

ECON 222
Macroeconomic Theory I
Fall Term 2010

Assignment 2

*Due: Drop Box 2nd Floor Dunning Hall by **noon October 15th** 2010*

No late submissions will be accepted
No group submissions will be accepted
No “Photocopy” answers will be accepted

*Remarks: Write clearly and concisely. Present graphs, plots and tables in a format that is easy to understand. The way you present your answers will be reflected in the final grade. Even if a question is mainly analytical, **briefly** explain what you are doing, stressing the economic meaning of the various steps.*

Question 1: Productivity, Output and Employment (30 Marks)

Suppose we have an economy with only one aggregate production function, given by:

$$Y = AK\sqrt{N}$$

where A is TFP, K represents capital and N represents labour. Set $A = 1.2$ and $K = 121$. The price level is 1.

1. Use the production function to derive an algebraic expression for the demand curve for labour, explaining your reasons. Assuming that the wage rate for the economy was constant at 8.55, what would be the level of employment? Show your results graphically.

Answer. Start by taking the derivative of the production with respect to labour to obtain the MPN curve. Set this equal to the real wage (w).

$$MPN = \frac{\partial Y}{\partial N} = \frac{1}{2}AKN^{-1/2} = w$$

Then solve this for N , which is the demand for labour, N^d .

$$N^d = \left(\frac{AK}{2w} \right)^2$$

Inserting values for TFP, $A = 1.2$, and the capital stock, $K = 121$, we get:

$$N^d = \frac{131769}{25w^2}$$

With $w = 8.55$, $N^d \approx 72.10$. The determination of labour demand is shown graphically in Figure 1.

□

2. Now suppose that the supply curve is upward sloping and has the following form:

$$NS = \frac{47}{50}w^2$$

where w is the real wage rate. Show how this addition affects your results by calculating the new wage rate and new demand for labour. What has been the effect?

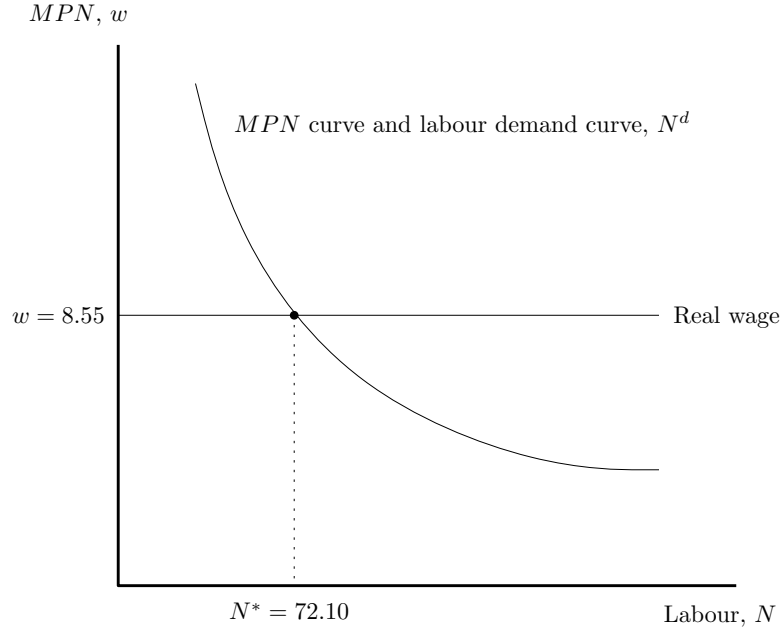


Figure 1: Question 1, part 1

Answer. In equilibrium, labour demand, N^d , has to be equal to supply, N^s .

$$N^d = \frac{131769}{25w^2} = \frac{47}{50}w^2 = N^s$$

We can rewrite this in terms of w , which yields

$$w = \left(\frac{263538}{47} \right)^{1/4} \approx 8.653$$

At this equilibrium wage rate, labour demand is $N^d = 131769/(25 \times 8.653^2) \approx 70.39$ (careful not to round too soon).

Compared to the case with a flat labour supply curve, the introduction of an upward sloping labour supply curve implies that workers require a higher wage just to supply the original amount of labour. As a consequence of the higher wage rate, firms now demand fewer workers. In equilibrium, fewer workers are employed (70.39 versus 72.10), and the market wage rate is higher (\$8.65 versus \$8.55). $\square$

3. Suppose that a minimum wage of $w = 9.00$ is imposed. What is the quantity of labour that households are willing to supply? If the tax rate on labour income, t , equals zero, what is the demand for labour? Plot labour supply, labour demand, and the impact of the imposed wage rate on the labour market. What is the resulting level of employment? What is the rate of unemployment? Does the introduction of the minimum wage increase the total income of workers, taken as a group?

Answer. A minimum wage of 9.00 is binding if the tax rate is zero. Then

$$N^s = \frac{47}{50}w^2 = \frac{47 \times 9.00^2}{50} \approx 76.14$$

and

$$N^d = \frac{131769}{25w^2} = \frac{131769}{25 \times 9.00^2} \approx 65.07$$

The graph should look something like Figure 2.

There are more people willing to work at the minimum wage than firms are willing to hire. Employment is therefore $N = 65.07$, and unemployment is $76.14 - 65.07 = 11.07$. The unemployment rate is $u = 11.07/76.14 = 0.145$ or 14.5 percent. Aggregate income of workers is $wN = 9 \times 65.07 = 585.6$, which is far lower than without a minimum wage ($8.65 \times 70.39 = 608.9$), because employment has declined so much.

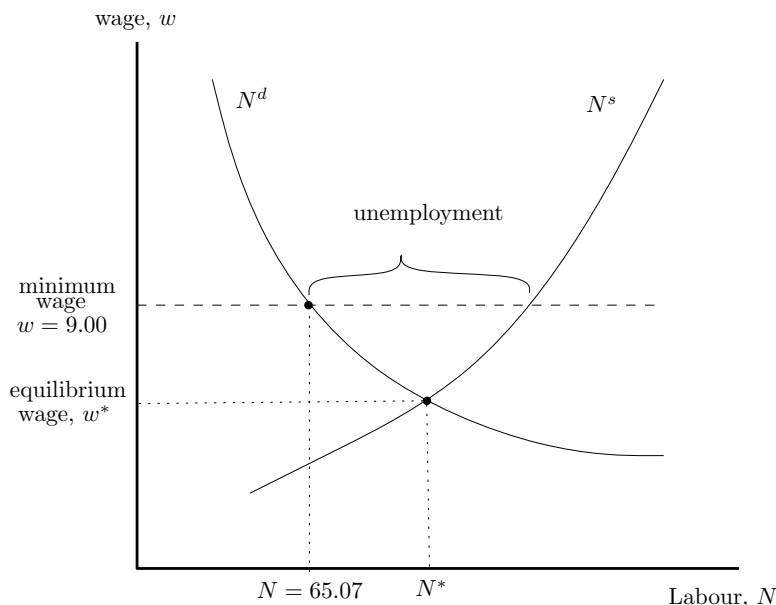


Figure 2: Question 1, part 4

□

4. A production function is said to exhibit constant returns to scale (CRS) if $cF(K, L) = F(cK, cL)$ for any constant c . Does the above production function exhibit CRS? What does that imply in terms of aggregation?

Answer. We have $F(cK, cL) = c^{3/2}F(K, L)$, so the function does not exhibit CRS. This makes aggregation difficult because the sum of all firms input decisions will not be equal the input decision of some hypothetical aggregate firm facing the same production technology. In other words, the sum of all output produced by individual firms will not equal the output predicted by the aggregate production function using the national capital stock and the aggregate level of employment. □

Question 2: The Consumption/Saving Decision (30 Marks)

This question studies the present value budgeting problem introduced in Appendix 4A. Suppose Michelle's life can be divided into two aggregate blocks of time: period 1 and 2. Michelle is a professional hand model. In period 1, Michelle's hands are featured in many fashion magazine ads for nail polish, and she earns income $y_1 = 40$. In period 2, Michelle's hands are old and full of wrinkles, and very few companies are willing to hire her for nail polish photo shoots. As a result, she only receives income $y_2 = 21$. The nominal interest rate is 7 percent ($i = 0.07$), and the expected inflation rate is 2 percent ($\pi^e = 0.02$). The real interest rate is therefore 5 percent ($r = i - \pi^e = 0.05$).

1. Derive an expression for the budget constraint by setting the present value of Michelle's lifetime consumption (P VLC) equal to the present value of her lifetime income (P VLI). Then rearrange the budget

constraint in terms of period 2 consumption. Interpret the expression. Use the notation

c_1 = consumption in period 1
 c_2 = consumption in period 2
 y_1 = income in period 1
 y_2 = income in period 2
 r = real interest rate

Answer. Setting PVLC equal to PVLI yields:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

After some manipulation, this can be rewritten as

$$c_2 = (y_1 - c_1)(1+r) + y_2$$

which says that period 2 consumption is equal to Michelle's income in period 2, plus whatever she saved from her income in period 1 including interest earned. If Michelle is a borrower, the interpretation is similar: period 2 consumption is equal to Michelle's income in period 2, less interest payments and whatever she borrowed in period 1. $\square$

2. Based on the information above, what is the present value of her lifetime income? What is the highest feasible level of consumption Michelle could enjoy in period 1? What is the highest feasible level of consumption Michelle could enjoy in period 2? Use this information to graph the budget line. What is the slope of the budget constraint and what is its interpretation?

Answer.

$$PVLI = 40 + \frac{21}{1.05} = 60$$

The highest feasible level of consumption Michelle could attain in period 1 is her PVLI, 60. Instead, if Michelle saves all of her period 1 income, she could consume $40(1.05) + 21 = 63$ in period 2. Figure 3 illustrates the budget line. The slope is $-(1+r) = -1.05$, which is the amount of period 2 consumption Michelle must give up in order to increase period 1 consumption by 1 dollar.

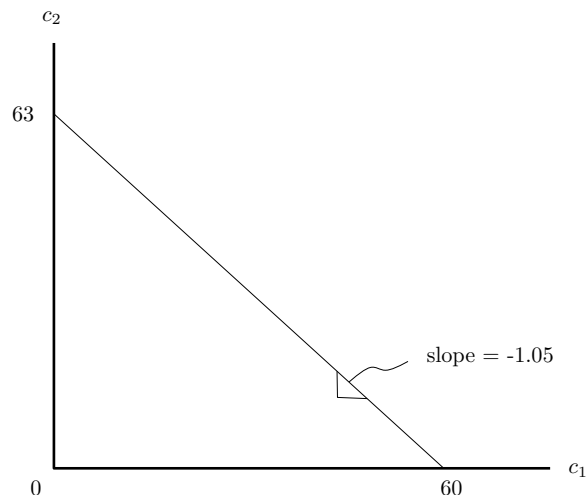


Figure 3: Question 3, part 2

$\square$

3. Michelle wishes to smooth her consumption over time, so that $c_1 = c_2 = c^*$. Find the optimal consumption in earn period, c^* , and the amount of saving/borrowing. Is Michelle a borrower or a lender? Plot the optimal consumption point along the budget line and the original no-borrowing, no-lending point on the graph. Also include in your graph an indifference curve representing Michelle's preferences that clearly identifies c^* as the optimal point.

Answer. Make use of consumption smoothing ($c_1 = c_2 = c$), and the lifetime budget constraint

$$c + \frac{c}{1.05} = PVLI = 60$$

We get $c = 30.73$, and saving $s^* = 40 - 30.73 = 9.27$. Since Michelle's saving is positive, she is a lender. The optimal consumption point is illustrated in Figure 4.

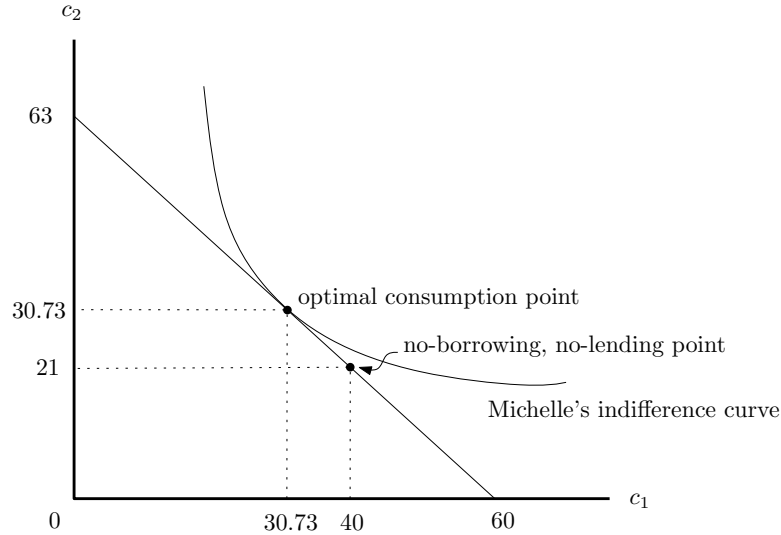


Figure 4: Question 2, part 3

□

4. Suppose that the government decides to institute a tax on interest earnings. Let $t = 1/7$ be the tax rate on interest earnings. The expected after-tax real interest rate is

$$r_{a-t} = (1 - t)i - \pi^e$$

Find the new optimal consumption and savings plan, and graphically show the effects of this policy change. Comparing with the result in part (b), explain which effect is stronger for Michelle, the substitution effect or the income effect?

Answer. With $r_{a-t} = (1 - t)i - \pi^e = 0.04$, Michelle's new PVLI is

$$PVLI_{a-t} = 40 + \frac{21}{1.04} = 60.192$$

The lifetime budget constraint becomes

$$c^* + \frac{c^*}{1.04} = PVLI_{a-t} = 60.192 \quad (1)$$

We get $c^* = 30.69$ and $s^* = 40 - 30.69 = 9.31$. The income effect dominates, since the decrease in the after-tax real interest rate causes Michelle to reduce current consumption and increase saving. Figure 5 illustrates the income and substitution effects. The substitution effect is an increase in current consumption and reduction in saving from point (1) to point (2) as a result of the decrease in the real interest rate. The income effect is the reduction in current consumption and increase in saving from point (2) to point (3).

□

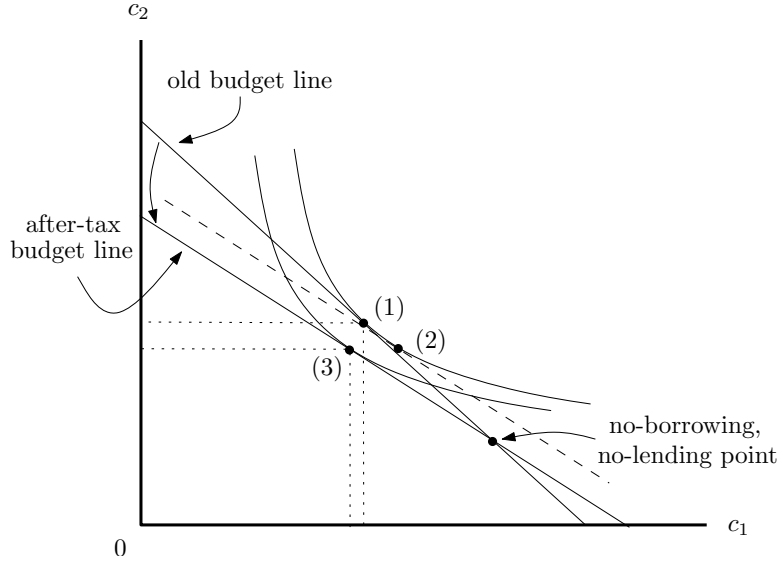


Figure 5: Question 2, part 3

Question 3: Consumption, Saving, and Investment (20 Marks)

Imagine a closed economy called Pabstania. The economywide expected future marginal product of capital is

$$MPK^f = 90 - 0.05K^f$$

where K^f is the future capital stock. The current capital stock in Pabstania is 1650 units, but capital depreciates at a rate of 20 percent per period ($d = 0.2$). The price of capital is 1 unit of output ($p_K = 1$). Firms in Pabstania pay taxes equal to 50 percent of their output ($\tau = 0.5$).

1. Suppose that the real interest rate is 5 percent per period. What are the values of the tax-adjusted user cost of capital, the desired future capital stock, and the desired level of investment?

Answer. The user cost of capital is

$$\frac{uc}{1 - \tau} = \frac{(r + d)p_K}{1 - \tau} = \frac{0.05 + 0.2}{1 - 0.5} = 0.5$$

In equilibrium, the future marginal product of capital is equal to the tax-adjusted user cost of capital.

$$90 - 0.05K^f = 0.5 \Rightarrow K^f = 1790$$

The desired level of investment can be found using the capital accumulation equation

$$K_{t+1} = K_t + I_t - dK_t \Rightarrow I_t = K_{t+1} - (1 - d)K_t = 1790 - (1 - .2) * 1650 = 470$$

□

2. Write the tax-adjusted user cost of capital as a function of the real interest rate r . Also write the desired future capital stock and desired investment as functions of r .

Answer. Substituting $d = .2$, $\tau = .5$ and $p_K = 1$ into the expression for the tax-adjusted user cost of capital yields $uc/(1 - \tau)$ as a function of r :

$$\frac{uc}{1 - \tau} = \frac{r + 0.2}{0.5} \text{ or equivalently } \frac{uc}{1 - \tau} = \frac{10r + 2}{5}$$

In equilibrium, $MPK^f = uc/(1 - \tau)$:

$$\begin{aligned} 90 - 0.05K^f &= \frac{10r + 2}{5} \\ \Rightarrow 0.05K^f &= \frac{452 - 10r}{5} \\ \Rightarrow K^f &= 1792 - 40r \end{aligned}$$

This expression for K^f is substituted into the rearranged capital accumulation identity. Using $K_t = 1650$ and $d = 0.2$, this yields:

$$I_t = 1792 - 40r - (1 - 0.2) * 1650 = 472 - 40r$$

□

The consumption function in Pabstania is

$$C = 1050 + 0.63Y - 4500r$$

Moreover, government purchases equal 220 ($G = 220$), and full employment output is 4000 ($\bar{Y} = 4000$).

3. Use the investment function derived in part 2 along with the consumption function and government purchases, to calculate the real interest rate that clears the goods market. What are the goods market-clearing values of consumption, saving, and investment? What are the tax-adjusted user cost of capital and the desired capital stock in this equilibrium? Show your results graphically.

Answer. One way to calculate the real interest rate is to start with the definition of desired national saving (S^d) and substitute for desired consumption (C^d):

$$\begin{aligned} S^d &= Y - C^d - G \\ S^d &= Y - (1050 + 0.63Y - 4500r) - G \\ S^d &= 210 + 4500r \end{aligned}$$

where the last line uses $\bar{Y} = 4000$ and $G = 220$.

Equilibrium in the goods market requires $S^d = I^d$. We have the equation for desired investment from part 2, which we can set equal to the equation for desired national saving that we just derived:

$$210 + 4500r = 472 - 40r \quad \Rightarrow \quad r \approx 5.771\%$$

We can use the equilibrium interest rate to compute the goods market-clearing values of consumption, saving, and investment:

$$\begin{aligned} C^d &= 1050 + 0.63Y - 4500r \approx 3310.31 \\ I^d &= 472 - 40r \approx 469.69 \\ S^d &= I^d \approx 469.69 \end{aligned}$$

The equilibrium after-tax user cost of capital is

$$\frac{uc}{1 - \tau} = \frac{r + 0.2}{0.5} \approx 0.5154$$

and the desired capital stock is

$$K^f = 1792 - 40r \approx 1789.69$$

Figure 6 illustrates the equilibrium graphically.

□

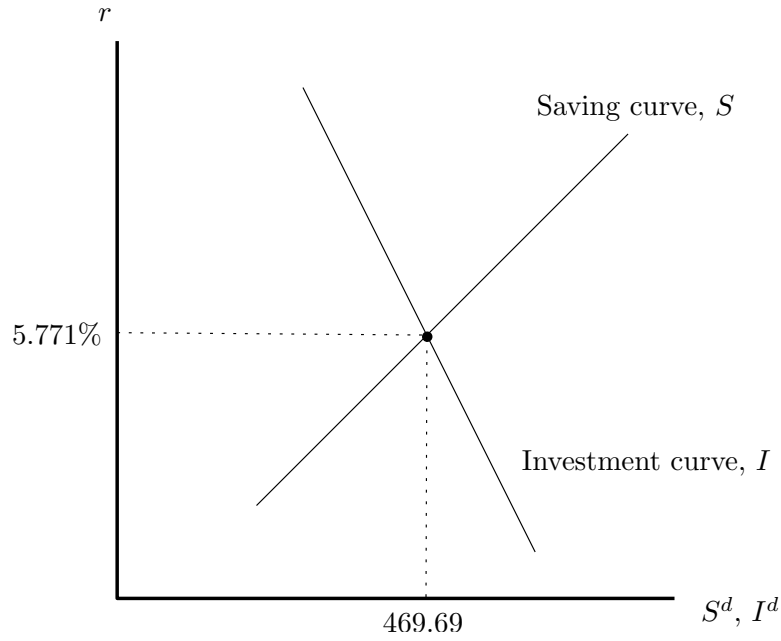


Figure 6: Question 3, part 3

Question 4: Saving and Investment in an Open Economy (20 Marks)

Consider the following information for a small open economy:

$$\text{output, } Y = 1200$$

$$\text{desired consumption } C^d = 125 + 0.7Y - 600r^w$$

$$\text{desired investment } I^d = 250 - 200r^w$$

$$\text{government purchases } G = 190$$

$$\text{net factor payments } NFP = 0$$

1. Write national saving for the economy as a function of the world real interest rate r^w .

Answer. First derive the expression for net exports using the goods market equilibrium condition for an open economy:

$$NX = Y - (C^d + I^d + G)$$

$$NX = Y - (125 + 0.7Y - 600r^w + 250 - 200r^w + 190)$$

$$NX = 0.3Y - 565 + 800r^w$$

$$NX = -205 + 800r^w$$

The expression for national saving as a function of r^w then comes the equilibrium condition $S^d = I^d + NX$:

$$S^d = 250 - 200r^w - 205 + 800r^w = 45 + 600r^w$$

□

2. Suppose the world real interest rate is $r^w = 7\%$. Find the economy's national saving, investment, current account surplus, net exports, desired consumption, and absorption.

Answer.

$$S^d = 45 + 600r^w = 87$$

$$I^d = 250 - 200r^w = 236$$

$$CA = NX = -205 + 800r^w = -149$$

$$C^d = 125 + 0.7Y - 600r^w = 923$$

$$\text{absorption} = C^d + I^d + G = 1349$$

□

3. Owing to a large number of hard-working graduates from the Queen's Economics program, the economy's output rises by 300 to 1500. Repeat parts 1 and 2.

Answer.

$$NX = Y - (C^d + I^d + G) = -115 + 800r^w$$

$$S^d = I^d + NX = 250 - 200r^w - 115 + 800r^w = 135 + 600r^w$$

At $r^w = 7\%$,

$$S^d = 135 + 600r^w = 177$$

$$I^d = 250 - 200r^w = 236$$

$$CA = NX = -115 + 800r^w = -59$$

$$C^d = 125 + 0.7Y - 600r^w = 1133$$

$$\text{absorption} = C^d + I^d + G = 1559$$

□