Institutions:
Why are They Persistent and
Why Do They Change?

Bryan Paterson*
Department of Economics
Queen’s University
paterson@econ.queensu.ca

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Abstract

This paper presents a model of institutional choice over time, where institutional arrangements arise endogenously in response to the need for protection of property. Variable returns to investment in new technology create economic inequality between communities, which in turn influences what institutional arrangement is chosen. The type of institution chosen in one period then affects subsequent economic outcomes in a way that tends to reinforce the existing institutional arrangement in the next period. This feedback effect makes institutions persistent over time. A change in the prevailing institution occurs only when particular realizations of the technological development process happen to counteract the institutional feedback effect. The longer a particular institution is in place, the less likely it is that this feedback effect can be overcome. Empirical evidence confirms this implied pattern of increasing institutional stability over time.

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1 Introduction

This paper examines two important questions regarding the nature and dynamics of institutions; why are institutions persistent and why do they change? The particular institution analyzed in this paper is the quality of constraints preventing governments and other elites from expropriating private property. Recent evidence suggests that this institution is particularly important for creating an economic environment conducive to investment and long-run growth.¹

Not only is there evidence regarding the importance of these property rights institutions, but there is also evidence of their persistence over time. For example, Acemoglu, Johnson and Robinson (2001) find a strong correlation in former European colonies between indices of the quality of current expropriation protection and those protection measures that were originally put in place by colonizers, often hundreds of years earlier. They also find that former colonies that had extractive institutions when they were formed are more likely to have extractive institutions today.

But this raises an important question. If property rights institutions are so important for growth, then why would poor quality institutions in particular be persistent over time? If property rights institutions never changed, one could explain this persistence by appealing to exogenous lock-in effects, path dependence, or large switching costs associated with institutional change. However, in spite of the evidence of institutional persistence, there are many historical examples of radical institutional change. In the 20th century alone there have been many coups and revolutions that have fundamentally altered the nature of property rights institutions. Hence the importance of the second question in the title of this paper is to emphasize that the answer to the first question is non-trivial.

This paper aims to address both of these questions in a unified theory of institutional persistence and change where agents are able to make frequent changes in the prevailing institution but most often they choose not to. The central idea of this paper is that both institutional persistence and change depend on the evolution of political and economic

¹Acemoglu and Johnson (2005) find that the quality of the protection of private property from elite expropriation has important effects on long-run economic growth. Protection from expropriation is also found to be more important for growth than other legal institutions governing private contracting.
inequality between different groups in a society. Unequal economic outcomes create an endogenous elite class of agents, who use their economic power to secure political power, and then use their political power to establish institutions that reinforce and protect their economic power. As a result, institutional persistence is driven by a feedback effect whereby economic inequality leads to political inequality which in turn maintains or increases economic inequality through the choice of institution. Institutional change occurs only when this feedback effect is sufficiently disrupted. An illustration of the institutional feedback effect is presented in figure 1.

Central to the theory described above is the notion that human societies can be divided into distinct communities which share some common features and who compete politically with other communities defined by different features. Many political economy models for example, impose a distinction between a community of rich agents and a community of poor agents. The interaction of these two communities then determines a political outcome between them. Following from the literature on ethnic fractionalization within countries, communities can also be divided along racial, linguistic and religious lines. Whichever way communities are divided, the important idea is that the nature of the political interaction between these communities determines the nature of institutions that span them.

First, consider the case where different communities share political power and control of institutions in a relatively equal way. In Mauritius for example, a Hindu majority, together with Franco-Mauritian, Creole and Muslim minorities coexist in relative equality. Economic inequality between these communities is low, all groups are represented politically and a well-functioning system of property rights extends across all groups equally. A similar situation can be seen in Botswana, one of the best performing economies in Sub-Saharan Africa. The Tswana tribes in the country are cooperative, relatively equal politically and economically, and property rights are well-established.

On the other hand, there are also examples of societies in which one community enjoys an elite status over other communities, an elite status that is reflected in economic and political inequality. In Bolivia for example, a white community representing roughly 10%
of the total population dominates other native communities that account for the rest of the population. The whites are significantly wealthier and better educated than the native groups, and they also dominate the political and legal system in the country. Similar dominance can be seen in Ethiopia, where the Amhara tribe, accounting for only 6% of the total population, dominates all other groups economically and politically. Non-elite communities in both Bolivia and Ethiopia are vulnerable to expropriation by elites.

Finally, there are examples of societies where the division between communities creates a stalemate where there is little political interaction and few institutions that span across the communities. In Nigeria for example, there is a sharp divide between the Muslims in the north of the country and the Christians in the south. Interaction between the groups is limited, and very different institutions exist in each of the two regions of the country.

All of the above examples suggest that the institutions in a society depend on the interaction between the communities within that society. But these examples can also help to illustrate how the institutional feedback effect can make these institutions persist over time. In Mauritius for example, there was some concern in the lead up to its independence from Britain that the Hindu majority would attempt to dominate the other minorities in the country. However, a constitutional conference in 1965 created a political compromise in which the rights of all groups would be protected. This political outcome established well-defined and well-enforced property rights which has reinforced relative economic and political equality between all groups in the country ever since. In Bolivia on the other hand, the economic and political dominance of the whites in the colonial period led to the establishment of institutionalized expropriation of labour effort of the native communities, land ownership restrictions and literacy requirements for voting (to exclude natives) such that economic and political inequality between whites and natives has been maintained ever since.4

In line with the examples cited above, this paper presents a model whereby two communities of agents interact politically to establish institutions that in turn affect future economic outcomes. In the model, agents inherit land and technological knowledge which

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4Native communities have made some political gains in recent years, but inequality between the groups is still high.
they combine with labour effort to produce output. However, before output can be harvested, agents have the opportunity to steal any unprotected output from agents in the opposing community (at zero cost.) It is this opportunity to steal output that motivates agents to spend resources to protect their property, either by defending it privately, or by establishing formal property rights through an institution that spans both communities.

Agents can also invest in the acquisition of additional technological knowledge that can be bequested to the next generation. Information sharing within communities ensures that technological knowledge advances at the same rate for all agents within the same community, but not necessarily across communities. In fact, idiosyncratic knowledge accumulation shocks in general lead to different paths of development and hence economic inequality between the two communities. This economic inequality then reflects the bargaining power of the two communities in the political arena, thereby influencing the choice of institution (if any).

The institution in turn affects the investment decisions of agents in both communities, creating a feedback loop whereby political outcomes result in institutions that affect economic outcomes, which in turn affect political outcomes and so on. The endogenous persistence of institutions in the model arises from the fact that an institution chosen in one period affects subsequent economic outcomes in a way that reinforces the existing institutional arrangement in the next period. For example, under an institution that promotes equality between the two communities, agents in both communities make the same investment in technological knowledge. This in turn increases the probability that the communities will remain relatively equal, so that they will choose the same equality-promoting institution in the next period. On the other hand, under an institution that promotes inequality, agents in one community are able to invest more in technological knowledge than the other. This reinforces (or even increases) the existing inequality such that the same inequality-promoting institution is chosen in the next period.

Even though this institutional feedback effect contributes to the persistence of the prevailing institution, change can still occur because of the uncertain return associated with the process by which new technological knowledge is acquired. Because of this variability,
it is still possible for the relative technologies of the two communities to diverge under an institution that promotes equality and to converge under an institution that promotes inequality. Hence a change in the prevailing institution occurs only when a particular realization of the technological development process is able to counteract the institutional feedback effect. However a key feature of the model is that this feedback is cumulative over time. In other words, the longer a particular institution is in place the less likely it is that this feedback effect can be overcome and institutional change can occur.

Given the link between property rights institutions and the political institutions implied by the model, this increasing persistence over time is in line with evidence from the Polity IV data on political institutions from 1800-2003. For example, one of the key measures in this dataset, constraints on executive power, provides an empirical measure of the quality of property protection from expropriation. By defining institutional change as a minimum change in these political measures within a short period of time, one can map out how long particular property rights institutions are in place within countries over time. This in turn allows for the calculation of survival probabilities of institutions, conditional on their current age, as presented in Table 1. Noting that these survival probabilities tend to increase in downward diagonal directions, one can see that institutions become more persistent the longer they are in place. For example, the probability that a 60 year old institution survives another 20 years is over 65% higher than the probability that a new institution survives an initial 20 years. This confirms the key feature of institutional persistence found in the model.

The remainder of the paper sets out a model that describes institutional formation, persistence and change within a single framework. Because the model is relatively simple, it also provides a promising platform from which to explore other related questions. For example, the effects of disease could affect the populations of different communities and thereby lead to different institutional outcomes between them. The model could also be extended to allow for communities that expand their land holdings over time, so that in time many communities could be interacting with each other at the same time, which would

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5Table 1 is an updated version of a similar table in Gurr(1974).
6Sachs (2003) argues for example, that the incidence of malaria is an important factor in explaining cross country income differences.
then require more complex institutional arrangements. The rest of the paper is set out as follows: section 2 presents the basic model, while sections 3 and 4 examine institutional choice and persistence respectively. Section 5 concludes.

2 Model

2.1 Description of Communities

To begin, suppose that there are two communities of equal size inhabiting a land region of size $2M$ that can be used for food production. Agents in both communities live for two periods and there is no population growth; young and old generations in each community are of equal size (normalized to 1) in all time periods $t$.\(^7\) Old agents in each community live as adults and make all economic decisions. Each old agent has one young agent who lives as a child and does nothing but consume. Land within each community is divided equally among all adult agents. Unless specified, all references to agents in the model refer to adult agents.

Community identity is observable so that all agents can discern from which community another agent is from. Community identity is important in the model because information disseminates perfectly among agents within the same community, but not between agents of different communities. Agents born into community $i$ at time $t$ inherit technological knowledge $a_{it}$ and land $M$ from the previous generation, and are endowed with a unit of labour effort. They make decisions to maximize utility given by

$$u_{it} = (1 - \gamma) \log(c_{it}) + \gamma E_t \log(a_{it+1}M) \tag{1}$$

where $c_{it}$ is consumption (shared between agents and their children) and $a_{it+1}$ is the level of technological knowledge available for use in the next period. Hence a bequest motive causes adult agents to pass on wealth $a_{it+1}M$ to their children.

Agents in community $i$ combine inherited land $M$ and technology $a_{it}$ together with labour effort $L_{it}$ to produce output. Output is produced according to

$$y_{it} = L_{it}^{1-\alpha}(a_{it}M)^\alpha$$

\(^7\)An exogenous rate of population growth could be added to the model without affecting any of the major results. A constant population is assumed for simplicity.
so that $y_{it}$ is zero unless both $L_{it}$ and $a_{it}$ are positive.

Agents can use their endowment of labour effort in three different activities. First as already mentioned, agents can work in production, supplying labour $L_{it}$ to produce output for consumption $c_{it}$. Second, agents can make an investment $\phi_{it}$ in education and research in order to acquire technological knowledge $a_{it+1}$ which can be bequested to the next generation. How much knowledge can be acquired by an agent depends on the magnitude of his investment $\phi_{it}$ and also on the technological frontier to which he is exposed. Because information disseminates perfectly among agents within the same community, the technological frontier faced by an agent in community $i$ is the highest technology among all agents in community $i$, denoted by $\bar{a}_{it}$. The technological knowledge that an agent in community $i$ carries into the next period is given by

$$a_{it+1} = (\epsilon_{it} A_t \phi_{it})^\mu (\bar{a}_{it})^{1-\mu}$$

(2)

where $\mu \in (0, 1)$ and $A_t$ denotes the worldwide technological knowledge frontier at time $t$.

The other key element in (2) is a community-specific knowledge accumulation shock $\epsilon_{it}$, that reflects the uncertainty that is inherent in the development of new technology; it is uniformly distributed on the interval $[\epsilon_L, \epsilon_H]$ with $E(\epsilon_{it}) = \bar{\epsilon}$. Because of perfect knowledge diffusion within a community, this shock is common among agents within a community, but differs across communities. Agents in each community know the expected value of the shock when they make their investment decisions, but $\epsilon_{it}$ is realized only after these decisions are taken.8

Another key feature of the model is that agents can steal any unprotected output from other agents before it is harvested at zero cost, yielding a return $\delta y_{it}$, where $\delta < 1$. Because information disseminates perfectly within a community, I assume that the certainty of capture and punishment within a community is sufficient to dissuade agents from stealing from agents within their same community.9 Hence, agents only have an incentive to steal

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8Note that agents will always choose a positive level of investment in technological knowledge since no investment means that $a_{it+1} = 0$. Implicitly, $\phi_{it}$ includes investment in educating young agents. If $\phi_{it} = 0$ then a totally uneducated younger generation would be unable to produce any output from the community’s land in the following period.

9The important point is that the mechanisms in place to protect private property within a community cannot operate between communities. For example, at a primitive level the fact that members of another community are strangers renders trust and accountability ineffective. At more advanced levels of develop-
output from agents outside of their community. This output theft can be averted only if an agent pays a protection cost $\tau_{it}$.

The exact form of this protection cost $\tau_{it}$ will depend on political interaction between the two communities. Following from this interaction the protection cost $\tau_{it}$ could be a private cost whereby agents actively guard their own output, or it could be a tax whereby agents contribute to an institution that protects private property (ie, police force, justice system etc.) An important point here is that no matter what the form of the protection cost, it must be paid in every period.\footnote{North (1990) for example, argues that it requires resources to enforce the rules of the game.} These costs imply that economic considerations will determine political outcomes, which is crucial for an institutional feedback effect.

Given an endowment of a unit of labour time, agents are then constrained according to

$$L_{it} + \phi_{it} + \tau_{it} \leq 1$$

### 2.2 Timing

The timing within each period is as follows. At the beginning of period $t$ within each community, agents enter adulthood with an endowment of labour effort and inherited land and technology from the previous period. There is no state uncertainty at the time when decisions are made.

**Stage 1:** Observing technological knowledge across both communities, agents can engage in political interaction regarding the protection of unharvested output. The outcome of this political interaction determines the protection cost $\tau_{it}$ that agents in each community must incur.

**Stage 2:** Agents allocate the remainder of their labour endowment between production and investment in new technological knowledge.

**Stage 3:** Agents from community $i$ have the opportunity to steal any unprotected output from community $j$ before it is harvested.

**Stage 4:** Output is harvested, agents consume, and the community-specific shocks $\epsilon_{it}$ are realized, determining the technological knowledge levels for the next period.
The timing within each period is summarized in figure 2.

2.3 Political Interaction

The ability to steal unharvested output from agents in the other community is the motivating factor for political interaction between the two communities in each period. The different outcomes that can result from this political interaction are illustrated in figure 3. As seen in the figure and in the detailed discussion below, there are four possible political outcomes, two of which result in the establishment of a property rights institution. How political interaction between the two communities results in one of these political outcomes can be described as follows.

In stage 1 of period $t$, agents in the technologically elite community make an offer to agents in community from the set of political outcomes denoted by $\{D, P, T, W\}$. Suppose without loss of generality that $a_{it} \geq a_{jt}$ so that community $i$ is the elite community and community $j$ is the non-elite community. Agents in community $j$ can then either accept the offer, reject the offer or counter with an offer of war. Hence given an offer by community $i$, the set of choices available to agents in community $j$ is given by $\{A, R, W\}$. If community $j$ accepts, then the offer made by community $i$ is the political outcome. If community $j$ counter offers with war, then the political outcome is $W$. Assumption 1 describes what happens if agents in community $j$ reject the offer from community $i$.

**Assumption 1:**

(i) If agents in community $j$ reject an offer of $D$ or $P$ from community $i$, then the political outcome is $D$.

(ii) If agents in community $j$ reject an offer of $T$ or $W$, then the political outcome is $W$.

Figure 4 provides an illustration of the intensive form of the political interaction game. At the end of each branch of the game tree is the resulting political outcome. The payoffs to agents in each community at the end of each branch of the tree are simply the expected utilities of agents in each community conditional on $\tau_{it}$ and $\tau_{jt}$ as determined by the political outcome. The nature of these protection costs under each political outcome are described

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11 If $a_{it} = a_{jt}$ then the preferences of agents in both communities will be identical. In this case the identity of the elite community is irrelevant.
below.

**Private Defense (D)**

First and most basic, agents in the two communities may not have any kind of formal institutional arrangement between them. In this case, the protection cost $\tau_{it}$ incurred by agents is the cost of guarding unharvested output. Formally, let $\tau_{it} = \tau_{it}(y_{it})$, where the cost of private defense depends positively on the amount of unharvested output. For simplicity, there is no option for agents to guard only some fraction of their output.

Note that because $y_{it}$ is an increasing function of $a_{it}$, the cost of private defense is increasing in the level of technology available to agents in community $i$. This in turn implies that technological progress makes private defense increasingly less attractive relative to other political alternatives.

**Power Sharing (P)**

In a power sharing arrangement, agents from each community construct an institution encompassing both communities, in which agents are taxed to pay for the protection of private property. In this case, agents agree to contribute equally to the costs required to set up and enforce a political and legal system which ensures this protection of property. Let the total cost of this system (including maintenance and enforcement costs) be given by $\tau^p$, so that the protection cost, which in this case is the share of the cost of the property rights institution paid by an agent, is equal to $\tau_{it} = \tau^p / 2$.

Under this arrangement, the two communities are equal in their contribution and in their control of the established formal system. Included within $\tau^p$ are the costs of various checks and balances on power that are necessary to prevent agents from one community from exploiting agents in the other community.\(^{12}\)

**Takeover (T)**

In a takeover arrangement, agents from the elite community construct and enforce an institution encompassing both communities, so that only agents from the elite community are taxed to pay for the protection of private property, at a cost of $\tau^T$. The formal system

\(^{12}\)Democracies for example require substantial costs for keeping checks and balances on power. For example, non-partisan costs for a federal election in Canada (wages for election officials, printing ballots etc.) run over $200 million. (Source: CBC (2004))
of property protection is the same as in power sharing, except that there are no checks or balances on the power of agents in the elite community; hence, $\tau^T < \tau^p$.

In this case, agents in the non-elite community who have no control of the political system are vulnerable to expropriative behaviour. Suppose without loss of generality, that community $i$ is the elite community and thereby contributes to and controls the system of property rights protection. Then let $\beta_{it}$ be the endogenous fraction of labour effort that agents in community $i$ can expropriate from community $j$.

The expropriation activity of community $i$ is costly in itself, so that for time $\beta_{it}$ extracted from community $j$, the additional resources available to community $i$ are given by $\beta_{it}(1 - q_{it})$, where the expropriation cost $q_{it} \in (0, 1)$ if $a_{it} \geq a_{jt}$ and $q_{it} = 1$ otherwise. This last assumption implies that expropriation is prohibitively costly unless there is an advantage in technological knowledge, as there is with agents in the elite community relative to agents in the non-elite community.

If community $i$ is the elite community, then the protection cost incurred by community $j$ is simply equal to the amount of resources that it loses through expropriation. Hence $\tau_{jt} = \beta_{it}$. The protection cost paid by community $i$ includes the institutional cost of the property rights system, minus the bonus resources that it is able to extract from community $j$. Hence $\tau_{it} = \tau^T - \beta_{it}(1 - q_{it})$. Both communities have full information about each other, and so community $i$ will extract the highest amount of resources from community $j$, up to an exogenously imposed maximum of $1/2$, so long as community $j$ is still better off in a takeover arrangement than if it were to exercise its outside option. This outside option is to fight a war, the fourth possible political outcome described below.

**War (W)**

The final political option available to agents in the two communities is to fight a war. The rationale for fighting a war is to solve the problem of protecting property from agents in the other community by eradicating them. In this case, agents in both communities pay a cost $\tau^w$ to fight each other. This fraction of time is exogenous and constant across the two communities.

Given that the two communities have the same population, and that both make the
same investment $\tau^w$ in fighting the war, the only factor that determines the probability of winning a war is the relative technology of the two communities. Let $p(\frac{a_i}{a_j})$ be the probability that community $i$ wins a war with community $j$ during time period $t$. Then I assume that $p(1) = 1/2$ and that $p(\frac{a_i}{a_j})$ is increasing in the technology ratio $\frac{a_i}{a_j}$. For simplicity, I assume that war is always decisive when it is fought so that $p_{it} + p_{jt} = 1$ (ie, there are no stalemates.)

The outcome of the war is known only at the end of the period, after output has been harvested and consumption has taken place. At this point, agents in the losing community suffer a technology loss, whereby current technology $a_{it}$ falls to some base level $a_0$. This technology loss reduces the level of technological knowledge that can be bequested to the next generation. At the end of the period, agents in the losing community are absorbed into the winning community and lose their community identity. As a result, there is no interaction between the two communities after a war is fought. Hence the primary purpose for war is as a threat point and an outside option to govern how much agents in the elite community would be able to expropriate from agents in the non-elite community without provoking a war instead.

In each of the above scenarios, the immediate outcome from political interaction between agents in the two communities is a cost parameter that must be incurred by agents in each community to address the problem of protecting unharvested output. These cost parameters are restricted such that all four political outcomes could be preferred by agents in the elite community under different circumstances. These restrictions are summarized as follows.

**Assumption 2:** The relative values of the different protection costs are as follows:

$$\tau^d(a_0M) < \tau^w < \frac{\tau_p}{2} < \tau^T < \tau^p$$

The first of the inequalities ensures that war is not preferred by the elite agents in the first period. Given that there is no interaction between the two communities after a war is fought, an initial choice for war is necessarily avoided. The second inequality ensures that

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13 This restriction is imposed only for simplicity and could be relaxed in principle. If war is always decisive, then the model as described stops once war is chosen. Although I allow for the possibility that war can occur, the key role of war in the model is as a threat point.

14 This feature of the model follows from Diamond (1997) who notes that war, and especially the threat of war, plays a key role in institutional development.
war could be preferred at some point by the elite agents. Since war risks the chance of a technology loss, its initial up front cost must be less than power sharing (a safe choice) or else agents would never prefer it.

The third inequality ensures that power sharing could be preferred to takeover. Since takeover incorporates the benefits of expropriation, it would always be preferred to power sharing if \( \tau^T \) were less than or equal to \( \tau^p / 2 \). The fourth inequality, as mentioned earlier simply incorporates the idea that checks and balances on power to prevent expropriative behaviour are costly.

### 2.4 Equilibrium

At the beginning of each period \( t \), agents in both communities take the inherited technology levels \( a_{it} \) and \( a_{jt} \) as given. These two technology levels in turn determine \( \beta_{it}, \beta_{jt}, q_{it}, q_{jt}, p_{it} \) and \( p_{jt} \). Given these values, an equilibrium in period \( t \) is a political outcome \( I_t \) drawn from the set \( \{D, P, T, W\} \) and a pair of investment levels \( (\phi_{it}(I_t), \phi_{jt}(I_t)) \) such that:

(i) Agents in both communities maximize expected utility in period \( t \) subject to constraints;

(ii) \( I_t \) is a sub-game perfect Nash equilibrium (SPNE) of the political interaction game;

(iii) A no-war constraint is satisfied: as of period \( t \), \( I_s \neq W_s \) for all \( s < t \).

### 2.5 Optimal Investment

If any output is left unprotected, this output will be stolen by agents in stage 3 of the period, leaving victimized agents with no output and ultimately no consumption. Hence, all agents will choose to incur the protection cost \( \tau_{it} \) in order to protect their output.

Moving backward to stage 2 of the period where investment decisions are made, political interaction has already taken place and the parameters \( \tau_{it} \) and \( \tau_{jt} \) resulting from the political outcome are known.

Solving the technology investment decision problem of agents in community \( i \) yields the optimal savings rule

\[
\phi^*_{it} = \frac{\mu\gamma}{1 + \mu\gamma}(1 - \tau_{it})
\]
This result shows the effect of the political interaction on the investment decisions of agents: the higher the protection cost incurred by the agents in a particular community, the less that they will invest in new technology.

3 Equilibrium Political Outcomes

In this section I examine stage 1 of the period, where agents in the two communities interact politically and make decisions about what institutional arrangement (if any) should exist between them. Assumption 3 allows for a simple characterization of equilibrium in the political interaction game.

Assumption 3: The cost of private property defense \( \tau^d \) is always sufficiently responsive to changes in technology such that

\[
\left| \frac{\partial \tau^d(a_{it}M)}{\partial a_{it}} \right| \geq \frac{(1 - \tau^d(a_{i0}M))(1 - \mu)\gamma}{ka_{i0}}
\]

Proposition 1: Given two communities \( i \) and \( j \) where \( a_{it} \geq a_{jt} \), and assumptions 1 - 3, the political outcome \( I_t \) between the two communities in any period \( t \) is given by:

(i) The preferred political outcome of agents in community \( i \) drawn from the set \( \{D, P, T, W\} \) when \( \tau^d(a_{jt}M) \geq \tau^p / 2 \);

(ii) The preferred political outcome of agents in community \( i \) from the set \( \{D, T, W\} \) when \( \tau^d(a_{jt}M) < \tau^p / 2 \).

Furthermore, if the political outcome is \( T \), then the labour effort extracted from community \( j \) will be such that agents in community \( j \) are just indifferent between accepting \( T \) and counter offering with \( W \).

Proof: See Appendix A.

Using proposition 1, define \( I_t \) to be a function of the state of the world from community \( i \)'s perspective. The state of the world at time \( t \) is given by \( (a_{it}, a_{jt}, \bar{W}) \), where \( \bar{W} \) is an indicator which takes the value of 1 if war has been fought in a past period, and 0 otherwise. Recall that a war is always decisive, so that \( \bar{W} = 1 \) means that the elite community is the only remaining community. In this case there is no property rights problem and the agents’ decisions are trivial. (For the rest of the paper I ignore this uninteresting case and assume that \( \bar{W} = 0 \).)
It is important to note that with the exception of $\tilde{W}$, no institutional details appear as state variables. This reflects the fact that there is no exogenous persistence in political outcomes, and so the political outcome is ultimately a choice variable for agents in the elite community in every period.\footnote{In fact, the absence of explicit institutional effects from prior periods or fixed costs to changing institutions both work against institutional persistence.}

At this point I partition the analysis into two separate stages: initial development and later development. I do this by noting that there exists some critical level of technology $a^{dp}$ such that $\tau^d(y(a^{dp} M)) = \tau^p / 2$. At this critical level of technology, agents are just indifferent between private defense and power sharing. Below this level communities will always choose private defense over power sharing, and above this level communities will always choose power sharing over private defense.

3.1 Phase 1: Initial Development ($a_{jt} < a^{dp}$)

At the beginning of period $t$, agents in community $i$ (the technologically elite community) compare the expected relative payoff of private defense with the expected relative payoffs of takeover and war with community $j$. Because of community $j$’s low level of technology, agents in community $i$ will always prefer private defense to a power sharing arrangement, so that community $i$’s political choice set is given by case (ii) in proposition 1.

To determine what political outcome will be chosen for any given combination of technologies, consider the following three functions. First define

$$a^{dw}_{it} = a^{dw}_{it}(a_{jt})$$

such that $u^D_{it} = u^W_{it}$. For any technology level $a_{jt} < a^{dp}$, this function gives the technology level $a_{it}$ such that community $i$ is just indifferent between private defense and fighting a war. Note that increases in $a_{it}$ decrease the payoff to both private defense and war, but by assumption 3 the payoff to private defense falls faster. Hence, for any technology level above $a^{dw}_{it}$, community $i$ prefers war to private defense, and for any technology level below $a^{dw}_{it}$, community $i$ prefers private defense to war.

Next define the function

$$a^{dt}_{it} = a^{dt}_{it}(a_{jt})$$
such that \( u^p_i = u^T_i \). For any technology level \( a_{jt} < a^{dp} \), this function gives the technology level \( a_{it} \) such that community \( i \) is indifferent between private defense and takeover. Note that for \( a_{it} > a^{dt}_{it} \), the payoff to private defense will fall and the payoff to takeover will rise. Hence, community \( i \) prefers takeover to private defense above \( a^{dt}_{it} \) and similarly prefers private defense to takeover when below \( a^{dt}_{it} \).

Finally, define the function

\[
a^{tw}_{it} = a^{tw}_{it}(a_{jt})
\]

such that \( u^T_i = u^W_i \). In other words, for any technology level \( a_{jt} < a^{dp} \), this function gives the technology level \( a_{it} \) such that community \( i \) is just indifferent between takeover and war. Note that for \( a_{it} > a^{tw}_{it} \), the payoff from both institutional choices will rise. However, higher levels of \( a_{it} \) have a negative level effect on the payoff to war. This means that community \( i \) will prefer takeover to war above \( a^{tw}_{it} \) and similarly prefer war to takeover when below \( a^{tw}_{it} \).

Using the critical values described above leads to the following lemma and proposition.

**Lemma 1:** For all technology levels \( a_{jt} \), the three critical values defined above are related in one of 3 possible ways: (i) \( a^{tw}_{it} > a^{dt}_{it} > a^{dw}_{it} \), (ii) \( a^{dw}_{it} > a^{dt}_{it} > a^{tw}_{it} \) or (iii) \( a^{tw}_{it} = a^{dt}_{it} = a^{dw}_{it} \).

**Proof:** See Appendix A.

**Proposition 2:** Given \( a_{jt} < a^{dp} \), assumptions 1 - 3 and Lemma 1, if a war has not occurred in any previous period, then the political outcome between communities \( i \) and \( j \) in period \( t \) is given by

\[
I_t = \begin{cases} T & a_{it} > \max\{a^{dt}_{it}, a^{tw}_{it}\} \\ W & a^{dw}_{it} < a_{it} < a^{tw}_{it} \\ D & \text{otherwise} \end{cases}
\]

(4)

**Proof:** See Appendix A.

Proposition 2 is illustrated in Figure 4. Since by assumption community \( i \) is always the technologically elite community, only the region above the 45 degree line is relevant. Also, since in the initial development case \( a_{jt} < a^{dp} \), the figure maps out only the region of possible technologies below this critical value.

The important result of proposition 2 and figure 4 is that the institution chosen depends on both the technology levels of the two communities and the technology ratio between them. In general, private defense continues as the institution of choice when the technology ratio is
close to 1. War occurs when the technology ratio becomes large at a low level of technological development. Household $i$ chooses to takeover community $j$ in the intermediate case, when technology is reasonably advanced and there is a sufficiently large technology ratio.

### 3.2 Phase 2: Later Development ($a_{jt} \geq a_{it}^{dp}$)

Once both communities’ technology reaches the level $a_{it}^{dp}$ or greater, a power sharing arrangement will always be preferred to private defense. In this case, power sharing becomes the benchmark against which war and takeover are compared.

First define the function

$$a_{it}^{pw} = a_{it}^{pw}(a_{jt})$$

such that $u_{it}^{P} = u_{it}^{W}$. For any technology level $a_{jt} \geq a_{it}^{dp}$, this function gives the technology level $a_{it}$ such that community $i$ is just indifferent between power sharing and fighting a war. Note that increases in $a_{it}$ decrease the payoff to war. Hence, for any technology level above $a_{it}^{pw}$, community $i$ prefers power sharing to war, and for any technology level below $a_{it}^{pw}$, community $i$ prefers war to power sharing.

Next define the function

$$a_{it}^{pt} = a_{it}^{pt}(a_{jt})$$

such that $u_{it}^{P} = u_{it}^{T}$. For any technology level $a_{jt} \geq a_{it}^{dp}$, this function gives the technology level $a_{it}$ such that community $i$ is just indifferent between power sharing and takeover. Note that for $a_{it} > a_{it}^{pt}$, the payoff to takeover will rise while the payoff to power sharing is unchanged. Hence, community $i$ prefers takeover to power sharing above $a_{it}^{pt}$ and similarly prefers power sharing to takeover when below $a_{it}^{pt}$.

Using these two critical values described above leads to proposition 3.16

**Proposition 3:** Given $a_{it}, a_{jt} \geq a_{it}^{dp}$, assumptions 1 - 3 and Lemma 1, if a war has not occurred in any previous period, then the political outcome between communities $i$ and $j$ in period $t$ is given by

$$I_t = \begin{cases} 
T & a_{it} > \max\{a_{it}^{pt}, a_{it}^{pw}\} \\
W & a_{it}^{pw} < a_{it} < a_{it}^{pw} \\
P & \text{otherwise}
\end{cases} \quad (5)$$

16I could also include a lemma at this point, which would be a simple restatement of Lemma 1 except with critical values $a_{it}^{pt}$ and $a_{it}^{pw}$ instead of $a_{it}^{dp}$ and $a_{it}^{dw}$. 
Proof: See Appendix A.

Proposition 3 is illustrated in Figure 5. Once again the political outcome chosen depends on both the technology levels of the two communities and the technology ratio between them. In general, power sharing is the outcome of choice as long as the technology ratio is small. Takeover on the other hand occurs when the technology ratio becomes sufficiently large. In the particular case shown in figure 4, war is never chosen in the later development stage. This is because in the case illustrated in figure 3, \(a^{dp}\) is so large that war is never optimal for community \(i\) no matter how large the technology ratio is.

Figure 6 combines propositions 2 and 3 by putting both the initial and later development cases together. Note that all four political outcomes can occur depending on the absolute and relative technology levels of the two communities. However, beyond a certain level of development, only power sharing and takeover are viable options. In general: private defense will be chosen given low levels of technological development and a small technology ratio, power sharing will be chosen given high levels of technological development and a small technology ratio, war will be chosen given low levels of technological development and a large technology ratio, and takeover will be chosen given high levels of technological development and a large technology ratio.

3.3 Probabilities of Different Political Outcomes

Given the existing technology levels of the two communities and the current institutional arrangement (if any), it is possible to calculate the probability that a particular arrangement will be chosen in the next period. To do this, we first define a joint cumulative distribution \(F\) over possible values of \(a_{it+1}, a_{jt+1}\), conditional on \(a_{it}, a_{jt}\). In other words, \(F(a^1, a^2)\) is the probability that \(a_{it+1} \leq a^1\) and \(a_{ij+1} \leq a^2\), conditional on \(a_{it}\) and \(a_{jt}\). This leads to the following proposition:

**Proposition 4:** Given the conditional joint cumulative distribution function \(F\) described above, the conditional probabilities of different political outcomes are as follows:

(i) Private defense:

\[
P(D_{t+1} | a_{it}, a_{jt}) = F(\min\{a_{it+1}^{dw}, a_{it+1}^{dt}\}, a^{dp})
\]
(ii) Power sharing:

\[ P(P_{t+1}|a_{it}, a_{jt}) = F(\min\{a_{it+1}^{pw}, a_{jt+1}^{pt}\}, \infty) - F(\min\{a_{it+1}^{pw}, a_{jt+1}^{pt}\}, a^{dp}) \]

(iii) War:

\[ P(W_{t+1}|a_{it}, a_{jt}) = F(a_{it+1}^{tw}, a^{dp}) - F(a_{it+1}^{dw}, a^{dp}) + F(a_{it+1}^{tw}, \infty) - F(a_{it+1}^{pw}, \infty) - F(a_{it+1}^{tw}, a^{dp}) + F(a_{it+1}^{pw}, a^{dp}) \]

(iv) Takeover:

\[ P(T_{t+1}|a_{it}, a_{jt}) = F(\infty, a^{dp}) - F(\max\{a_{it+1}^{tw}, a_{jt+1}^{dt}\}, a^{dp}) + 1 - F(\infty, a^{dp}) - F(\max\{a_{it+1}^{tw}, a_{jt+1}^{dt}\}, \infty) + F(\max\{a_{it+1}^{tw}, a_{jt+1}^{dt}\}, a^{dp}) \]

Figure 7 provides an illustration of Proposition 4. The small dashed box shown gives the possible combinations of technology levels in the next period given current technology levels \( a_{it} \) and \( a_{jt} \). This range of technologies then determines the probability that any particular political outcome is chosen in the next period. For example, the probability that takeover is chosen in period \( t + 1 \) is simply the probability that the two technology levels \( a_{it+1} \) and \( a_{jt+1} \) end up in the region \( T \). In the case illustrated in figure 7, the political outcome in period \( t \) is private defense (\( D \)) and that is also the institutional choice most likely to be chosen in period \( t + 1 \).\(^{17}\) Note that there is a small probability that takeover or power sharing will be chosen, and there is zero probability that war will be chosen.

4 Institutional Persistence

In this section I examine how the institutions created by both the power sharing and takeover outcomes in the political interaction game can persist over time, even in the absence of exogenous lock-in effects or large switching or setup costs. Specifically, I examine the advanced state of development in the model where only power sharing and takeover remain viable political options, to address the following two questions. First, how might either

\(^{17}\)Since the technology shock \( \epsilon \) is uniformly distributed, the two technology levels \( a_{it+1} \) and \( a_{jt+1} \) will be uniformly distributed within the range illustrated. Since most of the area of the dashed box is within the \( D \) range, private defense is most likely in the next period.
institution persist over time, even if every successive generation of communities has the opportunity to reevaluate the prevailing institution? Second, upon finding persistence of institutions over time, what, if anything, might cause a prevailing institution to change?

First define two critical values of the technology ratio between the two communities. Let \((\frac{a_i}{a_j})')\ be the technology ratio such that community \(i\) is just indifferent between power sharing and takeover, so that

\[
\tau^p/2 = \tau^T + \beta'_i(1-q_i)
\]

where \(\beta'_i = \beta_i((\frac{a_i}{a_j})')\) and \(q'_i = q_i((\frac{a_i}{a_j})')\). Note that when the technology ratio is below \((\frac{a_i}{a_j})')\, power sharing will be chosen, and when the technology ratio is above \((\frac{a_i}{a_j})')\, takeover will be chosen.

Next define \((\frac{a_i}{a_j})'')\ to be the highest sustainable technology ratio under a takeover arrangement. To solve for \((\frac{a_i}{a_j})'')\, note first that the evolution of the technology ratio under takeover is given by

\[
\frac{a_{it+1}}{a_{jt+1}} = \left(\frac{\epsilon_{it}}{\epsilon_{jt}}\right)\mu\left(1 - \tau^T + \beta''_i(1-q'_i)\right)\mu\left(\frac{a_{it}}{a_{jt}}\right)^{1-\mu}
\]

where \(\beta''_i = \beta_i((\frac{a_i}{a_j})'')\) and \(q''_i = q_i((\frac{a_i}{a_j})'')\). Since the ratio \((\epsilon_{it}/\epsilon_{jt})\) is equal to 1 (on average), the technology ratio will remain constant (on average) under takeover when

\[
(\frac{a_{it}}{a_{jt}})'' = \frac{1 - \tau^T + \beta''_i(1-q''_i)}{1 - \beta''_i} = \frac{\phi_{it}}{\phi_{jt}}
\]

Note that in the above equation, the left hand side is the current technology ratio and the right hand side is the current investment ratio. As long as the current investment ratio is larger than the current technology ratio, the technology ratio will continue to rise over time under takeover. If the investment ratio is smaller than the current technology ratio, the technology ratio will fall over time. Hence under takeover, there is convergence to the highest sustainable ratio \((\frac{a_i}{a_j})'')\, where the investment ratio and the current technology ratio are equal.

**Lemma 2:** For all time periods and at all levels of development

\[
(\frac{a_{it}}{a_{jt}})' < (\frac{a_{it}}{a_{jt}})''
\]
Lemma 2 is illustrated in figure 8. Note that the gap between \((\frac{a_{it}}{a_{jt}})')\) and \((\frac{a_{it}}{a_{jt}})'')\) grows as household \(j\)'s technology increases. This is because as community \(j\) becomes more advanced, war becomes a less attractive political alternative, so that community \(i\) is able to extract a larger \(\beta_{it}\) under takeover. This in turn creates a larger investment ratio under takeover (causing \((\frac{a_{it}}{a_{jt}})'')\) to rise) while at the same time making takeover more attractive relative to power sharing (causing \((\frac{a_{it}}{a_{jt}})'\) to fall). Once community \(j\) reaches a sufficiently high level of technological advancement, \(\beta_{it}\) is always at its maximum level \((1/2)\) under takeover and so both critical values are constant.

To simplify the analysis in the rest of this section, I examine institutional persistence given a sufficiently high level of development so that \((\frac{a_{it}}{a_{jt}})')\) and \((\frac{a_{it}}{a_{jt}})'')\) are constant over time (the time subscripts are dropped in these two expressions from this point). A more general analysis of persistence over the entire range of development is presented in Appendix B. In order to understand how institutional persistence is driven by the relationship between institutions and investment behaviour, consider two possible cases: joint technological development and independent technological development.

4.1 Joint Technological Development

In order to isolate the effect of institutions on investment decisions I first suppose that if the political outcome in a period is either \(P\) or \(T\) then the education and research effort of both communities in the current period is connected in the following way:

\[
\epsilon_{it} = \epsilon_{jt} = \max\{\epsilon_{it}, \epsilon_{jt}\}
\]

In other words, suppose that both communities operate a university where technological knowledge development takes place in each period. The above assumption simply says that both communities can conduct education and research at both universities, and then take the return from the more successful of the two learning centres. In this case the technological knowledge development of the two communities is connected, but not identical. Even with the same return to research activity both communities maintain their individual technological evolution.
First consider persistence in power sharing over time. If \( a_{it} < \left( \frac{a_i}{a_j} \right)' \), so that \( P \) is the prevailing institutional arrangement in period \( t \), then the technology ratio between the two communities evolves according to

\[
\frac{a_{it+1}}{a_{jt+1}} = \left( \frac{\epsilon_{it}}{\epsilon_{jt}} \right)^{\mu} \left( \frac{\phi_{it}}{\phi_{jt}} \right)^{\mu} \left( \frac{a_{it}}{a_{jt}} \right)^{1-\mu} = \left( \frac{a_{it}}{a_{jt}} \right)^{1-\mu}
\]

Given that both communities invest the same amount in new technology and receive the same return to research activity, the technology ratio will fall over time. This brings the technology ratio further below \( \left( \frac{a_i}{a_j} \right)' \), ensuring that power sharing will be chosen in period \( t+1 \).

Now consider persistence in takeover. If \( a_{it} \geq \left( \frac{a_i}{a_j} \right)' \), so that \( T \) is the prevailing institutional arrangement in period \( t \), then the technology ratio between the two communities evolves according to

\[
\frac{a_{it+1}}{a_{jt+1}} = \left( \frac{\epsilon_{it}}{\epsilon_{jt}} \right)^{\mu} \left( \frac{\phi_{it}}{\phi_{jt}} \right)^{\mu} \left( \frac{a_{it}}{a_{jt}} \right)^{1-\mu} = \left( \frac{1 - \tau_T + (1/2)(1 - q_{it})}{1/2} \right) \left( \frac{a_{it}}{a_{jt}} \right)^{1-\mu}
\]

Although both communities receive the same return to technological accumulation, under takeover they invest different amounts. This will cause convergence to \( \left( \frac{a_i}{a_j} \right)'' \) above \( \left( \frac{a_i}{a_j} \right)' \), ensuring that takeover will be chosen in period \( t+1 \).

**Proposition 5:** Given joint technological development at a sufficiently high level of technological advancement, both power sharing and takeover are persistent over time and there is zero probability of institutional change.

A key component of persistence in the model is the feedback between economic and institutional outcomes. Under power sharing, investment levels in new technology are equal, causing technology levels to converge, which in turn reinforces power sharing in the next period. Under takeover, investment levels in new technology are unequal, causing technology levels to diverge, which in turn reinforces takeover in the next period. With joint technological development, there is no other process that can counteract this institutional feedback effect. Hence the prevailing institution will persist over time with no chance of an institutional change.
4.2 Independent Technological Development

Now consider the case where the technological development in each community remains subject to idiosyncratic shocks regardless of what institutional arrangement exists. In this case, each community can only use its own university. As a result, the evolution of the technology ratio depends both on the relative investment levels of the communities and on the shocks $\epsilon_{it}$ and $\epsilon_{jt}$ that affect the returns to technological knowledge accumulation. In this case it is possible that particular realizations of the two community specific shocks can counteract the institutional effects on relative investment levels, triggering an institutional change.

First consider persistence in power sharing over time. If $P$ is the prevailing institutional arrangement in period $t$, then the technology ratio between the two communities evolves according to

$$\frac{a_{it+1}}{a_{jt+1}} = \left( \frac{\epsilon_{it}}{\epsilon_{jt}} \right)^{\mu} \left( \frac{a_{it}}{a_{jt}} \right)^{1-\mu}$$

since both communities invest the same amount in new technology. We see then that the institutional arrangement will switch to $T$ in the next period only if

$$\frac{\epsilon_{it}}{\epsilon_{jt}} > \left[ \left( \frac{a_i}{a_j} \right) \frac{(a_i/a_j)^{\prime}}{(a_i/a_j)^{(1-\mu)}} \right]^{\frac{1}{\mu}}$$

In other words, even though both communities invest the same amount in technological knowledge accumulation under power sharing, if the returns to investment are sufficiently different, the technology ratio could rise enough so that the elite community chooses to implement a takeover arrangement in the next period.

Next consider persistence in takeover. If $T$ is the prevailing institutional arrangement in period $t$, then the technology ratio between the two communities evolves according to

$$\frac{a_{it+1}}{a_{jt+1}} = \left( \frac{\epsilon_{it}}{\epsilon_{jt}} \right)^{\mu} \left( 1 - \tau^T + \frac{1}{2} \right) \left( 1 - q_{it} \right) \left( a_{it} / a_{jt} \right)^{1-\mu}$$

We see then that the institutional arrangement will switch to $P$ in the next period only if

$$\frac{\epsilon_{it}}{\epsilon_{jt}} < \frac{1/2}{1 - \tau^T + \frac{1}{2} \left( 1 - q_{it} \right) \left( a_{it} / a_{jt} \right)^{(1-\mu)}}$$

In this case, even though both communities invest different amounts in technological knowledge accumulation, if the returns to investment are sufficiently different, they could offset
the differences in investment, causing the technology ratio to fall. If the technology ratio falls far enough, the elite community could choose to implement a power sharing arrangement instead of takeover in the next period. The following proposition presents the probability of institutional change.

**Proposition 6:** Given a sufficiently advanced level of technological advancement

(i) the probability that power sharing changes to takeover in the next period is given by

\[
P(I_{t+1} = T | I_t = P) = \frac{1}{2s^p} \left( \frac{\epsilon_H - s^p \epsilon_L}{\epsilon_H - \epsilon_L} \right)
\]

where

\[
s^p = \left[ (\frac{a_i}{a_j})' \left( \frac{a_{it}}{a_{jt}} \right) \right]^{1-\mu}
\]

(ii) the probability that takeover changes to power sharing in the next period is given by

\[
P(I_{t+1} = P | I_t = T) = s^T \left( \frac{\epsilon_H - (1/s^T) \epsilon_L}{\epsilon_H - \epsilon_L} \right)
\]

where

\[
s^T = \frac{1/2}{1 - \tau^T + (1/2)(1 - q_d) \left[ (\frac{a_i}{a_j})' \left( \frac{a_{it}}{a_{jt}} \right) \right]^{1-\mu}}
\]

**Proof:** See Appendix A.

Another important implication of the model is that the likelihood of institutional change falls the longer either power sharing or takeover remains in place. To see this, note that under power sharing, the technology ratio converges to a value of 1, increasing the distance between the current technology ratio and the technology ratio necessary for institutional change \((\frac{a_i}{a_j})'\). Under takeover, the technology ratio rises to a value of \((\frac{a_i}{a_j})''\), again increasing the distance between the current technology ratio and the technology ratio necessary for institutional change. In either case, the larger the distance from \((\frac{a_i}{a_j})'\), the lower the probability of institutional change as stated in Proposition 7 and illustrated in Figure 10.

**Proposition 7:** The probability of institutional change falls the longer a particular institution is in place.

### 4.3 Numerical Simulation

Proposition 7 can be confirmed by simulating the model and examining the evolution of institutional persistence over time. Table 2 gives the values for parameters and functional
forms in the model. The parameters and functional forms are chosen so as to be consistent with the basic assumptions of the model and so that all institutional outcomes are possible in any given simulation.

Figure 11 illustrates how the institutional arrangement between two communities might evolve over the span of 20 periods (generations). The top panel shows the relative payoff of the elite community under each institutional arrangement. The bottom panel shows the technology ratio between the two communities, again from the perspective of the elite community.

Note that private defense is chosen in the first few generations, after which the communities switch to power sharing and then to takeover. By the eighth generation, both communities have advanced their technology to the point where power sharing and/or takeover dominate both private defense and war. From this point on, the institution with the highest relative payoff is directly dependent on the value of the technology ratio. When the ratio is high, takeover is chosen and when the ratio is low, power sharing is chosen.

Figure 12 illustrates how institutional persistence becomes stronger over time. Simulated data is generated from 5000 repetitions of the model over a time frame of 51 generations. The graph in figure 11 shows the average number of institutional changes in the last 5 generations. The initial peak is generated by the transition away from private defense (which always occurs). After this first transition period, the number of institutional changes declines quickly to a long run average of approximately 0.85 institutional changes every 5 periods. Table 3 shows that in the long run both power sharing and takeover are almost equally persistent.

As mentioned in the introduction, there is some empirical evidence that confirms this feature of increasing institutional stability over time. Table 1 presents survival probabilities of institutions conditional on age, where these survival probabilities tend to increase in downward diagonal directions. In other words, institutions become more persistent the longer they are in place. For example, the probability that a 60 year old institution survives another 20 years is over 65% higher than the probability that a new institution survives an initial 20 years. This confirms one of the key patterns of institutional persistence found in
the model.

5 Conclusion

In this paper I present a model of institutional formation, choice and persistence over time. I show that the need for protection of private property and the escalating cost of private defense of this property eventually gives rise to political interaction that generates institutional arrangements. Exactly what institutional arrangement is chosen depends on the relative paths of technological development of the two communities in the model. A high level of development and relative equality will generate a power sharing arrangement, while sufficiently unequal rates of technological development will lead to a takeover arrangement. I find that institutional arrangements are persistent over time because current institutions affect investment decisions in such a way as to reinforce the existing institutions in the future. Certain economic outcomes can overcome institutional effects and cause institutional change, but this possibility becomes less likely the longer that an institution is in place. In other words, the stability of a particular institution increases with age.
Appendix

A.1 Proof of Proposition 1:

Consider the four possible choices that agents in community \( i \) can make:

1. Community \( i \) offers \( W \):
   In this case it does not matter what community \( j \) would prefer to do. War occurs for certain.

2. Community \( i \) offers \( T \):
   In this case community \( j \) will only accept \( T \) if its expected utility under takeover is at least as high as it would be given war. Anticipating this, agents in community \( i \) can extract \( \beta_{it} \) from community \( j \) where \( \beta_{it} \) is the minimum of \( 1/2 \) or the level at which agents in community \( j \) are just indifferent between \( T \) and \( W \). Given this value for \( \beta_{it} \), community \( i \) will only offer \( T \) if this provides the highest expected utility conditional on the constraint on \( \beta_{it} \).

3. Community \( i \) offers \( D \):
   In this case, the political outcome is \( D \) unless community \( j \) chooses war. If community \( j \) prefers war then
   \[
   k \log(1 - \tau^w) - (1 - \mu) \gamma (1 - \mu)(1 - p_{jt}) \log(\frac{a_{jt}}{a_0}) > k \log(1 - \tau^d(a_{jt}M))
   \]
   But then since \( a_{it} \geq a_{jt} \), community \( i \) prefers \( D \) to \( W \) only if
   \[
   (1 - \mu) \gamma \log(\frac{a_{it}}{a_{i0}}) \frac{\partial p_{it}}{\partial a_{it}} < \frac{(1 - \mu) \gamma (1 - p_{it})}{1 - \tau^d(a_{it}M)} \frac{k}{1 - \tau^d(a_{it}M)} \frac{\partial \tau^d(a_{it}M)}{\partial a_{it}}
   \]
   But by assumption 3, the right hand side will be less than zero, which is a contradiction since \( p_{it} \) is non-decreasing in \( a_{it} \). Hence if community \( i \) prefers \( D \) to \( W \), so must community \( j \).

4. Community \( i \) chooses \( P \):
   Suppose first that \( \tau^d(a_{jt}M) < \tau^p/2 \). In this case community \( j \) prefers \( D \) to \( P \) in which case community \( j \) will reject the offer of \( P \) and \( D \) will be the political outcome. Anticipating this, agents in community \( i \) will not offer \( P \) in the first place.
   Next suppose that \( \tau^d(a_{jt}M) \geq \tau^p/2 \). In this case community \( j \) prefers \( P \) to \( D \). Community \( j \) will counter offer with war in this case because as seen in 3., if community \( i \) prefers \( D \) to \( W \), so must community \( j \).
A.2 Proof of Lemma 1:

(i) Suppose first that $a_{it}^{dt} > a_{it}^{dw}$. If community $i$ has a technology level equal to $a_{it}^{dt}$, then by definition of the given critical values, $u_{it}^W > u_{it}^D = u_{it}^T$. Since the relative payoff to war is greater than the payoff to takeover, community $i$’s technology must be below $a_{it}^{tw}$. Hence $a_{it}^{tw} > a_{it}^{dt} > a_{it}^{dw}$.

(ii) Suppose that $a_{it}^{dt} < a_{it}^{dw}$. If community $i$ has a technology level equal to $a_{it}^{dt}$, then by definition of the given critical values, $u_{it}^W < u_{it}^D = u_{it}^T$. Since the relative payoff to takeover is greater than the payoff to war, community $i$’s technology must be above $a_{it}^{tw}$. Hence $a_{it}^{dt} > a_{it}^{dw} > a_{it}^{tw}$.

(iii) Suppose that $a_{it}^{dt} = a_{it}^{dw}$. This implies then that $u_{it}^W = u_{it}^D = u_{it}^T$. Hence $a_{it}^{dt} = a_{it}^{dw} = a_{it}^{tw}$.

A.3 Proof of Proposition 2:

Given the properties of $a_{it}^{dw}$, $a_{it}^{tw}$ and $a_{it}^{dt}$

(i) $u_{it}^T > u_{it}^D$ when $a_{it} > a_{it}^{dt}$ and $u_{it}^T > u_{it}^W$ when $a_{it} > a_{it}^{tw}$. Hence community $i$ will choose takeover ($T_i$).

(ii) $u_{it}^W > u_{it}^D$ when $a_{it} > a_{it}^{dw}$ and $u_{it}^W > u_{it}^T$ when $a_{it} < a_{it}^{tw}$. Hence community $i$ will choose war ($W_i$).

(iii) $u_{it}^D > u_{it}^W$ when $a_{it} < a_{it}^{dw}$ and $u_{it}^D > u_{it}^T$ when $a_{it} < a_{it}^{dt}$. Hence community $i$ will choose private defense ($D_i$).

Figure 1 shows that $a_{it}^{dw}$ and $a_{it}^{dt}$ are increasing in $a_{jt}$. This is because:

1. Note that an increase in $a_{jt}$ decreases $u_{it}^W$ (because of a lower technology ratio) without affecting $u_{it}^D$. In order to restore equality of $u_{it}^W$ and $u_{it}^D$, $a_{it}$ must increase since this will increase the technology ratio and cause $u_{it}^D$ to fall by more than $u_{it}^W$ (by Assumption 3.)

2. Note that an increase in $a_{it}$ increases $u_{it}^T$ (because of a higher technology ratio) and decreases $u_{it}^D$. In order to restore equality of $u_{it}^T$ and $u_{it}^D$, $a_{jt}$ must increase, since this will lower the technology ratio, causing $u_{it}^T$ to fall until $u_{it}^T = u_{it}^D$. 

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A.4 Proof of Proposition 3:

Given the properties of \( a_{it}^{pw} \), \( a_{it}^{pw} \) and \( a_{it}^{tw} \).

(i) \( u_{it}^T > u_{it}^P \) when \( a_{it} > a_{it}^{pt} \) and \( u_{it}^W > u_{it}^T \) when \( a_{it} > a_{it}^{tw} \). Hence community \( i \) will choose takeover \((T_i)\).

(ii) \( u_{it}^W > u_{it}^P \) when \( a_{it} > a_{it}^{pw} \) and \( u_{it}^W > u_{it}^T \) when \( a_{it} < a_{it}^{tw} \). Hence community \( i \) will choose war \((W_i)\).

(iii) \( u_{it}^P > u_{it}^W \) when \( a_{it} < a_{it}^{pw} \) and \( u_{it}^P > u_{it}^T \) when \( a_{it} < a_{it}^{pt} \). Hence community \( i \) will choose power sharing \((P_i)\).

Figure 2 shows that \( a_{it}^{pt} \) is increasing in \( a_{jt} \). To see this, note that an increase in \( a_{it} \) increases \( u_{iT} \) (because of a higher technology ratio). In order to restore equality of \( u_{iT} \) and \( u_{iP} \), \( a_{jt} \) must increase, since this will lower the technology ratio, causing \( u_{iT} \) to fall until \( u_{iT} = u_{iP} \).

A.5 Proof of Lemma 2:

Since \( (\frac{a_{it}}{a_{jt}})' \) is decreasing in \( a_{jt} \) and \( (\frac{a_{it}}{a_{jt}})'' \) is increasing in \( a_{jt} \), if \( (\frac{a_{it}}{a_{jt}})'' > (\frac{a_{it}}{a_{jt}})' \) then \( (\frac{a_{it}}{a_{jt}})' > (\frac{a_{it}}{a_{jt}})'' \) for all \( t \).

If \( a_{jt} = a_{j0} \), then \( \beta_{it} = \tau^w \). Hence

\[
1 - \frac{\tau^p}{2} = 1 - \tau^T + \tau^w (1 - q_{it}(\frac{a_{it}}{a_{j0}}'))
\]

\[
(\frac{a_{it}}{a_{j0}})'' = \frac{1 - \tau^T + \tau^w (1 - q_{it}(\frac{a_{it}}{a_{j0}}''))}{1 - \tau^w}
\]

Combining the two equations above gives

\[
1 - \frac{\tau^p}{2} = (1 - \tau^w)(\frac{a_{it}}{a_{j0}})'' - \tau^w [(1 - q_{it}(\frac{a_{it}}{a_{j0}}'')) - (1 - q_{it}(\frac{a_{it}}{a_{j0}}'))]
\]

Since \( \tau^w < \tau^p/2 \) and \( (\frac{a_{it}}{a_{j0}})'' \geq 1 \), the equality above can only hold if

\[
q_{it}(\frac{a_{it}}{a_{j0}}') < q_{it}(\frac{a_{it}}{a_{j0}}'')
\]

which implies that \( (\frac{a_{it}}{a_{j0}})'' > (\frac{a_{it}}{a_{j0}})' \).

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A.6 Proof of Proposition 6:

(i) Given $I_t = P_t$, the probability that $I_{t+1} = T_{t+1}$ is the probability that

$$\epsilon_{jt} < \epsilon_{it}/s^p$$

where

$$s^p = \left[ \frac{(a_i/a_j)'}{(a_{it}/a_{jt})^{1-\mu}} \right]^{\frac{1}{2}}$$

Since $s^p > 1$, the probability that $\epsilon_{jt} < \epsilon_{it}/s^p$ is equal to the ratio of the area of the triangle below $\epsilon_{jt} = \epsilon_{it}/s^p$ contained within the square with side length $(\epsilon_H - \epsilon_L)$ to the area $(\epsilon_H - \epsilon_L)^2$. This ratio is

$$\frac{1}{2s^p} \left( \frac{\epsilon_H - s^p\epsilon_L}{\epsilon_H - \epsilon_L} \right)$$

(ii) Given $I_t = T_t$, the probability that $I_{t+1} = P_{t+1}$ is the probability that

$$\epsilon_{jt} > \epsilon_{it}/s^T$$

where

$$s^T = \left[ 1 - \frac{1}{2} + (1/2)(1 - q_{it}(\frac{a_{it}}{a_{jt}})) \right] \left[ \frac{(a_i/a_j)'}{(a_{it}/a_{jt})^{1-\mu}} \right]^{\frac{1}{2}}$$

Since $s^T < 1$, the probability that $\epsilon_{jt} > \epsilon_{it}/s^T$ is equal to the ratio of the area of the triangle above $\epsilon_{jt} = \epsilon_{it}/s^T$ contained within the square with side length $(\epsilon_H - \epsilon_L)$ to the area $(\epsilon_H - \epsilon_L)^2$. This ratio is

$$\frac{s^T}{2} \left( \frac{\epsilon_H - (1/s^T)\epsilon_L}{\epsilon_H - \epsilon_L} \right)$$

B Appendix

For a general characterization of institutional persistence at lower levels of development where $\beta_{it} \leq 1$ and $(\frac{a_{it}}{a_{jt}})'$ and $(\frac{a_{it}}{a_{jt}})''$ will vary over time.

B.1 Joint Technological Development

If $I_t = P_t$, so that $\frac{a_{it}}{a_{jt}} < (\frac{a_{it}}{a_{jt}})'$, then $I_{t+1} = P_{t+1}$ if

$$\left( \frac{a_{it}}{a_{jt}} \right)^{1-\mu} < \left( \frac{a_{it+1}}{a_{jt+1}} \right)'$$
If \( a_{jt+1} < a_{jt} \), then \( \frac{a_{it+1}}{a_{jt+1}}' > \frac{a_{it}}{a_{jt}}' \) and so power sharing will definitely be chosen in the next period. If \( a_{jt+1} \geq a_{jt} \), the persistence of power sharing in this case depends on how fast the critical value \( \frac{a_{it}}{a_{jt}}' \) falls as technology increases. If this critical value of the technology ratio falls too quickly, then the communities could revert to a takeover arrangement. In general, power sharing will be persistent so long as the functions \( p(\frac{a_{it}}{a_{jt}}) \) and \( q_{it}(\frac{a_{it}}{a_{jt}}) \) are responsive to changes in the technology ratio.

If \( I_t = T_t \), so that \( \frac{a_{it}}{a_{jt}} > (\frac{a_{it}}{a_{jt}})' \), then \( I_{t+1} = T_{t+1} \) if

\[
\frac{a_{it+1}}{a_{jt+1}} \geq (\frac{a_{it+1}}{a_{jt+1}})'
\]

If \( a_{jt+1} \geq a_{jt} \), then \( (\frac{a_{it+1}}{a_{jt+1}})' < (\frac{a_{it}}{a_{jt}})' \) and so takeover will definitely be chosen in the next period. If \( a_{jt+1} < a_{jt} \), then the persistence of takeover depends on how fast the critical value \( (\frac{a_{it}}{a_{jt}})' \) rises as technology falls. If this critical value of the technology ratio rises too quickly, then the communities could revert to a power sharing arrangement. In general, takeover will be persistent so long as the functions \( p(\frac{a_{it}}{a_{jt}}) \) and \( q_{it}(\frac{a_{it}}{a_{jt}}) \) are responsive to changes in the technology ratio.

**B.2 Independent Technological Development**

The probability of institutional change as described in Proposition 6 is modified in this case to allow for changes in the critical values \( (\frac{a_{it+1}}{a_{jt+1}})' \) and \( (\frac{a_{it+1}}{a_{jt+1}})'' \) at different levels of technological development. Here we have that:

(i) the probability that power sharing changes to takeover in the next period is given by

\[
P(I_{t+1} = T_{t+1}|I_t = P_t) = \frac{1}{a_{jt+1} - a_{jt+1}'} \int_{a_{jt+1}'}^{a_{jt+1}} \frac{1}{2s_t} \left( \frac{a_{it+1}}{a_{jt+1}'} \right)^{\frac{1}{\mu}} \frac{\epsilon_H - s_t \epsilon_L}{\epsilon_H - \epsilon_L} \frac{1}{da_{jt+1}}
\]

where

\[
s_t^P = \left[ (a_{it+1}/a_{jt+1})' \right]^{\frac{1}{\mu}}
\]

(ii) the probability that takeover changes to power sharing in the next period is given by

\[
P(I_{t+1} = P_{t+1}|I_t = T_t) = \frac{1}{a_{jt+1} - a_{jt+1}''} \int_{a_{jt+1}''}^{a_{jt+1}} \frac{1}{2s_t^T} \left( \frac{a_{it+1}''}{a_{jt+1}''} \right)^{\frac{1}{\mu}} \frac{\epsilon_H - (1/s_t^T) \epsilon_L}{\epsilon_H - \epsilon_L} \frac{1}{da_{jt+1}}
\]

where

\[
s_t^T = \frac{1 - \beta_t}{1 - \tau_t + \beta_t(1 - q_{it}(\frac{a_{it}}{a_{jt}})') \left[ (a_{it}/a_{jt})^{1-\mu} \right]^\frac{1}{\mu}}
\]
References


Table 1: Survival Probabilities of Institutions Conditional on Age

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<tr>
<th>Age</th>
<th>Total</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
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<tr>
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<td>646</td>
<td>0.32</td>
<td>0.14</td>
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Source: Polity IV database. Age is denoted in years. Polities receive a score between -10 and 10. A political change is defined as a change of 3 or more points in 3 years or less.

Table 2: Parameter Values for Numerical Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value/Function</th>
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<tbody>
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<td>$\epsilon_H$</td>
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<td>$\mu$</td>
<td>0.25</td>
<td>$\epsilon_L$</td>
<td>0.25</td>
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<tr>
<td>$\gamma$</td>
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<td>$A_t$</td>
<td>$6(a_{it} + a_{jt})/2$</td>
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<tr>
<td>$a_0$</td>
<td>1</td>
<td>$p_{it}$</td>
<td>$a_{it}/(a_{it} + a_{jt})$</td>
</tr>
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<td>$q_{it}$</td>
<td>$a_{jt}/a_{it}$</td>
</tr>
<tr>
<td>$\tau^r$</td>
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<td>$\tau^d(it)$</td>
<td>0.15$a_{it}$</td>
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<tr>
<td>$\tau^w$</td>
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Table 3: Long Run Institutional Transition Matrix

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<th>T</th>
<th>Freq.</th>
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<td>0.0794</td>
<td>0.4662</td>
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<td>-</td>
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Figure 1: Institutional Persistence and Change

Figure 2: Timing Within Each Period
Figure 3: Political Choices

Figure 4: Political Interaction Game
Figure 5: Institutional Choice in Initial Development

Figure 6: Institutional Choice in Later Development
Figure 7: Institutional Choice in Both Stages of Development

Figure 8: Probability that a Particular Institution is Chosen Next Period
Figure 9: Institutional Persistence

Figure 10: Institutional Feedback Effect
Figure 11: Relative Institutional Payoffs and the Technology Ratio

Figure 12: Average Number of Institutional Changes in Previous 5 Periods