ECON 222

Macroeconomic Theory I

Fall Term 2009

Answers to Assignment 3

Question 1: Neoclassical Growth Model (25 Marks)

(a) To get the production function in per-worker terms, divide by N and simplify:

$$\frac{Y}{N} = 1\frac{K^{ak}N1 - a_k}{N}$$
$$y = K^{a_k}N^{1-a_k-1}$$
$$y = K^{a_k}N^{-a_k}$$
$$y = (\frac{K}{N})^{a_k}$$
$$y = f(k) = k^{a_k}$$

b) From (6.10), $sf(k) = (n+d)k$. And from above: $sf(k) = sk^{a_k}$.
Combining we get $sk^{a_k} = (n+d)k$. Solving for k gives:

$$\frac{s}{(n+d)}k^{a_k} = k$$
$$\frac{s}{(n+d)} = k^{1-a_k}$$
$$(\frac{s}{(n+d)})^{\frac{1}{1-a_k}} = k.$$

With $a_k = \frac{1}{2}$, we get $k^* = (\frac{s}{(n+d)})^2$. Given $s\ = 0.2,\ n\ = 0.5,\ d\ = 0.05$, then $k^* = 4$.

$$y^* = (k^*)^{.5} = 2;$$
$$c^* = y^* - (n+d)k^* = 2 - .1(4) = 1.6;$$
$$i^* = (n+d)k^* = 0.1(4) = .4$$
$$(Alternatively, Y = C + I).$$

c) Plug new s into $k^* = (\frac{s}{(n+d)})^2$, to get $k^* = 16$. $y^* = 4$; $c^* = 2.4$; $i^* = 1.6$. All variables are higher in the new steady-state.

d) $k^* = 16, y^* = 4$, but $c^* = y^* - (n + d^{'})k^* = 3.2$ is higher than in (c). $k_d^* = 16 > k_b^* = 4$. All other variables are also higher in the new steady-state compared to (b).

e) Consider a 10 percent increase in savings, $s^{'} = .22$, versus a 10 percent fall in depreciation, $d^{'} = .045$. The savings change gives $k^* = 4.84 = (2.2)^2$; $y^* = 2.42$; $c^* = 1.94$. The comparable depreciation change only gives $k^* = 4.43 = (2.1)^2$; $y^* = 2.21$; $c^* = 1.80$.

Given equilibrium (b), a small push to increase savings gives a better return in the long-run consumption than an equivalent fall in depreciation. Notice, this result was overturned in (d) for large changes in s and d. Recall, however, there is a trade-off in the short-run: for savings per- worker to increase, consumption per-worker must temporarily fall. As an aside: finally, we can show that the golden rule k^*_{GR}, the capital- worker ratio which maximizes consumption is $f^{'}(k^*_{GR}) = (n + d)$. This comes from the first order condition from $max_k\ c(k) = f(k) - (n + d)k$. So if the economy got to $k^*_{GR} = 25$, then consumption would be maximized at $c^*_{GR} = 2.5; i^*_{GR} = 2.5; y^*_{GR} = 5$. This occurs when $s = .5$.

Question 2: Money Demand and Velocity (20 Marks)

(a)

$$\begin{aligned} eta_{Y,W} &= \frac{\partial \ln M^d_W}{\partial \ln Y_W} = 0.5 \\ eta_{i,W} &= \frac{\partial \ln M^d_W}{\partial \ln i_W} = -\frac{2}{3} \end{aligned}$$

(b)

$$P_W = \frac{4 i_W^{\frac{2}{3}} M^d_W}{\sqrt{Y_W}} = 14.36$$

(c)

$$V_W = \frac{P_W Y_W}{M^d_W} = 51.71$$

The quantity theory of money was not holding in West Germany. Real money demand is not proportional to real income since it depends on the interest rate.

(d)The quantity theory of money was not holding in East Germany either, it is not possible to express real money demand in this form kY where k is a constant.

(e) East Germany in 1980 (with $P_E = 100$):

$$\begin{aligned} eta_{Y,E} &= \frac{\partial \ln M^d_E}{\partial \ln Y_E} = \frac{0.8 Y_R}{0.7 + 0.8 Y_E} \\ Y_E &= 0.583 \\ M^d_E &= 116.67 \end{aligned}$$

East Germany in 1981 (with $P_E = 102$):

$$\begin{aligned} Y_E &= 0.471 \\ M^d_E &= 109.846 \end{aligned}$$

Growth of nominal money demand is then -5.846%.

Question 3: Business Cycles and Consumption (20 Marks) (b) All consumption series are pro-cyclical, but one can argue that non-durable goods are acyclical.

(c) Business cycles are not really smooth, so many answers can be accepted if a period of time is specified. The timing is mostly coincident for all four series.

Question 4: Monetary and Fiscal Policies (20 Marks)

(a)

$$\begin{aligned} Y &= C + I + G \\ Y &= 200 + 0.25Y - 50 + 150 + 0.25Y - 1,000i + 250 \\ Y &= 550 + 0.5Y - 1,000i \\ Y &= 1,100 - 2,000i \end{aligned}$$

(b)

$$\begin{aligned} 1,600 &= 2Y - 8,000i \\ i &= 0.00025Y - 0.2 \end{aligned}$$

(c)

$$\begin{aligned} Y &= 1,100 - 0.5Y + 400 \\ 1.5Y &= 1,500 \\ Y^* &= 1,000 \\ i^* &= 0.05 \end{aligned}$$

(d)

$$\begin{aligned} C^* &= 400 \\ I^* &= 350 \end{aligned}$$

(e)

$$\begin{aligned} i &= 0.00025Y - 0.23 \\ 1.5Y &= 1,560 \\ Y^* &= 1,040 \\ i^* &= 0.03 \\ C^* &= 410 \\ I^* &= 380 \end{aligned}$$

The expansionary monetary policy has the effect of reducing interest rates in the asset markets. Investment is greater at lower levels of the interest rate and this has the effect of increasing output.

(f)

$$\begin{aligned} Y &= 1,400 - 0.5Y + 400 \\ 1.5Y &= 1,800 \\ Y^* &= 1,200 \\ i^* &= 0.1 \\ C^* &= 450 \\ I^* &= 350 \end{aligned}$$

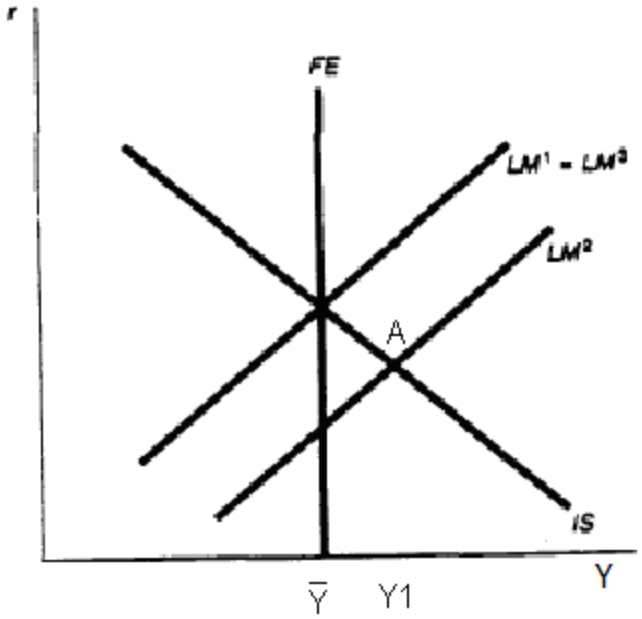
An increase in government spending affects the asset market, since money demand depends on real income. The amount of money supplied is fixed, therefore for this market to remain in equilibrium, interest rates must increase as well. Two opposites effects are affecting the investment, it is increasing because real income is increasing, but decreasing from the raise in the interest rate ("crowding-out effect").

Question 5: The IS-LM Model in Diagrams (15 Marks)

(a)

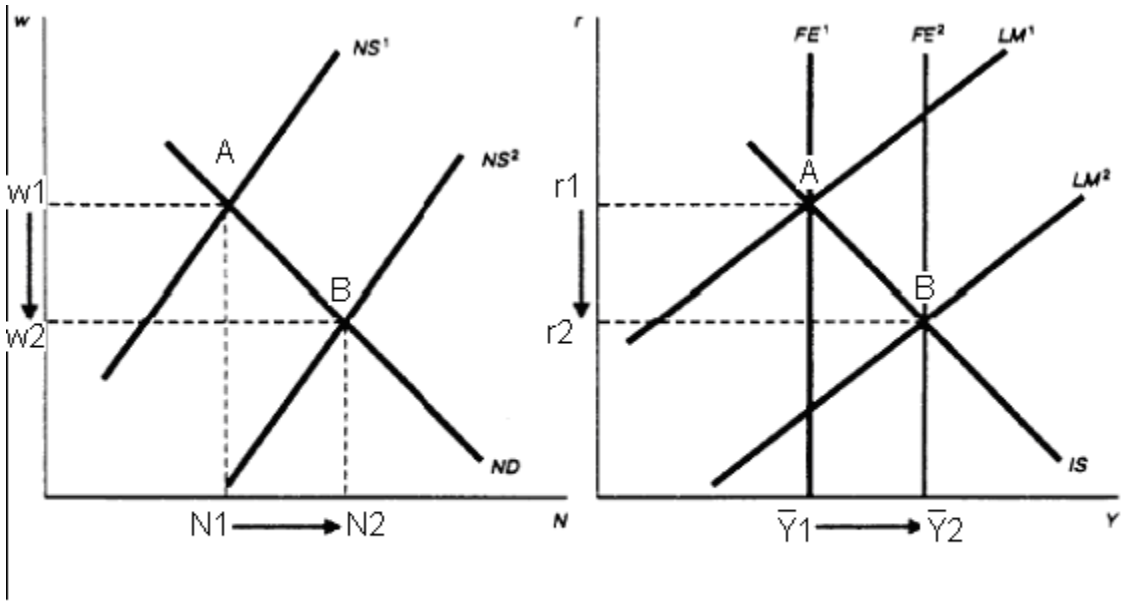
The rise in expected inflation shifts the money demand curve down since people do not want to hold as much money given that it will lose value overtime and so people want to hold more bonds. The price of bonds increases and so the interest rate declines.

The rise in expected inflation shifts the LM curve down as shown in Figure 5a. At point A, the economy is producing above full-employment. The price level rises, shifting the LM curve up to restore equilibrium since the real interest rate is unchanged. Consumption and investment are unchanged. In summary, there is no change in the real wage, employment, output, the real interest rate, consumption, or investment: and there is a rise in the price level.



(b)

The increase in labour supply shift the labour supply curve down to the right as shown in Fig. 5b1. This reduces the real wage rate from W1 to W2 and increases in employment from N1 to N2. The rise in employment causes an increase in output, shifting the FE line to the right as shown in Fig. 5b2. At point A we are below full-employment so there is downward pressure on prices. As prices drop, the LM curve than shifts down to the right until the LM intersects the IS and FE curves at point B.



Question 3: Money Demand and Velocity (20 Marks)

(a)

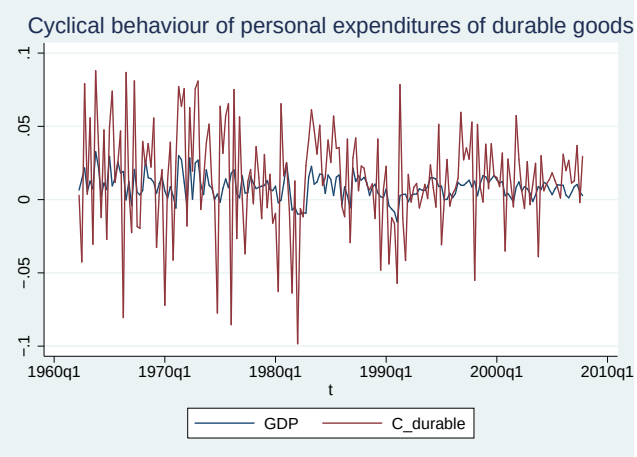
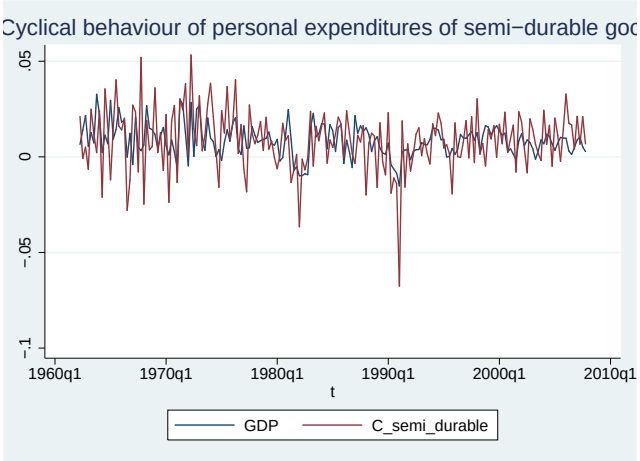
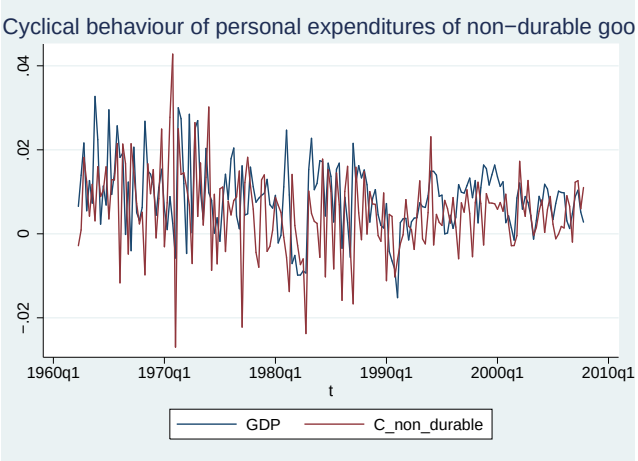
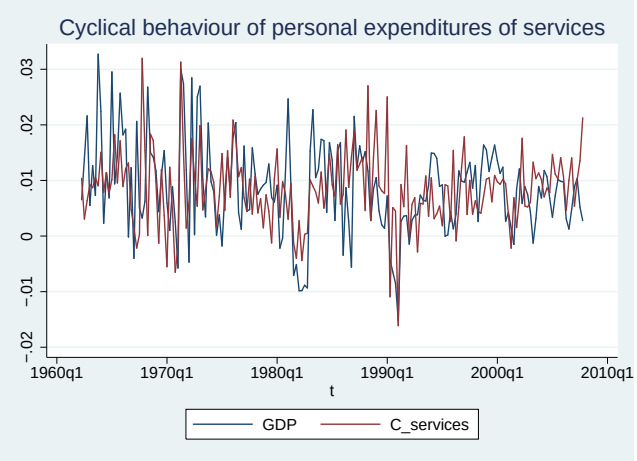


Table 1:

Durables	Semi-durables	Non-durables	Services
0.43	0.4	0.24	0.36


Cyclical behaviour of personal expenditures of semi−durable goo
.05
0
−.05
−.1
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
t
GDP
C_semi_durable


Cyclical behaviour of personal expenditures of non−durable goo
.04
.02
0
−.02
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
t
GDP
C_non_durable


Cyclical behaviour of personal expenditures of services
.03
.02
.01
0
−.01
−.02
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
t
GDP
C_services