The "Hot Potato" Effect of Inflation*

Lucy Qian Liu
Queen's University

Liang Wang
University of Pennsylvania

Randall Wright
University of Pennsylvania

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Abstract

Conventional wisdom is that inflation makes people try to spend their money faster, in order to pass it off like a "hot potato." Intuitively, this suggests that inflation increases velocity, which as we show is certainly true in the data. Search-based monetary theory seems ideal for analyzing this phenomenon – but it is hard to make it work. Since inflation is a tax, it lowers the expected surplus associated with monetary exchange, reducing buyers' incentive to search, and hence velocity, in standard models. What kind of a theory would predict that buyers trade faster when their surplus is lower? One with free entry. We prove buyers must spend their money more quickly when inflation increases to satisfy a natural equilibrium condition determining market participation.

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> The public discover that it is the holders of notes who suffer taxation [from inflation] and defray the expenses of government, and they begin to change their habits and to economize in their holding of notes. They can do this in various ways ... [such as] they can reduce the amount of till-money and pocket-money that they keep and the average length of time for which they keep it, even at the cost of great personal inconvenience ... By these means they can get along and do their business with an amount of notes having an aggregate real value substantially less than before.
>
> In Moscow the unwillingness to hold money except for the shortest possible time reached at one period a fantastic intensity. If a grocer sold a pound of cheese, he ran off with the roubles as fast as his legs could carry him to the Central Market to replenish his stocks by changing them into cheese again, lest they lost their value before he got there; thus justifying the prevision of economists in naming the phenomenon "velocity of circulation"! In Vienna, during the period of collapse ... [it] became a seasonable witticism to allege that a prudent man at a cafe ordering a bock of beer should order a second bock at the same time, even at the expense of drinking it tepid, lest the price should rise mean while. Keynes (1924, p. 51)

1 Introduction

It is an old idea that when inflation or nominal interest rates rise people spend their money more quickly – like a "hot potato" they want to get rid of it sooner rather than later – and this increases velocity.[1] Search-based monetary theory seems like an ideal laboratory to investigate this phenomenon, since it provides a genuine role for money as a medium of exchange, and one can easily introduce endogenous search intensity, as in standard job search theory (Mortensen 1985). Li (1994, 1995) builds such a model, assuming buyers search actively with endogenous intensity, while sellers wait passively for customers to come to them. He uses first-generation monetary search theory, following Kiyotaki and

[1] As Lucas (2000) describes the idea, "Since the opportunity cost of holding non-interest-bearing money is the nominal rate of interest, we would expect that the time people spend trying to economize on cash holdings should be an increasing function of the interest rate," and he says this is "consistent with much evidence." We will review the evidence in some detail below and show that velocity is indeed increasing in inflation or nominal interest rates, at both low and high frequencies.

Wright (1993), with indivisible goods and indivisible money. One cannot study inflation directly in such a model, but Li proxies for it with taxation. Among other interesting results, he shows that increasing this tax unambiguously makes buyers search harder and hence spend money faster.

These results do not easily generalize, however, to relaxing the assumptions of indivisible goods and money, which were made for convenience, and not meant to drive substantive results. Why? People cannot in general *avoid* the inflation tax by spending money more quickly – again like a "hot potato" buyers can only *pass it on* to sellers. Sellers are not inclined to absorb the incidence of the tax for free, obviously. Once we relax the restriction of indivisible goods and money, typically prices adjust with inflation and the net outcome is that buyers reduce rather than increase search effort. Intuitively, inflation is a tax on monetary exchange, and as such it reduces the return to this activity. When the return falls agents invest less in monetary activity, which means buyers search less and end up spending their money less quickly. It is only when we fix prices that agents automatically search harder when we increase the inflation tax.[2]

One can try to formalize this by making goods divisible in Li's model – i.e. by endogenizing search intensity in a second-generation monetary model, with bargaining, as in Shi (1995) or Trejos and Wright (1995). We present such a model, and show that while it is not impossible to get search effort to rise with inflation, it is somewhat difficult and certainly not robust. In any case, this is mainly a preliminary step before moving to third-generation models, with divisible money, as in Shi (1997) or Lagos and Wright (2005). Clearly, these

[2] This is reminescent of Gresham's law: good money drives out bad money when prices are fixed, but not necessarily when they are flexible. See e.g. Friedman and Schwartz (1963, fn. 27) for a discussion and Burdett et. al. (2001, Sec. 5) for a theoretical analysis of this idea.

models allow much more interesting qualitative and quantitative analyses, and they allow us to study inflation or nominal interest rate effects directly, instead of trying to proxy for them with taxes. Previously, Shi (1998) endogenized search intensity in the Shi (1997) model, and showed it can increase with inflation, due to general equilibrium effects, for some parameters. His model is quite complicated, however, partly because it was built to study a variety of different phenomena, but also because of technical issues concerning e.g. how to do bargaining in that environment (Rauch 1997; Zhu 2008).

Hence, we use here an environment closer to Lagos and Wright (2005). Previously Lagos and Rocheteau (2005) made a valiant effort to get the desired result – that buyers search more and spend their money faster when inflation rises – in this framework. It does not work, however, in their baseline model, when prices are determined by bargaining, for exactly the reason discussed above: inflation penalizes cash-based activity, reducing buyers' surplus from this activity, and leading them to invest (search) less.[3] They then go on to show that one can generate the result with price posting, rather than bargaining, although only for particular parameter values. The trick this: even though the total surplus from trade falls, if buyers' share of the surplus goes up enough, which is possible under posting if parameters are just right, buyers may get a higher return from monetary trade and hence increase search. This is clever, although not especially robust – one might think the result is so intuitive it ought not depend heavily on mechanisms (price posting vs. bargaining) or parameters.

Ennis (2008) evaluates the results in Lagos and Rocheteau, argues there is

[3] Thus, "The normative and positive implications of the model are not in line with the conventional wisdom ... that agents invest additional resources to get away from cash as the inflation rate increases and that these resources are part of the welfare cost of inflation. But in the model with bargaining, agents always *reduce* their search effort if the inflation rate increases." (Lagos and Rocheteau 2005, p. 506).

more to be done, and proposes an alternative theory. He assumes sellers have an advantage over buyers in terms of the frequency with which they can access a centralized market where they can off load their cash (exactly like the cheese merchant mentioned by Keynes in the epigram). In particular, Ennis assumes sellers access the central market every period, and buyers every other period. With inflation, buyers have an incentive to find sellers more quickly, because sellers can get the money out of their hands faster.[4] This is again clever. Still, we want to ask if there is an alternative, simple, and robust scenario under which inflation leads to buyers spending their money more quickly.[5] To this end, our idea is to proceed by studying the extensive rather than the intensive margin – i.e., by focusing on *how many* buyers are searching rather than on *how hard* they are searching.

The idea is obvious, once one sees it. Our goal is to get buyers to trade more quickly when the gains from trade are reduced by inflation. What kind of model of the goods market would predict that buyers spend their money faster when the gains from trade are lower? That would be like a model of the labor market predicting that firms hire more quickly when we tax recruiting. What kind of model could generate that? Well, the textbook model of search and recruiting in Pissarides (2000). It does so because it focuses on the extensive margin, or free entry. When recruiting is more costly and thus less profitable, generally, in this model some firms drop out, increasing the hiring rate for those remaining

[4]This is reminescent of the model of middlemen proposed by Rubinstein and Wolinsky (1995), where there are gains from trade between sellers and middlemen because the latter meet buyers more quickly than the former meet buyers.

[5]There is one more approach proposed in a recent paper by Nosal (2008). His buyers meet sellers with different goods and have to decide when to make a purchase – a standard search problem. Nosal shows buyers use reservation strategies, and as inflation rises, they get less picky, which increases the speed at which they trade. Again, this is fine; we simply want to consider a different (complementary) approach.

through the standard matching function. Of course firms hire faster when we tax them – that is the only way to keep profits constant. The same logic works here. Of course people spend their money faster when inflation rises – that is the only way to satisfy the analogous participation condition for consumers.

The rest of the paper is organized as follows. In Section 2 we begin by presenting the data to confirm the conventional wisdom that inflation increases velocity. In Section 3 we consider some rudimentary models, with indivisible money, in order to introduce some assumptions, notation, etc., and to discuss results in the earlier literature. In Section 4 we move to divisible money, and show the desired result does not obtain with endogenous search intensity, as in previous analyses, but does in our new model with free entry (or endogenous participation) by consumers. In Section 5 we present some alternative models, like ones with different free entry conditions, to see what assumptions are key for generating the desired results. In Section 6 we conclude.

2 The Evidence

We consider quaterly data between 1955 and 2008 (we could go back a little further, but there are some issues with these data that make 1955 a reasonable starting point). Figure 1a shows the behavior of inflation π and two measures of the nominal interest rate i, the T-Bill rate and the Aaa corporate bond rate.[6] The dotted lines are raw data and the solid lines are the HP trends. The models we will employ satisfy the Fisher equation, implying that $1+i = (1+\pi)/\beta$ where β is the discount factor. As one can see, assuming β is constant, this relationship is not literally true in the data, but it is not a bad approximation. All three

[6] All charts and tables are presented at the end of the paper.

of the variables on the chart are meant to capture the cost of holding money, and since it is not obvious which is the best measure, we will consider them all. Figure 1b shows for the same period velocity $v = PY/M$, where P is the price level and Y real output, so PY is nominal output, and M is the money supply, for three measure of money – $M0$, $M1$ and $M2$. Let us call these $v0$, $v1$ and $v2$. Obviously the braoder the definition of M the lower is v. Notice that v seems to have relatively small cyclical movements (deviations between raw data and HP trend) although there are significant and interesting movements in the trends of at least $v0$ and $v1$.

Figure 2 shows scatter plots for raw data on (all three measures of) v versus π and v versus i (we show only the T-bill rate in this chart, but the Aaa data look fairly similar). Figure 3 shows something similar after filtering out the higher frequency movements in the series, by plotting the scatter of the HP trends in the variables, and Figure 4 shows something similar after filtering out the low frequency movements, by plotting the scatter of the deviations between the raw data and the HP trends.[7] Table 1 gives the correlations. From the figures or the table, one can see that $v1$ and especially $v0$ move together with π or i in the raw data, while $v2$ does not. Notice however that $v2$ is strongly positively correlated with π or i at high frequencies, while the correlation between $v0$ and π or i is driven mainly by the low frequenc observations, and the correlations for $v1$ are positive at both high and low frequencies.

One may also see in the figures that there appears to be a strucutral break in velocity, especially the $v1$ series. Informally, looking at the charts, one might

[7] It is standard in macro to identify deviations between the raw data and the HP trend as business cycle movements, as discussed in several contributions to the Cooley (1995) volume. The method of looking at the relationship between filter trends as a way of teasing long-run properties out of time series data was used to good effect by Lucas (1980) and has recently been adopted by e.g. Berentsen et al. (2008) and Julien et al (2008).

say that sometime in the early 1980s things began to change in the sense that the inflation and the T-bill rate began to drop precipitously while $v1$ stayed flat. Or one might argue the big change was sometime in the mid 1990s when inflation and interest rates continued to fall but $v1$ started shooting upward. To control for this in a very crude way, Table 1 also reports the correlations when we stop the sample in 1982 and in 1995. In the former case, stopping the sample in 1982, we find that $v0$ moves about as strongly positively with π or i, but not both $v1$ and $v2$ move much more positively with π or i at both high and low frequency. In the latter case, stopping the sample in 1995, we find results somewhere in the middle, but the preponderance of evidence indicated clearly to us that all measures of v moves positively with π or i, although for some measures of either v or i this is mainly due to high for other measures this is more due to low frequency.

3 Indivisible Money

A $[0, 1]$ continuum of agents meet bilaterally and at random in a decentralized market. They consume and produce differentiated (nonstorable) goods, leading to a standard double coincidence problem: x is the probability a representative agent wants to consume what a random partner can produce. As agents are anonymous, credit is impossible, and money is essential. For now, money is indivisible and there is a unit upper bound on cash holdings. Given M units of money, at any point in time there are M agents each with 1 unit, called buyers, and $1 - M$ with 0, called sellers. Only sellers can produce, so if two buyers meet they cannot trade (one interpretation is that, after producing, agents need to consume before they can produce again). Only buyers search, so two sellers never meet (one interpretation is that sellers must produce at fixed locations). Hence, all trade involves a buyer giving 1 unit of money to a seller for q units of some good; there is no direct barter.

Each period, a buyer meets someone with a probability α. The probability he meets a seller that produces what he wants, a so-called *trade meeting*, is $\alpha_b = \alpha(1 - M)x$ (note this is also velocity, $v = \alpha_b$, the rate at which dollars turn over per period). As the total number of trade meetings is $M\alpha_b$, the probability of such a meeting for each seller is $\alpha_s = \alpha_b M/(1 - M) = \alpha M x$.[8] Buyers choose search intensity. Given M and x, we they can equivalently choose the underlying meeting rate α or α_b. We adopt the latter convention, and write search cost as $k(\alpha_b)$, where $k(0) = k'(0) = 0$, $k'(\alpha_b) > 0$ and $k''(\alpha_b) > 0$ for $\alpha_b > 0$. Policy is modeled as a tax on money holdings, but since it is indivisible, rather than taking away a fraction of your cash we take it all with probability τ each period (one interpretation is that buyers, in addition to meeting sellers, also meet government tax agents). To focus on steady state, we keep M constant by giving money to a seller each period with probability $\tau M/(1 - M)$. Like inflation, this tax has a greater impact the more money you hold and the longer

[8] This is how search is modeled in Li (1994, 1995), following the approach in Burdett et al. (1995). A story is this: Every agent has a location. If you are a buyer, each period you look for a location, and find one with probability α. That location entails a trade meeting (the agent is a seller and produces what you want) with probability $(1 - M)x$. Other buyers may contact you (stumble upon your location), with a probability depending on their search effort, but this is irrelevant since buyers do not trade with each other; and no seller contacts you since sellers do not search. Hence, your probability of a trade meeting when you are a buyer depends only on your own effort – there are no "search externalities" across buyers, although of course sellers are better off when buyers search harder. Lagos and Rocheteau (2005) use a different set up, as in Pissarides (2000), starting with an underlying matching function giving the number of meetings as a function of total search effort by buyers and the number of sellers, say $n(M\bar{e}, 1 - M)$, where $\bar{e}$ is average buyer effort. With this specification, the probability an individual buyer meets a seller is $en(M\bar{e}, 1 - M)/\bar{e}M$, where e is his own effort. Now search effort by one buyer affects the relevant meeting probability for other buyers (congestion). Our formulation is easier, and for the issues at hand delivers the same results. We return to matching functions below, when we switch from search effort to participation decisions.

you hold it.

Let $u(q)$ and $c(q)$ be the utility from consumption and disutility from production of q units of output, where in general $u(0) = c(0) = 0$, $u'(q) > 0$, $c'(q) > 0$, $u''(q) < 0$, $c''(q) \geq 0$, and $u'(0)/c'(0) = \infty$. Let q^* solve $u'(q^*) = c'(q^*)$. Let $\beta = 1/(1+r)$, $r > 0$, denote the discount rate. Let V_b and V_b be the value functions for buyers and sellers. Given that sellers are willing to trade goods for money, which we check below:[9]

$$
\begin{aligned}
(1+r)V_b &= -k(\alpha_b) + \tau V_s + \alpha_b[u(q) + V_s] + (1 - \tau - \alpha_b)\, V_b && (1) \\
(1+r)V_s &= \frac{\tau M}{1-M} V_b + \alpha_s[-c(q) + V_b] + \left(1 - \frac{\tau M}{1-M} - \alpha_s\right) V_s && (2)
\end{aligned}
$$

For now we take q as fixed, as in any first-generation search model of money, and write $u = u(q)$ and $c = c(q)$, assuming $c < u$. Then the necessary and sufficient FOC for α_b is then

$$k'(\alpha_b) = u + V_s - V_b. \qquad (3)$$

Solving (1) and (2) for V_s and V_b, and inserting these plus $\alpha_s = \alpha_b M/(1-M)$ into (3), we can reduce this to

$$
\begin{aligned}
T(\alpha_b) &= [r(1-M) + \tau + M\alpha_b]u - M\alpha_b c + (1-M)k(\alpha_b)] \\
&\quad - [r(1-M) + \tau + \alpha_b]\, k'(\alpha_b) = 0.
\end{aligned}
$$

It is easy to show $T(0) > 0$ and $T(\bar{\alpha}_b) < 0$, where $\bar{\alpha}_b$ is the natural upper bound, assuming $k'(\bar{\alpha}_b) = \infty$. Hence, there exists $\alpha_b^e \in (0, \bar{a}_b)$ with $T(\alpha_b^e) = 0$, and although we cannot guarantee uniqueness, in general, we can if $k''' > 0$

[9] We assume payoffs $-k(\alpha_b)$, $u(q)$ and $c(q)$ are all received next period, which is why the value functions V_b and V_s discount everything on the right by $1/(1+r)$; this affects nothing of substance, but makes for an easier comparison to models with divisible money. Thus, e.g., the value V_b is search cost $-k(\alpha_b)$, plus the probability of taxation times continuation value V_s, plus the probability of a trade meeting times $u(q) + V_s$, plus the remaining probability times V_b, all discounted.

(this makes T concave). To show α_b^e is an equilibrium, we have only to check sellers want to trade, $c \leq V_b - V_s$, which holds iff

$$(1-M)\alpha_b u - [(r+\alpha_b)(1-M)+\tau]\,c - (1-M)k(\alpha_b) \geq 0. \tag{4}$$

Assuming this holds with strict inequality at $\tau = 0$ (see the fn. below), monetary equilibrium exists for all $\tau \leq \bar{\tau}$ where $\bar{\tau} > 0$ satisfies (4) at equality. In terms of the effects of policy, suppose equilibrium is unique, a sufficient condition for which, we recall, is $k''' > 0$ (if there is multiplicity then, as always, the effects of parameters changes take the opposite sign in alternate equilibria). Then we have key result in Li, $\partial\alpha_b^e/\partial\tau > 0$, so a higher tax rate (read higher inflation) increases equilibrium search α_b^e and hence velocity v.

In terms of optimality, after simplification, average welfare $MV_b + (1-M)V_s$ is proportional to $\alpha_b(u-c) - k(\alpha_b)$. Hence the optimal α_b^* satisfies $k'(\alpha_b^*) = u-c$.Comparing this with the equilibrium condition (3), $\alpha_b^e = \alpha_b^*$ iff $c = V_b - V_s$. Hence, the optimal tax is the maximum feasible tax $\bar{\tau}$, which implies sellers get no gains from trade. This is a version of the standard Hosios (1990) condition, saying here that buyers should get all surplus, since they make all the investment in search effort. To put it another way, buyers equate the marginal cost of search to their private benefit, but unless they get all the gains from trade, sellers also get some benefit that is not internalized. In conclusion, in this model, monetary equilibrium exists iff $\tau \leq \bar{\tau}$; α_b and v are increasing in τ; and the optimal policy $\bar{\tau}$ maximizes α_b and v. Intuitively, we should tax buyers to make them spend their money faster, so that in an effort to avoid the tax they increase search effort towards the optimum.[10]

[10] In terms of technical details, we can now say $\bar{\tau} > 0$ iff $rc < \alpha_b^*(u-c) - k(\alpha_b^*)$. Also, for τ near $\bar{\tau}$ we have $T' < 0$, and uniqueness follows with no parameter restrictions near the

In the above model the price is fixed. To see how or if the results generalize, one approach is to keep money holdings in $\{0,1\}$, but make q divisible, and assume it determined in any trade meeting using the generalized Nash bargaining solution, with threat points given by continuation values and bargaining power for the buyer given by θ.[11] In fact, to make the main point, we start by assuming the buyer makes a take-it-or-leave-it offer, which corresponds to $\theta = 1$ and implies $c(q) = V_b - V_s$; this not only reduces the algebra, it also can be justified in terms of the Hosios condition discussed above. Inserting V_b and V_s, as above, this reduces to

$$c(q) = \frac{\alpha_b u(q) - k(\alpha_b)}{r + \alpha_b + \tau/(1-M)} \equiv f(q, \alpha_b).$$

A monetary equilibrium is now a pair (q, α_b) solving $c(q) = f(q, \alpha_b)$ and the usual FOC $k'(\alpha_b) = u(q) - c(q)$. The first relation defines a curve in (q, α_b) space we call BS (for bargaining solution) and second defines a curve we call SE (for search effort). Both are continuous; SE is concave, goes through $(0,0)$ and $(\hat{q}, 0)$, where $u(\hat{q}) = c(\hat{q})$, and is maximized at q^*, where $u'(q^*) = c'(q^*)$; and BS is convex and asymptotes to $q = 0$ and $q = \hat{q}$. Hence, there either exists no monetary equilibrium or (generically) exactly two monetary equilibria, say $q^L \in (0, \hat{q})$ and $q^H \in (q^L, \hat{q})$, with corresponding α_b^H and α_b^L. When τ increases BS shifts up, leading to $\Delta q^H < 0$ and $\Delta q^L > 0$. Notice from the FOC that α_b increases with τ iff the surplus $u(q) - c(q)$ increases with τ. Thus, the sign of $\partial\alpha/\partial\tau$ depends generally on which equilibrium we are in, q^L or q^H, and on whether the equilibrium is above or below the surplus maximizing q^* – e.g.

optimal policy. Regardless of uniqueness, in any equilibrium $\alpha_b^e < \alpha_b^*$ for $\tau < \bar{\tau}$. Finally, this all takes M as given, but it is a simple exercise to optimize over M as well as τ.

[11] This is the approach in Shi (1995) and Trejos-Wright (1995), although they only consider the case $\theta = 1/2$; see Rupert et al. (2001) for details of the generalization to any θ.

considering the equilibrium with the higher q^H, $\partial \alpha_b^H / \partial \tau < 0$ iff $q^H < q^*$.[12]

It is obvious that when there are multiple equilibria the effect of policy on α_b depends on which equilibrium we choose. One can argue that q^H is the natural equilibrium, but even then it depends on whether q^H is above or below q^*. So, it is possible for search effort to increase with the tax, but the effect is certainly not robust. There is also a sense in which this is purely an artifact of indivisible money. We show this explicitly in the next section, but we can also make the case here, as follows. In any trade meeting a buyer takes as much as he can get, which sometimes entails $q > q^*$. If money were divisible however, a buyer would prefer to spend less and accept a smaller q. Suppose we let him offer a *lottery* whereby he gets q and gives up the cash with probability λ, as in Berentsen et al. (2002), where λ proxies for divisibility. In that model, one can show we *never* get $q > q^*$ in equilibrium.[13] So, perhaps surprisingly, the result that seems so natural, $\partial \alpha_b / \partial \tau$, it is not easy get and not robust when prices are endogenous. Rather than dwelling on these results, we now move to models with divisible money.

4 Divisible Money

Relaxing the assumption of indivisible money is obviously a good idea, but can complicate the analysis because, in general, one has to keep track of the distribution of money across agents as an endogenous state variable. There are

[12] To be complete, there are three possible cases: (1) if $q^L < q^H < q^*$ then $\partial \alpha_b^H / \partial \tau < 0$ and $\partial \alpha_b^L / \partial \tau > 0$; (2) if $q^L < q^* < q^H$ then $\partial \alpha_b^H / \partial \tau > 0$ and $\partial \alpha_b^L / \partial \tau > 0$; and (3) if $q^* < q^L < q^H$ then $\partial \alpha_b^H / \partial \tau > 0$ and $\partial \alpha_b^L / \partial \tau < 0$.

[13] In fact, for general bargaining power θ, the following is true in that model: if θ is low then $q < q^*$ and $\lambda = 1$; as θ rises λ stays at 1 and q increases until we hit q^*; and once we hit q^*, λ falls with θ while q stays at q^*. There are other ways to argue that $q > q^*$ is not natural, or not likely, even without lotteries – e.g. the symmetric case, $\theta = 1/2$ and $M = 1/2$ always yields $q < q^*$ (Trejos and Wright 1995).

different approaches within the literature to this problem, including Green and Zhou (1998) or Molico (2005), who tackle distributional issues directly using either advanced analytical or computational methods, and Shi (1997) or Lagos and Wright (2005), who get around the problem with assumptions that render the distribution simple (conditional on an agents' type it is degenerate). We use the version of Lagos and Wright (2005) developed in Rocheteau and Wright (2005), because it is easy, and because it makes for a natural comparison with the models presented above since at any point in time we can partition the population into a set of buyers A_b and a set of sellers A_s with measure σ_b and σ_s.[14]

Before we discuss this in detail, we first describe a modification to the environment designed to deal with divisible money. Every period starts with a round of decentralized trade, as in the previous section. But after a round of trade, there convenes a centralized market, where agents consume and produce a different good X using labor H, and can also rebalance money holdings. This market is perfectly competitive, and ϕ to denotes the relative price of money in terms of X, while the real wage is normalized to 1, without loss of generality, because we assume linear technology, $X = H$. The aggregate money stock M is augmented via lumpsum transfers or taxes in the centralized market, and evolves according to $\hat{M} = (1+\gamma)M$. We focus on stationary monetary equilibrium, where $\phi > 0$ and ϕM is constant, which implies $\phi/\hat{\phi} = \gamma$ is the gross inflation rate. We restrict $\gamma > \beta$, although we can consider the limiting case $\gamma \to \beta$, which is the Friedman rule.

After the centralized market clears and closes, the period ends and we start

[14] It is usually understood that each agent is permanently a buyer or permanently a seller in this model, but everything works the same if type changes over time.

the next period with another round of decentralized trade. Let $W_j(m)$ and $V_j(m)$ be the centralized and decentralized market value functions for type $j \in \{b, s\}$, where b indicates buyer and s seller. The CM problem is[15]

$$\begin{aligned} W_j(m) &= \max_{X,H,\hat{m}} \left\{ U_j(X) - H + \beta \hat{V}_j(\hat{m}) \right\} \\ \text{st } X &= H + \phi(m - \hat{m} + \gamma M), \end{aligned}$$

where U_j is the utility over X for type j, satisfying the usual assumptions, and utility over H is linear. Given an interior solution, the FOC are the budget equation, $U_j'(X) = 1$, and $\phi \leq \beta \hat{V}_j'(\hat{m})$ with equality if $\hat{m} > 0$. In particular, $\hat{m}$ is independent of m, and so the choice of money to take out of the centralized and into the next decentralized market is the same for all agents of a given type. Also, $W_j'(m) = \phi$.

In the decentralized market, buyers want to consume but cannot produce, while sellers can produce but do not consume. This double coincidence problem, and anonymity, mean that buyers have to bring money to the market if they want to trade; sellers of course bring no money to the decentralized market. So like the indivisible money models, there will be a set of buyers with $m > 0$ and a set of sellers with $m = 0$ in the decentralized market, except here money is perfectly divisible. Also, buyers choose search effort exactly as before, but not in any trade meeting a buyer with m dollars pays $d \leq m$ and gets q units of the good. As above, (q, d) are determined by generalized Nash bargaining, with threat points given by continuation values and bargaining power for the buyer given by θ. Exactly as in Lagos and Wright (2005), one can easily show that in

[15] We assume here that the problem is convex, sufficient conditions for which are well known (see Wright 2008).

equilibrium $d = m$ and q solves $g(q) = \phi m$, where[16]

$$g(q) \equiv \frac{\theta c(q)u'(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}. \tag{5}$$

This implies $\partial q/\partial m = \phi/g'(q)$.

Now we have

$$V_b(m) = -k(\alpha_b) + W_b(m) + \alpha_b \left[u(q) + W_b(0) - W_b(m)\right],$$

where α_b is the probability of a trade meeting for a buyer, and the term is brackets is the buyer's surplus from trade, which reduces to $u(q) - \phi m$ since $W_j'(m) = \phi$. The probability of a trade meeting is given by

$$\alpha_b = \alpha\sigma_s x,$$

where α is an underlying meeting probability, σ_s is the fraction of the population that are sellers, and x is the probability a seller produces what a random buyer wants, corresponding to $\alpha_b = \alpha(1-M)x$ in the previous section (here we write the search cost k as a function of α_b instead of the underlying meeting probability α but this is obviously just notation). Then search effort is determined by

$$k'(\alpha_b) = u(q) - \phi m = u(q) - g(q), \tag{6}$$

after inserting the bargaining solution $\phi m = g(q)$.

To determine ϕm and hence q, compute

$$\begin{aligned} V_b'(m) &= (1-\alpha_b)W_b'(m) + \alpha_b u'(q)\partial q/\partial m \\ &= (1-\alpha_b)\phi + \alpha_b u'(q)\phi/g'(q), \end{aligned}$$

[16] The surplus for a buyer is $\Sigma_b = u(q) + W_b(m-d) - W_b(m) = u(q) - \phi d$, using $W_b' = \phi$. The surplus for a seller is $\Sigma_s = \phi d - c(q)$. It is easy to show that buyers do not bring more money than they spend, so $d = m$. Insert $d = m$ into the generalized Nash product $\Sigma_b^\theta \Sigma_s^{1-\theta}$, take the first-order condition with respect to q, and rearrange to get (5).

using $W_j'(m) = \phi$ and $\partial q/\partial m = \phi/g'(q)$. Update this to next period and insert into the FOC $\phi = \beta \hat{V}'(\hat{m})$ to get

$$\frac{1+\gamma}{\beta} = 1 - \alpha_b + \alpha_b \frac{u'(q)}{g'(q)},$$

using $\phi/\hat{\phi} = 1+\gamma$. By the Fisher equation, define the nominal interest rate $1+i = (1+\gamma)/\beta$, and reduce this to the usual condition

$$\frac{i}{\alpha_b} = \frac{u'(q)}{g'(q)} - 1. \tag{7}$$

Equilibrium now is a pair (q, α_b) solving (6) and (7).

Several remarks can be made about this model. For example, setting $\theta = 1$ implies $g(q) = c(q)$ and hence (6) guarantees search effort will be efficient, which as in the previous section is simply the Hosios condition. Given $\theta = 1$, we get the efficient q^* that solves $u'(q) = c'(q)$ by setting $i = 0$, which is the Friedman rule. It can be shown using (7) that $q < q^*$ for all $i > 0$ (it helps to note that q is increasing in θ). One can use routine methods to discuss existence, uniqueness vs. multiplicity, and so on. Thus, (6) and (7) define two curves in (q, α_b) space we can again call the SE and BS for search effort and bargaining solution (although the latter now really combines bargaining and money demand from the centralized market). Both start at the $(0,0)$; SE increases between as q increases from 0 to q^* and then decreases to $\alpha_b = 0$ again when $q = \hat{q}$; BS increases to $(\tilde{q}, 1)$ where $\tilde{q} \in (0, q^*]$. They could potentially intersect at multiple points, but it is easy to check that the SOC for the buyer's choice of q and α only holds when BS intersects SE from below.

When we increase the inflation rate γ or equivalently the nominal interest rate i, BS rotates up, which means at any point where BS intersects SE from below q and α_b both fall. More formally, differentiate (6) and (7) to get

$$\frac{\partial q}{\partial i} = -k''/D \text{ and } \frac{\partial \alpha_b}{\partial i} = -(u' - g')/D,$$

where $D = -\alpha_b \ell' k'' - (u' - g')(\ell - 1)$, with $\ell = \ell(q) \equiv u'(q)/g'(q)$. The SOC is $D > 0$, and since $u' > g'$ for all $i > 0$ by (7), we conclude that q and α_b fall with i. This is the (negative) result in Lagos and Rocheteau (2005), that inflation does not make buyers spend their money more quickly because, intuitively, it reduces q and the buyers surplus, which makes them less willing to invest in costly search.[17]

At this point we move to study the extensive rather than the intensive margin of search – i.e. instead of search intensity we consider a free entry or participation decision by buyers.[18] To this end we now impose a standard matching function $n = n(\sigma_b, \sigma_s)$, where n is the number of trade meetings and now we interpret σ_b and σ_s are the measures of buyers and sellers *in the decentralized market*, not the total population, as some agents may choose not to participate. An individual agent's probability of a trade meeting is then $\alpha_j = n(\sigma_b, \sigma_s)/\sigma_j$, for $j = b, s$. Assume n is twice continuously differentiable, homogeneous of degree one, strictly increasing, and strictly concave. Also $n(\sigma_b, \sigma_s) \leq \min(\sigma_b, \sigma_s)$, and $n(0, \sigma_s) = n(\sigma_b, 0) = 0$. Define the buyer-seller ratio, or market tightness, by $\delta = \sigma_b/\sigma_s$. Then $\alpha_b = n(1, 1/\delta)$, $\alpha_s = n(\delta, 1)$, and $\alpha_s = \delta\alpha_b$. Also, $\lim_{\delta\to\infty} \alpha_b = 0$ and $\lim_{\delta\to 0} \alpha_b = 1$.

[17] Lagos and Rocheteau (2005) is more complicated version of our model because of their matching technology, as mentioned above, and because their buyers get utility $\varepsilon u(q)$ from random sellers, where ε has some general distribution, where here buyers get either $u(q)$ or 0. These complications are not necessary for the main point.

[18] Notice that this is in some sense the opposite approach to the literature on limited participation in reduced form models of money (see e.g. Alvarez et al. 2008, Chiu 2006, Khan and Thomas 2007 and the references therein) or models with endogenous participation in the more search-based literature (e.g. Chiu and Molico 2007), where agents have to pay a cost to access something analogous to our centralized market, sometimes interpreted as the financial sector.

Participation decisions for now are made by buyers, who have to pay a fixed cost k_b to enter, while sellers get in for free and so all of them participate. We focus on the situation where the set A_b is sufficiently big that some but not all buyers go to the decentralized market, which means that in equilibrium they are indifferent between going and not going. Of course this means buyers get zero expected surplus from participating in the decentralized market, although those who actually trade do realize a positive surplus (just like the firms in the standard Pissarides 2000 model). If one does not like this, it is easy enough to assume all buyers draw a participation cost at random from some distribution $F(k)$ each period. Then instead of all buyers being indifferent, there will be a marginal buyer with cost k^* that is indifferent about going to the decentralized market, while all buyer with $k < k^*$ strictly prefer to go as they get a strictly positive expected surplus. Once this is understood, for ease of presentation we can focus on the case where k is the same for all buyers.

For a buyer who does not go to the decentralized market, $X = X^*$ and $\hat{m} = 0$. For a buyer who does go, we assume he pays the cost k_b next period, when he actually goes, but he has to acquire the money $\hat{m}$ in the current CM. Simple algebra implies wants to go iff $-\phi\hat{m} + \beta\left[-k_b + \alpha_b u(q) + (1-\alpha_b)\hat{\phi}\hat{m}\right] > 0$. Using (5) and again inserting the nominal rate i, this can be written $-ig(q) - k_b + \alpha_b\left[u(q) - g(q)\right] > 0$. In equilibrium, given A_b is big, this holds with equality:

$$\alpha_b = \frac{ig(q) + k_b}{u(q) - g(q)}. \tag{8}$$

Given q, this determines $\alpha_b = A(q)$. Then one gets the measure of buyers who participate σ_b from $\alpha_b = n(1, \sigma_s/\sigma_b)$, since σ_s is exogenous (all sellers participate). A monetary equilibrium with free entry by buyers is a solution (q, α_b) to (8) and condition (**??**) from the previous analysis. Let us call the curves defined by these relations BS and FE (now for free entry).

In (q, α_b) space, restricted to the region $(0, \tilde{q}) \times [0, 1]$, it is routine to verify the following: both curves are continuous, BS is upward sloping and goes through $(0, 0)$, while the FE curve is downward (upward) sloping to the left (right) of the FC curve, hitting a minimum where the curves cross.[19] This implies there is a unique equilibrium and that $\partial \alpha_b / \partial i > 0$ and $\partial q / \partial i < 0$. To see this, note that as i increases the BS and FE curves both shift up, so α_b increases. To see what happens to q, rewrite the model as two equations in q and α_b / i by dividing (8) by i. This new version of FE satisfies the same properties as before: is downward (upward) sloping to the left (right) of the FC curve. But now as i increases the FE curve shifts down while the FC curve does not shift (i.e. q as a function of α_b / i does not change when i changes). Hence q falls. Indeed, in addition to proving $\partial q / \partial i < 0$, this argument shows not only that α_b is increasing in i, but also that α_b / i is decreasing – which means α_b goes up by less than i, for whatever that is worth

Now consider velocity, $v = Y / \phi M$. While v is simple in the model in the previous section, and identically equal to the arrival rate α_b, things are more complicated becauese of the presence of the centralized market. Thus, total real output is now $Y = Y_C + Y_D$. Real centralized market output is $Y_C = X_b^* \bar{\sigma}_b +$

[19] Proof: The properties of FC are obvious. The slope of FE is given by

$$\frac{\partial \alpha_b}{\partial q} \simeq (u - g) i g' - (i g + k_b)(u' - g'),$$

where $\simeq$ means "equal in sign." Eliminating k using (8) and simplifying

$$\frac{\partial \alpha_b}{\partial q} \simeq i + \alpha_b - \alpha_b u' / g'.$$

From the centralized market problem, the derivative of the objective function $\phi = \beta \partial V^b / \partial \hat{m}$ can be rewritten in terms of q as $-(i + \alpha_b) + \alpha_b u'(q) / g'(q) = -\partial \alpha_b / \partial q$. There is a unique solution to this maximization problem, $\partial \alpha_b / \partial q$ is positive (negative) as q is less (greater) than the solution which is given by (**??**). Hence the FE curve is decreasing (increasing) to the left (right) of the BS curve. ■

$X_s^* \bar{\sigma}_s = \bar{X}^*$, where $\bar{\sigma}_b$ and $\bar{\sigma}_s$ are the total measures of buyers and sellers, and $U_j'(X_j^*) = 1$, and real decentralized market output is $Y_D = n(\sigma_b, \bar{\sigma}_s)\phi M/\sigma_b = \alpha_b \phi M$, since M/σ_b is the total amount of cash per buyer who participates in this market. Thus,

$$v = \frac{Y}{\phi M} = \frac{\bar{X}^* + \alpha_b \phi M}{\phi M} = \frac{\bar{X}^*}{\sigma_b g(q)} + \alpha_b,$$

using $M = \sigma_b g(q)/\phi$. Since $\partial \alpha_b/\partial i > 0$, $\partial \sigma_b/\partial i < 0$, and $\partial q/\partial i < 0$, we conclude that $\partial v/\partial i > 0$. Hence, this model unambiguously generates the result that velocity increases with inflation or nominal interest rates.

5 Alternative Specifications

Suppose now that $k_b = 0$ so that all buyers enter the decentralized market, but sellers have a nontrivial decision because $k_s > 0$. Condition (7) still holds, but we write it differently to work in (q, α_s) space. As σ_s increases, since σ_b is now fixed, α_s goes down and α_b up. Hence $\alpha_b = a(\alpha_s)$, with $a' < 0$, $a(0) = 1$ and $a(1) = 0$, and we have

$$1 + \frac{i}{a(\alpha_s)} = \frac{u'(q)}{g'(q)}. \tag{9}$$

Given α_s, this determines q, a new version of the BS curve. Also, (8) must be amended to

$$\alpha_s = \frac{k_s}{g(q) - c(q)}. \tag{10}$$

A stationary monetary equilibrium with free entry by sellers is a solution (q, α_s) to (9) and (10).

We can draw the new BS and FE curve in (q, α_s) space. Notice FC is now decreasing, goes through $(0, 1)$, and through $(q_1, 0)$ where q_1 solves $u'(q_1)/g'(q_1) =$

$1+i$. The FE curve is convex and decreasing for all $q < q^*$, and is above the FC curve for $q \approx 0$ or $q \approx q^*$. For big k_s FE will not cross FC at all, so there is no equilibrium with $q > 0$. For smaller k_s FE crosses FC mutiple times, since it lies above FC for $q \approx 0$ and $q \approx q^*$. Hence there are generically multiple equilibria with $q > 0$ if there are any equilibria. Intuitively, this multiplicity is due to a coordination issue or complementarity in the economy: when buyers bring more money, sellers surplus increases and so more sellers enter; and when more sellers enter α_b is higher so buyers bring more money. This does not happen when the free entry condition applies to buyers.

An increase in i in this version of the model rotates the FC curve around $(0, 1)$ towards the origin but does not affect FE. When there are multiple equilibria, consider first the one with the higher q. Clearly q falls and α_s rises, or in other words α_b falls and buyers spend their money less quickly. As is typically the case, with multiple equilibria, the opposite happens at the equilibrium with the next highest q. We therefore can get the "hot potato" effect in this model, but not in what we think of as the more natural equilibrium – the one generating the highest welfare. In the natural equilibrium, as inflation increases buyers bring less cash and q falls, so sellers exit the market, which makes α_s bigger and α_b smaller.

Another option is to fix the population and allow agents to choose whether to be a buyer or a seller. Normalizing the total population so that $\sigma_b + \sigma_s = 1$, free entry is now characterized by $W^b(m) = W^s(m)$ where $W^j(m)$ is the value function for type j planning to participate next period. This simplifies to

$$\alpha_s \left[g(q) - c(q)\right] = a(\alpha_s) \left[u(q) - g(q)\right] - ig(q). \tag{11}$$

Notice the complementarity effect still exists, so we expect multiple equilibria.

A stationary monetary equilibrium now is a solution (q, α_s) to (9) and (11). It is difficult to characterize equilibrium analytically, so we present an example in this case. Suppose $u(q) = Aq^{1-\mu}/(1-\mu)$ and $c(q) = Bq^2$. Consider the standard urn-ball matching function (see e.g. Burdett, Shi, and Wright 2001), $n(\sigma_b, \sigma_s) = \sigma_s \left[1 - (1 - \frac{1}{\sigma_s})^{\sigma_b}\right]$ which implies $\alpha_b = (1 - e^{-\delta})/\delta$ and $\alpha_s = 1 - e^{-\delta}$. Consider $A = 1.013$, $\mu = 0.04$, $\beta = 0.992$, $i = 0.02$, $k = 0.1$ and $B = 0.5$ (a normalization). We start with bargaining power $\theta = 0.5$ as a benchmark, and consider 0.3 and 0.8 for robustness. The following figures show the different models with different assumptions about entry.

In terms of velocity, for free entry by sellers, we have $v = \bar{X}^*/\bar{\sigma}_b g(q) + \alpha_b$, and

$$\frac{\partial v}{\partial i} = \frac{\partial \alpha_b}{\partial i} - Y_C \frac{g'(q)}{\bar{\sigma}_b g^2(q)} \frac{\partial q}{\partial i}.$$

Since $\partial \alpha_b / \partial i$ and $\partial q / \partial i$ have the same sign, e.g. both negative in the equilibrium with the highest q, the result is ambiguous. If Y_C is small, $\partial v / \partial i$ has the same sign as $\partial \alpha_b / \partial i$; if Y_C is large enough, $\partial v / \partial i$ the opposite is true. In numerical examples $\partial v / \partial i$ can be positive, negative or even nonmonotonic. In the case of free entry by both sides, $Y_D = n(\sigma_b, \sigma_s) g(q)$, and

$$v = \frac{X_b^* \sigma_b + X_s^* \sigma_s + n(\sigma_b, \sigma_s) g(q)}{\sigma_b g(q)} = \frac{X_b^*}{g(q)} + \frac{X_s^*}{\delta g(q)} + \alpha_b.$$

Again, in numerical examples $\partial v / \partial i$ can be positive, negative or even nonmonotonic.

6 Conclusion

This paper has analyzed the relationship between inflation or nominal interest rates, on the one hand, and the speed with which agents spend their money or velocity, on the other hand. We presented some evidence on the empirical relationship, showing that it is positive at both high and low frequencies. We then discussed some simple models, with indivisible money and endogenous search intensity, to see what they predicted. While it is possible for these models to generate a positive relationship between the variables in question, it is not easy, and in some sense any such result can be considered an artifact of indivisible money. We then moved to a modern search-based monetary model, with divisible money endogenous search intensity, and showed that it unambiguously predicts a negative relationship, counter to conventional wisdom and to our data. Then we changed the framework by focusing on the extensive rather than the intensive margin of search – i.e. on how many buyers are searching rather than on how hard they are searching. This model unambiguously predicts a rise in inflation or nominal interest rates leads to an increase in the speed with which agents spend their money and in velocity.

We think the results are fairly intuitive, even obvious once one sees them, but this does not mean they are uninteresting. Previous authors have worked hard to generate similar results, and we show it is actually easy and natural once one incorporates an endogenous participation condition for buyers. Moreover, in terms of methodology, we think the exercise makes the following useful point. Many times when one strives to do monetary economics with relatively explicit microfoundations, one is all too open to the critical question: "Why did we need a search- or matching-based model, when similar insights could be developed and similar predictions made with a reduced form model, with money-in-the-utility-function preferences or a cash-in-advance constraint?" The issues addressed here are all about search and matching – arrival rates are either determined by search intensity on the intensive margin or through the matching function on the extensive margin. It is not only for aesthetic reasons that we use a search model in this application; it is exactly the right tool for the job.

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Table 1: Correlations

	1955Q1-2008Q2			1955Q1-1982Q4			1955Q1-1995Q4		
Raw Data	V0	V1	V2	V0	V1	V2	V0	V1	V2
Inflation	0.620	0.172	-0.074	0.786	0.847	0.580	0.610	0.553	0.176
AAA	0.839	0.407	0.109	0.872	0.938	0.556	0.878	0.916	0.341
T-Bill	0.788	0.197	0.008	0.798	0.856	0.6230	0.774	0.748	0.247
Trend (Low Freq)	V0	V1	V2	V0	V1	V2	V0	V1	V2
Inflation	0.7403	0.197	-0.184	0.902	0.952	0.526	0.736	0.643	0.085
AAA	0.8854	0.416	0.080	0.923	0.981	0.568	0.929	0.945	0.325
T-Bill	0.9128	0.193	-0.127	0.915	0.974	0.570	0.902	0.847	0.167
Deviation (High Freq)	V0	V1	V2	V0	V1	V2	V0	V1	V2
Inflation	0.0603	0.167	0.502	-0.057	0.136	0.603	0.037	0.177	0.554
AAA	0.0128	0.324	0.429	-0110	0.219	0.459	-0.004	0.311	0.480
T-Bill	0.3425	0.393	0.662	0.264	0.309	0.612	0.301	0.339	0.620

Figure 1: