. Finally, the Ricardian equivalence holds in Derekland while it does not hold at all in Pabstania. Answer the following questions.

1. Assume first that both economies are closed. Find the interest rate and the level of consumption in each of the economies.

2. Assume now that both economies engage in free trade on the savings/investment market. Find the market clearing interest rate. What is the level of net exportations and consumption in the two countries in equilibrium ?

3. Assume the marginal propensity to consume is 0.6 in both countries. An election occurs at the same time in each country, so governments give tax breaks of 100 to consumers, increasing their revenues (while government spending remains the same). How does the reaction of consumers differ in the two countries? Find the new equilibrium interest rate. (*Hint:* the savings function change.)

Solution. 1. In both cases, we must have that savings equals demand :

$$\begin{aligned} 450 + 1000r &= 550 - 1500r_d \\ \Leftrightarrow 2500r_d &= 100 \\ \Leftrightarrow r_d &= \frac{1}{25} = 0.04 \\ 900 + 1000r_p &= 950 - 1400r_p \\ \Leftrightarrow 2400r_p &= 50 \\ \Leftrightarrow r_p &= \frac{1}{48} \approx 0.0208 \end{aligned}$$

Consumption is then given by $C = Y - I - G$:

$$\begin{aligned} C_d &= 2000 - 550 + 1500\frac{1}{25} - 400 \\ \Rightarrow C_d &= 1110 \\ C_p &= 1950 - 950 + 1400\frac{1}{48} - 400 \\ \Rightarrow C_p &\approx 629.2 \end{aligned}$$

2. We must now have $NX_p + NX_d = 0$:

$$\begin{aligned} 0 &= S_p - I_p + S_d - I_d \\ &= 900 + 1000r - 950 + 1400r + 450 + 1000r - 550 + 1500r \\ &= -150 + 4900r \\ \Leftrightarrow 150 &= 4900r \\ \Rightarrow r &\approx 0.031 \end{aligned}$$

The net level of exportations in Derekland is :

$$\begin{aligned} NX_d &= -100 + 2500r \\ &= -100 + 2500\frac{150}{4900} \\ &\approx -23.5 \end{aligned}$$

and trivially, the absolute value of this number represents NX_p.

Finally, consumption is given by $C = Y - I - G - NX$:

$$\begin{aligned} C_d &= 2000 - 550 + 1500\frac{150}{4900} - 400 + 23.5 \\ &\approx 1119.4 \\ C_p &= 1950 - 950 + 1400\frac{150}{4900} - 400 - 23.5 \\ &\approx 619.4 \end{aligned}$$

3. In Derekland, consumers anticipate future taxation to pay for the 100 spent and thus, they do not change their consumption behavior. Hence they save all their tax break, which compensates the loss of government savings. In Pabstania, they do not anticipate changes in future tax rates (or do not care about it) and thus they see this tax break as an increase in their income. They split it according to their marginal propensity to consume, eg, 60 units goes to consumption and 40 units to savings, while the government decreases savings by 100. Hence, the new savings functions are :

$$\begin{aligned} S_d &= 450 + 100 - 100 + 1000r \\ &= 450 + 1000r \\ S_p &= 900 + 40 - 100 + 1000r \\ &= 840 + 1000r \end{aligned}$$

Hence, there is a new interest rate that must satisfy :

$$\begin{aligned} 0 &= 840 + 1000r - 950 + 1400r + 450 + 1000r - 550 + 1500r \\ 0 &= -210 + 4900r \\ \Rightarrow r &\approx 0.043 \end{aligned}$$

□

Question 2: Growth theory (25 Marks)

Consider a closed economy in which the population declines at the rate of 1% per year. The per worker production function is given by $y = \frac{F(K,N)}{N} = 2k^{1/3}$ where k is capital per worker and y is output per worker. Capital depreciates at the rate of 3% per year. Households consume 70% of their income and save the remaining 30%.

1. Find the steady state values of y, k and i where i is investment per worker.
2. What is the growth rate of Y, K, I and C in the steady state?
3. Is the value of steady-state capital per worker you found in 1. bigger or smaller than the steady-state value of output per worker? Explain why this is the case in one or two sentences.
4. Suppose the country wants to increase its steady state output per worker by 10%. What level of k is necessary in order to achieve such a change? What savings rate would be necessary in order to achieve this?

Solution. 1. In equilibrium, investment is equal to depreciation. Hence, the per-worker condition is :

$$\begin{aligned} s2k^{1/3} &= (\delta + n)k \\ \Rightarrow k &= \left(\frac{s2}{\delta + n}\right)^{3/2} \\ &= \left(\frac{30}{100}\frac{100}{2}2\right)^{3/2} \\ &= 30^{3/2} \\ &\approx 164.32 \end{aligned}$$

This means that :

$$\begin{aligned} y &= 2\left(30^{3/2}\right)^{1/3} \\ &= 2\left(30\right)^{1/2} \\ &\approx 10.95 \\ i &= (\delta + n)k \approx 3.29 \end{aligned}$$

2. In the steady state, everything is constant in per-worker terms. Hence, the growth rate of Y, C, I is equal to the growth rate of the population, that is -1%. One can see this through using marginal changes :

$$\begin{aligned} Y &= yN \\ \Rightarrow \partial Y &= \underbrace{\partial y}_{=0} N + y\partial N \\ \Rightarrow \frac{\partial Y}{Y} &= \frac{y}{Y}\partial N \\ &= \frac{Y}{Y}\frac{\partial N}{N} \\ &= -0.01 \end{aligned}$$

and since all variables have $K = Nk$, $C = Nc$, we can deduce that they have the same growth rates.

3. Capital per worker is bigger than output per worker. This is so because capital builds up through time with investment. This illustrates the concept of flow variable vs. stock variable.

4. We need to satisfy the following equation :

$$\frac{11}{10}2(30)^{1/2} = 2k^{1/3}$$
$$\Rightarrow \left(\frac{11}{10}\right)^3 30^{3/2} = k$$

from which we deduce :

$$\left(\frac{11}{10}\right)^3 30^{3/2} = \left(\frac{2s}{\delta + n}\right)^{3/2}$$
$$\Rightarrow s = \left(\frac{11}{10}\right)^2 15(\delta + n)$$
$$\approx 0.36$$

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Question 3: The Asset Market, Money and Prices (20 Marks)

Using a diagram for the money demand and supply (Y on the vertical axis and M/P on the horizontal axis), discuss briefly what will happen if :

1. One cent coins are withdrawn from the market.
2. People expect a lower inflation rate.
3. Police seize a huge drug factory.
4. The Bank of Canada buys more bonds than usual during the open-market operations.

Solution. See graphs in figure 1.

1. Removing coins reduces the offer for money and thus shifts the vertical curve left. The effect is likely to be small.
2. Money becomes more valuable, meaning that demand shifts up (or the curve moves to the right).
3. Transactions for drugs are mostly made with cash money. Hence, we can deduce a reduction of demand for money, shifting the curve left.
4. Buying more bonds for currency raises the amount of currency offered on the market, while the demand curve remains the same.

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Question 4: Stylized facts of the Canadian Economy (30 Marks)

This question asks you to find and comment the main business cycle facts of the canadian economy. As such, it helps you to understand what are the "stylized facts" of economic fluctuations in Canada and helps you to relate the main economic variables. First, you are asked to show your understanding of the transformations used on the data. Second, you will compute some statistics based the data and comment them.
For the second part of the question, it will be easier if you work with a spreadsheet program such as Microsoft Excel, OpenOffice Calc or Apple Numbers.

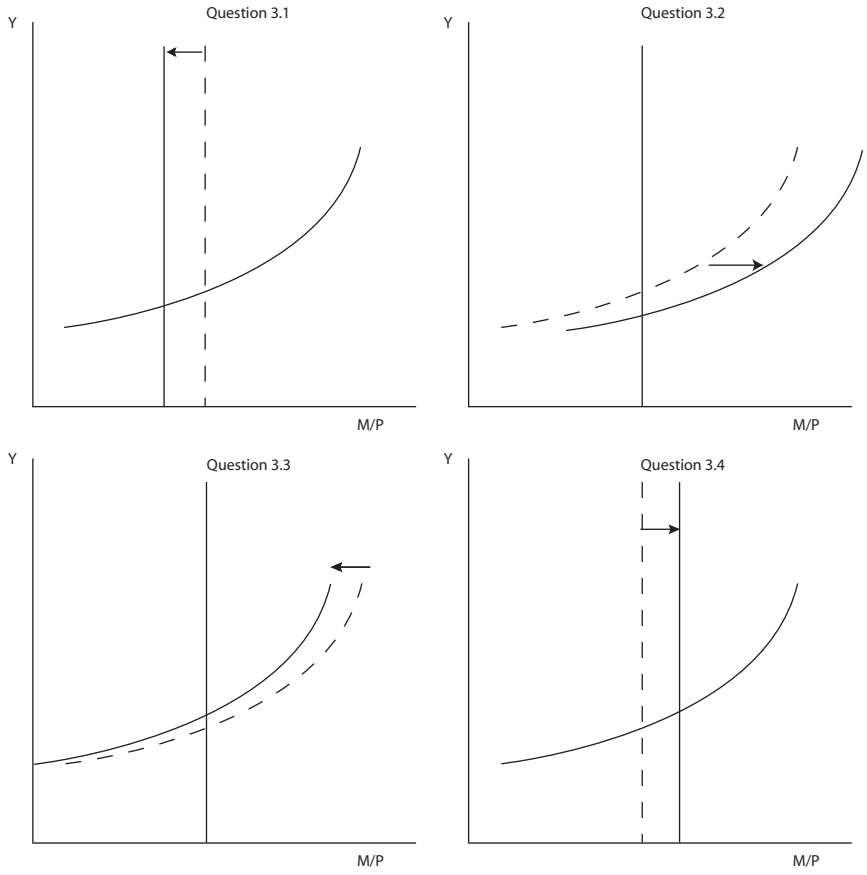

Figure 1: **Changes in money supply and demand**

Part a : Understanding Deviations From the Trend

1. In a spreadsheet, illustrate by a graph that for small values of x, (say $x \in [0.01, 0.1]$), the following approximation holds :

$$\log_e(1+x) \approx x$$

That is, the natural logarithm of $1+x$ is roughly equal to x. Report your graph in your homework.

2. It turns out that we can exploit this property of the natural logarithm to approximate the percentage deviation of an economic variable from its long-term trend. Use the property from part 1. to show that for some variable V_t and its long run trend V_t^* (see figure 2) :

$$\frac{V_t - V_t^*}{V_t} \approx \log_e(V_t) - \log_e(V_t^*).$$

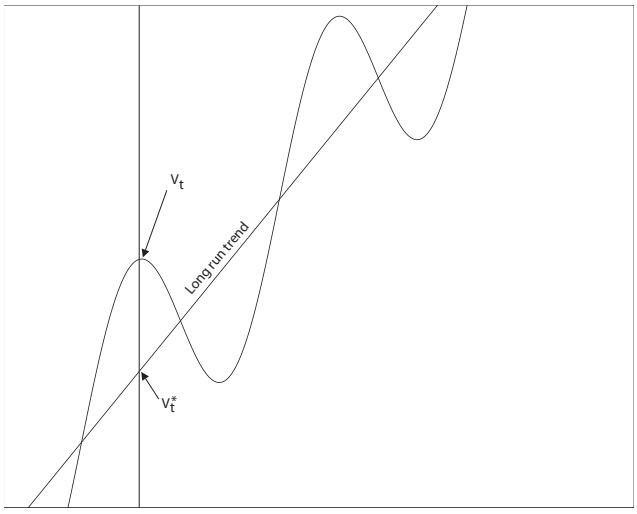

Figure 2: Current variable (V_t) and their long run value (V_t^*)

Solution. 1. The graph should look like figure 3.

2. The solution should look like :

$$\begin{aligned}
\log_e V_t - \log_e V_t^* &= \log_e\left(\frac{V_t}{V_t^*}\right) \\
&= \log_e\left(\frac{V_t - V_t^* + V_t^*}{V_t^*}\right) \\
&= \log_e\left(1 + \frac{V_t - V_t^*}{V_t^*}\right) \\
&\approx \frac{V_t - V_t^*}{V_t^*}
\end{aligned}$$

□

Part b : Business Cycles Facts

In the spreadsheet *businescycles.xls*, you will find 9 pairs of columns of data. Each pair is associated with one of the following macroeconomic real variables: Output (Y), capital (K), labour (N), investment (I), government expenses (G), unemployment (u), real wages (w), the stock of money (M) and consumption (C). Each of these variables range from 1976 to 2005. Within each pair, the first column is for the actual variable and the second column is for its long-run trend.

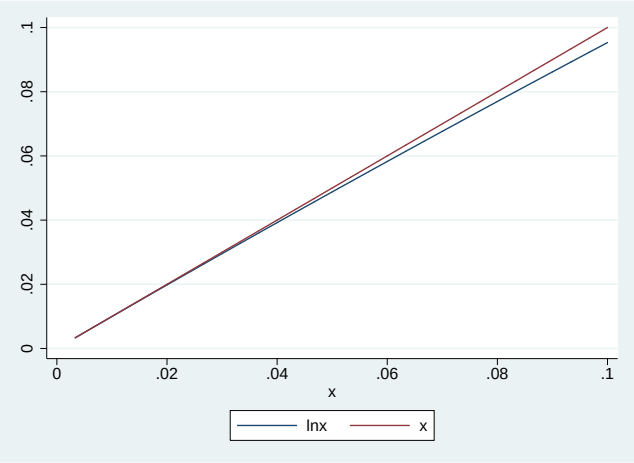

Figure 3: $\ln(1+x) \approx x$.

1. In nine different graphs, plot both each variables and its long-run trend. Answer briefly (one or two sentences) the following questions :
   (a) How would you heuristically describe the difference in fluctuations before and after the 90's ? Which period has higher fluctuations?
   (b) One graph should stand out in terms of trend. Which one is different from all the others?
   (c) What happened to government spending in the 90's ? Any idea of what might cause this ?
   (d) Which of consumption and investment has the highest fluctuations? What does this suggest in terms of consumption decisions?
2. For each variable, produce Δ_{V_t} as described in part a) 3. Compute the standard deviation (σ_V) and their correlation with output ($\rho_{V,Y}$), that is :

$$\sigma_V = \sqrt{\frac{1}{29}\sum_{t=1976}^{2005}(\Delta_{Vt} - \bar{\Delta}_{Vt})^2}$$

$$\rho_{Y,V} = \frac{1}{29}\frac{\sum_{t=1976}^{2005}(\Delta_{Yt} - \bar{\Delta}_{Yt})(\Delta_{Vt} - \bar{\Delta}_{Vt})}{\sigma_Y \sigma_V}$$

   Based on these calculations, report the following statistics in a table similar as Table 1. (*Hint:* you can use the functions *stdev* and *correl* in excel).
3. The first column is a measure of volatility of each of the variables. Name the two variables with the highest volatility and the two variables with the lowest volatilty.
4. Correlation measures by how much two variables move together. It is a number between minus one and one. A correlation of one means that two variables move perfectly together while a correlation of minus one means they move in an opposite manner. Which variables are cyclical? Which variables are *countercyclical* with respect to output?

Solution. 1. Graphs should look like figures 4 and 5. Some good observations about those graphs :
   (a) Fluctuations are smaller after the recession of the 90's. This seems to be true for all variables.
   (b) All graphs are trending upward, except unemployment which is hump-shaped.
   (c) There has been a dramatic cut in government spending in the 90's. This is likely due to post-recession pressure to stop deficits.

(d) Investment is much more volatile than consumption. This is evidence of consumption smoothing from consumers.

2. See table 1.

3. Unemployment and investment are the most volatile. Wages and government spending are the less volatile.

4. Government spending, stock of money and unemployment are coutercyclical. Otherwise variables are cyclical.

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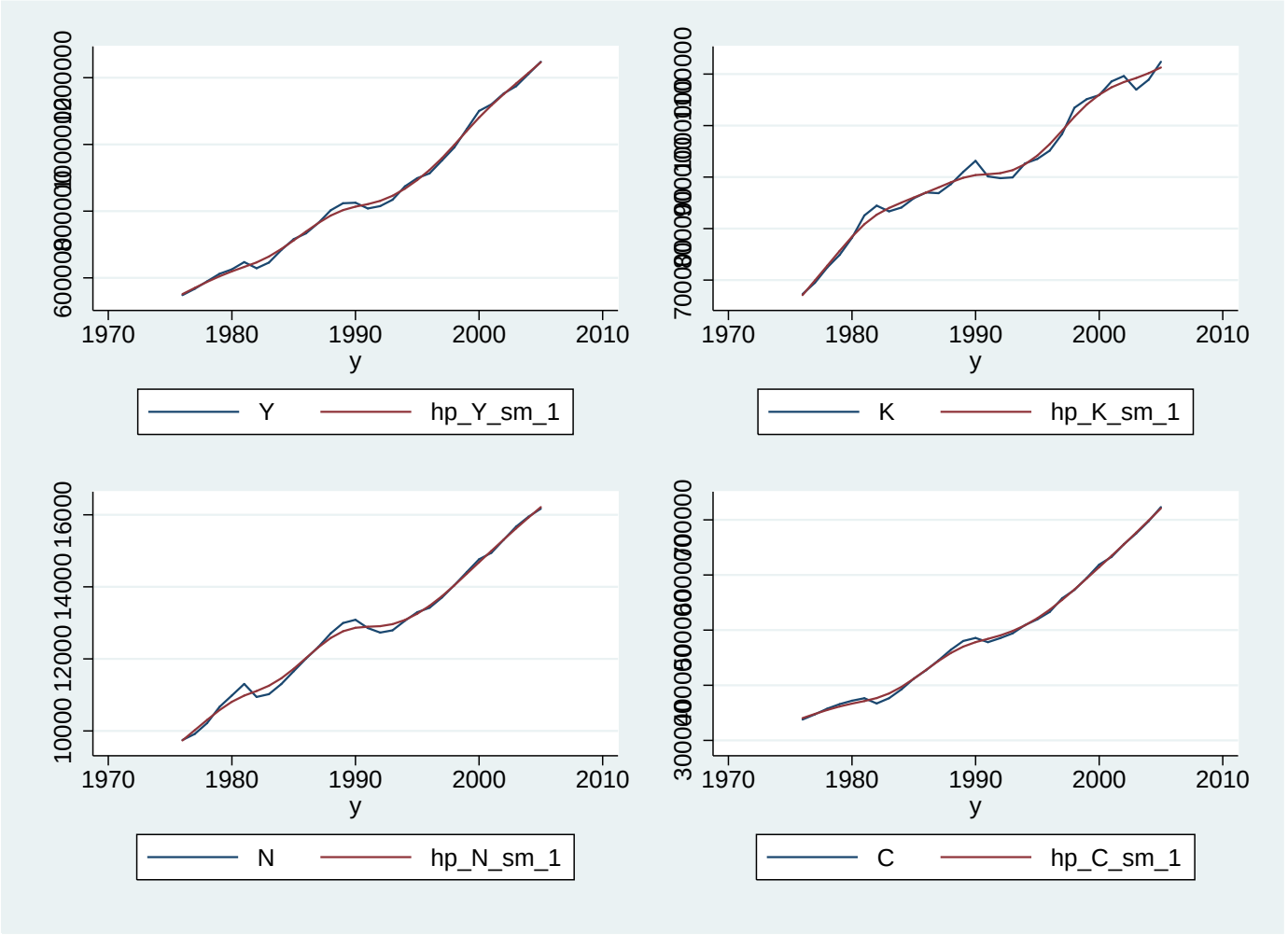

Figure 4: **Variables and their trend**

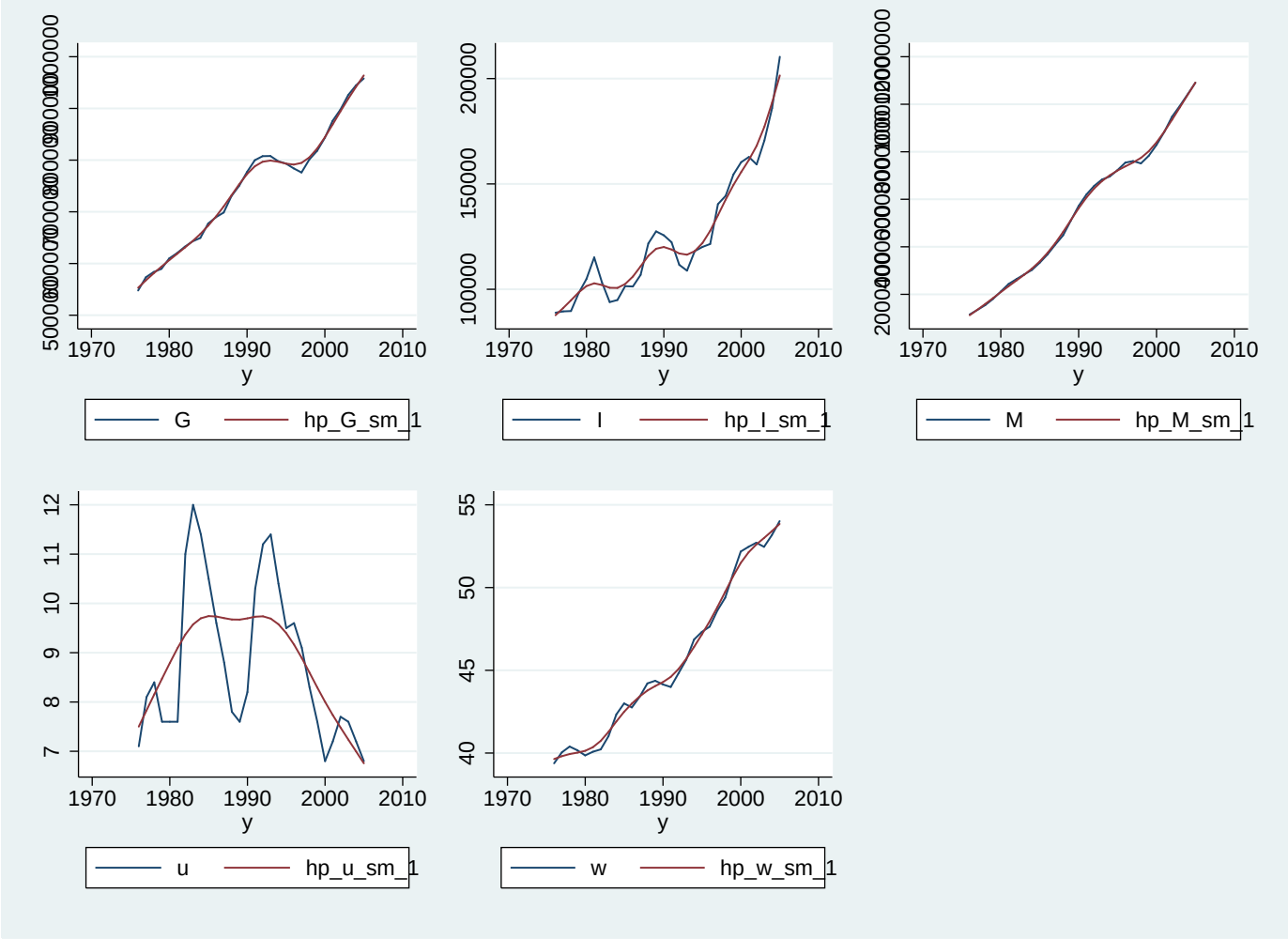

Figure 5: **Variables and their trend**

Variable (V)	σ_V	$\rho_{V,Y}$
Y	0.0134	1.000
K	0.0124	0.305
N	0.0108	0.821
C	0.0107	0.903
I	0.0449	0.601
u	0.1207	-0.845
G	0.0087	-0.113
w	0.0078	0.577
M	0.0187	-0.243

Table 1: Summary statistics, Question 1, part b)