

MA Methods  
Convexity and Relations Practice Problems

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1. Let  $A$  and  $B$  be convex sets. Show by counterexample that  $A \cup B$  need not be a convex set.

2. Let  $S$  and  $T$  be convex sets. Prove that each of the following is also a convex set:

(a)  $-S \equiv \{x \mid x = -s, s \in S\}$

(b)  $S - T \equiv \{x \mid x = s - t, s \in S, t \in T\}$

3. If  $A, B, X,$  and  $Y$  are sets, then for each of the following statements, provide an example and a proof

(a)  $(A \cup B) \times X = (A \times X) \cup (B \times X)$

(b)  $(A \cap B) \times (X \cap Y) = (A \times X) \cap (B \times Y)$

(c)  $(A - B) \times X = (A \times X) - (B \times X)$

4. Let  $S$  be the set of all people on earth. Let the relation  $\mathbf{R}$  be defined by the statement “loves.” Is  $\mathbf{R}$  complete? Transitive?

5. Let  $A$  and  $B$  be two sets in domain  $D$ , and suppose that  $B \subset A$ . Prove that  $f(B) \subset f(A)$  for any mapping  $f : D \rightarrow R$ .

6. Let  $A$  and  $B$  be two sets in range  $R$ , and suppose that  $B \subset A$ . Prove that  $f^{-1}(B) \subset f^{-1}(A)$  for any mapping  $f : D \rightarrow R$ .