

A New Approach to Estimating Tax Interactions in Fiscal Federalism*

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Abstract

The purpose of this paper is to propose a new approach to empirically analyze the vertical and horizontal tax competition of gasoline and cigarette taxation in the U.S. I explicitly estimate the structural parameters of consumer's utility and state government's objective functions. The slopes of the reaction functions, which represent the strategic interaction of state government taxation policies, are then computed given the estimated structural parameters. Empirical results show that there is very little horizontal tax interaction in both the gasoline and cigarette cases. On the other hand, there is a moderate positive vertical tax interaction on both gasoline and cigarettes, and the scale is bigger in the case of cigarette taxes. Furthermore, the value and sign of the slopes of the reaction function are very different across states. This suggests a new policy implication: as state governments respond differently to federal government fiscal policy, uniform fiscal policy is not appropriate for welfare maximization of the nation.

JEL classification : H11, H21, H71, H73, H77

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1 Introduction

The purpose of this paper is to explicitly estimate the structural parameters of consumer and government behavior, and to examine the existence of strategic interaction of taxation among state governments and between state and federal governments. The existence of strategic interaction between governments is evaluated by computing the slope of the reaction function given the estimated parameters. If the slopes of the reaction functions between state governments are positive, state governments' tax policies are strategic complements and a state government raises (reduces) its tax rate if other state governments raise (reduce) their tax rates. On the other hand, if the slopes are negative, tax policies are strategic substitutes and a state government reduces its tax rate if other state governments raise them. Horizontal commodity tax competition generally happens when state government's tax policy is strategic complements. State governments compete for tax resources each other, and each government reduces its tax rate below the optimal level to attract tax resource from other states. In this sense, the mobility of tax resources; i.e. cross border shoppers, is a crucial factor of horizontal commodity tax competition, and the scale of horizontal tax competition depends on the mobility of tax resources. On the other hand, vertical commodity tax competition happens either both state and federal government's tax policies are strategic complements or strategic substitutes. State and federal government share the common tax base, and either or both governments ignore that their tax would shrink the tax base of the other government, which usually results in the equilibrium tax rate to become higher than optimal. In this case, the tax elasticity of tax base; i.e. consumer's price elasticity of demand, is a crucial factor for the intensity of vertical commodity tax competition. Consequently, if there is a tax competition, there is a possibility that both tax rate and the amount of public good are not optimal, and tax coordination or intergovernmental transfer is necessary to raise the total welfare (Boadway and Keen (1996), Hoyt (2001), and Lucas (2004)). These fiscal policies depend on the scale and direction of this tax externality, and it is hard to know whether tax rates in equilibrium are lower or higher than optimal. Therefore, estimating the direction and the level of strategic interaction of taxes between governments becomes a very important policy question for countries under fiscal federalism, where both federal government and state governments co-exist.

In this paper, I use a structural approach to estimate the strategic interactions in tax policies. I first estimate the parameters of the household's utility function in a model of optimal consumption and cross border shopping. Then, using the estimated parameters

of household's specific utility function, I estimate the objective function of benevolent state governments in a model of optimal taxation¹. Finally, based on the estimated structural parameters of the individuals and state governments, I derive the slope of the reaction function of each state's tax with respect to other states, and federal government tax changes.

There is already a large body of literature² that discusses both vertical and horizontal strategic interactions of taxation, both theoretically and empirically. Besley and Rosen (1998) theoretically and empirically examine vertical excise tax competition, i.e. strategic interaction between state and federal government excise taxes. They find out that the theory of optimal consumer and government behavior does not put any restriction on the sign of the slope of the reaction function. Empirically, from their regression analysis, they find that federal tax rate has a positive effect on state taxes for both gasoline and cigarette taxes.

Devereux et al (2007) extend Besley and Rosen (1998) to include horizontal interaction in their model, i.e. strategic tax interactions between state governments. In order to analyze both horizontal and vertical tax interactions, they use a weighted matrix to approximate the complex strategic interaction between state governments. That is, they estimate a linear model where the dependent variable is state taxes and independent variables include the weighted average of other states' taxes, the federal tax and other socio-economic variables. The result is that for the cigarette tax, the coefficient of the weighted average state tax rate is estimated to be significantly positive but the coefficient of the federal tax rate is insignificant. For the gasoline tax, the former is insignificant and the latter is positive and weakly significant. Devereux et al (2007) argue that the difference in the estimated strategic interactions of gasoline and cigarette taxes could be attributed to the difference in the characteristic of the good, such as the difference in price elasticity of demand and transportation cost³.

While the above regression based on the approach with and without the weighted matrix have made us aware of the importance of the strategic interactions in

¹ I do not argue the strategic behavior of the federal government in this paper.

² In theoretical papers, both Mintz & Tulkens (1986) and Kanbur & Keen (1993) study horizontal commodity tax competition. Keen and Kotsogiannis (2002) analyze both horizontal and vertical capital tax competition. In empirical papers, Esteller-More and Sole-Olle(2001) examine vertical and horizontal tax competition on income and sales tax in U.S. Nelson (2002) considers horizontal tax competition on exercise taxes in U.S. Hayashi and Boadway (2001) analyze vertical tax competition on corporate income tax in Canada.

³ "When individual demand for the good is relatively price-inelastic, and incentives for inter state arbitrage are strong [because of lower transportation cost], the tax set in any state is likely to be strongly positively responsive to taxes set in neighboring states, but unresponsive to the federal tax. Conversely, when individual demand for the good is relatively price-elastic, and incentives for inter-state arbitrage are weak, the tax set in any state is likely to be unresponsive to taxes set in neighboring states, and responsive to the federal tax, although this response may be positive or negative. As argued below, the first case describes the market for cigarettes in the US well, and the second case the market for gasoline." ; extract from Devereux et al (2007) pp.452 line 16-24.

taxation, I argue that there are several difficulties in interpreting the estimation results, especially for the results that includes horizontal tax interaction where a weighted matrix is used.

First, the theory of state tax policy predicts that the slopes of the reaction functions, which measure the response of own taxes to the marginal change of other states' or federal taxes, depend on several variables, which are: difference between the own state tax rate and that of all the other states, transportation costs, own and other states' population, demand and price elasticity of demand. However, conventional construction of the weighted matrix allows the slope of the reaction function to depend on only one variable and also assumes the sign of the slope is same across states. Hence, the interaction terms of taxes and the variables not included in the weighted matrix are omitted from the independent variables, resulting in omitted variable bias. The direction and the magnitude of the bias are likely to depend on which variable is included in the weighted matrix. I suspect this is the reason why the results are not robust to the specification of the empirical model; i.e. different studies that use different variables in the construction of the weighted matrix often obtain very different parameter estimates of tax interaction.

Second, I argue that the weighted matrix approach is a poor approximation of the Nash equilibrium of state and federal governments' strategic taxation game. This is because the weighted matrix is a linear approximation around a symmetric Nash Equilibrium⁴ and only applicable when state governments are symmetric and consumer's utility function is Quasi-linear. Hence, the estimation result based on the weighted matrix is reliable only if the equilibrium is very close to being symmetric, i.e. if the states are very similar to each other. But in the data, states have very different population and distance to each other, and the weighted matrix is a poor approximation to the Nash equilibrium. Also, Quasi linear utility function means that demand is independent to income and this is not reasonable assumption considering consumption behavior vary across different income level⁵.

Lastly, the previous papers' results are not consistent with the usual idea of the relationship between price elasticity of demand and the scale of tax competition. Generally, in a Ramsey optimal taxation context, I would expect the government to avoid levying a heavier tax rate on the good whose price elasticity of demand is high to avoid losing tax base. Therefore, the slope of the reaction function should be small in the good whose price elasticity of demand is high. Nevertheless, both Besley and Rosen

⁴ Refer to the Proposition 3 in Devereux et al (2007)

⁵ This is based on the data of U.S. Bureau of Labour Statistics.

(1998) and Devereux et al (2007) report that the value of the slope of the reaction functions between state and federal government is larger in gasoline case than in cigarette case in spite of the fact that the price elasticity of demand of gasoline is higher than that of cigarette.

In this paper, I take a structural approach to analyze tax interaction. I first solve and estimate a model of optimal consumption and cross border shopping behavior of individuals, similar to the one analyzed in Devereux et al (2007). In this first stage, I recover the parameters of the representative consumer's utility function. In contrast to the weighted matrix approach, our estimation is based on the full solution of consumer's behavior subject to taxes. Hence, I take into account all the important factors that determine optimal consumption, such as differences in own state and other states' tax rate, transportation costs, population, demand and price elasticity of demand which affect state and federal taxation via consumer optimal behavior. I next estimate the parameter of the state government objective function by estimating the state government's first order condition with respect to taxes. After all the key structural parameters are estimated, I compute the slopes of the reaction functions and evaluate the strategic interaction between governments. Notice that the slope of the reaction function is derived from state government's first order condition, which maximizes the welfare of the representative household. This method fully captures the effect of other state or federal tax changes on consumer's cross border shopping and also takes into account the nonlinear functional forms of the reaction function.

The estimation results are the following. First, the slope of the reaction functions between state governments of both gasoline and cigarette taxes, which describes the horizontal tax interaction, is positive but very small. The reason why this value is small is that the share of gasoline and cigarette consumption to total income is small and the percentage of cross border shopping is estimated to be very small. Second, the slope of the reaction function between state and federal government, which describes the vertical tax interaction, is positive, and the value is higher for the cigarette tax than for the gasoline tax. This result supports the Ramsay idea of the relationship between price elasticity of demand and tax competition intensity. Third, the value of the slope of the reaction function of the tax interaction is positive on average, but its value and sign are very different among state and for some states it becomes negative. This is in contract to the results from the weighted matrix estimation, where the slopes have the same sign for all states and only change linearly with variable of the weighted matrix, such as population, distance, or the border population density. I also identify the structure of the slope of the reaction function. The scale of the slope of the reaction function mainly

depends on the share of commodity consumption to total income and share of own state consumption in horizontal tax externality case, while the utility function, price elasticity of demand and after tax price are important factors for vertical tax externality case, which are all different among states. This result casts some doubt on the validity of previous results which were obtained by assuming that the sign of the slope of the reaction function is all same among states, and the value of the slopes depends on only one factor.

The paper proceeds as follow. In section 2, I explain how to evaluate tax competition using a reaction function. In section 3, I introduce the model of household consumption and government taxation and spending. In section 4, I explain the estimation strategy, and section 5 explains the data. The 6th section discusses the results of the empirical analysis and section 7 explains the intuition of them. Section 8 discusses the relation with previous papers and section 9 concludes.

2 General framework of tax interaction

In this section, I briefly review the model of Devereux et al (2007).

Suppose there are two state governments, i and j who levy a excise tax on a good for which cross border shopping is possible. Assume that the state government i is Leviathan⁶, who maximizes the total tax revenue R_i . Total tax revenues is composed of tax rate t_i and tax base $X_i(t_i, t_j, d_{ij}, n_i, n_j)$, where t_j is another state's tax rate, and d_{ij} is the distance between state i and j , measuring the transportation cost of cross border shopping, and n_i, n_j are the population of state i and j . Tax base X_i can be divided into two components; per consumer demand $x_i(t_i)$ and the number of people who purchase the good in state i $s_i(t_i, t_j, d_{ij}, n_i, n_j)$. Then, the state government i 's problem is

$$\text{Max}_{t_i} R_i = t_i X_i(t_i, t_j, d_{ij}, n_i, n_j)$$

$$\text{where } X_i = x_i(t_i) s_i(t_i, t_j, d_{ij}, n_i, n_j)$$

The first order condition for maximization is

⁶ Here, I assume that state government is Leviathan only for explanation since the model is simple.

$$\frac{\partial R_i}{\partial t_i} = X_i(t_i, t_j, d_{ij}, n_i, n_j) + t_i \frac{\partial X_i(t_i, t_j, d_{ij}, n_i, n_j)}{\partial t_i} = 0.$$

Next, I derive the reaction function of state i 's tax in response to changes in state j 's tax. It is

$$\frac{\partial t_i}{\partial t_j} = - \frac{\partial^2 R_i}{\partial t_i \partial t_j} / \frac{\partial^2 R_i}{\partial t_i^2}$$

where

$$\frac{\partial^2 R}{\partial t_i \partial t_j} = \left\{ x_i(t_i) + t_i \frac{\partial x_i(t_i)}{\partial t_i} \right\} \frac{\partial s_i(t_i, t_j, d_{ij}, n_i, n_j)}{\partial t_j} + x_i(t_i) \frac{\partial s_i(t_i, t_j, d_{ij}, n_i, n_j)}{\partial t_i \partial t_j}$$

The denominator, which is a second derivative with respect to tax rate is $\frac{\partial^2 R}{\partial t_i^2} < 0$ and the sign of the reaction function depends on the sign of the numerator.

From the expression of the numerator, it is clear that per consumer demand and the price elasticity of per consumer demand enter in the reaction function. In addition, distance, which is related to transportation cost, and population affect the number of people who purchase the good $s_i(t_i, t_j, d_{ij}, n_i, n_j)$. Furthermore, both own tax and that of the other state enter in the reaction function as well. Many of these determinants of the slope of the reaction function are not included in the conventional weighted matrix specification. Moreover, we can see from the numerator that except for a very specific model specification and parameter values, the reaction function is a fundamentally nonlinear function of tax rate, and linear regression might not be appropriate.

To derive a specific expression of the reaction function, we need to give a specific functional form for household and government problem. In the next section, I will construct a more specific model of cross border shopping that I will then estimate.

3 Model Setting

There are N states ($i = 1, 2, \dots, N$), and a federal government. Federal and state governments levy a commodity tax on a good (x), and use this tax revenue to finance a

public good G . G represents the “per capita amount of the public good” in this model, and the context of the public good is different for each private good. In another words, I consider per capita Highway expenditure as a public good for gasoline consumption case and per capita Health expenditure as a public good for cigarette consumption case. This is because gasoline tax revenue and cigarette tax revenue are kinds of earmarked revenue for Highway and Health expenditure. I denote y to be the other composite consumption good. I also denote the tax for state i as t_i and the federal tax rate T .

They are both assumed to be per unit taxes. Then, the after tax commodity price in state i can be expressed as $P_i = p_i + t_i + T$, where p_i is the before tax price. State i has population n_i , and people can choose to cross border shop for the good that is taxed. State governments only consider the welfare of household for their own region, and the federal government’s purpose is to maximize the total welfare of people in the nation. I assume that state and federal governments are Nash Competitors, and state government determine their tax rate and public good with other state and federal governments tax policy as given. I do not discuss federal government’s behavior, and also do not think about the case where state government is Leviathan. Next, I describe the household’s problem.

3.1 The Household’s problem

A household in state i has income I_i , and gets utility from consumption of good x , the composite good y and the public good G . The household can buy good x either in her own state or in a neighboring state. In the household cross border shops, I assume that the transportation cost is independent of the amount of consumption. The price of the composite good y is assumed to be unity for simplicity. I omit any public good from federal government in this section for simplicity because, given the assumption of additive separability of the utility of private and public good, it will not affect the cross border shopping.

Utility Function and Demand Function

The utility function of a household in state A who chooses to purchase good x in

state i is expressed as follows⁷:

$$U_A^i = \alpha_A \log(x_A^i - r_x) + (1 - \alpha_A) \log y_A^i - \beta \cdot d_{Ai} + \phi_A \cdot G_A$$

where r_x is the subsistence level of the good x , and d_{Ai} is the distance between state A and state i . The household chooses x and y so as to maximize the above utility subject to the following budget constraint.

$$(p_i + t_i + T)x_A^i + y_A^i = I_A$$

The parameter α_i corresponds to the income share the household spends on the good x above the minimum consumption level r_x . β measures the transportation cost. ϕ_i is a weight between private good and public good utility. I assume that the value of α_i and ϕ_i are same for people in the same state but different across states. I allow heterogeneity for β within state by assuming β to be distributed randomly across households. The minimum consumption level r_x is assumed to be the same for all states.

The solution of the above problem gives us the following demand for good x and y

$$x_A^i = \frac{\alpha_A I_A}{(p_i + t_i + T)} + (1 - \alpha_A) r_x, \quad y_A^i = (1 - \alpha_A) \{I_A - (p_i + t_i + T) r_x\} \quad (1)$$

Substituting them into the utility function, I derive the indirect utility function as follows.

$$\begin{aligned} V_A^i(P_i, I_A, d_{Ai}, G_A) & \quad (2) \\ &= \alpha_A \log \alpha_A + (1 - \alpha_A) \log(1 - \alpha_A) - \alpha_A \log(p_i + t_i + T) + \log(I_A - (p_i + t_i + T) r_x) - \beta \cdot d_{Ai} \\ &+ \phi_A G_A \end{aligned}$$

Next, I derive the proportion of consumer who crosses border shop. Since the utility

⁷ I choose Stone = Geary utility function in my model for the following two reasons. First, according to previous papers, a price elasticity of demand for gasoline and cigarettes is about -0.8~1, and -0.5 for each. Stone=Geary utility function is flexible to these values of price elasticity of demand. Second, this Stone =Geary utility function fit well with per capita consumption data of cigarette and gasoline. Other utility function, for example Cobb-Douglas utility function and Quasi-Linear utility function don't satisfy these two points.

from public goods is exogenous and does not depend on cross-border shopping, I exclude it from the indirect utility function. Furthermore, I also add a random component to the indirect utility function, which measures the unobserved utility the consumer gets from shopping in state i . Then, it becomes

$$V_A^i(P_i, I_A, d_{Ai}, \varepsilon_{Ai}) = -\alpha_A \log(p_i + t_i + T) + \log(I_A - (p_i + t_i + T)r_x) - \beta \cdot d_{Ai} + \varepsilon_{Ai} \quad (2')$$

where ε_{Ai} is an error term if people in state A choose state i for shopping. I assume that people only cross border shop in neighboring states that share the same border with their own state. Suppose that state A is surrounded by states B and C, and people in state A make a choice among three states A, B and C for shopping. Then a household chooses the state to shop that gives the highest indirect utility. That is, if a household in state A chooses state A for shopping, it means $V_A^A > V_A^B, V_A^C$. The share of households in state A that purchase products in their own state A is equal to the probability that state A is chosen for shopping among these three states. If the error term ε_{Ai} is independent and identically distributed with an extreme value distribution, the probability that state A is chosen by households in state A can be expressed as

$$s_A^A = \frac{\exp(-\alpha_A \log(p_A + t_A + T) + \log(I_A - (p_A + t_A + T)r_x) - \beta \cdot d_{AA})}{\sum_{i=A}^C \exp(-\alpha_A \log(p_i + t_i + T) + \log(I_A - (p_i + t_i + T)r_x) - \beta \cdot d_{Ai})} \quad (3)$$

where s_i^j is the share of households in state i who shop in state j . It can be calculated given the parameters α_i, β , and r_x and data on income, after tax price and distance. Remember that household can cross border shop only in neighboring states, and if state i does not share the border with state j , both s_i^j and s_j^i are zero.

Price elasticity of demand

From the demand function, the price elasticity of demand becomes

$$\varepsilon = \frac{\partial x}{\partial P} \frac{P}{x} = \frac{-\alpha_i \cdot I_i}{\alpha_i I_i + P_i(1 - \alpha_i)r_x} \quad \text{where } P_i = p_i + t_i + T \quad (4)$$

From this equation, it is clear that the model restricts the price elasticity of demand to

lie between -1 and 0, ($-1 < \varepsilon < 0$). Most estimates of price elasticity of demand in the previous literature satisfy the above restriction. Given the parameters α_i , r_x and data for income and after tax price, price elasticity of demand can easily derived.

3.2 State government's problem

I assume that the state government is benevolent and maximizes the aggregate indirect utility of all households in the state.⁸

$$W_A = \int V_A^{i^*}(P_{i^*}, I_A, d_{Ai^*}, \varepsilon_{Ai^*}) d\varepsilon$$

where i^* is the optimal choice of states that a household in state A goes to shop. Given that the unobserved utility term ε is assumed to be i.i.d extreme valued, the above integral can be expressed analytically as follows (N_A are neighboring states for state A).

$$W_A = \log \left\{ \sum_{i=A}^{N_A} \exp[\alpha_A \log(x_A^i - r_x) + (1 - \alpha_A)y_A^i - \beta \cdot d_{Ai} + \phi \cdot G_A] \right\}$$

The state government's budget constraint is as follows.

$$G_A = TR_{GA} + TR_{OA} + g_A \quad ^9$$

where TR_{GA} is per capita revenue from the gasoline tax (in cigarette case, it is per capita revenue from the cigarette tax), TR_{OA} is per capita tax revenue from other sources and g_A is a per capita grant from the federal government. Gasoline tax revenue TR_{GA} can be expressed as follows.

$$TR_{GA} = \frac{1}{n_A} (n_A s_A^A x_A^A + n_B s_B^A x_B^A + n_C s_C^A x_C^A) t_A.$$

Where the term in parenthesis is the tax base, i.e. the amount of gasoline that is

⁸ I assume that state government determine gasoline and cigarette tax rate separately. It is impossible to include both gasoline and cigarette consumption in one utility function because it prevents estimating the share function s_i^j separately for both goods.

⁹ Some people might think that federal transfer depends on the federal tax rate, and the amount g_A is also a function of federal tax rate T to derive the reaction function, especially for gasoline tax case. When I research the history, there are several times when federal government raise gasoline tax rate but most of increased tax revenue is used to finance other things, like war expenditure, decreasing fiscal deficit and so on. Therefore, the increase in federal gasoline tax rate doesn't necessarily mean increase in federal grant and I assume that increasing federal tax rate will not affect the federal grant for simplicity.

purchased in state A. Notice that the tax base consist of not only households in state A but also households in neighboring states B and C that decide to purchase gasoline in state A. It is also important to notice that the gasoline tax revenue not only depends on per capita consumption x_i^A , but also on the number of shoppers from state i , $n_i s_i^A$, and per unit tax t_A . Next, I derive the first order condition of the optimal taxation.

First Order Condition

The state government A determines the optimal tax rate to maximizes W_A with other state and federal governments' tax policy as given. The first order condition with respect to its state tax t_A is,

$$\begin{aligned} \frac{\partial W_A}{\partial t_A} &= \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1 - \alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) s_A^A + \phi_A \frac{\partial TR_{GA}}{\partial t_A} = 0 \\ &= - \left(\frac{\alpha_A I_A + (1 - \alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\} (p_A + t_A + T)} \right) s_A^A + \phi_A \frac{\partial TR_{GA}}{\partial t_A} = 0 \end{aligned} \quad (5)$$

where

$$\begin{aligned} \frac{\partial TR_{GA}}{\partial t_A} &= n_A s_A^A \frac{\partial x_A^A}{\partial t_A} t_A + n_B s_B^A \frac{\partial x_B^A}{\partial t_A} t_A + n_C s_C^A \frac{\partial x_C^A}{\partial t_A} t_A \\ &+ n_A \left(s_A^A + \frac{\partial s_A^A}{\partial t_A} t_A \right) x_A^A + n_B \left(s_B^A + \frac{\partial s_B^A}{\partial t_A} t_A \right) x_B^A + n_C \left(s_C^A + \frac{\partial s_C^A}{\partial t_A} t_A \right) x_C^A = 0 \end{aligned}$$

Reaction Function

The reaction function is derived from differentiating the first order condition above with respect to the tax rate. The slope of the reaction function measuring the effect of state B taxes change on state A government's tax is

$$\frac{\partial t_A}{\partial t_B} = - \frac{\partial^2 W_A}{\partial t_A \partial t_B} / \frac{\partial^2 W_A}{\partial t_A^2} \quad (6)$$

and the slope of the reaction function measuring the effect of federal tax change on

taxes of state A¹⁰ is

$$\frac{\partial t_A}{\partial T} = -\frac{\partial^2 W_A}{\partial t_A \partial T} / \frac{\partial^2 W_A}{\partial t_A^2} \quad (7)$$

Where (for more details, see Appendix A)

$$\begin{aligned} \frac{\partial^2 W_A}{\partial t_A^2} = & \left(\frac{-\alpha_A}{(x_A^A - r_x)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A^2} + \frac{-(1-\alpha_A)}{y_A^A} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A^2} \right) S_A^A \\ & - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial S_A^A}{\partial p_A^S} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A^2} \right) < 0 \end{aligned}$$

$$\frac{\partial^2 W_A}{\partial t_A \partial t_B} = - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial S_A^A}{\partial t_B} + \phi \left(\frac{\partial TR_{GA}^2}{\partial t_A \partial t_B} \right)$$

$$\begin{aligned} \frac{\partial^2 W_A}{\partial t_A \partial T} = & \left(\frac{-\alpha_A}{(x_A^A - r_x)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right) \left(\frac{\partial x_A^A}{\partial T} \right) + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A \partial T} + \frac{-(1-\alpha_A)}{y_A^A} \left(\frac{\partial y_A^A}{\partial t_A} \right) \left(\frac{\partial y_A^A}{\partial T} \right) + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A \partial T} \right) S_A^A \\ & - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial S_A^A}{\partial T} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \right) \end{aligned}$$

Strategic Interactions between governments

I would like to explain where the strategic interactions of taxation between governments are represented in the reaction function. In horizontal tax interaction case, state governments compete for cross border shoppers to increase tax revenue for public good, and how much cross border shoppers are sensitive to the tax rate change of other state governments is an important factor. In the model, the term $\frac{\partial S_A^A}{\partial t_B}$ represents this

sensitivity which shows up in the term $\frac{\partial^2 W_A}{\partial t_A \partial t_B}$ and $\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B}$, and the scale of horizontal

tax competition depends on this factors. If this value is small, the scale of horizontal tax competition is small and vice versa.

On the other hand, in vertical tax interaction case, state and federal government share the common tax base and how much this tax base (consumer's demand) is sensitive to tax rate change of the federal government is a crucial factor. In the equation,

¹⁰ If the state government is Leviathan, $\frac{\partial t_A}{\partial t_B} = -\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B} / \frac{\partial^2 TR_{GA}}{\partial t_A^2}$ and $\frac{\partial t_A}{\partial T} = -\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} / \frac{\partial^2 TR_{GA}}{\partial t_A^2}$ for each.

the term $\frac{\partial x_A^A}{\partial T} = \frac{x_A^A}{P^2} \varepsilon$ represents the tax elasticity of tax base which enter in the term $\frac{\partial^2 W_A}{\partial t_A \partial T}$ and $\frac{\partial^2 TR_{GA}}{\partial t_A \partial T}$. In short, the price elasticity of demand and after tax price are key factors for vertical tax competition. I will come back to this issue again in section 7 and section 8.2.

4 Empirical Analysis

4.1 Estimating the parameters of the Household Utility Function

Moment Condition

One difficulty in estimating the parameters of consumers utility is that the data on how much gasoline or cigarettes are consumed by households in different states are unavailable in state level¹¹. The only available data are total sales, tax revenue, per unit tax rate and population in each state level. In other words, I do not know how much of tax revenue comes from in-state consumers or out of state consumers. Considering this data restriction, I match the data of total sales in each state with the predicted total sales based on the model. For example, consider the case where there are only 3 states: state A, B and C, and they are all neighbors to each other. Then, the predicted total sales in state A, B and C can be expressed as follows.

$$C_A = n_A \cdot s_A^A \cdot x_A^A + n_B \cdot s_B^A \cdot x_B^A + n_C \cdot s_C^A \cdot x_C^A$$

$$C_B = n_A \cdot s_A^B \cdot x_A^B + n_B \cdot s_B^B \cdot x_B^B + n_C \cdot s_C^B \cdot x_C^B$$

$$C_C = n_A \cdot s_A^C \cdot x_A^C + n_B \cdot s_B^C \cdot x_B^C + n_C \cdot s_C^C \cdot x_C^C$$

I then assume that the actual total sales C_i^d is the sum of the predicted total sales

C_i plus an error term e_{C_i} . That is,

$$C_i^d = C_i + e_{C_i}, i = A, B, C$$

¹¹ I confirmed this point with the U.S. Bureau of Labour Statistics.

The Moment Condition, which minimize the difference between total sales in the data C_i^d and the total sales predicted based on the model C_i is

$$E(C_i - C_i^d | Z_i) = E(e_{C_i} | Z_i) = 0 \quad (8)$$

where Z_i is a vector of instruments. From this Moment Condition, I can estimate the parameters of the household utility function α_i , β and r_x which show up in the share function s_i^j and demand function x_i^j .

Endogeneity Issues

The tax policy of the state government creates a potential endogeneity problem in the above moment condition estimation in equation (8). Since the state i government maximizes welfare taking into account the consumer's behavior, its tax rate should be a function of the demand, and the error term should affect its tax. Hence, the error term e_{C_i} and tax rate t_A will be correlated, resulting in the bias of the coefficient estimates. To deal with this issue, I use per capita federal grants as an instrument variable; i.e. per capita federal grants to Highway departments for the gasoline consumption case and per capita federal grants to Health departments for the cigarette consumption case. Grants from the federal government are related with state tax policy but I believe that it is reasonable to assume that it is not related to the error term, i.e. the unobservable component of the state gasoline and cigarette sales.

Next, I discuss in more detail the parameterization of the empirical model. The parameter α_i means how much share of their income a household spends on the consumption of the good x . This is likely to depend on the household's preferences and economic conditions. Hence, I assume that α_i for gasoline consumption is a linear function of log population density and log per capita income, and α_i for cigarette consumption is a linear function of log ratio of females in population and log per capita income¹². That is,

$$\alpha_i = \alpha_0 + \alpha_1 [\log(density) - \overline{\log(density)}] + \alpha_2 [\log(income) - \overline{\log(income)}] \text{ for gasoline}$$

$$\alpha_i = \alpha_0 + \alpha_1 [\log(female) - \overline{\log(female)}] + \alpha_2 [\log(income) - \overline{\log(income)}] \text{ for cigarettes}$$

¹² I also subscribe from the average, which is represented by the bar term.

It is also natural to think that the cost of cross border shopping is different between people who live in the center of the state and people who live in the border of the state. In order to fully deal with this issue, one needs to accurately measure the geography of each state and the distribution of consumers over its area, which is infeasible. Instead, I address the issue by applying the idea of “the random coefficient model” from Bajari et al (2007) and Berry et al (1995), and allow the transportation cost parameter β to take different values for different households in the same state, but restrict the distribution of β to be same across states. I assume that $\beta = \eta\beta^*$ where β^* is taken to be chi-squared distributed with one degree of freedom. The parameter η is estimated. The minimum consumption r_x is assumed to be the same for all states.

4.2 Estimating the parameter of State Government Objective Function

After estimating the parameters of the household utility, $\hat{\alpha}_i$, $\hat{\eta}$ and \hat{r}_x , I then estimate the remaining parameter of the state government objective function ϕ_i , which determines the weight between utility from private goods and utility from public goods. It is estimated using the first order condition of the state government choosing the optimal tax level as a moment condition. That is,

$$\frac{\partial W_i}{\partial t_i} = - \left(\frac{\alpha_i I_i + (1 - \alpha_i) r_x (p_i + t_i + T)}{\{I_i - (p_i + t_i + T) r_x\} (p_i + t_i + T)} \right) s_i^i + \phi_i \frac{\partial TR_{Gi}}{\partial t_i} = 0 \quad . \quad (9)$$

Similar to the idea of the parameter α_i , I consider the parameter ϕ_i is likely to depend on the economic environment of each state. Therefore, I assume that ϕ_i is a linear function of log per capita income for both gasoline and cigarette case.

$$\phi = \phi_0 + \phi_1 \cdot \log(\text{income})$$

I estimate ϕ using the above moment condition in equation (9), given the parameter of household utility and data on price, per unit tax, income, and population. As instrumental variables, I use the previous year’s per capita federal grant to Highway department, CO2 emission, per capita car registration and population for gasoline case, while for cigarette case, I use the previous year’s per capita federal grant to Health

department, the percentage of smoker, the number of death caused by cancer and population.

5 Data

I use data on 48 U.S. states from 1998 to 2001. I exclude Hawaii and Alaska since both do not share the border with other states. I downloaded the unit tax rate of gasoline from the webpage of Federal Highway Administration (U.S Department of Highway). I used the gasoline price and consumption data in official Energy Statistics from the U.S. Government, which can be obtained from the website of Energy Information Administration. For cigarettes, I use cigarette price, tax rate, tax revenue, and consumption from the Report “The Tax Burden on Tobacco” by Orzechowski and Walker¹³. There are two candidates for state consumption data. One is consumption data itself, and another is calculated by dividing total tax revenue by unit tax price. I find that the original consumption data seems to be more accurate because of its small variance of per capita consumption across states. I use population and per capita disposal income data from Bureau of Economic Analysis, and per capita government expenditure for Highway or Health from U.S Census of Bureau. Per capita federal grants to Highway departments are available from the webpage of Federal Highway Administration while per capita federal grants to Health departments are available from Statistics of Abstract (National data book from U.S. Census of Bureau). I derive the population density by dividing the population by the land area which is available from Statistics of Abstract. Ratio of female to total population is accessible from the webpage of Center of Disease Control and Prevention. CO2 emission is obtained from the webpage of U.S. Environment Protection Agency. The number of car registration is available from the webpage of Federal Highway Administration. The percentage of smoker and the number of death caused by cancer come from Statistics of Abstract. I also computed the distance data from Google map. All the details about data resource are explained in Appendix B.

¹³ This report is available by request. Please refer http://www.srnt.org/pubs/nl_05_06/spotlight.html.

6 Estimation Results

6.1 Moment Estimation

All the parameter estimates for household utility are in Table 1. Recall that α_i measures the share of private good consumption to total income after excluding minimum amount of consumption r_x . η measures the disutility from transportation cost. High transportation cost discourages consumer to purchase goods in other states. r_x measures the minimum amount of consumption.

Table 1.

$$\alpha_i = \alpha_0 + \alpha_1 [\log(density) - \overline{\log(density)}] + \alpha_2 [\log(income) - \overline{\log(income)}] \text{ for gasoline}$$

$$\alpha_i = \alpha_0 + \alpha_1 [\log(female) - \overline{\log(female)}] + \alpha_2 [\log(income) - \overline{\log(income)}] \text{ for cigarettes}$$

	Gasoline	Cigarette
α_0	0.0212	0.00573
α_1	-0.00149	0.1
α_2	-0.022	-0.005
r_x	100	32
η	1.803	1.238
Demand price elasticity	-0.793	-0.556

From Table 1, for gasoline the coefficient α_1 is estimated to be -0.00149. This means that people in states where population density is high spend lower share of their income on gasoline, which seems to be reasonable, since the high population density states would be more urban. Furthermore, the coefficient α_2 is estimated to be -0.022. This means that state that have higher per household income spend lower share of their income on gasoline, which again seems reasonable. The total share of gasoline consumption to income, including the minimum consumption r_x is calculated to be 0.0266 on average. This is very close to the value 0.0275 in the data.

The parameter value of r_x is 100. The per capita demand for gasoline is 495 (gallon) on average in states, where minimum amount is 294 in New York and maximum amount is 690 in Wyoming. Considering these values, I believe the value of

r_x to be reasonable. η is estimated to be 1.803. This parameter is used to explain the relative importance of cross border shopping. The estimated average share of households that purchase products in their own state is 95.8%. That is, about 4.2% of people do cross border shopping across states, which I believe to be sensible. Using the model and the parameter estimates, I also calculated the price elasticity of demand, which is -0.793 on average. This is close to the values obtained in the literature, which range from -0.8 to -1.

Next, I discuss the estimation results when data on cigarette consumption are used. α_1 is estimated to be 0.1. Interestingly, this means that states whose share of female population is higher consume more cigarettes. This result is acceptable considering the recent trend¹⁴ that a decline in the percentage of smoker is much larger for males than for females and the previous papers' result that the price elasticity of cigarette demand is less for female than for male. α_2 is estimated to be -0.005, which again means that higher income states spend lower share of their income on cigarettes. This again is consistent with the literature on smoking in health economics and in consumption estimation. The total share of cigarette consumption to income including the minimum consumption r_x is 0.0102 on average. This again is very close to the share of cigarette consumption 0.0115 in the data.

r_x is estimated to be 32¹⁵. The per capita demand for cigarette is 84 (package) on average. The minimum per capita demand is 35 in California and maximum is 156 in New Hampshire. Considering these values, I again believe the estimated value of r_x to be reasonable. η is estimated to be 1.238. As before, I can figure out whether this value is reasonable from the value of the share function. The estimated average share of within state consumption is 94.9%. That is, about 5% of people cross border shop for cigarettes. Flenor (1998) shows that the percentage of cross border purchase of cigarettes to be approximately 3.6% in 1997. Considering the recent increase of cigarette price and tax rate from 1997, I believe the value 5% to be consistent with his result. I also compute the price elasticity of demand to be -0.556, which is close to the value -0.5 obtained in the literature.

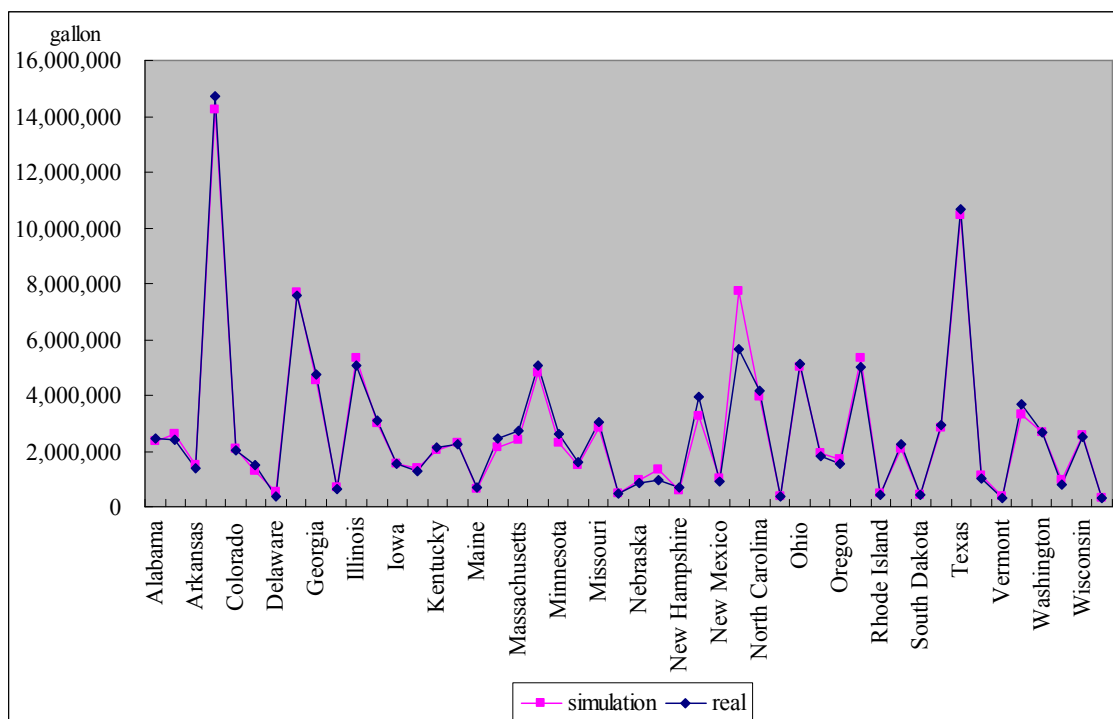
To demonstrate my model's fitness to actual data, I compare the real total sales and total sales predicted based on my model (call "simulation"). Graph 1a and Graph 1b compare real data and simulation data for gasoline case. From Graph 1a, we can see that

¹⁴ McGinnis, M., (1987) "TOBACCO AND HEALTH: Trends in Smoking and Smokeless Tobacco Consumption in the United States.", Annual Review Public Health, 8, pp.442-467

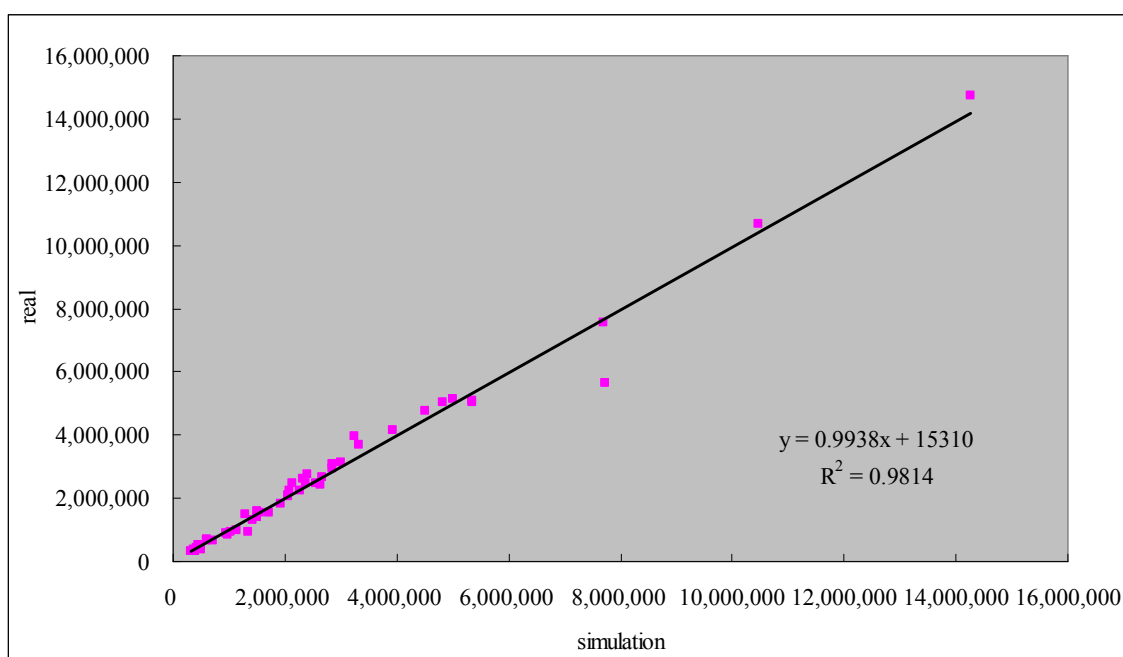
¹⁵ The idea of "minimum consumption" for cigarette consumption seems odd considering that not everyone consume cigarette, and only about 20% people smoke. The value r_x is a kind of number when we assume that everyone smoke, and this amount is minimum amount of consumption of the representative person.

simulation data fit very well across 48 states except New York. Graph 1b show the correlation between simulation data and real data, and the value of the correlation is 0.9938, which is very close to 1, and R-squared is 0.9814. When I draw the same Graph like Graph 1b excluding New York, the value of the correlation is 1.02 and R-squared is 0.994, and simulation data fit almost perfectly with real data. The reason why only New York's simulation data does not fit well is that per capita demand for gasoline is extremely low in New York. On average, per capita gasoline consumption across states is around 500 gallon, but in New York it is less than 300 gallon. Even though geographic factor like the population density is taken into account for household preference, many alternative transportation methods in New York are not captured, and the model could not fully capture this lower consumption in New York. Nevertheless, the fitness of simulation data with real data is very well in other states, and these two Graphs support that my model is very appropriate.

Graph 1a: Average (4years) Total Gasoline Sales



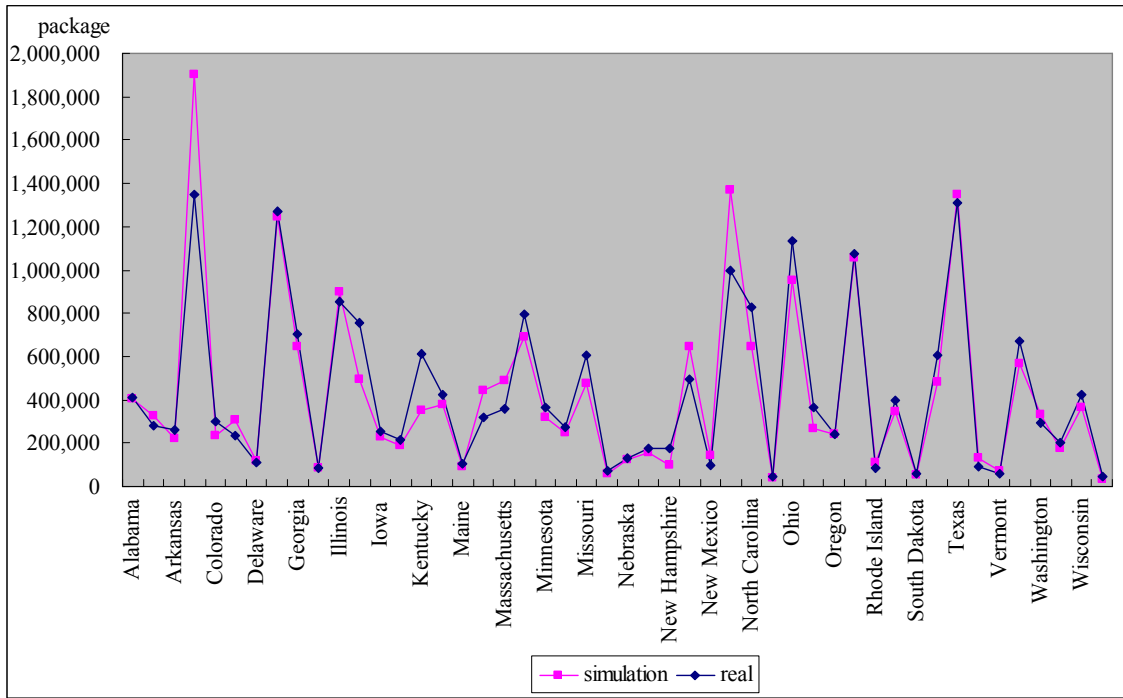
Graph 1b: The relation between simulation and real data (Gasoline)



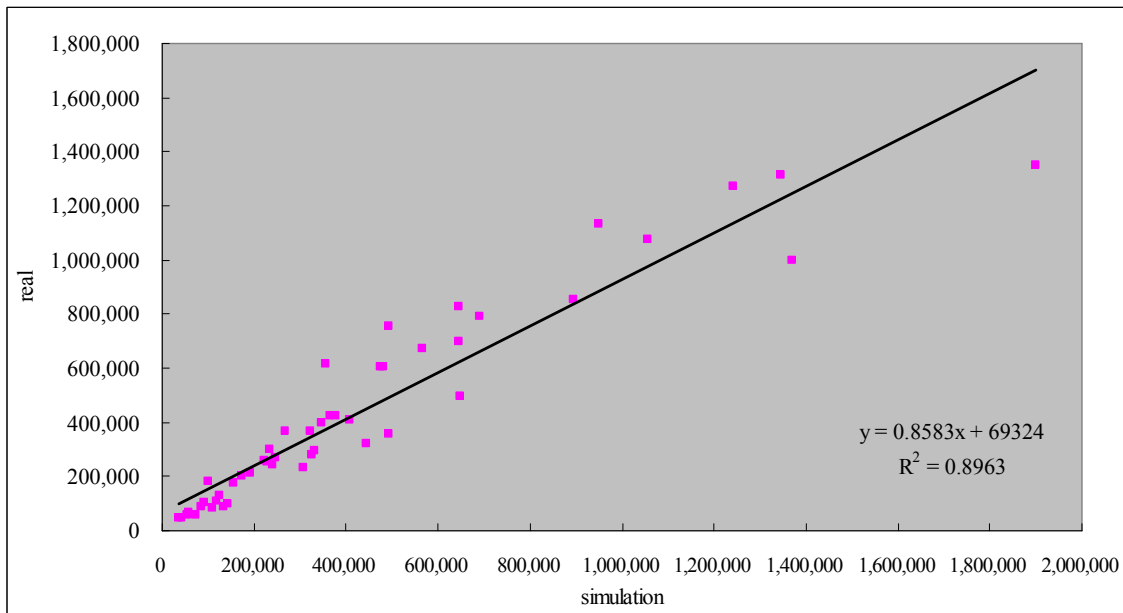
Graph 2a and Graph 2b show the similar graph for cigarette case. Graph 2a shows that the simulation data fit very well with real data in most of states. Only California and New York's simulation data overestimate cigarette consumption. Graph 2b shows the correlation between simulation data and real data, and the value of the correlation is 0.858, and R-squared is 0.893. If I exclude New York and California, the value of the correlation is 1.03 and R-squared is 0.936, and simulation's fitness improves a lots. The reason why California and New York's fitness are not good is per capita consumption in two states are extremely low. In California, per capita cigarette consumption is 35 package and 46 package in New York, although the average per capita consumption across states is 80 package. The reason why cigarette consumption are too low in these two states is that these two state concern much about younger generation's smoking and health problem, and increase cigarette tax rate drastically and spend much money for preventing smoking¹⁶. For cigarette's consumption behavior, gender and income are taken into account for preference different across state, but these political concerns are not taken into account. Nevertheless the simulation data fit very well with actual data on the whole, and these two Graphs confirms that my model capture consumer's consumption behavior very well.

¹⁶ Refer the website of Campaign for tobacco-free kids. <http://www.tobaccofreekids.org/reports/settlements/>

Graph 2a : Average (4years) Total Cigarette Sales



Graph 2b: The relation between simulation and real data (Cigarette)



To sum up, in my estimated model, households use about 3% of their income on gasoline consumption, and 4.2% of households cross the state border to purchase

gasoline. Similarly, about 1% of income is used for cigarette consumption, and 5% of households cross border to buy cigarettes. It is also important to notice that η is estimated to be higher for gasoline than for cigarettes, which results in households cross border shopping more for cigarettes than for gasoline. I consider the above result to be reasonable since the transportation cost of gasoline should be higher than that of cigarettes.

6.2 Estimation of First Order Condition of State Government

The parameter ϕ means the weight between utility from the private good and utility from the public good, and this parameter is estimated from the first order condition of state government with respect tax rate. The result is shown in Table 2.

Table 2.

$$W_A = \log \left\{ \sum \exp \left[\alpha_A \log(x_A^i - r_x) + (1 - \alpha_A) y_A^i - \eta \cdot \beta^* \cdot d_{Ai} + (\phi_0 + \phi_1 \log(\text{income})) G_A \right] \right\}$$

	Gasoline	Cigarette
ϕ_0	0.00054126	0.00042643
ϕ_1	-4.9116 e-05	-3.8090 e-05

ϕ_1 is estimated -4.9116 e-05 for gasoline and -3.8090 e-05 for cigarette¹⁷. This means that the higher income states weight utility from the private good higher than utility from the public good. This result seems natural since in the richer state, the private sector offers similar or alternative service in place of public services and state government does not need to offer these services by themselves.

6.3 Reaction Function

Given the model and the estimated parameters, per unit taxes and other variables in the data, I compute the slope of the reaction function following equations (6) and (7).

¹⁷ This value seems to be very small, but unit changeable.

Both the horizontal and vertical reaction functions for each 48 states are derived. Notice that household can do cross border shopping only in the neighboring states, and the value of the slope of reaction function between state governments is 0 if two states are not neighbors or states do not compete for the same cross border shoppers. The average slope¹⁸ of the horizontal reaction function between state governments is 0.00015801 for gasoline tax and 0.00011065 for cigarette tax (for more details, see Appendix C). Based on those results, we see almost no horizontal tax interaction among state governments. One reason for this result is that the share of gasoline or cigarette consumption to income is very small; 3% for the former and 1% for the latter. This small share will not give households enough incentive for cross border shopping. Also, in the data, state sales and state population roughly correspond, and only very small fraction of the households is estimated to cross border shop (4.2% for gasoline and 5% for cigarette). Since cross border shopping is the only reaction to taxes in other states, the small horizontal reaction seems to be reasonable. It is also important to notice that the value of the slope of the reaction function between non-neighboring states is not always estimated to be zero. This is because tax changes in non-neighboring state can have an effect through the cross-border shopping by consumers who live in states between those two¹⁹.

In contrast, the value of the slope of the reaction function between state and federal governments is much higher (for more details, see Appendix D). The average value is 0.19173 for gasoline tax and 0.17732 for cigarette tax. It means that state and federal taxes are strategic complements. An increase in federal tax reduces the tax base of the state government and makes it necessary for state government to increase taxes to pay for the spending of public goods. Nevertheless, the true criterion for the tax externality is the absolute value of the slope of reaction function. When absolute average value is computed, it is 0.199 for gasoline case and 0.221 for cigarette case. It means that the scale of vertical externality is bigger in cigarette case than in gasoline case.

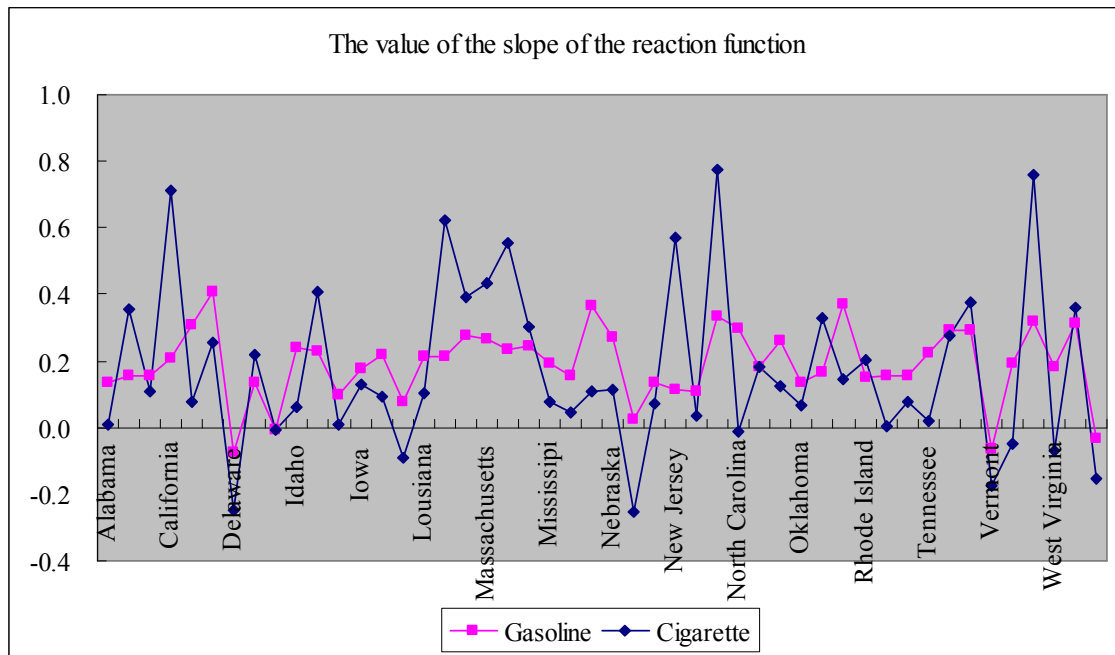
From Graph 3, it is clear that the scale of the slope of reaction function is bigger in cigarette case than in gasoline case. Graph 4 of the Histogram also shows that the scale and variance of the slopes are different between the gasoline and cigarette cases. This result is consistent with the general idea of the relation between price elasticity of demand and intensity of tax competition. Generally, government is reluctant to levy a heavier tax rate on a good whose price elasticity of demand is high to avoid losing tax base. For the good whose price elasticity of demand is high, consumer's demand change

¹⁸ The average is the average value for neighbors which take non zero value.

¹⁹ This is the case where non neighbors state compete for the same cross border shoppers.

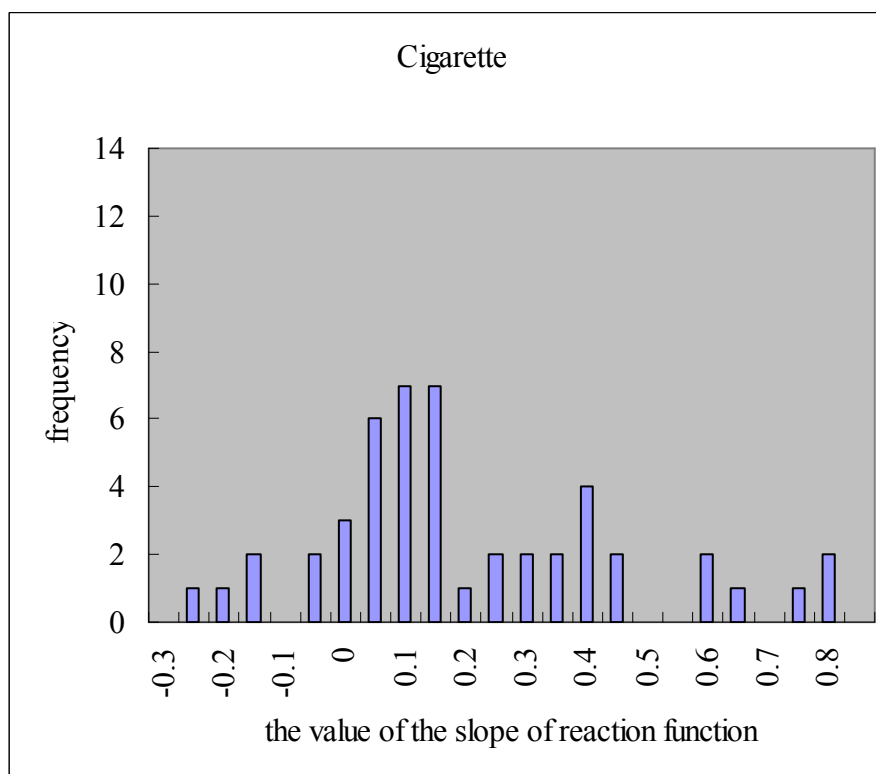
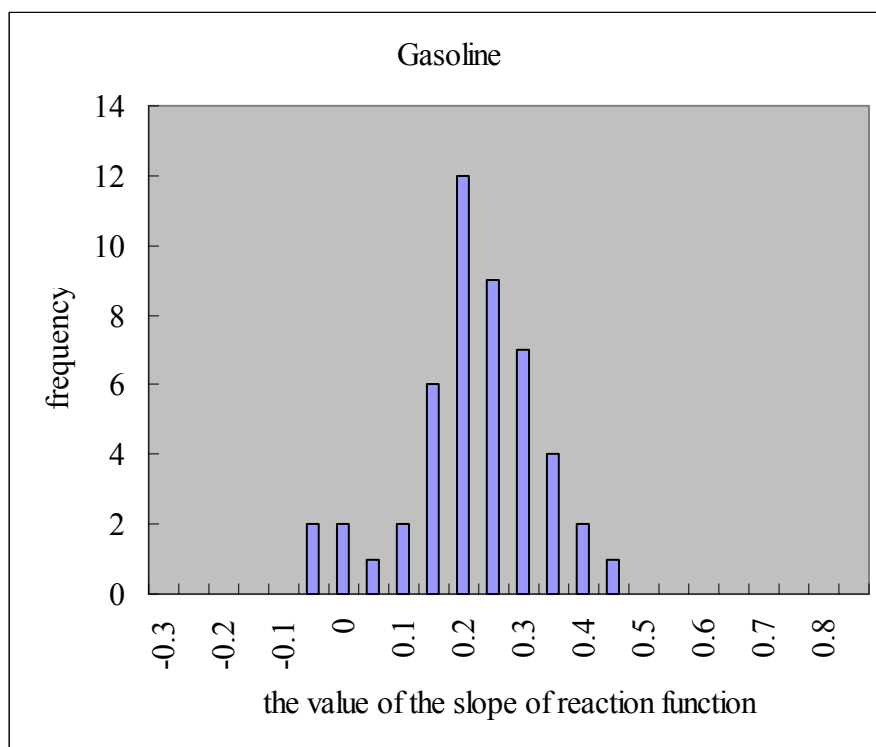
drastically with the change of tax rate. If consumer's demand is very sensitive to tax rate change, state governments are reluctant to change tax rate to be afraid of losing tax resources. In short, the response of state government tax policy to other state or federal tax changes must be bigger for a good whose price elasticity of demand is low since the state government does not need to be afraid of losing tax base even though they change their own tax rate following the other government's taxation change. Graph 3 also shows a high correlation between the gasoline and cigarette cases. I will explain this correlation by analyzing the relation between the scale of the vertical externality and the price elasticity of demand or after tax price in section 8.2.

Graph 3: The value of the slope of the reaction function (Vertical Externality case)



It is also important to notice that even though on average, the slopes of the reaction function are positive, it is negative for many states. This result underscores my main point that the slopes of the reaction function are highly nonlinear functions of variables such as the share of consumption to income α_i , price elasticity of demand ε , after tax price P_i , the share function s_i^j and income I_i . As these variables show sizeable variation across state, it is very natural that the slopes of the reaction function vary across state in ways that cannot be approximated very well by the weighted matrix, which imposes the same sign, and slope in vertical externality case.

Graph 4: Histogram of the value of the slope of the reaction function (Vertical Externality case)



7 Intuition

In this section, I would like to explain the factors which determine the sign and the scale of the slope of the reaction function based on the model and the intuition of it.

Horizontal tax interaction is attributed to consumers' cross border shopping and the scale of the slope of the horizontal reaction function is mainly determined by the share of private good consumption to income and the share function. We have seen from the estimation results that the slope of the horizontal reaction function is estimated to be small. The model indicates that the slope of the reaction function crucially depends on the cross border shopping behavior of households, since that is the only way that tax changes of other states affect consumers. We now present how share of own state consumers of state A (s_A^A) change due to changes in taxes in a neighbor state B (t_B).

$$\frac{\partial s_A^A}{\partial t_B} = - \left(\frac{\alpha_A}{x_A^B - r_x} \frac{x_A^B}{P_B} \varepsilon + \frac{1 - \alpha_A}{y_A^B} \frac{\partial y_A^B}{\partial t_B} \right) s_A^A s_A^B$$

We can see that it depends on the parameter α_A roughly measuring the share of private goods consumption to income, price elasticity of demand ε and share function s_A^A and s_A^B . First, the share of private good consumption to income α_A is small. The ratio of gasoline or cigarette consumption to income is 3% and 1% for each, and this small ratio does not give people enough motivation to do cross border shopping. In addition, only small percentage of people do cross border shopping in both gasoline (4.2%) and cigarette (5%) case and the share function s_i^j (in this example case s_A^B) is very small. Therefore, people in state A who are affected by state government (B)'s tax rate change are very small. Because of these two reasons, the value of the slope of the horizontal reaction function is small in both gasoline and cigarette cases.

Vertical tax interaction results from the fact that federal and state government share the common tax base and the scale of the slope of the vertical reaction function depends on the utility function, price elasticity of demand and after tax price. If the federal government increases its tax rate, households reduces their demand for the private good. Tax revenue in state A decreases, and utility from both the private good and the public good decline. If state government (A) increase its tax rate with the federal government, utility from the private good and the public good move in opposite directions. Households have to reduce their demand more for the private good, and utility from the

private good decreases further. On the other hand, tax revenue from the private good increases, and utility from the public good increases. For simplicity, consider the no cross border shopping case and express the utility function as,

$$W = u(x) + f(G)$$

where x is private good and G is public good. The numerator of the vertical reaction function is expressed as follow.

$$\begin{aligned} \frac{\partial W}{\partial t_A \partial T} &= \frac{\partial^2 u}{\partial x^2} \left(\frac{\partial x}{\partial t_A} \right) \left(\frac{\partial x}{\partial T} \right) + \frac{\partial u}{\partial x} \left(\frac{\partial^2 x}{\partial t_A \partial T} \right) + \frac{\partial^2 f}{\partial G^2} \left(\frac{\partial G}{\partial t_A} \right) \left(\frac{\partial G}{\partial T} \right) + \frac{\partial f}{\partial G} \left(\frac{\partial^2 G}{\partial t_A \partial T} \right) \\ &= \frac{\partial^2 u}{\partial x^2} \left(\frac{x}{P} \varepsilon \right)^2 + \frac{\partial u}{\partial x} \left(-2 \frac{x}{P^2} \varepsilon \right) + \frac{\partial^2 f}{\partial G^2} \left(\frac{x \cdot t_A}{P} \varepsilon + x \right) \left(\frac{x \cdot t_A}{P} \varepsilon \right) + \frac{\partial f}{\partial G} \left(-2 \frac{x \cdot t_A}{P^2} \varepsilon + \frac{x}{P} \varepsilon \right) \end{aligned}$$

State Government (A) compares “the extent of change of disadvantage (additional decrease in utility from a private good)” which is represented by first and second term, and “the extent of change of advantage (increase in utility from a public good)” which is represented by third and fourth term, and tries to equalize these two values to maximize the welfare of people. The scale of state government (A)’s response to federal government tax rate change hinges on the difference between these two scales in increasing its tax rate. If this difference is big, the state government has to respond considerably to equalize the marginal benefit and cost of increasing the tax rate, and if not, the state government does not need to react much to federal government’s tax policy change. It is clear from this equation that this difference is mainly determined by the utility function ($u(x)$ and $f(G)$), price elasticity of demand (ε) and after tax price(P).

The sign of the slope of the reaction function depends on the relative scale of “advantage” and “disadvantage” of increasing the tax rate. In the horizontal tax competition case, “advantage” is increased tax revenue to finance the public good and “disadvantage” is disutility from reducing consumption of the private good. If state government (B) increases its tax rate, some people not only in state A but also in other state shift the place for shopping from state B to A. Then, the tax base of state A expands, and if state government (A) raises its tax rate, tax revenue increases. On the other hand, if state government (A) increases tax rate at this time, not only people who

originally purchase their own region's good but also people who stop cross border shopping to state B have to reduce consumption of the private good, and utility from the private good decreases. State government (A) has to compare this advantage and disadvantage. If the advantage is bigger, the sign of the slopes is positive, and the state government increases its tax rate to increase tax revenue for the public good. If the disadvantage is bigger, the sign of the slopes is negative, and the state government decreases its tax rate to protect utility from the private good.

Similarly, in the vertical tax competition case, "advantage" is the increase in the utility from a public good and "disadvantage" is the disutility from additionally reducing private good consumption. If the federal government increases its tax rate, people reduce the demand for private good and tax revenue in state A decreases. If state government (A) increases its tax rate, utility from the private good and the public good move in opposite directions. Households have to reduce more the demand for the private good and utility from the private good decreases further. On the other hand, tax revenue from the private good increases, and utility from the public good increases. If the scale change of advantage (utility from a public good) is bigger than the scale change of disadvantage (utility from a private good), the sign of the slopes is positive and the state government increase its tax rate to finance public good. If the scale change of disadvantage is bigger than that of advantage, the sign of the slopes is negative and the state government decreases its tax rate to protect utility from the private good. In summary, the share of consumption to total income and the percentage of cross border shopping are important factors for horizontal tax competition while the utility function, price elasticity of demand and after tax price are important factors for vertical tax competition.

8 Discussion with previous papers

In this section, I would like to emphasize the contribution in this paper from two different aspects. One is comparing the weighted matrix method with the structural estimation method. Another is clarifying the sign of the slope of the reaction function and the relationship between the scale of the slope of the reaction function and the price elasticity of demand or after tax price in the vertical tax competition case. I refer Besley and Rosen (1998) and Devereux et al (2007) for the first argument and Keen (1998) for the latter argument.

8.1 Comparison between the weighted matrix method and the structural estimation method

In previous papers, the weighted matrix method is commonly used for estimating horizontal tax competition. The idea of the weighted matrix method is calculating a weighted average of other state tax rates using a weighted matrix and regressing each state's tax rate with this weighted average tax rate as an independent variable. In short, this method approximates the complex strategic interactions between state governments. Table 3 shows the comparison between previous papers and this paper's result.

Table 3.

	Besley & Rosen (98)	Devereux et al (07)			This Paper
		Uniform	Neighbor	Density ²⁰	
Gasoline State		0.131	0.191	-0.099	0.000158
Federal	0.413 **	0.033	0.077	0.122 **	0.192
Cigarette State		0.2	0.277 ***	0.156 **	0.000111
Federal	0.277 **	0.103	0.082	0.081	0.177

** mean 5% significant, *** means 1% significant

Previous papers results show that (1) Devereux et al (2007) estimate the positive and significant horizontal tax externality in cigarette case, but not in gasoline case. (2) Besley and Rosen (98)'s paper estimate the positive and significant vertical tax externality both in gasoline and cigarette, while Devereux et al (07) find only in gasoline case. The scale of vertical externality is bigger in gasoline case than in cigarette case. (3) The sign of the slope of the reaction function is all positive both in the horizontal and vertical externality case. On the other hand, my result derives different results. First, there is little horizontal tax externality in both the gasoline and the cigarette case. Second, there is a positive vertical tax externality in both the gasoline and the cigarette case. The scale of the tax externality is larger in the cigarette case than the gasoline case, which meet the general idea that government is reluctant to levy a higher tax rate on a good whose price elasticity is high. Third, the sign and value of the slope of the reaction function is very different across states, and some states take negative values.

²⁰ These are the factors of the weighted matrix.

There are some reasons why my results are different from previous papers. The time span for the empirical analysis is different²¹. Also, socio economic factors used as independent variables are different. But the most important difference is a method of estimation; reduced form weighted matrix method or structural estimation. This weighted matrix method has some draw backs. First, tax response depends only on one variable which is used as a factor of the weighted matrix, and the sign of the slope of the reaction function is assumed to be same across states. Other important factors (difference between own state tax and that of other state, transportation cost, own and other state's population, demand and price elasticity of demand) are all excluded, resulting in bias and unstable results. Because of this instability, results are very different, depending on which variable is used for the weight. Second, the weighted matrix approach is a poor approximation of the Nash equilibrium of state and federal governments' strategic taxation game. This is because the weighted matrix is a linear approximation around a symmetric Nash Equilibrium which is applicable only if state governments are symmetric and consumer's utility function is Quasi linear. Hence, the estimation result based on the weighted matrix is reliable only if the equilibrium is very close to being symmetric, i.e. if the states are very similar to each other, and people's demand for the private good is independent to income, which is not true. This results in a misspecification problem.

To demonstrate the problem of the weighted matrix method, I simulate state tax rates based on my model under the condition of no cross border shopping, and replicate the weighted matrix method following Devereux et al (2007)²². The simulated data²³ fit well with real state tax rate (please refer Appendix E) and this supports that my model is appropriate. The estimation result is shown at Table 4. The estimation result shows the positive and significant horizontal tax externality both for the gasoline and the cigarette case. These results are very strange since state tax rate are simulated under the condition that there is no cross border shopping, and the state government determine its tax rate without taking into account other state's taxation. From this analysis, it is no exaggeration to say that the estimated coefficient does not necessarily mean the slope of the reaction function and reduced form weighted matrix method is not appropriate for assessing tax externality.

²¹ Besley and Rosen (1998) paper use data from 1975 to 1989 while Devereux et al (2007)'s paper use data from 1977 to 1997. My paper's time span is 1999 to 2002.

²² I am grateful for Michael Devereux for letting me use his data.

²³ Of course, the coefficients used for simulation is different form the result of Table 2 since I assume no cross border shopping. But the coefficients estimated under no cross border shopping are almost same as those of Table 2.

Table 4.

	Simulated Data (No Cross Border Shopping)		
Gasoline	Uniform	Neighbor	Density
State	0.984 ^{***} (13.17)	0.701 ^{***} (10.31)	0.629 ^{***} (9.29)
Federal	-0.495 (-1.22)	-0.739 (-1.64)	-0.732 (-1.57)
Cigarette			
State	0.639 [*] (1.77)	0.364 ^{***} (3.37)	0.194 ^{***} (2.66)
Federal	0.234 (0.47)	0.566 ^{**} (2.1)	0.763 ^{***} (2.98)

The value in parentheses is t statistics. *mean 10% significant, ** mean 5% significant, *** means 1% significant.

Contrary to these defaults, my method has the following virtues. First, my estimation is based on an optimal behavior of household consumption and state government's welfare maximization and fully captures all the important factors for taxation in the model. In addition, the slope of the reaction function is computed directly from the first order condition of the state government, and non linear functional form is taken into account. All the slopes of the reaction functions of state governments are derived for each state and federal government, and different values and sign are allowed across states. Concretely, my structure estimation method overcomes all the problems of previous weighted matrix method, and my result is more appropriate considering this analysis.

8.2 The Reaction Function in the Vertical tax competition case

Keen (1998)'s paper examines vertical tax competition and analyzes the sign of the slope of the reaction function. According to his explanation, the sign of the slope of the reaction function depends on the demand function in the Leviathan case. If the demand function is log convex in after tax price, the sign is negative and if not, the sign is positive. This idea is consistent with my paper. My demand function is log convex in after tax price and if I calculate the slope of the reaction function in the Leviathan case

$\frac{\partial t_A}{\partial T} = -\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} / \frac{\partial^2 TR_{GA}}{\partial t_A^2}$, the value becomes negative. Also, under the extreme

assumption of no cross border shopping, this value becomes almost -1/2 from the equation (C') in Appendix F. On the other hand, he explains that the sign of the slope of the reaction function is positive in benevolent government case. He believed that the cost of additional reduction of utility from the private good is less than the benefit of increase in utility from the public good when both federal and state government increase tax rate, and state government increase its tax rate to finance public good when federal government increase its tax rate. Unfortunately, this is not true as I argued. The sign depends on the relative scale of “advantage (utility increase from the public good)” and “disadvantage (utility decrease from the private good)” in increasing tax rate, which hinges on utility function, price elasticity of demand and after tax price, and some state takes negative value. My results show that some state’s slope of the reaction function is positive, and these state governments increase its tax rate when the federal government raise it tax rate, while some state’s slope of the reaction function is negative, and these state government decrease it tax rate when the federal government raise it tax rate.

I also clarify the relationship between the scale of the slope of the vertical reaction function and price elasticity of demand (ε) or after tax price (P). I derive

$\frac{\partial t_A}{\partial T} = -\frac{\partial^2 W_A}{\partial t_A \partial T} / \frac{\partial^2 W_A}{\partial t_A^2}$ in the equation (C) in Appendix F under the assumption of no

cross border shopping. This is an extreme case but gives some sense how the scale and direction of tax competition is determined. From this equation, it is clear that the slope of the reaction function depends on price elasticity of demand and after tax price. If I differentiate the equation (C) with respect to price elasticity of demand (ε), the value becomes negative²⁴. On the other hand, if I differentiate the equation (C) with respect to after tax price (P), the value becomes positive²⁵.

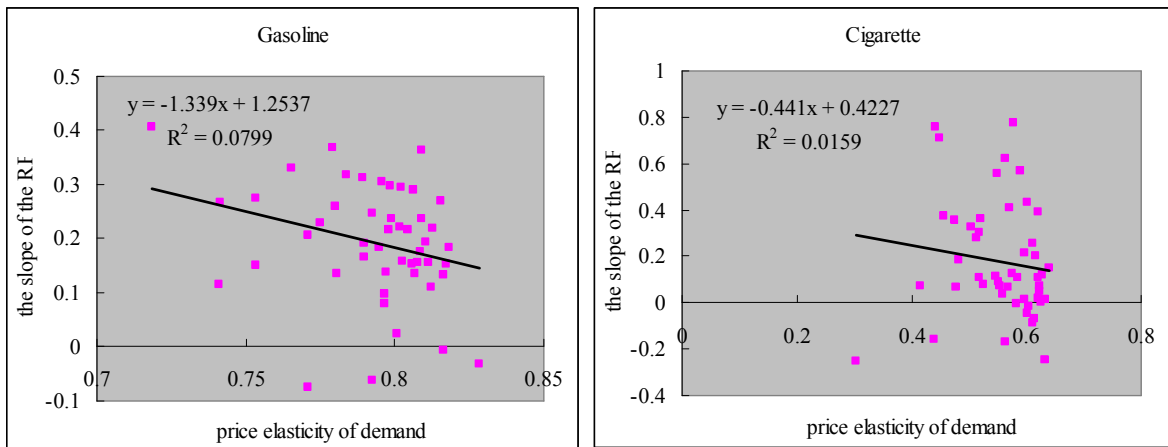
²⁴ the value is $\frac{-2P}{(P+t_A\varepsilon)} \left\{ -2 - \left(\frac{x}{x-r_x} + 2 \frac{P-t_A}{P+t_A} \right) \varepsilon \right\} < 0$. This value is derived assuming that x and P is constant

for simplicity.

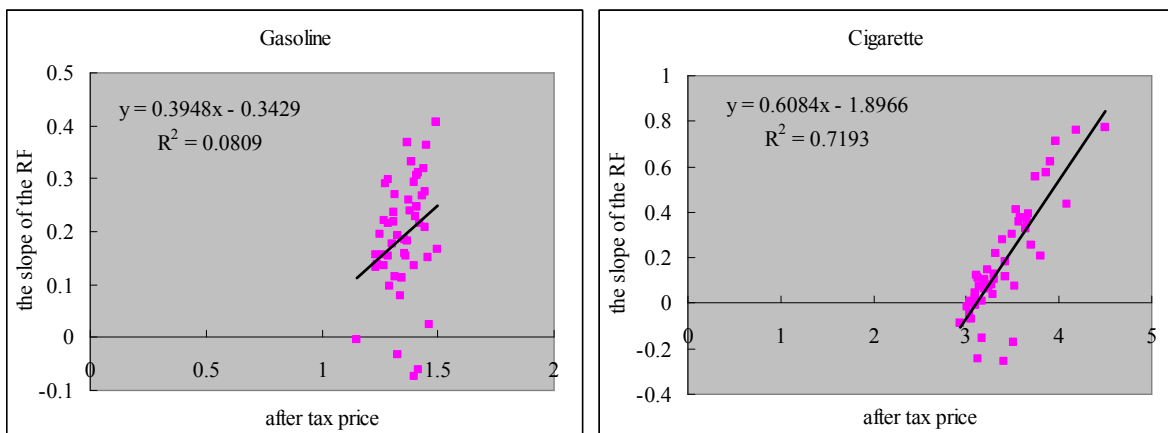
²⁵ the value is $\frac{2t_A}{(P+t_A\varepsilon)^2} \left\{ -\frac{2}{\varepsilon} - \frac{x}{x-r_x} - 2 \left(\frac{P-t_A}{P+t_A} \right) \right\} > 0$. This value is derived assuming that ε and x is constant for

In short, the slope of the reaction function has a negative relation with price elasticity of demand and a positive relation with after tax price. If price elasticity of demand is large, consumer's demand response to tax rate change is large, and state government is reluctant to change its tax rate to avoid losing tax base. This is why there is a negative relation between price elasticity of demand and the scale of tax competition. There is a positive relation between price elasticity of demand and after tax price²⁶. If after tax price is big, the price elasticity of demand becomes small. If price elasticity of demand is small, state government's response to other state government's tax change becomes large, and this is why there is a positive relation between after tax price and the scale of tax externality. This idea is consistent with my result, as two Graph 5 and 6 show.

Graph 5: The relation between the slope of the reaction function and price elasticity of demand



Graph 6: The relation between the slope of the reaction function and price elasticity of demand

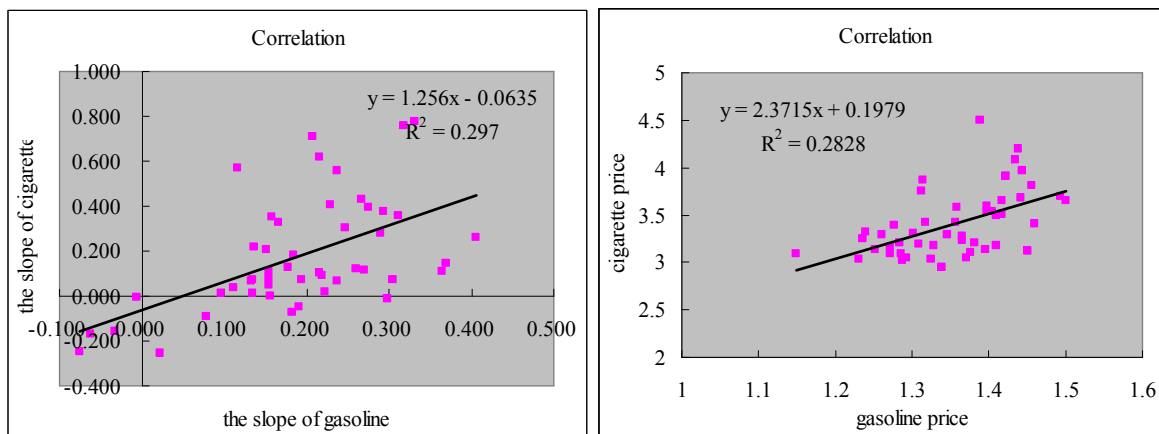


simplicity.

²⁶ Please refer equation (4).

Comparing the gasoline and cigarette tax, the price elasticity of demand is bigger in gasoline case than in cigarette case, and the scale of vertical externality is larger in cigarette case than in gasoline. Also I find a strong correlation between the gasoline and cigarette case in Graph 4, and it is because there is a high positive correlation between after price tax of gasoline and cigarette, as Graph 7 shows.

Graph 7: The correlation between Gasoline case and Cigarette case



From this argument, I can conclude that the price elasticity of demand and after tax price are important factors for vertical externality in benevolent government case, while it is not much important in the Leviathan case.

9 Conclusion

In this paper, I propose a structural estimation approach to analyze the question whether vertical and horizontal tax competition exist for gasoline and cigarette taxes, which overcome all the problems of the commonly used weighted matrix approach. Through the process of analysis, I estimate the structure parameters of the household utility function. Given the parameters of household utility, I recover the parameter of benevolent state government's objective function. Using all the estimated structure parameters, I compute the value of the slope of reaction function for each state which represents strategic interaction of taxation between governments.

From this analysis, I have the following results. First the value of the slope of the reaction function among state government is very small and there is barely an interaction among state tax policies in both the gasoline and the cigarette case. This is because both gasoline and cigarette's consumption share to total income is very small,

and the percentage of cross border shopping estimated to be very small. Second, the value of the slope of the reaction function between the state and the federal government is positive on average for both cases, and the state government increases its tax rate when the federal government increases. The value of the slope of the reaction function is bigger in the cigarette tax case than that in the gasoline tax case, and this result is consistent with the general idea of the relation between price elasticity of demand and tax competition intensity. Third, on average the value of the slope of the reaction function is positive in both case, the value and sign of the slope is very different among states. The scale of the slope is determined mainly by the share of commodity consumption to total income and the percentage of cross border shopping in the horizontal externality case, while the utility function, the price elasticity of demand and the after tax price are important factors in the vertical externality case. The sign of the slope is determined by the relative scale of advantage and disadvantage of increasing the tax rate, which depends on the utility function, the price elasticity of demand, the share function and after tax price and the tax rate itself.

My result is different from previous paper Besley and Rosen (1998) and Devereux et al (2007), and I am convinced that my structural estimation approach is more appropriate than their method considering the weak point of the weighted matrix method. Also, this paper is a complement of Keen's (1998) paper. The estimation result also has an important policy implication. The different value and sign of the slope of the reaction function tell us that state governments respond to federal government tax policy differently, and the federal government should not use same policy for all state to maximize the total welfare in the nation. In this view, it is significant to investigate more how each state government reacts to federal government policy and what is the reason for it. Through these analyses, I expect more practical policies are proposed for the welfare in the nation.

Appendix A: The factor of the reaction function

$$\begin{aligned}
\frac{\partial^2 W_A}{\partial t_A^2} &= \left(\frac{-\alpha_A}{(x_A^A - r_x)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A^2} + \frac{-(1-\alpha_A)}{y_A^A} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A^2} \right) s_A^A \\
&\quad - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial t_A} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A^2} \right) < 0 \\
&= \left(\frac{\alpha_A I_A^2 - 2\alpha_A I_A (p_A + t_A + T) r_x - (1-\alpha_A) (p_A + t_A + T)^2 r_x^2}{\{I_A - (p_A + t_A + T) r_x\}^2 (p_A + t_A + T)^2} \right) s_A^A \\
&\quad - \frac{\alpha_A I_A + (1-\alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\} (p_A + t_A + T)} \frac{\partial s_A^A}{\partial t_A} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A^2} \right) < 0
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial^2 TR_{GA}}{\partial t_A^2} &= n_A s_A^A \frac{\partial^2 x_A^A}{\partial t_A^2} t_A + n_B s_B^A \frac{\partial^2 x_B^A}{\partial t_A^2} t_A + n_C s_C^A \frac{\partial^2 x_C^A}{\partial t_A^2} t_A \\
&\quad + 2n_A \left(s_A^A + \frac{\partial s_A^A}{\partial t_A} t_A \right) \frac{\partial x_A^A}{\partial t_A} + 2n_B \left(s_B^A + \frac{\partial s_B^A}{\partial t_A} t_A \right) \frac{\partial x_B^A}{\partial t_A} + 2n_C \left(s_C^A + \frac{\partial s_C^A}{\partial t_A} t_A \right) \frac{\partial x_C^A}{\partial t_A} \\
&\quad + n_A \left(2 \frac{\partial s_A^A}{\partial t_A} + \frac{\partial^2 s_A^A}{\partial t_A^2} t_A \right) x_A^A + n_B \left(2 \frac{\partial s_B^A}{\partial t_A} + \frac{\partial^2 s_B^A}{\partial t_A^2} t_A \right) x_B^A + n_C \left(2 \frac{\partial s_C^A}{\partial t_A} + \frac{\partial^2 s_C^A}{\partial t_A^2} t_A \right) x_C^A < 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 W_A}{\partial t_A \partial t_B} &= - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial t_B} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B} \right) \\
&= - \left(\frac{\alpha_A I_A + (1-\alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\} (p_A + t_A + T)} \right) \frac{\partial s_A^A}{\partial t_B} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B} \right)
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial^2 TR_{GA}}{\partial t_A \partial t_B} &= n_A \frac{\partial s_A^A}{\partial t_B} \frac{\partial x_A^A}{\partial t_A} t_A + n_B \frac{\partial s_B^A}{\partial t_B} \frac{\partial x_B^A}{\partial t_A} t_A + n_C \frac{\partial s_C^A}{\partial t_B} \frac{\partial x_C^A}{\partial t_A} t_A \\
&\quad + n_A \left(\frac{\partial s_A^A}{\partial t_B} + \frac{\partial^2 s_A^A}{\partial t_A \partial t_B} t_A \right) x_A^A + n_B \left(\frac{\partial s_B^A}{\partial t_B} + \frac{\partial^2 s_B^A}{\partial t_A \partial t_B} t_A \right) x_B^A + n_C \left(\frac{\partial s_C^A}{\partial t_B} + \frac{\partial^2 s_C^A}{\partial t_A \partial t_B} t_A \right) x_C^A
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 W_A}{\partial t_A \partial T} &= \left(\frac{-\alpha_A}{(x_A^A - r_x)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A^2} + \frac{-(1-\alpha_A)}{y_A^A} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 + \frac{1-\alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A^2} \right) s_A^A \\
&\quad - \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1-\alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right) \frac{\partial s_A^A}{\partial T} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \right) \\
&= \left(\frac{\alpha_A I_A^2 - 2\alpha_A I_A (p_A + t_A + T) r_x - (1-\alpha_A) (p_A + t_A + T)^2 r_x^2}{\{I_A - (p_A + t_A + T) r_x\}^2 (p_A + t_A + T)^2} \right) s_A^A \\
&\quad - \frac{\alpha_A I_A + (1-\alpha_A) r_x (p_A + t_A + T)}{\{I_A - (p_A + t_A + T) r_x\} (p_A + t_A + T)} \frac{\partial s_A^A}{\partial T} + \phi \left(\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \right)
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} &= n_A s_A^A \frac{\partial^2 x_A^A}{\partial t_A \partial T} t_A + n_B s_B^A \frac{\partial^2 x_B^A}{\partial t_A \partial T} t_A + n_C s_C^A \frac{\partial^2 x_C^A}{\partial t_A \partial T} t_A \\
&\quad + n_A \left(s_A^A + \left(\frac{\partial s_A^A}{\partial t_A} + \frac{\partial s_A^A}{\partial T} \right) t_A \right) \frac{\partial x_A^A}{\partial t_A} + n_B \left(s_B^A + \left(\frac{\partial s_B^A}{\partial t_A} + \frac{\partial s_B^A}{\partial T} \right) t_A \right) \frac{\partial x_B^A}{\partial t_A} + n_C \left(s_C^A + \left(\frac{\partial s_C^A}{\partial t_A} + \frac{\partial s_C^A}{\partial T} \right) t_A \right) \frac{\partial x_C^A}{\partial t_A} \\
&\quad + n_A \left(\frac{\partial s_A^A}{\partial T} + \frac{\partial^2 s_A^A}{\partial t_A \partial T} t_A \right) x_A^A + n_B \left(\frac{\partial s_B^A}{\partial T} + \frac{\partial^2 s_B^A}{\partial t_A \partial T} t_A \right) x_B^A + n_C \left(\frac{\partial s_C^A}{\partial T} + \frac{\partial^2 s_C^A}{\partial t_A \partial T} t_A \right) x_C^A
\end{aligned}$$

Appendix B: Data sources

Data	resource	webpage
Gasoline unit tax rate	Federal Highway Administration	http://www.fhwa.dot.gov/policy/ohpi/hss/index.cfm
Gasoline price	Energy Information	http://www.eia.doe.gov/emeu/states/_seds.html
Gasoline consumption	Energy Information	http://www.eia.doe.gov/emeu/states/_seds.html
Cigarette unit tax rate	Report:Tax Burden on Tabocco	http://www.srnt.org/pubs/nl_05_06/spotlight.html
Cigarette price	Report:Tax Burden on Tabocco	
Cigarette consumption	Report:Tax Burden on Tabocco	
Expenditure of State	U.S Census of Bureau	http://www.census.gov/govs/www/index.html
Per capita federal grant to Highway department	Federal Highway Administration	http://www.fhwa.dot.gov/policy/ohpi/hss/index.cfm
Per capita federal grant to Health department	Statistics of Abstract	http://www.census.gov/compendia/statab/
Population	Bureau of Economic Analysis	http://www.bea.gov/regional/spi/
Per capita disposal Income	Bureau of Economic Analysis	http://www.bea.gov/regional/spi/
Land area	Statistics of Abstract	http://www.census.gov/compendia/statab/
Ratio of female	Center of Disease Control and Prevention	http://wonder.cdc.gov/Bridged-Race-v2006.HTML
CO2 emission	U.S. Enviroment Protection	http://www.epa.gov/climatechange/emissions/state_energyco2inv.html
the number of car registration	Federal Highway Administration	http://www.fhwa.dot.gov/policy/ohpi/hss/index.cfm
the percentage of smoker	Statistics of Abstract	http://www.census.gov/compendia/statab/
the number of death cancer	Statistics of Abstract	http://www.census.gov/compendia/statab/
Distance between center of city	Google map	

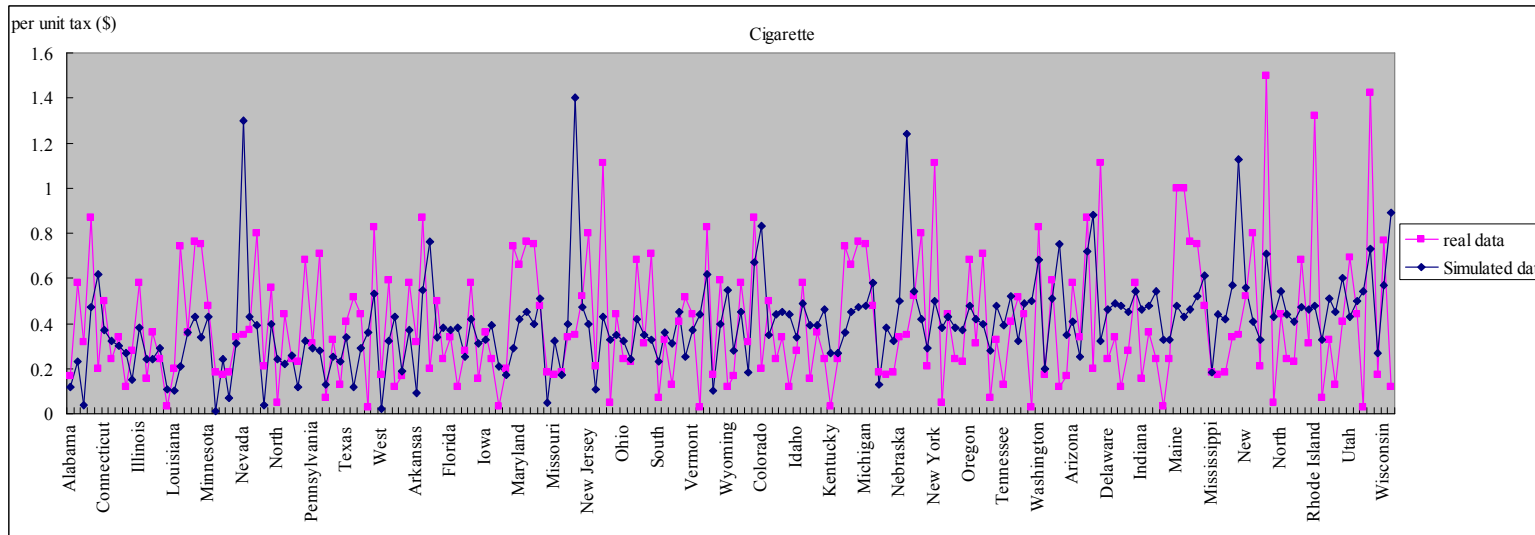
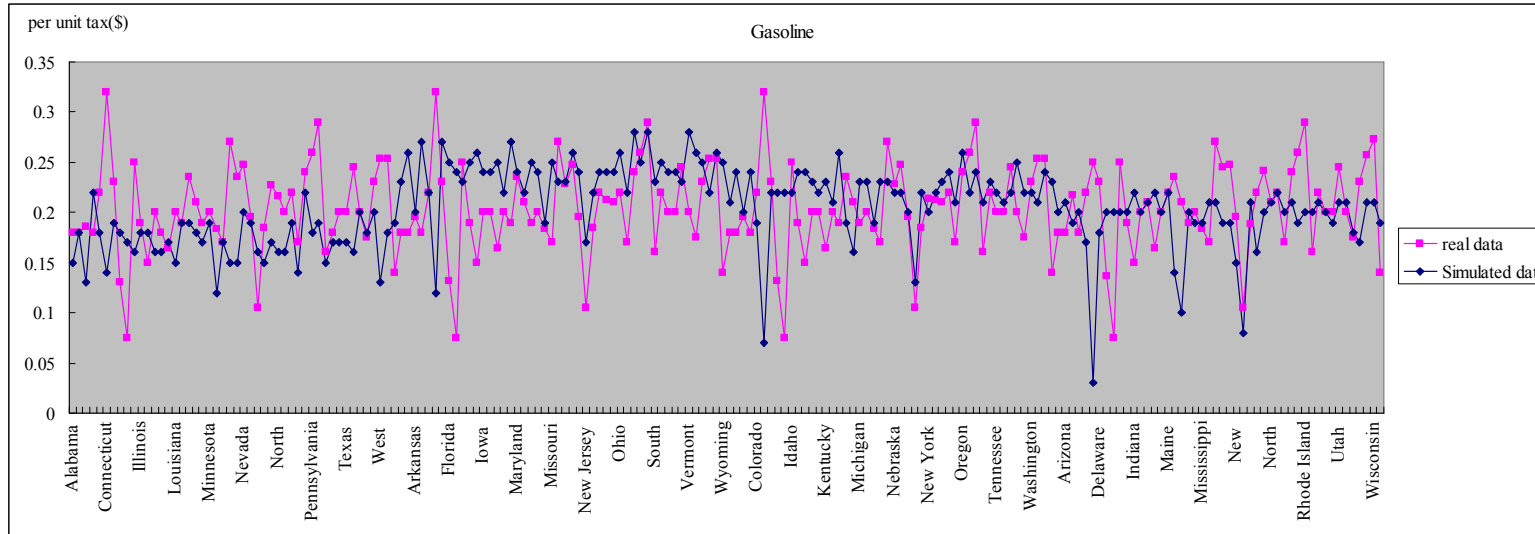
Data summary statistics

Data	Observation	Mean	Standard Deviation	Min	Max
Gasoline unit tax rate	192	0.20	0.04	0.075	0.32
Gasoline price	192	0.97	0.14	0.66	1.23
Gasoline consumption	192	493.7	61.7	293.8	690.4
Cigarette unit tax rate	192	0.40	0.29	0.03	1.5
Cigarette price	192	2.68	0.24	2.275	4.035
Cigarette consumption	192	83.6	24.8	35.3	164.8
Highway Expenditure	192	330.5	109.6	154.9	739.2
Health Expenditure	192	115.2	65.3	44.9	416.2
Per capita federal grant to Highway	192	142.9	60.5	51.9	323.5
Per capita federal grant to Health	192	605.3	169.4	238.8	1294.6
Population	192	5843	6232	492	34526
Per capita disposal Income	192	24719	3421	18038	35868
Land area	192	168340	122466	4002	695621
Ratio of female	192	0.51	0.01	0.49	0.52
CO2 emission	192	37.2	40.9	1.6	223.2407
the number of car registration	192	2790933	3035170	195762	18400000
the percentage of smoker	192	23.2	3.1	12.7	32.6
the number of death cancer	192	3.25	3.09	0.2	13.5
Distance between center of city	1159	1692.5	991.4	66.2	4292.0

Appendix D: The slope of the reaction function in the case of Vertical Externality

	Gasoline	Cigarette
Alabama	0.134	0.011
Arizona	0.158	0.354
Arkansas	0.155	0.107
California	0.207	0.710
Colorado	0.306	0.075
Conneticut	0.406	0.258
Delaware	-0.075	-0.247
Florida	0.137	0.216
Georgia	-0.005	-0.007
Idaho	0.238	0.064
Illinois	0.228	0.408
Indiana	0.096	0.011
Iowa	0.177	0.128
Kansas	0.219	0.092
Kentucky	0.079	-0.089
Lousiana	0.216	0.105
Maine	0.215	0.621
Maryland	0.275	0.391
Massachusetts	0.266	0.433
Michigan	0.237	0.556
Minnesota	0.247	0.305
Mississipi	0.194	0.075
Missouri	0.154	0.046
Montana	0.364	0.109
Nebraska	0.270	0.114
Nevada	0.023	-0.255
New Hampshire	0.135	0.072
New Jersey	0.116	0.571
New Mexico	0.111	0.038
New York	0.332	0.774
North Carolina	0.298	-0.014
North Dakota	0.184	0.182
Ohio	0.260	0.122
Oklahoma	0.133	0.067
Oregon	0.167	0.328
Pennsylvania	0.369	0.148
Rhode Island	0.151	0.205
South Carolina	0.156	0.002
South Dakota	0.154	0.079
Tennessee	0.222	0.018
Texas	0.290	0.278
Utah	0.294	0.375
Vermont	-0.062	-0.172
Virginia	0.192	-0.046
Washigton	0.319	0.759
West Virginia	0.183	-0.071
Wisconsin	0.312	0.360
Wyoming	-0.031	-0.156

Appendix E: Real data and Simulated data



Appendix F

Here, I would like to express the slope of the reaction function in the vertical externality case. For simplicity, I assume there is no cross border shopping; that is $s_i^i = 1$ and $s_i^j = 0$. This is an extreme example, but gives a clear idea what are important factors for vertical tax competition. I estimated that the percentage of cross border shopping is around 4.2% for gasoline and 5% for cigarette, and this extreme assumption is not inappropriate. The numerator and denominator of the reaction function are expressed as follows.

$$\frac{\partial^2 W_A}{\partial t_A^2} = \gamma_A^2 s_A^A - \gamma_A^1 \frac{\partial s_A^A}{\partial t_A} + \phi \frac{\partial^2 TR_A}{\partial t_A^2} \quad (\text{A})$$

$$\frac{\partial^2 W_A}{\partial t_A \partial T} = \gamma_A^2 s_A^A + \gamma_A^1 \frac{\partial s_A^A}{\partial T} + \phi \frac{\partial^2 TR_A}{\partial t_A \partial T} \quad (\text{B})$$

where

$$\gamma_A^2 = \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial^2 x_A^A}{\partial t_A^2} - \frac{\alpha_A}{(x_A^A - r_x)^2} \left(\frac{\partial x_A^A}{\partial t_A} \right)^2 + \frac{1 - \alpha_A}{y_A^A} \frac{\partial^2 y_A^A}{\partial t_A^2} - \frac{1 - \alpha_A}{y_A^{A2}} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2 \right)$$

$$\text{or} = -2 \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A^2} \varepsilon - \frac{\alpha_A}{(x_A^A - r_x)^2} \frac{x_A^{A2}}{P_A^2} \varepsilon^2 - \frac{1 - \alpha_A}{y_A^2} \left(\frac{\partial y_A^A}{\partial t_A} \right)^2$$

$$\gamma_A^1 = \left(\frac{\alpha_A}{x_A^A - r_x} \frac{\partial x_A^A}{\partial t_A} + \frac{1 - \alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A} \right)$$

$$\text{or} = \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A} \varepsilon + \frac{1 - \alpha_A}{y_A^A} \frac{\partial y_A^A}{\partial t_A}$$

From the first order condition,

$$\phi = \frac{\gamma_A^1}{x_A^A \left\{ \varepsilon \frac{t_A}{P_A} + 1 \right\}}$$

If we omit the part of y_A^A term for simplicity,

$$\frac{\partial^2 W_A}{\partial t_A^2} = \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A^2} \left(-2\varepsilon - \frac{x_A^A}{x_A^A - r_x} \varepsilon^2 - 2 \left(\frac{P_A - t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2 \right) \quad (\text{A}')$$

$$\frac{\partial^2 W_A}{\partial t_A \partial T} = \frac{\alpha_A}{x_A^A - r_x} \frac{x_A^A}{P_A^2} \left(-2\varepsilon - \frac{x_A^A}{x_A^A - r_x} \varepsilon^2 - \left(\frac{P_A - 2t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2 \right) \quad (\text{B}')$$

Then, the value of the slope of the reaction function becomes,

$$\frac{\partial^2 W_A}{\partial t_A \partial T} \bigg/ \frac{\partial^2 W_A}{\partial t_A^2} = \frac{-2\varepsilon - \frac{x_A^A}{x_A^A - r_x} \varepsilon^2 - \left(\frac{P_A - 2t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2}{-2\varepsilon - \frac{x_A^A}{x_A^A - r_x} \varepsilon^2 - 2 \left(\frac{P_A - t_A}{P_A + t_A \varepsilon} \right) \varepsilon^2} \quad (\text{C})$$

It is also interesting to see the value of the slope of the reaction function in vertical tax competition for the Leviathan case.

$$\frac{\partial^2 TR_{GA}}{\partial t_A \partial T} \bigg/ \frac{\partial^2 TR_{GA}}{\partial t_A^2} = \frac{P_A - 2t_A}{-2(P_A - t_A)} \approx -\frac{1}{2} \quad (\text{C}')$$

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