# Illiquidity, Fire-sales and Capital Structure*

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**Abstract**

This paper investigates the industry dynamics and capital structure of firms that hold illiquid assets and face potential fire-sales. Firms enter the industry by paying a fixed cost financed by equity and debt. Once in operation, shareholders have the option to adjust their asset holdings at a cost but also retain the option to exit the industry by defaulting on their debt. While costly, asset sales allow the shareholders to boost dividends or to service debt payments rather than defaulting. However, as shareholders make their decisions once debt is in place, the resulting conflict with bondholders entails over-investment and early liquidation due to debt-overhang. A substantial number of firms also exit following an exogenous financial shock. In the stationary industry equilibrium, firms selling assets but are not defaulting find it more costly to reduce capacity due to the price effects of fire-sale liquidations. This price feedback effect results in lower industry leverage but a higher default rate. Capital regulation reduces leverage ex-ante but at the cost of inducing more default. Restricting asset sales mitigates fire-sales and also reduce leverage ex-ante.

**JEL Classification Codes:** G28, G32, G33
**Keywords:** Capital Structure, Fire-Sales, Illiquidity, Macro-prudential Regulation

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# 1 Introduction

The recent crisis has called into question the effectiveness of regulation for financial institutions. These regulations were designed to limit the probability of failure of individual institutions. They require problem institutions to either sell declining assets or raise equity. However, the indifference shown to the method of adjustment may be problematic when a large number of financial institutions are in difficulty as this may precipitate excessive balance-sheet shrinkage leading to a credit crunch.

This has led to an alternative approach to regulation, termed "macro-prudential" regulation, aimed at preventing these types of problems. Hanson, Kashyap, and Stein (2010) define this approach "as an effort to control the social costs associated with excessive balance-sheet shrinkage on the part of multiple financial institutions hit with a common shock." They further identify two costs of generalized balance-sheet shrinkage: credit-crunches and fire-sales. When financial intermediaries reduce their assets by making reductions in new lending to operating firms, the result is a reduction in "investment and employment with contractionary consequences for the macroeconomy." Fire-sales, the simultaneous attempt by many firms to shed assets, may lead to large declines in prices as described by Shleifer and Vishny (1992, 2011). These two effects are closed linked since a decline in asset prices may necessitate further asset reductions, leading to further declines in prices.

The challenge is to understand why financial institutions don't take adequate steps to mitigate these effects. Hanson, Kashyap, and Stein (2010) point to the debt-overhang problem as the reason financial institutions are incapable of raising fresh equity once a crisis is underway. Moreover, financial institutions also fail to take adequate steps prior to a crisis when there is a preference for debt. The reason is that they don't take into account the negative price externality their fire-sales impose on the value of collateral of other institutions during a crisis. Stein (2010) offers an account along these lines.

In this paper, I extend the framework of Stein (2010) to analyze the dynamic capital structure choice of financial intermediaries in an industry where firms compete in an a market for financial intermediation services but also in a market for capital or assets that make these services feasible. Firms enter the industry by purchasing an initial stock of assets at a fixed cost financed by equity and debt. Once in operation, shareholders have the option to adjust their asset holdings at a cost but also retain the option to exit the industry by defaulting on their debt. While costly, capacity reductions allow the shareholders to boost dividends or to service debt payments rather than defaulting. However, as shareholders make their decisions once debt is in place, the resulting conflict with bondholders entails over-investment and early liquidation due to debt-overhang. Moreover, following Hanson, Kashyap, and Stein (2010), this paper proscribes a role for equity injections by outsiders. Nevertheless, capital injections from existing shareholders are permitted and default only results when shareholders choose to forgo such injections in favour of liquidation.

Meanwhile, firms also face the prospect of immediate liquidation, to be interpreted as the realization of an adverse financial shock. At any given moment, a significant number of firms exit the industry and liquidate their asset holdings in the process. The paper examines the impact potential fire-sales have on the determination of ex-ante capital structure and also the impact on the default decision of firms not directly impacted by the financial shock. In the stationary industry equilibrium, firms selling assets that are not defaulting find it more costly to reduce capacity due to the price effects of liquidations by defaulting firms. This price feedback effect results in lower industry leverage but a higher default rate.

In terms of the implications for regulation, the findings here stress the importance of preventing disorderly liquidation and the need for reducing leverage ex-ante. While reducing leverage, capital regulations appear counter-productive as they tend to reinforce the debt-overhang problem. Here, the requirement to maintain the value of equity at or above a pre-determined level is equivalent to a fixed cost, much like the cost of debt service. This leads shareholders to over-invest in unproductive assets and exit earlier, but as bondholders anticipate this, leverage is lower.

This paper is related to the large literature on dynamic contingent analysis beginning with Black and Scholes (1973) and Merton (1974). Specifically, it draws on the real options valuation literature. Brennan and Schwartz (1984), Mello and Parsons (1992) as well as Titman and Tsyplakov (2007) analyze the interaction between financing decisions and default through the use of numerical methods. Dixit (1992) and Caballero (1991) as well as Bertola and Caballero (1994) study industry equilibria of all equity financed firms that make entry and exit decisions. Leland (1994, 1998), and Morellec (2001) analyze the optimal capital structure choices of a single firm. Miao (2005) studies investment and optimal capital structure capital decisions are costless to reverse.

This paper also relates to the literature on macro-prudential regulation. In an early contribution, Blum and Hellwig (1995) show that rigid capital adequacy regulation for banks may reinforce macroeconomic fluctuations. Repullo and Suarez (2010) offer important modifications to current capital regulations that help mitigate the severity of credit crunches following an aggregate shock. Hanson, Kashyap, and Stein (2010) analyze qualitatively the effectiveness of several proposed macro-prudential regulatory measures. Their analysis underlines the need to proceed cautiously in raising capital requirements. However, they emphasize the need to reduce leverage ex-ante through the imposition of hair-cuts on asset purchases. The findings here largely support these conclusions.

This paper is organized as follows. The following section details the main elements of the model. Section 3 analyzes the investment and optimal capital structures choices of a firm that may only *reduce* capacity at a cost. Section 4 extends these results to also permit costly capacity expansions. Then, in Section 5, the industry equilibrium is analyzed, prior to concluding.

# 2 Model

I consider a financial intermediation industry with a large number of financial intermediaries or firms. All investors and firms are risk-neutral and discount future cash flows at a constant risk-free rate $\rho > 0$. Time varies continuously over $[0, \infty)$. All stochastic processes are defined over the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ which describes the uncertainty in the economy.

## 2.1 Financial Intermediaries

There are a continuum of ex-ante identical firms. At each instant, every firm holds a stocks of assets $a$ that generate instantaneous operating profit given by:

$$\pi(x, a) = pxa^{1-\gamma} - c_f$$

where $\gamma \in (0, 1)$, $c_f > 0$, $p$ is the price of intermediation services prevailing in the market, and $x$ represents firm-specific asset productivity. In above expression, $xa^{\gamma}$ is the quantity of intermediation services provided so that $pxa^{\gamma}$ are the flow revenues, while $c_f$ is a fixed operating cost. In addition, $\{x_t\}_{t \geq 0}$ follows a geometric Brownian motion

$$dx_t/x_t = \mu_x dt + \sigma_x dW_t \tag{1}$$

where $\mu_x, \sigma_x > 0$ and $\{W_t\}_{t \geq 0}$ is a standard Brownian motion representing firm-specific shifts in productivity. Growth in firm-specific productivity captures financial innovation that expands the intermediation capabilities of the firm.

Financial intermediaries often adjust their asset stocks in response to market conditions.[1] Here, the firms can reduce their intermediation capacity by

[1] Adrian and Shin (2010) document changes in asset levels and leverage for large financial intermediaries.

selling assets (disinvesting) to outsiders at a price $q$ per unit. I consider first the case where firms can only sell assets so that sales are effectively irreversible reductions in capacity as in Morellec (2001). Then, in Section 4, I relax this assumption by allowing firms to also expand capacity through additional asset purchases.

## 2.2 Intermediation and Resale Markets

Firms compete in an intermediation market, with the quantity of assets held by a firm determining the level of intermediation provided by the firm. Competition entails that individual firms act as price-takers in the intermediation market. The price that each firm faces at every instant $t$ is then given by:

$$p = D(Y) \tag{2}$$

where $D(Y)$ is the inverse aggregate demand for intermediation services. $D(Y)$ is a decreasing function of the aggregate quantity of intermediation services $Y$. For tractability, I assume that $D(Y)$ is of the following functional form:

$$D(Y) = Y^{-\frac{1}{\epsilon}} \tag{3}$$

where $\epsilon$ is the elasticity of demand.

Firms can sell units of their assets to outsiders. The price prevailing at any time in the resale market is a function of aggregate assets $A_l$ for sale at that time, and individual firms act as price-takers in the resale market. Assuming an aggregate demand function of the iso-elastic form in the resale market, the resale price $q$ is given by:

$$q = A_l^{-\frac{1}{\epsilon'}} \tag{4}$$

where $\epsilon'$ is the elasticity of demand in the resale market. While the assets across various firms need not be identical, I assume they are treated as such by outside investors as the latter are insufficiently informed to distinguish them.

## 2.3 Unlevered Firm

The unlevered firm's objective is to choose an investment policy and abandonment rule to maximize the expected value of the stream of discounted profits. The instantaneous profits of the firm are:

$$\pi(x, a; p) = pxa^{1-\gamma} - c_f \tag{5}$$

where it takes the intermediation price $p$ as given.

Now, let $u_t$ denote cumulative gross disinvestment (i.e. sales of assets) up to date $t$. The stochastic process for assets is:

$$da_t = -du_t. \tag{6}$$

Then, given an intermediation price $p$ and a resale price $q$, the unlevered firm starting with an initial stock $a$ and demand $x$ solves the following problem:

$$v_u(x, a; p, q) = \max_{T \in \mathcal{T}, \{u_t, t \in [0,T]\}} (1-\tau)\mathbb{E}^x \left[ \int_0^T e^{-(\rho+\eta)t} \left\{ \pi(x_t, a_t; p) dt + q du_t \right\} \right] \tag{7}$$

where $\{u_t\}_{0 \leq t \leq T}$ is a nondecreasing continuous stochastic process with $u_0 = 0$, $\mathcal{T}$ is the set of all stopping times relative to the filtration generated by the Brownian motion $\{W_t\}_{t \geq 0}$, $E^x$ is the expectation taken with respect to the process $\{x_t\}_{t \geq 0}$ when the starting value is $x$, $\tau$ is the tax rate on corporate income, and $\eta$ is a Poisson death rate. The parameter $\eta$ captures shock to the financial system affected a positive measure of firms that subsequently liquidate their asset holdings and exit. This exogenous exit channel exacerbates selling pressure in the market for assets resulting in equilibrium prices below their fundamental value. I interpret the sales of these firms as "fire-sales."

The abandonment decision is an option to abandon that is exercised the first time asset productivity falls below a threshold level $x_u(a_u; p, q)$, where $a_u$ is the asset stock at abandonment. The investment decision corresponds to a *continuum* of options that are exercised the first time asset productivity falls below a threshold curve $x_l(a_u; p, q)$ for $a > a_u$. This curves traces out the combinations of asset productivity, $x$ and stock $a$ that equate the marginal benefit from selling to the marginal product of assets in place.

## 2.4 Debt and Liquidation Value

Firms issue debt because interest payments to debt are tax deductible. Debt is issued at par with infinite maturity following Leland (1994) and Duffie and Lando (2001). The firm is obligated by the debt contract to pay a coupon $b$ to bondholders as long as it is in operation. The residual profit-flow is distributed among shareholders. Upon default the firm is liquidated immediately,[2] at which time the bondholders receive the liquidation value and the shareholders are wiped out.

[2]The underlying assumption here is that debt restructuring is very costly. If one considers the case of large financial intermediaries, the The failures of Bear Stearns and Lehman Brothers provide

Mello and Parsons (1992), Morellec (2001) and Miao (2005), model liquidation value as a fraction of the unlevered firm value $v_u(x, a; p, q)$. The unlevered firm value is equal to the after-tax present value of the flow of profits, plus the value associated with the options to alter asset levels and the option to abandon the assets. Financial intermediaries typically own assets that are marked-to-market. The liquidation value of a financial intermediary is then typically the market value of its asset holding. For this reason, I specify the abandonment value of the unlevered firm to be the market value of its assets, namely $qa$ if the firm has $a$ assets on hand at the time of abandonment.

## 2.5 Equity, Investment and Liquidation

Given a coupon $b$, the shareholders make investment and liquidation decisions to maximize the value of equity:

$$
\begin{aligned}
&e(x, a, b; p, q) \\
&\quad = \max_{T \in \mathcal{T}, \{u_t, t \in [0,T]\}} (1-\tau)\mathbb{E}^x \left[ \int_0^T e^{-(\rho+\eta)t} \left\{ [\pi(x_t, a_t; p) - b]dt + q du_t \right\} \right]. \quad (8)
\end{aligned}
$$

Note that the value of equity is increasing in the resale price $q$. Moreover, costs of debt service operate in the same manner as the fixed costs $c_f$.

The default decision corresponds to an option to abandon which is exercised the first time demand falls below a threshold level $x_d(a_d, b; p, q)$ that depends on the coupon $b$, and where $a_d$ is the asset stock at default. The investment decision again corresponds to a continuum of options to reduce capacity. These options are exercised whenever asset productivity falls below the threshold curve $x_l(a; p, q)$ as long as $a > a_d$. Notice that as the coupon essentially imposes a fixed cost on shareholders, it does not impact the selling threshold $x_l$ as the latter is pinned down by the *marginal* costs and benefits of reducing capacity.

## 2.6 Debt Value

Debt holders are entitled to the coupon payments $b$ while the firm is in operation along with the abandonment value. Bankruptcy is costly and thus the abandonment value is a is a fraction $(1 - \zeta) \in (0, 1)$ of the market value $qa_d$ of the firm's assets at default. Hence, the arbitrage-free value of debt

examples of costly liquidations in the case of financial intermediaries.

$d(x, a, b; p, q)$ is given by:

$$d(x, a, b; p, q) = \mathbb{E}^x \left[ \int_0^{T_{x_d}} e^{-(\rho+\eta)t} b dt \right] + (1 - \zeta) q a_d E^x [e^{-(\rho+\eta)T_{x_d}}] \tag{9}$$

where $T_{x_d}$ denotes the first time firm-specific demand falls below the default threshold $x_d$.

## 2.7 Firm Value

The value of the firm $v(x, a, b; p, q)$ is the sum of the value of the equity and debt. Hence,

$$v(x, a, b; p, q) = e(x, a, b; p, q) + d(x, a, b; p, q) \tag{10}$$

## 2.8 Entry

There are a continuum of potential firms that can enter the industry at each instant by incurring a fixed sunk cost of entry $c_e$. This cost is financed by debt and equity. The initial productivity and size are drawn uniformly from the set $[\underline{x}, \overline{x}] \times [\underline{a}, \overline{a}]$. The draws across firms are assumed to be independent. Upon entering, firms are not obliged to begin providing intermediation services immediately. Entry is to be viewed as a costly mechanism to observe the initial firm-specific asset productivity, and acquire a stock of assets. As a result, firms may choose to wait till asset productivity is sufficiently high to begin operations.

I assume that $\underline{x} > x_d(a_d, b; p, q)$ and $\underline{a} > a_d$, assumptions that will need to be verified in equilibrium as $x_d$ and $a_d$ are determined endogenously. As in Miao (2005), these assumptions serve to avoid the situation where firms enter and exit immediately. After entry, asset productivity of an entrant is given by the process $\{x_t\}_{t \geq 0}$ that is identical to existing firms. However, given the different starting values of $x$, firms face different sequences of productivity levels $\{x_t\}_{t \geq 0}$.

As firms are all identical prior to entering the industry, the ex-ante value to entering must equal the entry cost in equilibrium. Thus, the free-entry condition can be written as:

$$\int_{\underline{x}}^{\overline{x}} \int_{\underline{a}}^{\overline{a}} v(x, a, b; p, q) F(da \times dx) = c_e \tag{11}$$

where $F$ is the uniform distribution on $[\underline{x}, \overline{x}] \times [\underline{a}, \overline{a}]$. The optimal choice of the coupon is made by firms prior to entering. They select $b^*(a; p, q)$ to maximize the expected value $\int_{\underline{x}}^{\overline{x}} \int_{\underline{a}}^{\overline{a}} v(x, a, b; p, q) F(da \times dx)$. As all firms are ex-ante identical they choose the same coupon in equilibrium.

## 2.9 Timing

The timing of the decisions is shown in the figure below:

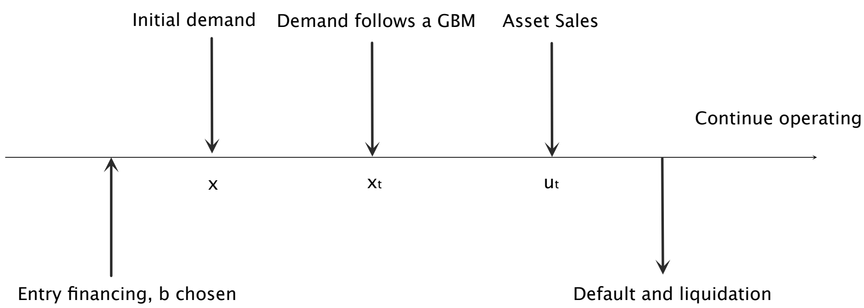


Figure 1: Timing

## 2.10 Aggregates

The long-run steady state is characterized by a stationary distribution of surviving firms $\nu$, and a constant entry rate $N$. Note that if $\nu$ is stationary, so will the equilibrium asset holdings as long as the asset levels of entrants can be chosen appropriately. Then, given a Borel set $B$ in $\mathbb{R}^2$, $\nu(B)$ is the number of surviving firms with $(x, a)$ in $B$. The support of $\nu$ is $[x_d, \infty) \times [a_d, \infty)$. Moreover, we can compute aggregates as follows:

$$Y = \int_{x_d}^{\overline{x}} \int_{a_d}^{\overline{a}} [x a^{1-\gamma}] \nu(dx \times da) \tag{12}$$

$$A_l = \int_{x_d}^{x_l} \int_{a_d}^{\overline{a}} [a - x_l^{-1}(x_l(a; p, q) - x; p, q)] \nu(dx \times da) \tag{13}$$

where $x^{-1}(\cdot)$ is the inverse of the selling threshold function $x_l$. It is important to emphasize that $\nu$ is not a probability distribution.

### 2.11 Equilibrium

In the case where only capacity contractions are permitted, a stationary industry equilibrium is a constant intermediation price $p^*$, a constant resale price $q^*$, an exit/default asset stock $a_d^*$, an exit/default threshold $x_d(a_d^*, b^*; p^*, q^*)$, an entry rate $N^*$, and a stationary distributions $\nu^*$ such that:

(i) $b^*$ is determined via $\arg\max_b \int_{\underline{x}}^{\overline{x}} \int_{\underline{a}}^{\overline{a}} v(x, a, b; p, q) F(da \times dx)$

(ii) shareholders solve (8)

(iii) markets clear:

$$p^* = Y(\nu^*, a_d^*, b^*)^{-\frac{1}{\epsilon}} \tag{14}$$

$$q^* = A_l(\nu^*, a_d^*, b^*)^{-\frac{1}{\epsilon'}} \tag{15}$$

(iv) the free-entry condition (11) holds

(v) the distribution $\nu^*$ is an invariant measures over $[x_d^*, \infty) \times [a_d^*, \infty)$

The stationarity of the equilibrium distributions implies that the distribution of firms is constant over time. Implicitly, due to the idiosyncratic nature of shocks a law of large numbers is assumed. Nevertheless, considerable dynamics underlie this stationary equilibrium. At any point in time, a large number of firms are entering while an equal amount are exiting. Similarly, a large number of firms are selling assets, either to service debt payments or because of liquidations following default, while a large number of firms are holding capacity fixed. Finally, the above definition of equilibrium can naturally be modified to apply when capacity expansions are also permitted.

## 3 Optimal Capital Structure

In this section, I solve the model where only capacity reductions are possible. This case is instructive as closed-form expressions for the optimal abandonment and investment decisions, along with firm value can be obtained. These results closely mirror those in Morellec (2001). In the next section, these results are extended to the case when capacity expansions are also possible.

Shareholders make investment and default decisions after debt is in place and are protected by limited liability, so they seek to expropriate bondholders.

The latter anticipate this and as a result, the optimal capital structure is inefficient. The resulting debt-overhang problem leads shareholders to over-invest in unproductive assets and exit early, as in Myers (1977). Moreover, debt-overhang causes increased delays in investing as

## 3.1 Unlevered Firm

As noted earlier, the objective of shareholders in the unlevered firm is to make abandonment and investment decisions that maximize the expected stream of discounted profits. Formally, the problem is as follows:

$$\begin{aligned} &v_u(x, a; p, q) \\ &\quad = \max_{T \in \mathcal{T}, \{u_t, t \in [0,T]\}} (1-\tau)\mathbb{E}^x \left[ \int_0^T e^{-(\rho+\eta)t} \left\{ \pi(x_t, a_t; p)dt + q du_t \right\} \right] \end{aligned}$$

Before presenting a formal solution to this problem, I proceed heuristically to derive the optimal decisions and firm value relying on intuition from the real-options literature. This approach makes the economic tradeoffs involved in the decisions more transparent.

The value of the unlevered firm stems from three sources. The first source of value is the right to the discounted profits from operating the firm's assets forever. The second source of value comes from the continuum of irreversible options to sell assets, or reduce capacity. Finally, the third source of value derives from the value associated with the (irreversible) option to default. Thus, to determine the value of the firm, it suffices to sum the value of each of these options.

The value from operating the firm's assets forever without altering capacity when the initial asset level is $a$ and initial asset productivity is $x$ can be written as:

$$\Pi(x, a; p) = (1-\tau)\mathbb{E}^x \left[ \int_0^\infty e^{-(\rho+\eta)t} \left\{ \pi(x_t, a_t; p)dt \right\} \right].$$

This is of course just the value of the discounted stream of profits in perpetuity. Given that instantaneous profits are linear in asset productivity, the expectation above can be explicitly computed so that $\Pi(x, a; p)$ can be written as

$$(1-\tau)\left( \frac{pxa^{1-\gamma}}{(\rho+\eta)-\mu_x} - \frac{c}{\rho+\eta} \right). \tag{16}$$

This expression is simply the difference between the after-tax present-values of revenues and costs.

The value from capacity reductions can be viewed as a *continuum* of options available to the firm. The firm starts with an initial stock of assets $a$, and can reduce capacity to every level below $a$. Each possible reduction constitutes an option, and since there are continuum of possible reductions, the firm holds a continuum of options. The optimal capacity reduction decision trades off the marginal product of assets against the sale price. Variability in asset productivity naturally leads to changes in the marginal product of assets. When the marginal product falls below the resale price, assets are sold.

As the costs of adjusting capacity are linear, the optimal investment policy is defined by a continuous increasing threshold function $x_l(a; p, q)$. Thus, given an initial asset level $a$, if productivity falls below $x_l(a; p, q)$ assets are sold. The proceeds from these sales are paid out as dividends to shareholders. The firm undertakes these reductions in capacity because they boost dividends when asset productivity is low. On the other hand, if asset productivity is above $x_l(a; p, q)$, the firm does nothing, as the option to sell is not "in the money." Following Morellec (2001), the value of the option to reduce capacity can be written as:

$$\int_a^{a_u} \Big( qE^x \big[ e^{-(\rho+\eta)T_{x_l(\tilde{a};p,q)}} \big] - \\ (1-\tau)\mathbb{E}^x \left[ \int_{T_{x_l(\tilde{a};p,q)}}^{\infty} e^{-(\rho+\eta)t} \pi_a(x_t, \tilde{a}_t; p) dt \right] d\tilde{a} \Big) \quad (17)$$

where $\pi_a(\cdot)$ is the marginal product of assets, and $a_u$ is the firm size at abandonment. In the expression above, the first term captures the expected discounted proceeds from sales. The second term captures the loss in profits resulting from a permanent reduction in capacity. The optimal selling threshold is found using the super-contact condition:

$$\frac{\partial^2 v_u}{\partial x \partial \tilde{a}}\Big|_{x = x_l(\tilde{a};p,q)} = 0 \quad (18)$$

The option to abandon the firm's assets is a single irreversible option. The optimal abandonment policy trades off the value of continuing operations against the liquidation value. The former fluctuates due to changes in asset productivity. The firm then chooses to liquidate when the value generated by assets in place fall below the liquidation value. The firm's abandonment policy is defined by another continuous threshold function $x_u(a; p, q)$ such that given an asset stock $a$, the firm liquidates the first time asset productivity falls below $x_u(a; p, q)$. Firm size at abandonment, $a_u$, is determined by equating the two thresholds, $x_l(a_u; p, q) = x_u(a_u; p, q)$. That is, the firm is abandoned when the liquidation value of the firm exceeds the value from continuing operations at a lower capacity.

The value from abandonment is the sum of the liquidation value discounted till abandonment minus the discounted cash flows from continuing operations at abandonment:

$$qa_u \mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}] - (1-\tau)\mathbb{E}^{x_u}\left[\int_0^{T_{x_u}} e^{-(\rho+\eta)t}\pi(x_t, a_t; p)dt\right].$$

Exploiting the strong Markov property of stopping times to rewrite the second term the value of the abandonment option is:

$$qa_u \mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}] - \mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}]\left[\int_0^{\infty} e^{-(\rho+\eta)t}\pi(x_t, a_t; p)dt\right]$$

Then, noting that $\mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}] = (x/x_u)^{-\lambda}$, the value is

$$[qa_u - \Pi(x_u, a_u; p)]\,(x/x_u)^{-\lambda}. \tag{19}$$

The optimal abandonment threshold $x_u$ is found via the following smooth-pasting condition:

$$\left.\frac{\partial v_u}{\partial x}\right|_{x=x_u} = 0. \tag{20}$$

The optimal decisions of the firm along with its value are described in the following proposition:

**Proposition 1.** *Assume $(\rho+\eta) > \mu_x > 0$. Denote by $-\lambda$ be the negative root of the fundamental quadratic:*

$$(\rho+\eta) - \mu_x\vartheta - \frac{1}{2}\sigma^2(\vartheta-1)\vartheta = 0. \tag{21}$$

*Then,*

$$\begin{aligned} v_u = \Pi(x, a; p) + \frac{q}{(1+\lambda)(1+\gamma\lambda)}&(a(x/x_l(a;p,q))^{-\lambda} - a_u(x/x_l(a_u;p,q))^{-\lambda} \\ &+ [qa_u - \Pi(x_u, a_u; p)]\,(x/x_u(a_u;p,q))^{-\lambda} \end{aligned} \tag{22}$$

*where the capacity reducation and abandonment thresholds are:*

$$x_l(a;p,q) = \frac{\lambda q((\rho+\eta)-\mu_x)}{(1+\lambda)(1-\gamma)(1-\tau)p}a^{\gamma} \tag{23}$$

$$x_u(a_u;p,q) = \frac{\lambda((\rho+\eta)-\mu_x)\,[(1-\tau)(c/(\rho+\eta)+qa_u]\,a_u^{\gamma-1}}{(1+\lambda)(1-\tau)p} \tag{24}$$

*Finally, the firm size upon abandonment is:*

$$a_u = \frac{c(1-\gamma)(1-\tau)}{q(\rho+\eta)\gamma} \tag{25}$$

*Proof.* See Appendix A. □

In the above expression for the value of the firm, $v_u$, the first term is the value from operating the firm's assets forever, the second corresponds to the value from capacity reductions while the final term captures the value from the option to abandon. The optimal investment and abandonment policies are illustrated in the figure below: The selling threshold $x_l$ is increasing with

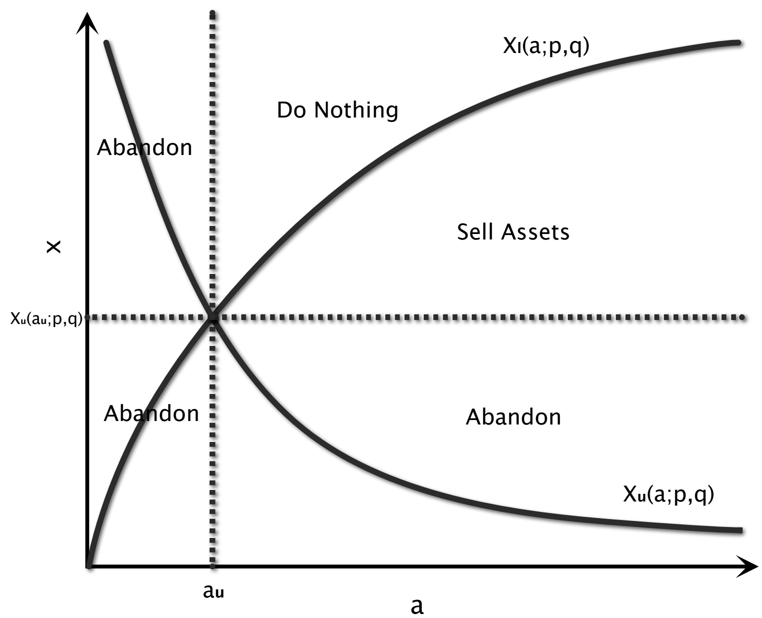


Figure 2: The optimal investment and abandonment policies.

$a$. Above $x_l$, no action is taken as capacity expansions are not possible, while below $x_l$ capacity reductions are undertaken. On the other hand, the abandonment threshold $x_u$ is decreasing in $a$. Below this threshold the liquidation value of the firm exceeds the value from continuing to operate. The intersection of the two thresholds pins down the size upon abandonment $a_u$.

The above expressions allow us to deduce an important difference in how changes in the intermediation price $p$ and the resale price $q$ affect the value of the firm. First consider changes in the intermediation price $p$. Clearly, increasing $p$ raises the value from operating the assets, $\Pi(x, a; p)$. However, by substituting the expression for $x_l$ in $v_u$, it is clear that capacity reductions are less valuable as $p$ is raised. This is because the marginal product of assets goes up as $p$ increases, reducing the need for reducing capacity. Similarly, abandonment is less valuable as $p$ is raised because the foregone profits from continuing operations are higher. The overall effects from changes in $p$ on the value of the firm are ambiguous because increases in operating profits are offset by declines in the values of the options.

Now consider the effect of changing the resale price $q$. Clearly, changing $q$ does not affect the profits from operating the firm's assets in perpetuity, $\Pi(x, a; p)$. However, raising $q$ increases the value from capacity reductions. This is because a higher $q$ results in more proceeds from asset sales. Similarly, the value of the option to default (given by the last term in $v_u$) is also increasing with the resale price as increasing $q$ raises the liquidation value $qa_u$. Thus, while increasing $p$ decreases the values of the options available to the firm, increasing $q$ raises them. The overall effects from changes in $q$ on value are unambiguous: raising $q$ increases the value of the firm.

Table 3.1 below summarizes the effects discussed in the above, along with a few additional comparative statics. The base parameters were chosen to be comparable to those of Morellec (2001) and Miao (2005) as follows: $\gamma = 0.53, a = 100, x = 1, \rho = 0.05, \eta = 0.01, \sigma_x = 0.1, \tau = 0.15, c = 1, p = 1, q = 1, \eta = 0.01$.

| Comparative Statics | | | | |
|---|---|---|---|---|
| Parameter Value | Value without capacity changes | Value with capacity changes | Value of selling option (%) | Size upon default $a_d$ |
| Baseline | 133.9 | 144.5 | 7.8% | 12.6 |
| $\gamma = 0.5$ | 155.8 | 160.5 | 3.0% | 14.2 |
| $\gamma = 0.6$ | 93.1 | 163.4 | 75.3% | 9.4 |
| $\sigma_x = 0.05$ | 133.9 | 160.5 | 19.9% | 12.6 |
| $\sigma_x = 0.15$ | 132.9 | 145.2 | 9.3% | 12.6 |
| $p = 0.5$ | - | 238.3 | - | 12.6 |
| $p = 1.5$ | - | 209.9 | - | 12.6 |
| $q = 0.5$ | - | 134.0 | - | 25.1 |
| $q = 1.5$ | - | 218.0 | - | 8.4 |
| $\tau = 0.05$ | 149.7 | 156.2 | 4.3% | 14.0 |
| $\tau = 0.55$ | 70.9 | 212.8 | 200.1% | 6.7 |

The results show that firm value is no longer monotonically increasing in $p$. For instance, when $p$ is increased from 0.5 to 1, value drops from 238.3 to 144.5, whereas the value then jumps back up to 209.9 as $p$ is raised to 1.5. By contrast, increases in the resale price result in significant increases in value (nearly 50% compared with the baseline), and also lead to much lower abandonment levels.

The effect of changes in the tax rate on firm value are also not monotone. In fact, firm value does increase as taxes as lowered, however it also jumps up for very high tax rates as capacity reductions and abandonment become much more valuable. This can be seen in the last line of the table above.

It is also interesting to note that changes in the volatility in asset productivity do not affect firm value monotonically. This is because more volatility raises the value from the ability to reduce capacity. Indeed, the results above show an increase in value from either increasing or decreasing volatility.

## 3.2 Levered Firm

The firm has a preference for issuing debt over raising equity because of the inherent tax advantages of debt. I consider infinite maturity debt contracts that consist of a coupon $b$ to be paid to bondholders in perpetuity along with a commitment to liquidate the firm upon default. Moreover, absolute priority is enforced in that the proceeds from liquidation are used to first pay bondholders before paying shareholders. Default is triggered when shareholders choose to abandon the firm's assets rather than inject new capital to meet debt obligations.

Formally, given a coupon $b$ shareholders solve the following problem:

$$\begin{aligned} &e(x, a, b; p, q) \\ &\quad = \max_{T \in \mathcal{T}, \{u_t, t \in [0,T]\}} (1-\tau)\mathbb{E}^x \left[ \int_0^T e^{-(\rho+\eta)t} \left\{ [\pi(x_t, a_t; p) - b]dt + q du_t \right\} \right] \end{aligned}$$

When managers operate the firm in the interest of shareholders, they make capacity and default decisions to maximize the value of equity. Equity is valuable because shareholders have residual income rights along with the option to alter the firm's capacity, and the option to default. The option to reduce capacity by selling assets is valuable for two reasons. Selling assets when they are less productive allows the firm to either pay additional dividends or meet debt obligations without raising additional equity.

Following the same real-options approach to compute the value of equity, the value to shareholders arising from their right to operate the firm's assets forever is:

$$\Pi(x, a, b; p) = (1-\tau)\mathbb{E}^x \left[ \int_0^\infty e^{-(\rho+\eta)t} \left\{ [\pi(x_t, a_t; p) - b]dt \right\} \right] \tag{26}$$

As is clear from the above expression, the cost of servicing debt reduces the discounted flow of profits available to shareholders.

The option to default trades off the value from continuing operations against the liquidation value. Debt obligations reduce the profits from operations while bankruptcy costs reduce the liquidation value. When the reduction in profits dominates the reduction in liquidation value, shareholders default earlier than in the unlevered firm. Essentially, there is inefficient liquidation due to debt-overhang as in Myers (1977).

The value to shareholders from the options to reduce capacity is similar to the value of these options in the unlevered case. This is because the fixed coupon $b$ does not alter either the marginal costs of reducing capacity nor the resale price $q$. Hence, the value of the options to reduce capacity is:

$$\int_a^{a_d} \Big( qE^x[e^{-(\rho+\eta)T_{x_l(\tilde{a};p,q)}}] - \\ (1-\tau)\mathbb{E}^x \left[ \int_{T_{x_l(\tilde{a};p,q)}}^{\infty} e^{-(\rho+\eta)t} \pi_a(x_t, \tilde{a}_t; p) dt \right] d\tilde{a} \Big). \quad (27)$$

where $a_d$ is the size of the firm upon default. Again, the optimal capacity reduction threshold is found via the super-contact condition:

$$\frac{\partial^2 v_u}{\partial x \partial \tilde{a}} \Big|_{x=x_l(\tilde{a};p,q)} = 0 \quad (28)$$

The value of equity is then the sum of the values described above. Formally, the problem is similar to the problem of the unlevered firm and is solved using the same method. The optimal policies chosen by shareholders and the value of equity are summarized in proposition below:

**Proposition 2.** *Assume $(\rho + \eta) > \mu_x > 0$. Again denote by $-\lambda$ the negative root of the fundamental quadratic:*

$$(\rho + \eta) - \mu_x \vartheta - \frac{1}{2}\sigma^2(\vartheta - 1)\vartheta = 0. \quad (29)$$

*Then,*

$$e(x, a, b; p, q) = \Pi(x, a, b; p) + \frac{q}{(1+\lambda)(1+\gamma\lambda)} (a(x/x_l(a; p, q))^{-\lambda} - a_d(x/x_l(a_d; p, q))^{-\lambda} \\ - \Pi(x_d, a_d, b; p) \left( \frac{x}{x_d} \right)^{-\lambda} \quad (30)$$

*where the capacity reduction and default thresholds are:*

$$x_l(a; p, q) = \frac{\lambda q a^{\gamma}((\rho + \eta) - \mu_x)}{p(1-\gamma)(1+\lambda)(1-\tau)} \quad (31)$$

$$x_d(a, b; p, q) = \frac{\lambda a^{\gamma-1}((\rho + \eta) - \mu_x)(c + b)}{p(\rho + \eta)(1+\lambda)} \quad (32)$$

*Finally, the firm size upon default is:*

$$a_d = \frac{(c + b)(1-\gamma)(1-\tau)}{q(\rho + \eta)} \quad (33)$$

*Proof.* See Appendix A. □

**Corollary 1.** *When debt service costs are sufficiently higher than the fixed costs, i.e.* $c < \frac{\gamma b}{1-\gamma}$*, then* $a_d > a_u$*.*

The expression for equity above differs from the unlevered value of the firm in two aspects. When maximizing the value of equity with debt in place, shareholders choose a default threshold $x_d$ rather than $x_u$. In addition, shareholders do not receive the liquidation value $qa_d$ upon default.

Most importantly, the above results show that the conflict between shareholders and bondholders leads to debt-overhang. To see this, first note that by examining (32) and (33), it is clear that both the default threshold $x_d(a, b; p, q)$ and the firm size upon default $a_d$ are both increasing in the coupon $b$. In other words, higher leverage increases the probability of default and investment in unproductive assets. Then, when the fixed operating cost is sufficiently low, higher leverage induces shareholders to exit early and over-invest in unproductive assets relative to the unlevered firm.

The optimal decisions of shareholders are depicted below:

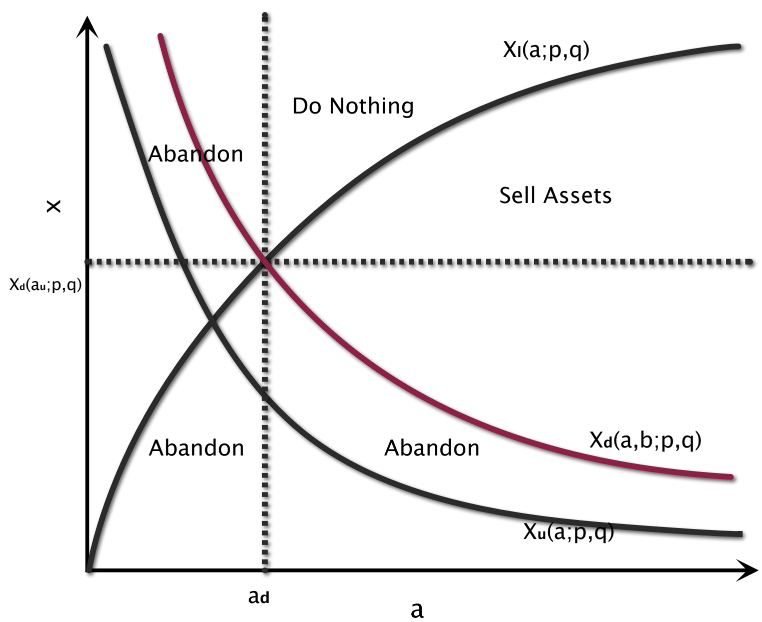


Figure 3: Optimal policies chosen by shareholders.

The figure clearly exhibits the debt-overhang problem: the default threshold curve is above the abandonment threshold curve of the unlevered firm. This implies a higher probability of default or early exit. Furthermore, as the exit size $a_d$ is higher than the unlevered abandonment size $a_u$, shareholders over-invest in unproductive assets than are abandoned upon default.

Given the optimal decisions of shareholders, the value of debt is:

$$d(x, a, b; p, q) = \mathbb{E}^x \left[ \int_0^{T_{x_d}} e^{-(\rho+\eta)t} b dt \right] + (1-\zeta) q a_d E^x [e^{-(\rho+\eta)T_{x_d}}] \tag{34}$$

$$= \frac{b}{(\rho+\eta)} \left( 1 - \left( \frac{x}{x_d} \right)^{-\lambda} \right) + (1-\zeta) q a_d \left( \frac{x}{x_d} \right)^{-\lambda}. \tag{35}$$

As the above expression makes clear, the value of debt is essentially the discounted value of the coupon $b/(\rho+\eta)$ till default plus the liquidation value discounted appropriately.

The value of the levered firm can then be written simply as the sum of the values of equity and debt:

**Proposition 3.** *The value of the levered firm is:*

$$\begin{aligned} &v(x, a, b; p, q) \\ &= \Pi(x, a, b; p) + \frac{q}{(1+\lambda)(1+\gamma\lambda)} (a(x/x_l(a; p, q))^{-\lambda} - a_d(x/x_l(a_d; p, q))^{-\lambda} \\ &\quad + [(1-\zeta) q a_d - \Pi(x_d, a_d, b; p)] \left( \frac{x}{x_d} \right)^{-\lambda} + \frac{b}{(\rho+\eta)} \left( 1 - \left( \frac{x}{x_d} \right)^{-\lambda} \right) \end{aligned} \tag{36}$$

In order to clarify the role of the tax-shield, we can re-write the value of the levered firm using the expressions for $\Pi(x, a, b; p)$ and $\Pi(x_d, a_d, b; p)$ as follows:

$$\begin{aligned} &v(x, a, b; p, q) \\ &= \Pi(x, a; p) + \frac{q}{(1+\lambda)(1+\gamma\lambda)} (a(x/x_l(a; p, q))^{-\lambda} - a_d(x/x_l(a_d; p, q))^{-\lambda} \\ &\quad + [(1-\zeta) q a_d - \Pi(x_d, a_d; p)] (x/x_d)^{-\lambda} + \frac{\tau b}{(\rho+\eta)} \left( 1 - \left( \frac{x}{x_d} \right)^{-\lambda} \right) \end{aligned} \tag{37}$$

In comparison with the expression for the value of the unlevered firm, clearly the default and abandonment thresholds are different. In addition, the last term is new. This term captures the value of the tax shield. It is simply equals the value of the taxes on expected discounted stream of coupon payments until default. Finally, $\zeta$ captures the bankruptcy costs.

In terms of comparative statics, the effects of changes in $p$ and $q$ are again quite different. Just as in the unlevered case, raising $p$ increases profits from operations but also decreases the value from capacity reductions. However, leverage renders the effects of raising $p$ on the value of the default option ambiguous. This is because the increase in profits foregone upon default from a higher $p$ is offset by a lower probability of default. Overall, raising $p$ is more likely to boost the value of equity when the firm is levered. Moreover, the changes in the value of debt are also ambiguous. This is because raising $p$ reduces the probability of default and so increases the expected dividend payments yet reduces the expected liquidation payout.

As for the effects of changes in $q$, it is clear that the value of equity is increasing in the resale price it raises the value of capacity reductions while leaving everything else unaltered. Similarly, the value of debt is also increasing in $q$ as it raises the liquidation value of the firm. Hence, the value of the levered firm is monotonically increasing in the resale value $q$.

Table 3.2 summarizes the comparative statics of the model. The base parameters were again chosen to be comparable to those of Morellec (2001) as follows: $\gamma = 0.53, a = 100, x = 1, \rho = 0.05, \eta = 0.01, \sigma_x = 0.1, \tau = 0.15, c = 1, p = 1, q = 1, \eta = 0.01, \zeta = 0.2$.

| Comparative Statics | | | | | | | |
|---|---|---|---|---|---|---|---|
| Parameter Value | Unlevered firm value $v(x, a, 0; p, q)$ | Levered firm value $v(x, a, b^*; p, q)$ | Value of tax shield (%) | Optimal Leverage (%) | Size upon default $a_d$ | Optimal Coupon $b^*$ | Severity of debt-overhang $\gamma b/(1-\gamma)$ |
| Baseline | 144.5 | 150.7 | 4.3% | 35.6% | 29.9 | 3.5 | 3.9 |
| $\gamma = 0.5$ | 160.5 | 168.8 | 5.4% | 44.0% | 41.4 | 4.8 | 4.8 |
| $\gamma = 0.6$ | 163.4 | 167.2 | 2.3% | 16.5% | 15.9 | 1.8 | 2.7 |
| $\sigma = 0.05$ | 144.6 | 164.5 | 4.7% | 13.8% | 17.4 | 1.6 | 1.8 |
| $\sigma = 0.15$ | 146.8 | 147.1 | 0.2% | 6.1% | 10.2 | 0.5 | 0.6 |
| $p = 0.5$ | 238.3 | 241.2 | 1.2% | 5.2% | 12.1 | 0.8 | 0.9 |
| $p = 1.5$ | 209.9 | 222.7 | 6.1% | 54.7% | 59.2 | 7.9 | 8.9 |
| $q = 0.5$ | 134.0 | 145.0 | 8.2% | 69.7% | 100.0 | 6.5 | 7.3 |
| $q = 1.5$ | 218.0 | 222.7 | 2.2% | 16.1% | 14.9 | 2.4 | 2.7 |
| $\tau = 0.05$ | 156.2 | 158.2 | 1.3% | 24.7% | 25.4 | 2.4 | 2.7 |
| $\tau = 0.55$ | 212.8 | 229.3 | 7.1% | 15.7% | 13.7 | 2.9 | 3.3 |

The results demonstrate the ambiguous effects of changes in $p$ as levered firm value increases when $p$ is both increased and decreased. The levered value is nevertheless monotonically increasing in the resale price $q$. More importantly, given the choice of parameters, increases in the intermediation price and declines in the resale price are important drivers of leverage, debt-overhang and over-investment in unproductive assets. In addition, reductions in volatility, decreasing the returns to the intermediation technology or raises taxes are also important in exacerbating the agency problem.

# 4 Asset Purchases

Financial intermediaries often expand capacity by purchasing assets.[3] Capacity expansions increase the value of the firm directly because additional capacity boosts operating profit flows. Moreover, the value of the firm's options to sell assets become more valuable as sales are no longer irreversible. Nevertheless, assets are not completely reversible because the asset market is assumed to be illiquid, that is $Q > q$. This illiquidity may arise for example from informational asymmetries between the firm and outside buyers and sellers. Varying degrees of illiquidity are common in markets for assets traded by financial intermediaries, especially in over-the-counter (OTC) markets. The degree of illiquidity in the asset market as measured by $Q-q$, is then a measure of the degree to which a firm's capacity changes are reversible.

## 4.1 Unlevered Firm

Let $L_t$ denote the cumulative gross investment (i.e. purchases of assets) up to date $t$. The stochastic process for assets is then:

$$da_t = dL_t - du_t. \tag{38}$$

The unlevered firm's objective is to maximize the expected discounted profits or

$$v_u^L(x, a; p, q, Q) = \max_{T\in\mathcal{T},\{L_t,u_t,t\geq 0\}} \mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\left\{\pi(x_t, a_t; p)dt - QdL_t + qdu_t\right\}\right] \tag{39}$$

where $\{L_t\}_{t\geq 0}$ and $\{u_t\}_{t\geq 0}$ are nondecreasing continuous stochastic processes that describe the investment decisions of the firm with $L_0 = u_0 = 0$.

The value of the firm now derives from four sources: the value of operating the firm's assets forever, the values associated with the ability to either expand or contract capacity and finally the value associated with the option to abandon and liquidate the firm.

[3] Evidence collected by Adrian and Shin (2010) points to FIs altering their asset holdings through repurchase agreements. I abstract from the contractual form used to acquire additional assets and assume that FIs purchase them directly.

The value derived from operating the firm's assets forever is again the value of the stream of discounted profit flows from operations:

$$\Pi(x, a; p) = (1-\tau)\mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\left\{\pi(x_t, a_t; p)dt\right\}\right]$$

The option to abandon trades off the value of the assets in place against the liquidation value of the firm. As in Section 3.1, we can write the value of this option as

$$\left[qa_u^L - \Pi(x_u^L, a_u^L; p)\right](x/x_u^L)^{-\lambda}. \tag{40}$$

where the abandonment threshold is now $x_u^L$ and the new the firm size upon abandonment is $a_u^L$.

The key difficulty arises in deducing the value of the options to alter capacity. This is because changes in capacity are no longer permanent. For example, consider determining the value of the option to reduce capacity. Since capacity reductions are no longer permanent because assets may be purchased later, the value of capacity reductions can no longer be written as the difference between revenues and permanent changes in future profit flows as in (17). In fact, the value of the two capacity altering options must be determined jointly because the ability to expand capacity reduces the irreversibility associated with asset sales and vice-versa.

The optimal investment decisions are now characterized by a pair of thresholds $x_L(a; p, q, Q) > x_l(a; p, q, Q)$ such that investment occurs whenever asset productivity exceeds the threshold $x_L(a; p, q, Q)$ while disinvestment occurs whenever returns are below $x_l(a; p, q, Q)$. When asset productivity is between $x_L(a; p, q, Q)$ and $x_l(a; p, q, Q)$ no investment is undertaken.

In order to value the capacity alteration options, I use an approach that relies on the HJB equation. It can be shown that the value of capacity options can be written as:

$$\alpha(a; x_L, x_l)x^{-\lambda} + \beta(a; x_L, x_l)x^{\theta}$$

where $\alpha(a)$ and $\beta(a)$ are functions of initial asset level $a$ and the optimal thresholds $x_L$ and $x_l$, and $-\lambda$, and $\theta$ are solutions to the fundamental quadratic. The functions $\alpha(\cdot)$ and $\beta(\cdot)$ can then be determined in terms of the thresholds using the following smooth-pasting conditions:

$$\frac{\partial v_u^L}{\partial \tilde{a}}\Big|_{x=x_l(\tilde{a};p,q,Q)} = q \text{ when } \tilde{a} \in [a_u, a] \tag{41}$$

$$\frac{\partial v_u^L}{\partial \tilde{a}}\Big|_{x=x_L(\tilde{a};p,q,Q)} = Q \text{ when } \tilde{a} \in [a_u, a] \tag{42}$$

Furthermore, the optimal thresholds can be determined using the following super-contact conditions:

$$\left.\frac{\partial^2 v_u^L}{\partial x \partial \tilde{a}}\right|_{x=x_l(\tilde{a};p,q,Q)} = 0 \text{ when } \tilde{a} \in [a_u, a] \tag{43}$$

$$\left.\frac{\partial^2 v_u^L}{\partial x \partial \tilde{a}}\right|_{x=x_L(\tilde{a};p,q,Q)} = 0 \text{ when } \tilde{a} \in [a_u, a] \tag{44}$$

While it can be shown that the thresholds $x_L$ and $x_l$ are continuous and increasing functions, closed-form expressions for the thresholds cannot be obtained.

The optimal investment and abandonment decisions in the unlevered case are depicted in the figure below. As is made clear in the figure, allowing the firm

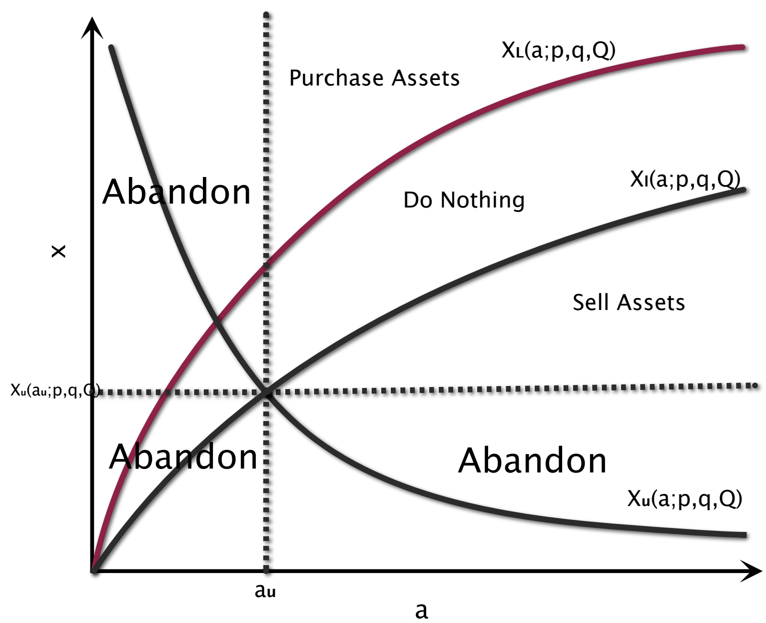


Figure 4: Optimal policies in the unlevered case.

to expand capacity narrows the inaction region. In fact, the inaction region shrinks as the asset market becomes more liquid, that is as $Q - q$ decreases. In this sense, one can interpret the earlier results without capacity expansions as the case where $Q$ is very large, yielding a large inaction region.

In addition, as capacity contractions are more valuable in the presence of the capacity expansion option, the threshold $x_l(a; p, q, Q)$ lies above the corresponding threshold $x_l(a; p, q)$. As a result, the firm size upon abandonment is smaller.

The following proposition provides expressions for the value of the options and the value of the unlevered firm.

**Proposition 4.** *Assume $(\rho + \eta) > \mu_x > 0$. Let $-\lambda, \theta$ be the negative and positive roots of the fundamental quadratic*

$$(\rho + \eta) - \mu_x \vartheta - \frac{1}{2}\sigma^2(\vartheta - 1)\vartheta = 0. \tag{45}$$

*Then, the value of the unlevered firm is:*

$$\begin{aligned} v_u^L(x, a; p, q, Q) = \Pi(x, a; p) + \alpha(a; x_L, x_l)x^{-\lambda} + \beta(a; x_L, x_l)x^{\theta} \\ + \left[q a_u^L - \Pi(x_u^L, a_u^L; p)\right] (x/x_u^L)^{-\lambda} \end{aligned} \tag{46}$$

*where*

$$\alpha(a; x_L, x_l) = \int_a^{a_u^L} \left[ \frac{x_l^{-\theta}\left(q - \pi_a(x_l, \tilde{a}; p)\right) + x_L^{-\theta}\left(\pi_a(x_L, \tilde{a}; p) - Q\right)}{x_l^{-(\theta+\lambda)} - x_L^{-(\theta+\lambda)}} \right] d\tilde{a} \tag{47}$$

$$\beta(a; x_L, x_l) = -\int_{a_u^L}^{a} \left[ \frac{x_l^{\lambda}\left(q - \pi_a(x_l, \tilde{a}; p)\right) + x_L^{\lambda}\left(\pi_a(x_L, \tilde{a}; p) - Q\right)}{x_L^{\theta+\lambda} - x_l^{\theta+\lambda}} \right] d\tilde{a} \tag{48}$$

*and the optimal thresholds $x_L(a; p, q, Q)$ and $x_l(a; p, q, Q)$ are implicitly defined by the following system of equations:*

$$\begin{aligned} \lambda \left[ \frac{x_l^{-\theta}\left(q - \pi_a(x_l, a; p)\right) + x_L^{-\theta}\left(\pi_a(x_L, a; p) - Q\right)}{x_L^{\lambda+1}(x_l^{-(\theta+\lambda)} - x_L^{-(\theta+\lambda)})} \right] \\ - \theta \left[ \frac{x_l^{\lambda}\left(q - \pi_a(x_l, a; p)\right) + x_L^{\lambda}\left(\pi_a(x_L, a; p) - Q\right)}{x_L^{1-\theta}(x_L^{\theta+\lambda} - x_l^{\theta+\lambda})} \right] = \pi_{xa}(x_L, a; p) \end{aligned} \tag{49}$$

$$\begin{aligned} \lambda \left[ \frac{x_l^{-\theta}\left(q - \pi_a(x_l, a; p)\right) + x_L^{-\theta}\left(\pi_a(x_L, a; p) - Q\right)}{x_l^{\lambda+1}(x_l^{-(\theta+\lambda)} - x_L^{-(\theta+\lambda)})} \right] \\ - \theta \left[ \frac{x_l^{\lambda}\left(q - \pi_a(x_l, a; p)\right) + x_L^{\lambda}\left(\pi_a(x_L, a; p) - Q\right)}{x_l^{1-\theta}(x_L^{\theta+\lambda} - x_l^{\theta+\lambda})} \right] = \pi_{xa}(x_l, a; p). \end{aligned} \tag{50}$$

*Proof.* See Appendix A. □

In the above expression, $\alpha(a; x_L, x_l)x^{-\lambda}$ represents the value of the option to contract capacity whereas $\beta(a; x_L, x_l)x^{\theta}$ represent the value of the option to expand capacity. The expressions for $\alpha$ and $\beta$ above underscore the interdependence of the values of the two capacity altering options. Each is an appropriately weighted sum of the marginal benefits associated with selling assets, $(q - \pi_a(x_l, a; p))$ and purchasing assets, $(\pi_a(x_L, a; p) - Q)$. In brief, the value of the option to contract is now higher because of the ability to expand capacity in the future. Similarly, the value of the option to expand capacity is higher because of the ability to contract capacity at a later point in time.

The optimal abandonment threshold is found as before using the following smooth-pasting condition:

$$\left.\frac{\partial v_u^L}{\partial x}\right|_{x=x_u^L} = 0.$$

which gives the following implicit equation for the abandonment threshold $x_u^L$:

$$\Pi_x(x_u^L, a_u^L; p) - \lambda\alpha(a_u^L, x_u^L, x_u^L)\left(x_u^L\right)^{-\lambda-1} + \theta\beta(a_u^L, x_u^L, x_u^L)\left(x_u^L\right)^{\theta-1} - \frac{\lambda}{x_u^L}[qa_u^L - \Pi_x(x_u^L, a_u^L; p)] = 0 \quad (51)$$

However, since the capacity altering options are worth zero upon exit, the above equation can be solved to yield the following expression for the default threshold:

$$x_u^L(a; p, q) = \frac{\lambda((\rho+\eta) - \mu_x)\left[(1-\tau)(c/(\rho+\eta) + qa_u^L\right] a^{\gamma-1}}{(1+\lambda)(1-\tau)p} \quad (52)$$

The firm is abandoned when it is preferable to liquidate the assets rather reduce capacity further. Therefore, in keeping with the last section, the abandonment size is determined at the intersection of the thresholds $x_u^L$ and $x_l$. However, in contrast with the last section, the threshold $x_l$ also depends on the option value of future capacity expansions. Therefore, the liquidation decision fully takes into account the value from the option to expand future capacity.

The comparative statics of the model are summarized in Table 4.1. The base parameters were chosen as follows: $\gamma = 0.53, a = 100, x = 1, \rho = 0.05, \eta = 0.01, \sigma_x = 0.1, \tau = 0.15, c = 1, p = 1, q = 1, Q = 1.1, \eta = 0.01$.. The value of the firm is certainly higher when capacity can be increased. The level of increase depends of course upon the degree of illiquidity in the asset market. In Table 4.1, we see an increase in firm value on the order of 16% to 0.8% as the purchase price is raised.

One important source for this increase in value is that capacity reductions are now more valuable. This can be seen in the decline in firm size upon abandonment as capacity is reduced more aggressively when asset productivity falls because these reductions are partially reversible.

Similarly, changes in $q$ affect not only the value of reducing capacity but also the value of increasing capacity. In Table 4.1 this is most easily seen as $q$ is increased from 1 to 1.05. The increase in value over and above the case when capacity expansions are not possible is roughly 20%.

| Comparative Statics | | | | |
|---|---|---|---|---|
| Parameter Value | Value with contraction option only | Value with contraction and expansion | Value of expansion option (%) | Size upon default $a_u$ |
| Baseline | 144.5 | 168.2 | 16.4% | 9.7 |
| $\gamma = 0.5$ | 160.5 | 190.5 | 18.7% | 11.0 |
| $\gamma = 0.6$ | 163.4 | 203.4 | 24.5% | 7.1 |
| $p = 0.5$ | 238.3 | 322.5 | 35.3% | 9.7 |
| $p = 1.5$ | 209.9 | 273.6 | 30.3% | 9.7 |
| $q = 0.5$ | 134.0 | 142.0 | 6.0% | 25.1 |
| $q = 1.05$ | 147.5 | 175.7 | 19.1% | 8.7 |
| $Q = 1.5$ | 144.5 | 155.1 | 7.3% | 11.2 |
| $Q = 2.0$ | 144.5 | 150.0 | 3.8% | 11.8 |
| $Q = 4.0$ | 144.5 | 145.6 | 0.8% | 12.4 |
| $\tau = 0.05$ | 156.2 | 183.2 | 17.3% | 10.9 |
| $\tau = 0.55$ | 212.8 | 282.4 | 32.7% | 4.6 |

## 4.2 Levered Firm

As in Section 3.2, shareholders in the levered firm make abandonment and investment decisions to maximize the expected value of the stream of discounted profits:

$$e_u^L(x,a,b;p,q,Q) \\ = \max_{T\in\mathcal{T},\{L_t,u_t,t\geq 0\}} \mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\left\{[\pi(x_t,a_t;p)-b]dt - QdL_t + qdu_t\right\}\right]$$

Again, the cost of servicing debt reduces the profit flow available to shareholders. On the other hand, the ability to expand capacity raises the value of equity. This problem is similar to one facing shareholders in the unlevered case above, and is solved using the same methods. The following proposition characterizes the optimal decisions made by shareholders and the value of the levered firm.

**Proposition 5.** *Assume $(\rho + \eta) > \mu_x > 0$. Let $-\lambda, \theta$ be the negative and positive roots of the fundamental quadratic*

$$(\rho + \eta) - \mu_x \vartheta - \frac{1}{2}\sigma^2(\vartheta - 1)\vartheta = 0. \tag{53}$$

*Then, the value of equity in the levered firm is:*

$$\begin{aligned} e_u^L(x, a, b; p, q, Q) = \Pi(x, a, b; p) + \alpha(a, b; x_L, x_l, b)x^{-\lambda} + \beta(a, b; x_L, x_l, b)x^{\theta} \\ - \Pi(x_d^L, a_d^L, b; p)(x/x_d^L)^{-\lambda} \end{aligned} \tag{54}$$

*where*

$$\alpha(a, b; x_L, x_l) = \int_a^{a_d^L} \left[ \frac{x_l^{-\theta}\left(q - \pi_a(x_l, \tilde{a}, b; p)\right) + x_L^{-\theta}\left(\pi_a(x_L, \tilde{a}, b; p) - Q\right)}{x_l^{-(\theta+\lambda)} - x_L^{-(\theta+\lambda)}} \right] d\tilde{a} \tag{55}$$

$$\beta(a, b; x_L, x_l) = -\int_{a_d^L}^{a} \left[ \frac{x_l^{\lambda}\left(q - \pi_a(x_l, \tilde{a}, b; p)\right) + x_L^{\lambda}\left(\pi_a(x_L, \tilde{a}, b; p) - Q\right)}{x_L^{\theta+\lambda} - x_l^{\theta+\lambda}} \right] d\tilde{a} \tag{56}$$

*and the optimal thresholds $x_L(a, b; p, q, Q)$ and $x_l(a, b; p, q, Q)$ are implicitly defined by the following system of equations:*

$$\begin{aligned} &\lambda \left[ \frac{x_l^{-\theta}\left(q - \pi_a(x_l, a, b; p)\right) + x_L^{-\theta}\left(\pi_a(x_L, a, b; p) - Q\right)}{x_L^{\lambda+1}(x_l^{-(\theta+\lambda)} - x_L^{-(\theta+\lambda)})} \right] \\ &\quad - \theta \left[ \frac{x_l^{\lambda}\left(q - \pi_a(x_l, a, b; p)\right) + x_L^{\lambda}\left(\pi_a(x_L, a, b; p) - Q\right)}{x_L^{1-\theta}(x_L^{\theta+\lambda} - x_l^{\theta+\lambda})} \right] = \pi_{xa}(x_L, a, b; p) \end{aligned} \tag{57}$$

$$\lambda\left[\frac{x_l^{-\theta}\left(q-\pi_a(x_l,a,b;p)\right)+x_L^{-\theta}\left(\pi_a(x_L,a,b;p)-Q\right)}{x_l^{\lambda+1}(x_l^{-(\theta+\lambda)}-x_L^{-(\theta+\lambda)})}\right]$$
$$-\theta\left[\frac{x_l^{\lambda}\left(q-\pi_a(x_l,a,b;p)\right)+x_L^{\lambda}\left(\pi_a(x_L,a,b;p)-Q\right)}{x_l^{1-\theta}(x_L^{\theta+\lambda}-x_l^{\theta+\lambda})}\right]=\pi_{xa}(x_l,a,b;p). \tag{58}$$

*Proof.* See Appendix A. □

The value of equity is simply the value of the discounted profit flow accruing to shareholders plus the value of the capacity altering and default options. This differs from the unlevered firm value in that the default threshold $x_d^L$ differs from the abandonment threshold $x_u^L$, and in that shareholders are not entitled to the liquidation value $qa_d^L$.

Moreover, the purchasing and selling thresholds $x^L$ and $x_l$ are unaltered by debt, as instantaneous profits $\pi$ are linear in debt so that $\pi_a$ and $\pi_{xa}$ are actually independent of $b$. The optimal abandonment threshold is found as before using the following smooth-pasting condition:

$$\left.\frac{\partial v^L}{\partial x}\right|_{x=x_d^L}=0.$$

Noting that the capacity altering options are worthless upon default, the above equation can be solved for the default threshold:

$$x_d^L(a,b;p,q)=\frac{\lambda a^{\gamma-1}((\rho+\eta)-\mu_x)(c+b)}{p(\rho+\eta)(1+\lambda)}. \tag{59}$$

The shareholders default when it is preferable to liquidate the assets rather than reduce capacity further. Debt shifts the default threshold towards the right, increasing the probability of default. However, recall that with capacity expansions shareholders reduce capacity more aggressively in response to declines in productivity. This is reflected in a higher selling threshold $x_l$. Hence, firm size upon default is higher than in the unlevered case but lower than in the case without capacity expansions.

The conflict between shareholders and bondholders results nevertheless results in early exit due to debt-overhang and over-investment in unproductive assets. However, these problems are mitigated with capacity expansions because they reduce the probability of default and reduce firm size upon default.

The optimal decisions, along with the debt-overhang problem are shown in the figure below:

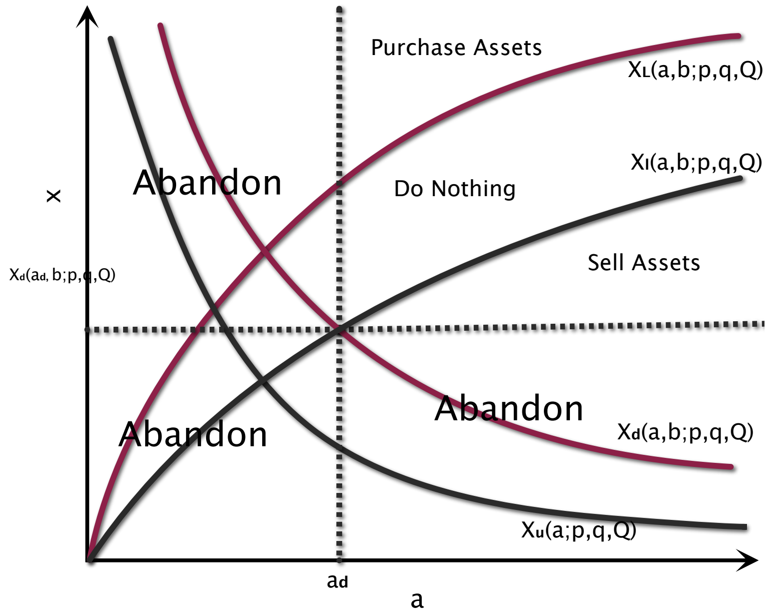


Figure 5: Optimal policies chosen by shareholders.

The value of debt is simply:

$$d^L(x, a, b; p, q, Q) = \mathbb{E}^x \left[ \int_0^{T_{x_d}} e^{-(\rho+\eta)t} b dt \right] + (1-\zeta) q a_d^L E^x [e^{-(\rho+\eta)T_{x_d}}] \quad (60)$$

$$= \frac{b}{(\rho+\eta)} \left( 1 - \left( \frac{x}{x_d^L} \right)^{-\lambda} \right) + (1-\zeta) q a_d^L \left( \frac{x}{x_d^L} \right)^{-\lambda} \quad (61)$$

It is essentially the discounted value of the coupon $b/(\rho+\eta)$ till default plus the liquidation value discounted appropriately.

The value of the levered firm can then again be written simply as the sum of the values of equity and debt:

**Proposition 6.** *The value of the firm is:*

$$
\begin{aligned}
v^L(x, a, b; p, q, Q) \\
&= \Pi(x, a, b; p) - \Pi(x_d^L, a_d^L, b; p)(x/x_d^L)^{-\lambda} \\
&+ \alpha(a; x_L, x_l, b)x^{-\lambda} + \beta(a; x_L, x_l, b)x^{\theta} \\
&+ \frac{b}{(\rho+\eta)}\left(1 - \left(\frac{x}{x_d^L}\right)^{-\lambda}\right) + (1-\zeta)qa_d^L\left(\frac{x}{x_d^L}\right)^{-\lambda} \qquad (62)
\end{aligned}
$$

Table 4.2 summarizes the comparative statics. The base parameters were chosen as follows: $\gamma = 0.53, a = 100, x = 1, \rho = 0.05, \eta = 0.01, \sigma_x = 0.1, \tau = 0.15, c = 1, p = 1, q = 1, Q = 1.1, \eta = 0.01, \zeta = 0.2$.

| Comparative Statics | | | | | |
|---|---|---|---|---|---|
| Parameter Value | Unlevered firm value $v(x, a, 0; p, q)$ | Levered firm value $v(x, a, b^*; p, q)$ | Optimal Leverage (%) | Size upon default $a_d$ | Optimal Coupon $b^*$ |
| Baseline | 168.2 | 178.4 | 22.2% | 21.1 | 2.0 |
| $\gamma = 0.5$ | 162.1 | 165.8 | 27.8% | 28.4 | 2.84 |
| $\gamma = 0.6$ | 163.9 | 165.6 | 9.9% | 11.9 | 1.01 |
| $p = 0.5$ | 238.6 | 239.7 | 3.4% | 10.3 | 0.5 |
| $p = 1.5$ | 213.7 | 218.5 | 36.4% | 41.0 | 4.9 |
| $q = 0.5$ | 134.5 | 140.4 | 51.8% | 76.8 | 4.5 |
| $q = 1.5$ | 222.2 | 224.3 | 7.8% | 9.5 | 1.1 |
| $Q = 1.5$ | 145.3 | 148.2 | 22.8% | 21.5 | 2.1 |
| $Q = 2.0$ | 146.0 | 148.7 | 21.4% | 20.5 | 2.0 |
| $Q = 4.0$ | 145.3 | 148.2 | 22.8% | 21.5 | 2.1 |
| $\tau = 0.05$ | 157.6 | 159.1 | 2.6% | 7.7 | 0 |
| $\tau = 0.55$ | 213.3 | 228.1 | 15.8% | 13.8 | 2.8 |

Again, these results largely parallel those obtained in the last section. However, allowing the shareholders to expand capacity raises the value of equity. In addition, the ability to expand capacity also boosts the value of debt as the higher capacity generates additional profit flow that can be use to service debt payments. In the simulations shown above, the rise in debt value is not completely mitigated by the rise in the value of equity and thus higher leverage is generally exhibited. In addition, the increase in the value of equity leads shareholder to exit later, mitigating debt-overhang, as they are more willing to inject new capital into the firm.

In the following section, in the industry equilibrium, the resale prices $q$ and $Q$ are determined by the buying and selling behaviour of surviving firms but also by liquidation of assets from defaulting firms, and liquidations from firms exiting exogenously or fire-sales. As a result, the industry will be more highly leveraged and will exhibit greater over-investment in unproductive assets.

# 5 Industry Equilibrium

In this section, I show that a stationary industry equilibrium exits. Also, I provide intuition for the key steps in the proof while relegating the details to the appendix. Finally, I draw on a set of comparative statics to explain the key implications of the model.

**Proposition 7.** *Assume that*

1. $(\rho + \eta) - \mu_x > 0$
2. $\lambda > \gamma$
3. $\eta > \sigma_x^2 - \mu_x$
4. $\mu_x - \sigma_x^2/2 > 0$

*where* $-\lambda$ *is the negative root of the fundamental quadratic. Then, there exists a stationary equilibrium* $(p^*, q^*, b^*, x_l, x_d, N^*, \nu^*)$*, such that* $\underline{x} > x_d$ *and* $\underline{a} > a_d$*.*

*Proof.* See Appendix A □

The first assumption above is required to bound the payoffs of the firm. The second is required to bound the higher moments of the stationary distributions as they have infinite support. Finally, the last two assumptions are needed to ensure the existence of the stationary distributions.

I briefly outline the intuition behind the construction of the stationary equilibrium. Shareholders first select the coupon $b$ given industry prices $p$ and $q$ prior to entry. Since all firms are ex-ante identical they choose an identical coupon. The relation between the equilibrium prices $p^*$ and $q^*$ is then determined from the free entry condition. This condition can be understood as follows. Given a resale price $q$, whenever the intermediation price is above the equilibrium price $p^*$, there is a positive benefit to entry and thus firms enter. However, entry drives down the intermediation price. Similarly, given an intermediation price $p$, whenever the resale price is above $q^*$, there is again a positive benefit to entry and so firms enter. However, entry drives down the resale price $q^*$ as some firms that enter wish to reduce capacity upon entry. Analogously, when prices are below equilibrium levels, the value to entry falls below the entry cost $c_e$ and firms choose not to enter. Therefore, in order for there to be positive and finite entry in equilibrium, the value to entering must equal the entry cost $c_e$.

I then solve compute the optimal selling and default thresholds, $x_l$ and $x_d$ in terms of the optimal coupon $b^*$ and equilibrium output prices $p^*$, $q^*$. I then compute the invariant distribution $\nu^*$ up to a scale factor that is the entry rate. The derivations make use of a conditions that matches the incoming flows and outgoing flows of firms and assets in terms of the density of the stationary distribution. This condition can then be solved for the distribution by adapting the procedure outlined in Miao (2005).

Finally, the market clearing conditions are used to pin down the entry rate and the equilibrium prices.

## 5.1 Results

Comparative Statics

| Parameter Value | Industry Intermediation (output) | Intermediation Price $p$ | Resale Price $q$ | Average Leverage (%) | Size upon default $a_d$ |
|---|---|---|---|---|---|
| Baseline | 164 | 1.00 | 1.00 | 20.1% | 25.3 |
| $\gamma = 0.5$ | 156 | 0.94 | 0.95 | 23.5% | 26.4 |
| $\gamma = 0.6$ | 160 | 1.07 | 1.04 | 19.4% | 23.2 |
| $\sigma = 0.05$ | 153 | 0.95 | 0.96 | 36.2% | 36.4 |
| $\sigma = 0.2$ | 161 | 1.04 | 1.06 | 9.7% | 19.2 |
| $\zeta = 0.1$ | 166 | 1.02 | 1.01 | 21.2% | 24.6 |
| $\zeta = 0.3$ | 161 | 0.98 | 0.99 | 19.4% | 25.9 |
| $\tau = 0.10$ | 178 | 0.93 | 0.95 | 18.2% | 21.4 |
| $\tau = 0.20$ | 152 | 1.08 | 1.04 | 25.6% | 28.6 |
| $\eta = 0.02$ | 178 | 0.93 | 0.95 | 18.2% | 21.4 |
| $\eta = 0.04$ | 152 | 1.08 | 1.04 | 25.6% | 28.6 |

These results are intuitive to interpret. The key set of results are the comparative statics with respect to the exogenous probability of default $\eta$. These suggest that as the possibility of a fire-sale is raised, industry leverage decreases. This occurs becomes bondholders anticipate a decline in the value of firm and extend credit on less generous terms. This effect dominates the decline in equity from the reduced expected cash-flows. While not shown in table above, this result appears to be robust to further changes in $\eta$.

In terms of regulatory measures, minimum capital requirements have essentially operate as follows. They place a floor on the value of equity that must be maintained. Given that the shareholders have the option to abandon the firm, this type of regulation simply alters their choice of default policy in favour of abandoning earlier. Again, in anticipation of such behaviour, lenders tighten the terms of the debt contract, leading to lower leverage. Nevertheless, clearly minimum capital requirements to not work as intended as they serve to increase the probability of default, rather than decreasing it.

Reducing the ability of firms to sell assets when solvent would possibly be more effective. Limiting the quantity of assets on the market, limits the price feedback effect triggered by the liquidation of defaulting firms. This would result in ex-ante lower value for the firm as the value of the option to reduce capacity is devalued. This would again lead to lower leverage levels, however this time the increase in default rates is more limited. Indeed, only those firms that are on the marginally solvent and are selling to service debt would be affected.

# 6 Conclusion

This paper investigated the optimal capital structure and default decisions of firms in an industry competing in both product and asset markets, with the possibility of fire-sales in the asset market. Fire-sales impact firm financing through two channels. One is a direct channel as creditors take into account the possibility that each firm may need to resort to a fire-sale of its assets. The other is through a negative price externality that liquidating firms impose on other firms simply seeking to reduce capacity. Naturally, as the possibility of fire-sales is reduced, industry leverage increases while default rates fall.

As for regulation, imposing minimum capital requirements reduces the ex-ante leverage of firms but at the cost of inducing higher default rates. Restrictions on asset sales reduce the value of the options to increase firm capacity, thereby reducing the value of firm. This reduction in value leads bondholders to raise borrowing costs anticipating a more severe debt-overhang problem.

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# A Proofs

***Proof of Proposition 1.*** The proof follows Miao (2005). The value of the unlevered firm is the sum of the expected discounted profits from operating the assets forever plus the value of the option to abandon and the value of the option to default:

$$\begin{aligned}
v_u(x,a;p,q) &= \max_{T\in\mathcal{T},\{u_t,t\in[0,T]\}}(1-\tau)\mathbb{E}^x\left[\int_0^T e^{-(\rho+\eta)t}\left\{\pi(x_t,a_t;p)dt+qdu_t\right\}\right] \\
&= \Pi(x,a;p)+qa_u\mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}]-(1-\tau)\mathbb{E}^{x_u}\left[\int_0^{T_{x_u}} e^{-(\rho+\eta)t}\pi(x_t,a_t;p)dt\right] \\
&+\int_a^{a_u}\left(qE^x[e^{-(\rho+\eta)T_{x_l(\tilde{a};p,q)}}]-(1-\tau)\mathbb{E}^x\left[\int_{T_{x_l(\tilde{a};p,q)}}^\infty e^{-(\rho+\eta)t}\pi_a(x_t,a_t;p)dt\right]d\tilde{a}\right)
\end{aligned}$$

The firm term on the second line is:

$$\begin{aligned}
\Pi(x,a;p) &= (1-\tau)\mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\left\{\pi(x_t,a_t;p)dt\right\}\right] \\
&= (1-\tau)\left(\frac{pxa^{1-\gamma}}{(\rho+\eta)-\mu_x}-\frac{c}{\rho+\eta}\right)
\end{aligned}$$

where the second line follows from computing the expectation directly. Using the strong-Markov property of stopping times of Brownian motion, the second-term can be written as:

$$\begin{aligned}
&qa_u\mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}]-\mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}]\left[\int_0^\infty e^{-(\rho+\eta)t}\pi(x_t,a_t;p)dt\right] \\
&=[qa_u-\Pi(x_u,a_u;p)]\,(x/x_u)^{-\lambda}
\end{aligned}$$

where the second line following from noting that $\mathbb{E}^x[e^{-(\rho+\eta)T_{x_u}}]=(x/x_u)^{-\lambda}$ (see Karatzas and Shreve (1991), p. 191). Combining the above points, the final term can be written as:

$$\begin{aligned}
&\int_a^{a_u}\left(q(x/x_l)^{-\lambda}-(1-\tau)\mathbb{E}^x[e^{-(\rho+\eta)T_{x_l}}]\mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\pi_a(x_t,\tilde{a}_t;p)dt\right]d\tilde{a}\right) \\
&=\int_a^{a_u}\left(q(x/x_l)^{-\lambda}-(1-\tau)\mathbb{E}^x[e^{-(\rho+\eta)T_{x_l}}]\mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\pi_a(x_t,\tilde{a}_t;p)dt\right]d\tilde{a}\right) \\
&=\int_a^{a_u}(x/x_l)^{-\lambda}\left(q-(1-\tau)\mathbb{E}^x\left[\int_0^\infty e^{-(\rho+\eta)t}\pi_a(x_t,\tilde{a}_t;p)dt\right]d\tilde{a}\right) \\
&=\int_a^{a_u}(x/x_l)^{-\lambda}\left(q-(1-\tau)\left[\frac{(1-\gamma)px\tilde{a}^{-\gamma}}{(\rho+\eta)-\mu_x}\right]\right)d\tilde{a}
\end{aligned}$$

Then, the optimal threshold (23) is found using the super-contact conditon (see Dumas (1991)):

$$\frac{\partial^2 v_u}{\partial x \partial \tilde{a}}\Big|_{x=x_l(\tilde{a};p,q)} = 0$$

and the optimal abandonment threshold (24) is found using the smooth-pasting condition (see Dixit and Pindyck (1994)):

$$\frac{\partial v_u}{\partial x}\Big|_{x=x_u} = 0.$$

Then, using (23), and (24) in the expressions above yields (22). Finally, noting that the above thresholds meet at $a_u$, equating (23), and (24) and solving for $a_u$ yields (25).

□

***Proof of Proposition 2.*** The expression for the value of equity is:

$$\begin{aligned} & e(x,a,b;p,q) \\ & \quad = \max_{T\in\mathcal{T},\{u_t,t\in[0,T]\}} (1-\tau)\mathbb{E}^x\left[\int_0^T e^{-(\rho+\eta)t}\left\{[\pi(x_t,a_t;p)-b]dt + q du_t\right\}\right] \end{aligned}$$

which is identical to the expression for the unlevered value minus the liquidation value. As a result, the method of proof is identical to the method used in proving Proposition 1. Applying the super-contact and smooth-pasting conditions, yields (31) and (32). The firm size at abandonment can again be found by equating (31) and (32) at $a_d$. As in Morellec (2001), existence and uniqueness follow from the fact that $\partial x_l/\partial a > \partial x_d/\partial a$, and for $a > a_d$, $x_l > x_d$.

□

***Proof of Corollary 1.*** The expressions for $a_u$ and $a_d$ are given respectively by (25) and (33). Then, $a_d > a_u$ when:

$$\frac{(c+b)(1-\gamma)(1-\tau)}{q(\rho+\eta)} > \frac{c(1-\gamma)(1-\tau)}{q(\rho+\eta)\gamma} \tag{63}$$

$$\implies \frac{b\gamma}{1-\gamma} > c \tag{64}$$

□

***Proof of Proposition 3.*** The expression (36) follows from adding the expressions for equity (30) and debt (34).

□

***Proof of Proposition 4.*** Then, the value of the unlevered firm is:

$$v_u^L(x,a;p,q,Q) = \Pi(x,a;p) + \alpha(a;x_L,x_l)x^{-\lambda} + \beta(a;x_L,x_l)x^{\theta} \\ + \left[qa_u^L - \Pi(x_u^L,a_u^L;p)\right](x/x_u^L)^{-\lambda}$$

To write the above expression in terms of the thresholds $x_l$ and $x_L$ we use the smooth-pasting conditions:

$$\frac{\partial v_u^L}{\partial \tilde{a}}\Big|_{x=x_l(\tilde{a};p,q,Q)} = q \text{ when } \tilde{a} \in [a_u, a]$$
$$\frac{\partial v_u^L}{\partial \tilde{a}}\Big|_{x=x_L(\tilde{a};p,q,Q)} = Q \text{ when } \tilde{a} \in [a_u, a]$$

These yield:

$$\alpha' x^{-\lambda} + \beta' x^{\theta} + \frac{\partial \Pi(x,a;p)}{\partial a}\Big|_{x=x_L} = Q \tag{65}$$
$$\alpha' x^{-\lambda} + \beta' x^{\theta} + \frac{\partial \Pi(x,a;p)}{\partial a}\Big|_{x=x_l} = q. \tag{66}$$

These equations can be solved for $\alpha'$ and $\beta'$ in terms of the thresholds $x_l$ and $x_L$ to give (47) and (48). The optimal thresholds can be determined using the following super-contact conditions:

$$\frac{\partial^2 v_u^L}{\partial x \partial \tilde{a}}\Big|_{x=x_l(\tilde{a};p,q,Q)} = 0 \text{ when } \tilde{a} \in [a_u, a]$$
$$\frac{\partial^2 v_u^L}{\partial x \partial \tilde{a}}\Big|_{x=x_L(\tilde{a};p,q,Q)} = 0 \text{ when } \tilde{a} \in [a_u, a]$$

which yield:

$$-\lambda \alpha' x^{-\lambda} + \theta \beta' x^{\theta} + \frac{\partial \Pi(x,a;p)}{\partial a \partial x}\Big|_{x=x_L} = 0 \tag{67}$$
$$-\lambda \alpha' x^{-\lambda} + \theta \beta' x^{\theta} + \frac{\partial \Pi(x,a;p)}{\partial a \partial x}\Big|_{x=x_l} = 0. \tag{68}$$

Replacing $\alpha'$ and $\beta'$ in the equations above by the expressions in (47) and (48), we obtain the two equations (49) and (50) that define the optimal thresholds. □

***Proof of Proposition 5.*** The expression for equity is:

$$e_u^L(x,a,b;p,q,Q) = \Pi(x,a,b;p) - \Pi(x_d^L,a_d^L,b;p)(x/x_d^L)^{-\lambda} \\ + \alpha(a;x_L,x_l,b)x^{-\lambda} + \beta(a;x_L,x_l,b)x^{\theta} \quad (69)$$

which is similar to the expression for the value of the firm in Proposition 4. The expression for equity (54), as well as the optimal thresholds, are found using the same method as in Proposition 4.

□

***Proof of Proposition 6.*** The expression (62) follows from adding the expressions for equity (54) and debt (60).

□

***Proof of Proposition 7.*** The proof follows Dixit and Pindyck (1994) and Miao (2005). The first step consists of deriving a condition linking the intermediation price $p^*$ and the resale price $q^*$ using the free-entry condition. The capacity reduction and default thresholds can then be found from Proposition 2 above. Then, the equilibrium distribution of firms $\nu^*$ is derived. Finally, the equilibrium prices can be determined using the market clearing conditions.

To show the existence of a stationary distribution, it is convenient to work with the logarithm of $x$. Let $z = \log x$. Then, $\{z_t\}_{t\geq 0}$ is a Brownian motion with growth rate $\mu_z = \mu_x - \frac{1}{2}\sigma_x^2$, and volatility $\sigma_z = \sigma_x$. Then, the initial draw of $z = \log(x)$ has an exponential distribution over $[\underline{z}, \overline{z}]$ where $\underline{z} = \log \underline{x}$ and $\overline{z} = \log \overline{x}$. This is because the initial draw of $x$ is uniform over $[\underline{x}, \overline{x}]$. The density of this distribution is given by:

$$g_1(z) = \exp(z - \hat{z}) \quad (70)$$

where $\hat{z} = log(\overline{x} - \underline{x})$. As shown in Harrison (1985), p90-92, the distribution of assets also follows a geometric Brownian motion with growth rate $[\mu - \frac{1}{2}\sigma_x^2]\gamma$. Then, letting $\dot{a} = \log a$, the above logic tells us that the density of the distirbution of $\dot{a}$ is given by:

$$g_2(\dot{a}) = \exp(\dot{a} - \hat{a}), \quad (71)$$

where $\hat{a} = log(\overline{a} - \underline{a})$, where $\dot{a}$ has growth rate $\mu_{\dot{a}} = [\mu - \frac{1}{2}\sigma_x^2]\gamma$ and volatility $\sigma_{\dot{a}} = \sigma_x$. As the draws are independent across $x$ and $a$, we can write the density of the joint distribution of $z, \dot{a}$ as:

$$g(z,\dot{a}) = \exp(z - \hat{z})\exp(\dot{a} - \hat{a}). \quad (72)$$

Now, denote by $N^*\phi(z, \dot{a})$ the stationary distribution of incumbent firms with support on $[z_d, \infty) \times [\dot{a}_d, \infty)$, where $z_d = \log x_d$, $\dot{a}_d = \log a_d$ and $N^*$ is the entry rate which will be determined later. Following Dixit and Pindyck (1994), for stationarity the density $\phi$ must satisfy the following partial differential equation:

$$\frac{1}{2}\phi_z^2\phi_{zz} - \mu_z\phi_z - \eta\phi + g(z, \dot{a}) = 0 \tag{73}$$

A particular solution to this equation is:

$$\phi_0(z, \dot{a}) = \frac{\exp(z - \hat{z})\exp(\dot{a} - \hat{a})}{\eta + \mu_z - \sigma_x^2/2} \tag{74}$$

The general solution is: $A_1(\dot{a})\exp\delta_1 z + A_2(\dot{a})\exp\delta_2 z$ where $A_1(\dot{a})$ and $B_1(\dot{a})$ are functions to be determined using the boundary conditions, and $\delta_1$, $\delta_2$ are roots of the corresponding fundamental quadratic. The functions differ in the following three cases: $\underline{z} \leq z < \overline{z}$, $z_d \leq z < \underline{z}$, and $z \geq \overline{z}$. These functions can then be found using an appropriate set of boundary conditions as in Miao (2005).

$\square$