

# A Unified Framework for Monetary Theory and Policy Analysis

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Search-theoretic models of monetary exchange are based on explicit descriptions of the frictions that make money essential. However, tractable versions of these models typically make strong assumptions that render them ill suited for monetary policy analysis. We propose a new framework, based on explicit micro foundations, within which macro policy can be studied. The framework is analytically tractable and easily quantifiable. We calibrate the model to standard observations and use it to measure the cost of inflation. We find that going from 10 percent to 0 percent inflation is worth between 3 and 5 percent of consumption—much higher than previous estimates.

## I. Introduction

Most monetary models in macroeconomics are *reduced-form* models. By this we mean that they make assumptions, such as putting money in the

Many people provided extremely helpful input to this project, including S. B. Aruoba, A. Berentsen, V. V. Chari, L. Christiano, N. Kocherlakota, D. Krueger, G. Rocheteau, S. Shi, N. Stokey, N. Wallace, and C. Waller. Research support from the C. V. Starr Center for Applied Economics at New York University, the Suntory-Toyota International Centre for Economics and Related Disciplines at the London School of Economics, Equipe de Recherche sur les Marches, l'Emploi et la Simulation at University of Paris II, and the National Science Foundation is gratefully acknowledged. In addition to the usual disclaimer, we note that the views expressed here are those of the authors and not necessarily those of the Federal Reserve Banks of Minneapolis or Cleveland or the Federal Reserve System.

[*Journal of Political Economy*, 2005, vol. 113, no. 3]  
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utility function or imposing cash-in-advance constraints, that are presumably meant to stand in for a role of money that is not made explicit; for example, it helps overcome spatial, temporal, or informational frictions. There are models that provide micro foundations for monetary economics based on search theory, with explicit descriptions of meetings, specialization, information, and so on, but they are ill suited for the analysis of monetary policy as it is usually formulated. The reason is that to make the models tractable, people typically adopt extreme restrictions on how much cash agents can hold.

In this paper we propose a new framework, with explicit micro foundations and without these extreme restrictions, that is easy to use for policy analysis. To illustrate, we use the model to measure the welfare cost of inflation. Calibrating parameters to standard observations for the U.S. economy, we estimate that the gain to reducing inflation from 10 percent to 0 percent is worth between 3 percent and 5 percent of consumption. This is much higher than earlier estimates based on reduced-form models. Our interpretation of the results is that building monetary models with explicit micro foundations not only is theoretically appealing but also can make a significant difference for quantitative results.

There are previous attempts to study search models without extreme restrictions on money holdings. Trejos and Wright (1995) present a model in which agents can hold any  $m \in \mathbb{R}_+$ , but like Shi (1995), they have analytic results only for the case  $m \in \{0, 1\}$ . Papers that make some progress on the general case include Green and Zhou (1998), Camera and Corbae (1999), Zhou (1999), and Zhu (2003), but they are sufficiently complicated that it is difficult to get substantive analytic results. One can also study the model numerically, as in Molico (1997), but we think that it is useful to have a framework that delivers analytic results. For one thing, simple models with sharp results are often a better vehicle than computer output for developing economic understanding. Also, numerical methods make it difficult to say much about issues such as existence, uniqueness or multiplicity, and dynamics—issues that are especially relevant in monetary economics. Finally, it is useful to have a benchmark that is analytically tractable even if (especially if) one ultimately wants to complicate things for the sake of realism, policy relevance, and so forth.

What complicates the analysis in previous search models is the endogenous distribution of money holdings,  $F(m)$ . Our assumptions make  $F(m)$  degenerate. We accomplish this by assuming quasi-linear preferences and giving agents periodic access to centralized markets in addition to the decentralized markets that make money essential as in the typical search model. Quasi linearity means that there are no wealth effects in the demand for money, so all agents in the centralized markets

choose the same  $m$ . Hence  $F(m)$  is degenerate across agents in the decentralized market. This allows us to get sharp analytic results. It also makes the framework about as easy to use as standard reduced-form models for addressing policy questions such as the cost of inflation, although, as we shall see, the answers are different.<sup>1</sup>

The rest of the paper is organized as follows. Section II describes the environment. Section III defines equilibrium and derives some basic results. Section IV introduces monetary policy considerations. Section V uses a calibrated version to quantify the welfare cost of inflation. Section VI presents conclusions and discusses some extensions to the model.

## II. The Environment

Time is discrete. There is a  $[0, 1]$  continuum of agents who live forever with discount factor  $\beta \in (0, 1)$ . Each period is divided into two sub-periods, say day and night. Agents consume and supply labor in both subperiods. In general, preferences are  $\mathcal{U}(x, h, X, H)$ , where  $x$  and  $h$  ( $X$  and  $H$ ) are consumption and labor during the day (night). Although in principle we do not need any special restrictions to define or discuss equilibrium in this model, to get the distribution of money degenerate we need  $\mathcal{U}$  to be linear in either  $X$  or  $H$ . Here we use

$$\mathcal{U}(x, h, X, H) = u(x) - c(h) + U(X) - H. \quad (1)$$

Assume that  $u$ ,  $c$ , and  $U$  are twice continuously differentiable with  $u' > 0$ ,  $c' > 0$ ,  $U' > 0$ ,  $u'' < 0$ ,  $c'' \geq 0$ , and  $U'' \leq 0$ . Also,  $u(0) = c(0) = 0$ , and suppose that there exists  $q^* \in (0, \infty)$  such that  $u'(q^*) = c'(q^*)$  and  $X^* \in (0, \infty)$  such that  $U'(X^*) = 1$  with  $U(X^*) > X^*$ .

As in a typical search model, during the day agents interact in a decentralized market with anonymous bilateral matching, where  $\alpha$  is the probability of a meeting. The day good  $x$  comes in many varieties, of which each agent consumes only a subset. Each agent can transform labor one for one into one of these *special goods* that he himself does not consume. For two agents  $i$  and  $j$  drawn at random, there are four possible events. The probability that both consume what the other can produce (a double coincidence) is  $\delta$ . The probability that  $i$  consumes

<sup>1</sup> A related model is developed by Shi (1997), who also gets  $F$  degenerate but by different means: he assumes that the fundamental decision-making unit is not an individual, but a family with a continuum of agents, and appeals to the law of large numbers. In our working paper (Lagos and Wright 2004), we provide a detailed discussion of the two approaches; suffice it to say here that our model avoids some technical problems in the infinite-family model and seems, at least to us, more natural for many applications. Also, independent of making the money distribution degenerate, there are many other reasons why it may be interesting to integrate some competitive markets into search models; see Sec. VI for some examples.

what  $j$  produces but not vice versa (a single coincidence) is  $\sigma$ . Symmetrically, the probability that  $j$  consumes what  $i$  produces but not vice versa is  $\sigma$ . And the probability that neither wants what the other produces is  $1 - 2\sigma - \delta$ . In a single-coincidence meeting, if  $i$  wants the special good  $j$  produces, we call  $i$  the *buyer* and  $j$  the *seller*.

At night agents trade in a centralized (Walrasian) market. With centralized trade, specialization does not lead to a double-coincidence problem, and so it is irrelevant whether the night good  $X$  comes in many varieties or one; hence we assume that at night all agents produce and consume a *general good*. Agents at night can transform one unit of labor into one unit of the general good. The general goods produced at night and the special goods produced during the day are perfectly divisible and nonstorable. There is another object, called *money*, that is perfectly divisible and storable in any quantity  $m \geq 0$ . For now the total money stock is fixed at  $M$ , but later we allow it to change over time.

Given our assumptions, during the day the only feasible trades are barter in special goods and the exchange of special goods for money, and at night the only feasible trades involve general goods and money. Special goods cannot be traded at night nor general goods during the day because they are produced in only one subperiod and are not storable.<sup>2</sup> Money is essential in this model for the same reason it is essential in the typical search model: since meetings in the day market are anonymous, there is no scope for trading future promises in this market, so exchange must be quid pro quo (see Kocherlakota [1998] and Wallace [2001] for detailed discussions).

### III. Equilibrium

Let  $F(\tilde{m})$  be the measure of agents starting the decentralized day market at  $t$  holding  $m \leq \tilde{m}$ . Similarly let  $G_t(\tilde{m})$  be the distribution at the start of the centralized night market. The initial distribution—either  $F_0$  or  $G_0$ , depending on whether we start the model at  $t = 0$  during the day or night—is given exogenously. Since for now the total money stock is fixed,  $\int m dF(m) = \int m dG(m) = M$  for all  $t$ . Let  $\phi_t$  be the price of money in the centralized market (i.e.,  $1/\phi_t$  is the nominal price of general goods). There is no uncertainty in the basic model except for random matching. Hence, an individual's decisions in a given period depend on only his money holdings,  $m$ . That is, at each  $t$ , since aggregate variables such as  $F_t$ ,  $G_t$ , and prices are taken as given by individuals, we can characterize

<sup>2</sup> Some extensions that allow general goods to be storable are discussed in Sec. VI. Of course, whether or not goods are storable, we can allow the exchange of intertemporal claims across centralized markets. Since agents are homogeneous in this market, such claims will not trade, but we can still price assets like real or nominal bonds.

their decisions in terms of common value functions with  $m$  as the only argument.<sup>3</sup>

Let  $V_i(m)$  be the value function for an agent with  $m$  dollars when he enters the decentralized market and  $W_i(m)$  the value function when he enters the centralized market. Since trade is bilateral in the day market, a seller's production  $h$  must equal a buyer's consumption  $x$ . Hence, we denote their common value by  $q_i(m, \tilde{m})$  and use  $d_i(m, \tilde{m})$  to denote the dollars the buyer pays; these may depend on the money holdings of the buyer  $m$  and seller  $\tilde{m}$ . Also, in double-coincidence meetings, let  $B_i(m, \tilde{m})$  be the payoff for an agent holding  $m$  who meets someone with  $\tilde{m}$ . Then

$$\begin{aligned} V_i(m) = & \alpha\sigma \int \{u[q_i(m, \tilde{m})] + W_i[m - d_i(m, \tilde{m})]\}dF_i(\tilde{m}) \\ & + \alpha\sigma \int \{-c[q_i(\tilde{m}, m)] + W_i[m + d_i(\tilde{m}, m)]\}dF_i(\tilde{m}) \\ & + \alpha\delta \int B_i(m, \tilde{m})dF_i(\tilde{m}) + (1 - 2\alpha\sigma - \alpha\delta)W_i(m), \end{aligned} \quad (2)$$

where the four terms represent the expected payoffs to buying, selling, bartering, and not trading.

A version of (2) appears in Trejos and Wright (1995) and Molico (1997), except  $\beta V_{t+1}(m)$  replaces  $W_i(m)$ , because in those papers there is no centralized market and all agents can do with money is carry it forward to the next round of decentralized trade. Here, they get to go to the centralized market, where they solve

$$W_i(m) = \max_{X, H, m'} \{U(X) - H + \beta V_{t+1}(m')\} \quad (3)$$

subject to  $X = H + \phi_i m - \phi_i m'$ ,  $X \geq 0$ ,  $0 \leq H \leq \bar{H}$ , and  $m' \geq 0$ , where  $\bar{H}$  is an upper bound on hours,  $m'$  is money taken out of the market, and again  $\phi_i$  is the price of money. We assume an interior solution for  $X$  and  $H$ . This is guaranteed for  $X$  under standard assumptions, but things are more problematic for  $H$  because of quasi linearity. Our approach is to assume interiority, characterize equilibria, and then check that

<sup>3</sup> As editor Nancy Stokey emphasized to us, our equilibrium concept is a blend of traditional Arrow-Debreu components describing aggregates as functions of time  $t$  and recursive components describing individuals' problems as functions of  $t$  and individual state variables. Equilibrium also specifies the distributions of the individual state for all  $t$ . As is usual in monetary models, there may be multiple equilibria, some of which are nonstationary (there may also be sunspot equilibria, but they are ignored here; see Lagos and Wright [2003]). The  $t$  index indicates the prices and distributions that are relevant, but there are no aggregate state variables. In Lagos and Wright (2004), we stay closer to conventional recursive methods, but the results are less general; see nn. 6 and 8 below.

$0 < H < \bar{H}$  is satisfied. Thus, when we say equilibrium here, we always mean equilibrium with  $0 < H < \bar{H}$ .

Given the value functions, we now consider the terms of trade in the decentralized market. In double-coincidence meetings, we use the symmetric Nash bargaining solution with threat point given by the continuation value  $W_t(m)$ . It is easy to show that this implies that, regardless of the money holdings of the agents, each gives the other  $q^*$  as defined by  $u'(q^*) = c'(q^*)$ , and no money changes hands (see Lagos and Wright 2004). Hence,  $B_t(m, \tilde{m}) = u(q^*) - c(q^*) + W_t(m)$ . In single-coincidence meetings, we use the generalized Nash solution in which the buyer has bargaining power  $\theta > 0$  and threat points are again given by continuation values. Hence,  $(q, d)$  maximizes

$$[u(q) + W_t(m - d) - W_t(m)]^\theta [-c(q) + W_t(\tilde{m} + d) - W_t(\tilde{m})]^{1-\theta} \quad (4)$$

subject to  $d \leq m$  and  $q \geq 0$ , where  $m$  and  $\tilde{m}$  are the buyer's and seller's money holdings.

This leads us to the following definition.

**DEFINITION.** An equilibrium is a list  $\{V_t, W_t, X_t, H_t, m'_t, q_t, d_t, \phi_t, F_t, G_t\}$ , where, for all  $t$ ,  $V_t(m)$  and  $W_t(m)$  are the value functions;  $X_t(m)$ ,  $H_t(m)$ , and  $m'_t(m)$  are the decision rules in the centralized market;  $q_t(m, \tilde{m})$  and  $d_t(m, \tilde{m})$  are the terms of trade in the decentralized market;  $\phi_t$  is the price in the centralized market; and  $F_t$  and  $G_t$  are the distributions of money holdings before and after decentralized trade. The equilibrium conditions are as follows. For all  $t$ , (i) given prices and distributions, the value functions and decision rules satisfy (2) and (3); (ii) the terms of trade in the decentralized market maximize (4), given the value functions; (iii)  $\phi_t > 0$  (i.e., we focus on monetary equilibria); (iv) centralized money markets clear,  $\int m'(m) dG_t(m) = M$  (goods markets clear by Walras' law); and (v)  $\{F_t, G_t\}$  is consistent with initial conditions and the evolution of money holdings implied by trades in the centralized and decentralized markets.<sup>4</sup>

We now characterize equilibria. Here is an outline of what will follow. We begin by deriving some properties of the solution to the centralized market problem. We use these properties to solve the bargaining problem in the decentralized market and then use these results to simplify  $V_t$  and to solve an individual's problem of choosing  $m'_t(m)$ . In particular, we show that under certain conditions,  $m'_t = M$  for all agents regardless of the value of  $m$  with which they entered the centralized market (i.e.,

<sup>4</sup> To see what this last condition entails, consider an agent with  $m$  entering the decentralized market at  $t$ . With probability  $\alpha\sigma$ , he buys and leaves with  $m - d_t(m, \tilde{m})$ , where  $\tilde{m}$  is a random draw from  $F_t$ . Similarly, with probability  $\alpha\sigma$ , he sells and leaves with  $m + d_t(m, \tilde{m})$ , and with probability  $1 - 2\alpha\sigma$ , he neither buys nor sells (although he might barter) and leaves with  $m$ . This maps  $F_t$  into  $G_t$ . Later that period, in the centralized market, an agent with  $m$  chooses  $m'_t(m) = m_{t+1}$ , and this maps  $G_t$  into  $F_{t+1}$ .

$F_{t+1}$  must be degenerate) in any equilibrium. Finally, we combine the solutions to the centralized and decentralized market problems to reduce the model to a single difference equation.

To begin, substitute for  $H$  from the budget equation to write (3) as

$$W_t(m) = \phi_t m + \max_{X, m'} \{U(X) - X - \phi_t m' + \beta V_{t+1}(m')\}. \quad (5)$$

This immediately implies several things. First,  $X_t(m) = X^*$ , where  $U'(X^*) = 1$ . Also,  $m'_t(m)$  does not depend on  $m$ .<sup>5</sup> Third,  $W_t$  is linear in  $m$  with slope  $\phi_t$ . Given this linearity, the bargaining problem (4) simplifies to

$$\max_{q, d} [u(q) - \phi_t d]^\theta [-c(q) + \phi_t d]^{1-\theta} \quad (6)$$

subject to  $d \leq m$  and  $q \geq 0$ .

We claim that the solution to (6) is

$$\begin{aligned} q_t(m, \tilde{m}) &= \begin{cases} \hat{q}_t(m) & \text{if } m < m_t^* \\ q^* & \text{if } m \geq m_t^*, \end{cases} \\ d_t(m, \tilde{m}) &= \begin{cases} m & \text{if } m < m_t^* \\ m^* & \text{if } m \geq m_t^*, \end{cases} \end{aligned} \quad (7)$$

where  $\hat{q}_t(m)$  is the  $q_t$  that solves  $\phi_t m = z(q_t)$ , with

$$z(q) = \frac{\theta c(q)u'(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}, \quad (8)$$

and  $m_t^* = z(q^*)/\phi_t$ . To verify this, notice that if we ignore the constraint  $d \leq m$ , then necessary and sufficient conditions for a solution are

$$\theta[-c(q_t) + \phi_t d_t]u'(q_t) = (1-\theta)[u(q_t) - \phi_t d_t]c'(q_t) \quad (9)$$

and

$$\theta[-c(q_t) + \phi_t d_t] = (1-\theta)[u(q_t) - \phi_t d_t]. \quad (10)$$

Thus  $u'(q_t) = c'(q_t)$ , or  $q_t = q^*$ , and  $d_t = m_t^* = [\theta c(q^*) + (1-\theta)u(q^*)]/\phi_t$ . If  $m \geq m_t^*$ , the constraint is not binding. If  $m < m_t^*$ , the solution is given by (9) with  $d_t = m$ , which easily yields  $\phi_t m = z(q_t)$ . This verifies the claim.

In words, if the buyer's cash is at least  $m_t^*$ , the constraint  $d \leq m$  is not binding and he gets  $q^*$  for  $m_t^*$  dollars; otherwise the constraint is bind-

<sup>5</sup> The result that  $m'$  does not depend on  $m$  suggests that it is reasonable to look for equilibrium where  $F_t$  is degenerate; but we are after bigger game. We want to show that  $F_t$  is degenerate in *all* equilibria, and even if  $m'$  does not depend on  $m$ , it is possible that there are multiple solutions to (5) and different agents select different  $m'$ . Below we give conditions implying that  $V_t$  is strictly concave in the relevant range, and hence there is a unique solution to (5).

ing, and he spends all of his money to get  $\hat{q}(m)$ . Notice that the solution does not depend on the seller's money holdings  $\tilde{m}$  at all, and so we write  $q_t(m, \tilde{m}) = q_t(m)$  and  $d_t(m, \tilde{m}) = d_t(m)$  in what follows. For all  $m < m_t^*$ ,  $q'_t(m) = \phi_t/z'(q_t)$ , where

$$z' = \frac{u'c'[\theta u' + (1 - \theta)c'] + \theta(1 - \theta)(u - c)(u'c'' - c'u'')}{[\theta u' + (1 - \theta)c']^2} > 0. \quad (11)$$

It is a simple matter to check that  $\hat{q}(m) \rightarrow q^*$  as  $m \rightarrow m_t^*$ . Hence,  $q_t(m) = \hat{q}(m)$  is strictly increasing for  $m < m_t^*$ , is continuous at  $m_t^*$ , and is constant at  $q_t(m) = q^*$  for all  $m \geq m_t^*$ .

We can now use what we know about the bargaining solution and  $V_t(m)$  to simplify (2) to

$$V_t(m) = v_t(m) + \phi_t m + \max_{m'} \{-\phi_t m' + \beta V_{t+1}(m')\}, \quad (12)$$

where

$$\begin{aligned} v_t(m) \equiv & \alpha\sigma\{u[q_t(m)] - \phi_t d_t(m)\} + \alpha\sigma \int \{\phi_t d_t(\tilde{m}) - c[q_t(\tilde{m})]\} dF(\tilde{m}) \\ & + \alpha\delta[u(q^*) - c(q^*)] + U(X^*) - X^*. \end{aligned} \quad (13)$$

By repeated substitution we have

$$\begin{aligned} V_t(m_t) = & v_t(m_t) + \phi_t m_t \\ & + \sum_{j=t}^{\infty} \beta^{j-t} \max_{m_{j+1}} \{-\phi_j m_{j+1} + \beta[v_{j+1}(m_{j+1}) + \phi_{j+1} m_{j+1}]\}. \end{aligned} \quad (14)$$

This reduces the choice of the sequence  $\{m_{t+1}\}$  to a sequence of simple problems defined in terms of primitives, since  $v_{t+1}$  is a known function.<sup>6</sup>

Notice that

$$v'_{t+1}(m_{t+1}) = \alpha\sigma\{u'[q_{t+1}(m_{t+1})]q'_{t+1}(m_{t+1}) - \phi_{t+1}d'_{t+1}(m_{t+1})\} \quad (15)$$

is zero for all  $m_{t+1} \geq m_{t+1}^*$  by (7). Hence,  $\phi_t < \beta\phi_{t+1}$  implies that the problem of choosing  $m_{t+1}$  in (14) has no solution, since the objective function is strictly increasing for all  $m_{t+1} \geq m_{t+1}^*$ . This means that any equilibrium

<sup>6</sup> This is not quite true since  $v_{t+1}$  depends on  $F_{t+1}$ , which we do not know yet; but this merely influences the intercept and not the choice  $m_{t+1}$ . The point is that we eliminated  $V_{t+1}$  from the problem. In Lagos and Wright (2004), we used standard dynamic programming methods by writing  $V$  as a stationary function of  $(m, \phi)$  and showing that, even though  $V$  is unbounded, one can apply the contraction mapping theorem to prove existence and uniqueness. That is more work and is less general because there are equilibria that cannot be represented in terms of time-independent functions of  $(m, \phi)$ , or any other aggregate state variable; see n. 8 for an example. However, if one is willing to sacrifice generality (e.g., to focus on stationary equilibria), then the results in Lagos and Wright (2004) are useful because one can study the problem  $\max_{m'} \{-\phi m' + \beta V(m', \phi')\}$  in (12) directly.

must satisfy  $\phi_t \geq \beta\phi_{t+1}$ . Therefore, the minimum inflation rate consistent with equilibrium is  $\phi_t/\phi_{t+1} = \beta$ , which is the Friedman rule. Given  $\phi_t \geq \beta\phi_{t+1}$ , for all  $t$ , the objective function in (14) is nonincreasing in  $m_{t+1}$  for  $m_{t+1} \geq m_{t+1}^*$ .

The slope of the date  $t$  objective function in (14) as  $m_{t+1} \rightarrow m_{t+1}^*$  from below is proportional to  $-\phi_t + \beta\phi_{t+1} + \beta\alpha\sigma\phi_{t+1}\Sigma$ , where

$$\Sigma \equiv \frac{u'(q^*)^2}{u'(q^*)^2 + \theta(1-\theta)[u(q^*) - c(q^*)][c''(q^*) - u''(q^*)]} - 1 \quad (16)$$

is the buyer's marginal gain from bringing an additional dollar into a single-coincidence meeting evaluated at  $q = q^*$ . Note that  $\Sigma \leq 0$ , and the inequality is strict except when  $\theta = 1$ . So, unless  $\phi_t = \beta\phi_{t+1}$  and  $\theta = 1$ , the slope of the objective function in (14) as  $m_{t+1} \rightarrow m_{t+1}^*$  is strictly negative, and therefore any solution must satisfy  $m_{t+1} < m_{t+1}^*$ . In the extreme case in which  $\phi_t = \beta\phi_{t+1}$  and  $\theta = 1$ , the slope of the objective function at  $m_{t+1}^*$  is zero; in this case, however, we consider only solutions that are limits when either  $\phi_t - \beta\phi_{t+1} \rightarrow 0$  from above or  $\theta \rightarrow 1$  from below. Hence,  $m_{t+1} < m_{t+1}^*$  as long as we are not in the extreme case  $\phi_t = \beta\phi_{t+1}$  and  $\theta = 1$ , and in this case we take the limit.<sup>7</sup>

For  $m_{t+1} < m_{t+1}^*$ , we have  $v''_{t+1} = \alpha\sigma[u''(q_{t+1})(q'_{t+1})^2 + u'(q_{t+1})q''_{t+1}]$ . We would like to be able to conclude that  $v''_{t+1} < 0$ , since then there will be a unique solution  $m_{t+1}$ . In numerical work one can check it directly, but it seems useful to have some conditions to guarantee it more generally. For example, suppose that we set  $c(q) = q$ , mainly to reduce notation. Then if we insert  $q'$  and  $q''$ , one can check that  $v''$  takes the sign of  $\Gamma + (1-\theta)[u'u''' - (u'')^2]$ , where  $\Gamma < 0$  but is otherwise of no concern. The problem in general is the presence of  $u'''$ , but this vanishes if  $\theta \approx 1$ . Alternatively, for an arbitrary  $\theta$ , we can make assumptions on preferences; a sufficient condition is  $u'u''' \leq (u'')^2$ , which holds if  $u'$  is log concave.

In summary, we now have simple conditions either on  $\theta$  or on preferences to guarantee  $v''_{t+1} < 0$ , and this implies that there is a unique choice of  $m_{t+1}$  in any equilibrium. That is,  $F_{t+1}$  must be degenerate at  $m_{t+1} = M$ . In any monetary equilibrium, the first-order condition evaluated at  $m_{t+1} = M$  is  $\phi_t = \beta[v'_{t+1}(M) + \phi_{t+1}]$ , or

$$\phi_t = \beta\{\alpha\sigma u'[q_{t+1}(M)]q'_{t+1}(M) + (1 - \alpha\sigma)\phi_{t+1}\}. \quad (17)$$

<sup>7</sup> It is standard in monetary theory to consider only equilibria at the Friedman rule that are the limit of equilibria as inflation approaches the Friedman rule. Here we can actually do more, by considering any equilibria at the Friedman rule as long as  $\theta < 1$ , or as long as  $\theta = 1$ , but we consider only equilibria that correspond to a limit as  $\theta \rightarrow 1$ .

Inserting  $\phi_t = z(q_t)/M$  and  $q'_t(M) = \phi/z'(q_t)$ , from the bargaining solution, we arrive at

$$z(q_t) = \beta z(q_{t+1}) \left[ \alpha\sigma \frac{u'(q_{t+1})}{z'(q_{t+1})} + 1 - \alpha\sigma \right], \quad (18)$$

a simple difference equation in  $q_t$ . A monetary equilibrium is now characterized by any path for  $\{q_t\}$  satisfying (18) that stays in  $(0, q^*)$ , since  $q_t < q^*$  follows from the result that  $m_t < m_t^*$ .

For the rest of this paper we focus on stationary equilibria, or steady states.<sup>8</sup> Notice, however, that it was important not to restrict attention to such equilibria earlier, since the argument that  $F_{t+1}$  is degenerate did not use stationarity. In any case, in steady state (18) yields

$$\frac{u'(q)}{z'(q)} = 1 + \frac{1 - \beta}{\alpha\sigma\beta}. \quad (19)$$

Consider first the limiting case  $\theta = 1$ , which means  $z(q) = c(q)$ . Then it is easy to see that a unique solution  $q > 0$  to (19) exists under standard conditions, such as  $u'(0) = \infty$ . For  $\theta < 1$ , a steady state also exists under this condition, but we cannot be sure of uniqueness because  $u'(q)/z'(q)$  may not be monotone. One can show that it is monotone under certain additional conditions, such as  $\theta \approx 1$ , or  $c$  linear and  $u'$  log concave (Lagos and Wright 2004). One can also show that  $u'(q)/z'(q)$  is increasing in  $\theta$ ; hence when the solution is unique, we know that  $\partial q/\partial\theta > 0$  (Lagos and Wright 2004). Similarly, we know that  $\partial q/\partial\alpha > 0$ ,  $\partial q/\partial\sigma > 0$ , and  $\partial q/\partial\beta > 0$ . Also note that at  $\theta = 1$ , (19) implies  $q \rightarrow q^*$  as  $\beta \rightarrow 1$ ; for  $\theta < 1$ , however,  $q < q^*$  even in the limit as  $\beta \rightarrow 1$ .

To summarize the results, we have shown that in any equilibrium the distribution of money is degenerate across agents exiting the centralized and entering the decentralized market. Also, from the choice of  $m_{t+1}$  at date  $t$ , we know that  $m_{t+1} \leq m_{t+1}^*$  with strict inequality except in the extreme case of  $\phi_t/\phi_{t+1} = \beta$  and  $\theta = 1$ . This means that in single-coincidence meetings at  $t+1$ , the buyer exchanges all his money,  $d_{t+1} = M$ , for  $q_{t+1} = \hat{q}_{t+1}(M)$ . A steady state exists under the usual conditions, and although it may not be unique, in general, it will be under more stringent conditions on preferences or  $\theta$ . Steady states have natural

<sup>8</sup> Dynamics are studied in detail in Lagos and Wright (2003), including cyclic, chaotic, and sunspot equilibria, but we do want to mention one thing here. Equilibrium condition (18) defines a function  $q_t = Q(q_{t+1})$  that may not be invertible—i.e.,  $q_{t+1} = Q^{-1}(q_t)$  may be a correspondence—as is standard in monetary models. In this case there is nothing that precludes equilibria in which we select different  $q_{t+1}$  at different dates given the same  $q_t$ . For example, under certain conditions, we can construct equilibria in which we select  $q_{t+1}$  from  $Q^{-1}(q_t)$  so that  $q_t$  cycles between  $q_L$  and  $q_H$  for all  $t < T$  and then select  $q_{T+1} \notin \{q_L, q_H\}$  from  $Q^{-1}(q_T)$ . There is no way to represent such an equilibrium in terms of a time-invariant function of any aggregate state.

comparative static properties, and in particular  $q$  is increasing in both  $\beta$  and  $\theta$ . One thing we want to emphasize is that the steady state is efficient if and only if  $q = q^*$ , which requires both  $\beta = 1$  and  $\theta = 1$ .

To close this section, recall that so far we have simply assumed an interior solution for  $H$ . Suppose that we want to guarantee  $H > 0$ . Beginning at  $t = 0$  with the centralized market, let  $\bar{m}_0$  be the upper bound of the initial  $G_0$  distribution. In the candidate equilibrium,  $X = X^*$  and  $m_{t+1} = M$ , so agents endowed with  $m$  work  $H(m) = X^* + \phi_0(M - m)$  hours. Since in any equilibrium  $\phi_t \leq \phi^* = z(q^*)/M$ , we have  $H_0(m) > 0$  for all  $m$  as long as

$$\bar{m}_0 < M + \frac{X^*}{\phi^*} = M \left[ 1 + \frac{X^*}{z(q^*)} \right].$$

One can similarly show that if the lower bound satisfies

$$\underline{m}_0 > M \left[ 1 + \frac{X^* - \bar{H}}{z(q^*)} \right],$$

then  $H_0(m) < \bar{H}$  for all  $m$ . Finally, for  $t \geq 1$ , one can show that  $X^* > z(q^*)$  and  $X^* < \bar{H} - z(q^*)$  guarantee  $0 < H_t < \bar{H}$ . Hence, simple conditions imply that the constraint  $0 \leq H_t \leq \bar{H}$  will be slack for all  $t$ .

#### IV. Changes in the Money Supply

We now allow  $M$  to change over time, with new money injected as lump-sum transfers in the centralized market. The generalization of (18) is

$$\frac{z(q_t)}{M_t} = \beta \frac{z(q_{t+1})}{M_{t+1}} \left[ \alpha\sigma \frac{u'(q_{t+1})}{z'(q_{t+1})} + 1 - \alpha\sigma \right]. \quad (20)$$

If  $M_{t+1} = (1 + \tau)M_t$  with  $\tau$  constant, it makes sense to consider steady states in which  $q$  and real balances  $\phi M = z(q)$  are constant, that is, in which  $\phi_t/\phi_{t+1} = 1 + \tau$ . As in the previous section,  $\phi_t/\phi_{t+1} \geq \beta$  is necessary for equilibrium to exist; hence, we have  $\tau \geq \tau^F = \beta - 1$ , and a lower bound on feasible  $\tau$  is given by the Friedman rule.

The steady-state condition is now

$$\frac{u'(q)}{z'(q)} = 1 + \frac{1 + \tau - \beta}{\alpha\sigma\beta}. \quad (21)$$

As is standard, we can also state this in terms of the nominal interest

rate  $i$ , defined by  $1 + i = (1 + r)(1 + \pi)$ , where  $\pi = \tau$  is the equilibrium inflation rate and  $r = (1 - \beta)/\beta$  the equilibrium real interest rate:

$$\frac{u'(q)}{z'(q)} = 1 + \frac{i}{\alpha\sigma}. \quad (22)$$

Assuming a unique monetary steady state, we have  $\partial q/\partial \tau < 0$  or, equivalently,  $\partial q/\partial i < 0$ .

If  $\theta = 1$ , then  $z(q) = c(q)$ , and the efficient outcome  $q^*$  obtains if and only if  $\tau = \tau^F$  or, equivalently,  $i = 0$ . If  $\theta < 1$ , however, then  $q < q^*$  at  $\tau^F$ . Since a necessary condition for monetary equilibrium is  $\tau \geq \tau^F$  or, equivalently,  $i \geq 0$ , the Friedman rule is always optimal here; but when  $\theta < 1$ , it does not achieve  $q^*$ . The reason is that in this model there are two types of inefficiencies: one due to  $\beta$  and one to  $\theta$ . The  $\beta$  effect is standard: when you acquire cash, you can turn it into future consumption; but because  $\beta < 1$ , you are willing to produce less than the  $q^*$  you would produce if you could turn the proceeds into immediate consumption. The Friedman rule corrects this by generating a real return on money that compensates for discounting. Notice that although this effect is standard, the model does generate some novel insights about it, since the frictions show up explicitly: (21) or (22) makes it clear that the effect depends on search and specialization through the term  $\alpha\sigma$ .

The more novel effect is the wedge due to  $\theta < 1$ . One intuition for this effect is the notion of a *holdup problem*. An agent who carries a dollar into next period is making an investment with cost  $\phi$ , since he could have spent the cash on general goods. When he uses the money in the future, he reaps the full return on his investment if and only if  $\theta = 1$ ; otherwise the seller bargains away part of the surplus. Thus  $\theta < 1$  reduces the incentive to invest, which lowers the demand for real balances and hence  $q$ . Therefore,  $\theta < 1$  implies  $q < q^*$  even at the Friedman rule. The Hosios (1990) condition for efficiency says that the bargaining solution should split the surplus so that each party is compensated for his contribution to the surplus in a match. Intuitively, the surplus in a single-coincidence meeting is all due to the buyer, since the outcome depends on  $m$  but not on  $\tilde{m}$ . Hence, efficiency requires  $\theta = 1$  here.<sup>9</sup>

The wedge due to  $\theta < 1$  is important for issues such as the welfare cost of inflation. Measuring welfare by  $V$ , when  $\theta = 1$  we know that  $V$  is maximized at  $\tau^F$  and achieves the efficient outcome  $V^*$ , where

$$(1 - \beta)V^* = \alpha(\delta + \sigma)[u(q^*) - c(q^*)] + U(X^*) - X^*, \quad (23)$$

as shown in figure 1. With  $\theta = 1$ , small deviations from  $\tau^F$  have very

<sup>9</sup> This holdup problem does not arise in the paper by Shi (1997) because of the way he solves the bargaining problem, although it would if he used a more standard bargaining solution, as Rauch (2000) points out. See Berentsen and Rocheteau (2003) for a discussion.

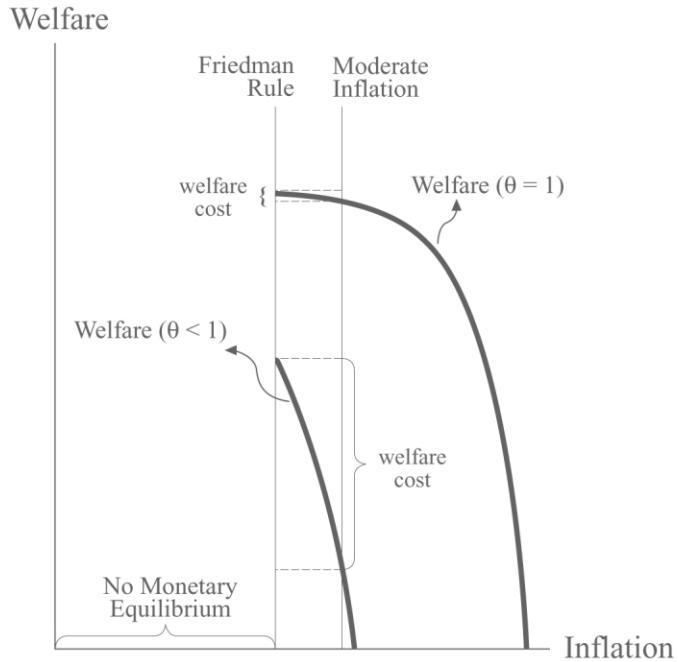


FIG. 1.—Welfare cost of moderate inflation

small effects on  $V$  because of the envelope theorem, just as in the typical reduced-form (e.g., cash-in-advance) model. When  $\theta < 1$ ,  $\tau^F$  is a constrained optimum:  $\tau < \tau^F$  would achieve a higher  $q$  and  $V$  if it were feasible, but it is not. Hence the slope of  $V$  with respect to  $\tau$  is steep at  $\tau^F$  and the envelope theorem does not apply. A moderate inflation therefore has a larger welfare cost when  $\theta < 1$ . We quantify this statement in the next section.

## V. Quantitative Analysis

We parameterize the model as follows. Assume that  $u(q) = [(q + b)^{1-\eta} - b^{1-\eta}]/(1 - \eta)$ , where  $\eta > 0$  and  $b \in (0, 1)$ . This generalizes standard constant relative risk aversion preferences by including  $b$ , which forces  $u(0) = 0$  (a maintained assumption); this does not matter much quantitatively, however, because we set  $b \approx 0$  for this exercise. Since  $q^* = 1 - b$ , this means that  $q^* \approx 1$ . Next, assume that  $U(X) = B \log(X)$ . Notice that this implies  $X^* = B$ . Finally, assume that  $c(h) = h$ , which makes the disutility of labor the same in the two markets.

We begin with a yearly model, mainly to facilitate comparison with

the existing literature (below we show that a monthly model yields similar results). The annual rate of time preference is  $r = 0.04$ . We can normalize  $\alpha = 1$  since results depend only on the products  $\alpha\delta$  and  $\alpha\sigma$ . We set  $\delta = 0$ , but this actually matters little for the results. We shall take two approaches to setting  $\sigma$ : first we shall estimate it along with the preference parameters, using a procedure discussed below; then, since it does not matter much for the results, we shall simply fix it at  $\sigma = 0.5$ , which means that every agent always has an opportunity to either buy or sell in each meeting of the decentralized market.<sup>10</sup>

We now describe the method we use to fit the parameters  $(\eta, B, \sigma)$ . The idea, exactly as in Lucas (2000), is to look at the relationship between the nominal rate  $i$  and  $L \equiv M/PY$ . This relationship represents “money demand” in the sense that “desired” real balances  $M/P$  are proportional to  $Y$ , with a factor of proportionality  $L$  that depends on the cost of holding cash,  $i$ . To construct  $L$  in the model, note that nominal output in the centralized market is  $X^*/\phi = B/\phi$ , and nominal output in the decentralized market is  $\sigma M$ . Hence,  $PY = (B/\phi) + \sigma M$  and  $Y = B + \sigma\phi M$ . In equilibrium,  $M/P = \phi M = z(q)$ , and so

$$L = \frac{M/P}{Y} = \frac{z(q)}{B + \sigma z(q)}. \quad (24)$$

Condition (22) gives  $q$  and hence  $L$  as a function of  $i$ ; (24) is the “money demand” curve implied by theory.

We want to fit this relationship to the data by choosing  $(\eta, B, \sigma)$ . We again follow Lucas (2000) and let  $i$  be the commercial paper rate and let  $M$  be M1 (there are issues concerning how to measure all these variables, perhaps especially  $M$ ; again our choices are made mainly to facilitate comparison with previous studies). The sample period is 1900–2000. The fitted values of  $(\eta, B, \sigma)$  are described below. Before we can proceed, however, we need to discuss the bargaining power parameter,  $\theta$ . Our method is to try three alternatives:  $\theta = 1$ , which eliminates the holdup problem and makes our setup closer to previous studies;  $\theta = 0.5$ , which means symmetric bargaining; and  $\theta = \theta_\mu$ , where  $\theta_\mu$  is the value that generates a markup  $\mu$  (price over marginal cost) consistent with the evidence, which we take to be  $\mu = 1.1$ .<sup>11</sup>

Given  $\theta = 1$  or  $0.5$ , we fit the parameters to the “money demand” data; given  $\theta_\mu$ , we fit them subject to the constraint  $\mu = 1.1$  at a bench-

<sup>10</sup> Setting  $\sigma = 0.5$  maximizes the importance of the decentralized market, but it is still fairly small in the calibrations reported below: at a benchmark inflation rate of 4 percent, it contributes less than 10 percent to aggregate output.

<sup>11</sup> See Basu and Fernald (1997) for the evidence. To compute  $\mu$  in the model, note that price over marginal cost in the decentralized market is  $\phi M/q$ , whereas in the centralized market it is one. Aggregate  $\mu$  averages these markups using the shares of output from each sector.

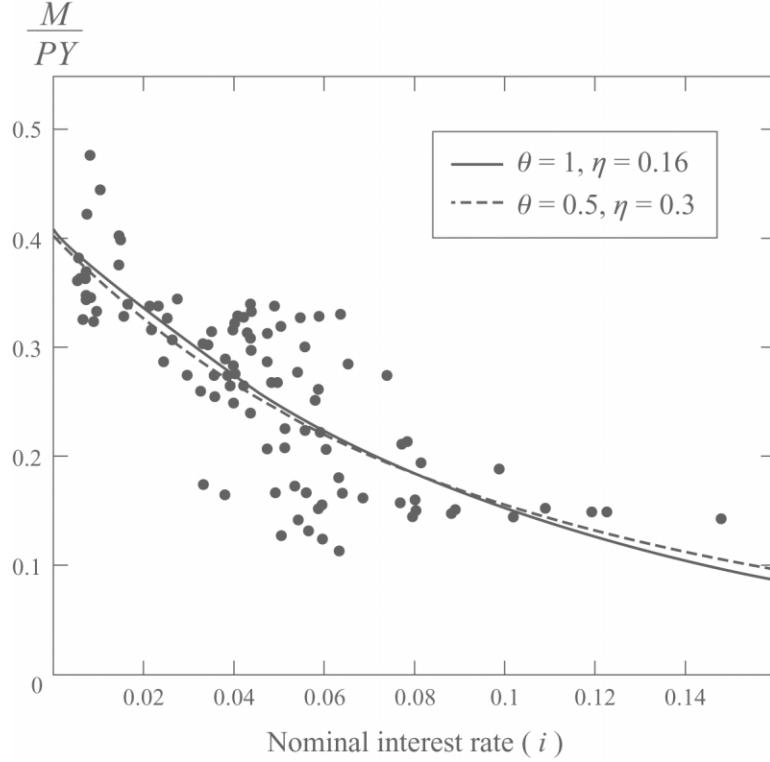


FIG. 2.—Model and data

mark inflation rate of 4 percent. For example, given  $\theta = 1$ , the best fit is  $(\eta, B, \sigma) = (0.266, 2.133, 0.311)$ . However, things are not precisely identified; if we fix  $\sigma = 0.5$ , we estimate  $(\eta, B) = (0.163, 1.968)$  with virtually no sacrifice in fit and no change in the welfare implications, as we shall see below. Hence we often simply set  $\sigma = 0.5$ . Figure 2 shows the fitted relationship for the case  $\theta = 1$  as the solid line and for the case  $\theta = 0.5$  as the dashed line, where in each case we set  $\sigma = 0.5$  and fit the preference parameters; clearly there is little in these data to recommend one  $\theta$  over another.

Our measure of the cost of inflation asks how much agents would be willing to give up in terms of total consumption to have inflation zero instead of  $\tau$ . For any  $\tau$ , steady-state utility is

$$(1 - \beta)V(\tau) = U(X^*) - X^* + \alpha\sigma\{u[q(\tau)] - q(\tau)\}. \quad (25)$$

TABLE 1  
ANNUAL MODEL (1900–2000)

| $\theta = 1$   | $\theta = .5$  | $\theta_\mu = .343$ | $\theta = 1$   |
|----------------|----------------|---------------------|----------------|
| $\sigma = .31$ | $\sigma = .50$ | $\sigma = .50$      | $\sigma = .50$ |
| $\eta = .27$   | $\eta = .16$   | $\eta = .30$        | $\eta = .39$   |
| $B = 2.13$     | $B = 1.97$     | $B = 1.91$          | $B = 1.78$     |
| (1)            | (2)            | (3)                 | (4)            |
| $q(\tau)$      | .243           | .206                | .143           |
| $q(0)$         | .638           | .618                | .442           |
| $q(\tau^F)$    | 1.000          | 1.000               | .779           |
| $1 - \Delta_0$ | .014           | .014                | .032           |
| $1 - \Delta_F$ | .016           | .016                | .041           |
|                |                |                     | .094           |
|                |                |                     | .296           |
|                |                |                     | .568           |
|                |                |                     | 1.000          |
|                |                |                     | .046           |
|                |                |                     | .012           |
|                |                |                     | .068           |
|                |                |                     | .013           |

If we reduce  $\tau$  to zero but also reduce consumption of both general and special goods by a factor  $\Delta$ , utility becomes

$$(1 - \beta)V_\Delta(0) = U(X^*\Delta) - X^* + \alpha\sigma\{u[q(0)\Delta] - q(0)\}. \quad (26)$$

We measure the cost of  $\tau$  as the value  $\Delta_0$  that solves  $V_{\Delta_0}(0) = V(\tau)$ ; agents would give up  $1 - \Delta_0$  percent of consumption to have zero rather than  $\tau$ . We also consider  $\Delta_F$ , which is how much they would give up to have the Friedman rule  $\tau^F$  rather than  $\tau$ . Our experiments use  $\tau = 0.1$  (i.e., 10 percent inflation), but we also report the costs of a wide range for inflation at the end of the section.

In table 1, column 1 presents results for the case  $\theta = 1$  and the fitted  $(\eta, B, \sigma)$ . To focus on one number, we find that going from 10 percent to 0 percent inflation is worth 1.4 percent of consumption. The column 2 results pertain to the case in which we fix  $\sigma = 0.5$  and refit  $(\eta, B)$ . As mentioned, the results are very similar, especially for welfare. The main point we want to make is that these numbers are similar to, if slightly larger than, typical estimates in the literature, including those in Lucas (2000), which reports a range for  $1 - \Delta_0$ , depending on the exact specification, but typically slightly under 1 percent. We interpret the results with  $\theta = 1$  as being in line with, if slightly higher than, the consensus view in the literature.<sup>12</sup>

Since  $\sigma$  does not matter much for the results, the remaining columns in table 1 vary  $\theta$ , while fixing  $\sigma = 0.5$  and reestimating  $(\eta, B)$  for each  $\theta$ . The goodness of fit is basically the same for each  $\theta$ , but the welfare costs increase and  $q$  decreases sharply with  $\theta$ . This indicates that the

<sup>12</sup> Lucas actually uses  $r = 0.03$  rather than  $r = 0.04$ , but this has a small effect on our estimates. Cooley and Hansen (1989, 1991) report even smaller numbers. Molico (1997) gets small numbers, and sometimes negative numbers, because inflation in his model beneficially redistributes liquidity to those who need it most; this effect is absent in our model, of course, because  $F$  is degenerate. Wu and Zhang (2000) get bigger numbers because they assume monopolistic competition, and they also provide references to some other related studies.

TABLE 2  
ANNUAL MODEL (1959–2000)

|                | $\theta = 1$   | $\theta = .5$  | $\theta_\mu = .404$ | $\theta = 1$   |
|----------------|----------------|----------------|---------------------|----------------|
|                | $\sigma = .50$ | $\sigma = .50$ | $\sigma = .50$      | $\sigma = .50$ |
|                | $\eta = .27$   | $\eta = .45$   | $\eta = .48$        | $\eta = .48$   |
|                | $B = 3.19$     | $B = 2.92$     | $B = 2.71$          | $B = 2.71$     |
|                | (1)            | (2)            | (3)                 | (4)            |
| $q(\tau)$      | .392           | .192           | .135                | .590           |
| $q(0)$         | .752           | .409           | .307                | .852           |
| $q(\tau^f)$    | 1.000          | .602           | .478                | 1.000          |
| $1 - \Delta_0$ | .008           | .025           | .031                | .007           |
| $1 - \Delta_F$ | .009           | .035           | .046                | .008           |

holdup problem is serious and is in fact what generates a relatively large cost of inflation. In the case of  $\theta = 0.5$ , the aggregate markup is  $\mu = 1.04$ . Column 4 generates  $\mu = 1.10$  at 4 percent inflation and implies that the welfare costs are  $1 - \Delta_0 = 0.046$  and  $1 - \Delta_F = 0.068$ —substantially larger than the consensus view. To verify that it is indeed the holdup problem that lies at the heart of these effects, as opposed to the differences in other parameters across the columns, column 5 uses the same parameters as column 4 but sets  $\theta = 1$ . This yields fairly low costs. Hence, it is  $\theta < 1$  and not the other parameters that generates the big effects.<sup>13</sup>

Table 2 reports similar experiments fitting the model to a shorter sample, 1959–2000. Although the welfare costs are slightly lower, the main conclusion is the same: decreasing  $\theta$  from 1 to 0.5 or to  $\theta_\mu$  increases the welfare costs considerably.<sup>14</sup> Table 3 reports a final robustness check by recalibrating so that the period is a month; that is, we transform the data for  $Y$  and  $i$  to make them monthly. For these results we estimate  $\sigma$  along with the preference parameters in every case. While the estimates change when we go from an annual to a monthly model, the overall fit is about the same.<sup>15</sup> What we want to emphasize is that the welfare costs

<sup>13</sup> There is a sense in which we may be overestimating the cost of inflation by using  $\theta_\mu$ , since  $\theta_\mu$  is calibrated to generate an average markup of 10 percent under the assumption that the centralized market is perfectly competitive; if there were a noncompetitive markup in the centralized market, we would not need such a low  $\theta$  to match the average markup.

<sup>14</sup> Intuitively, the cost of inflation is lower in the shorter sample because the estimated “money demand” curve has a much flatter slope in the shorter sample. This is not meant to be a rigorous explanation, however, especially since we shall argue below that the area under the “money demand” curve is not necessarily the right way to measure the cost of inflation.

<sup>15</sup> In particular, estimates of  $B$  and  $\sigma$  are smaller in the monthly data, for the following simple reason. First, in equilibrium, centralized market consumption is  $X^* = B$ , and obviously monthly consumption is less than annual. Second,  $\sigma$  is the probability of a single coincidence, and obviously this probability is lower per month than per year. Intuitively, this last point is important because it means that we can match velocity equally well when we vary the period length.

TABLE 3  
MONTHLY MODEL (1900–2000)

|                | $\theta = 1$ | $\theta = .5$ | $\theta_\mu = .315$ | $\theta = 1$ |
|----------------|--------------|---------------|---------------------|--------------|
| $\sigma$       | .033         | .052          | .052                | .052         |
| $\eta$         | .20          | .23           | .33                 | .33          |
| $B$            | .17          | .15           | .14                 | .14          |
|                | (1)          | (2)           | (3)                 | (4)          |
| $q(\tau)$      | .230         | .151          | .101                | .552         |
| $q(0)$         | .623         | .476          | .329                | .831         |
| $q(\tau^*)$    | 1.000        | .845          | .644                | 1.000        |
| $1 - \Delta_0$ | .013         | .030          | .049                | .010         |
| $1 - \Delta_F$ | .015         | .038          | .069                | .011         |

here are very similar to those in table 1, and so the main conclusion is robust to changing the period length as well as the sample. That conclusion is that bargaining power seems to be a quantitatively important consideration in estimating the welfare cost of inflation, and one that previous analyses have missed entirely.

To illustrate the effects of less moderate inflations, figure 3 shows the welfare cost  $1 - \Delta_0$  for inflation rates ranging from 0 to 150 percent. The upper curve pertains to  $\theta_\mu$  and the parameters from column 4 of table 1, whereas the lower curve pertains to  $\theta = 1$  and the same parameters. The difference in the curves is due to the holdup problem. Notice that the difference gets smaller at big inflation rates, because  $q$  gets very small for big  $\tau$  regardless of  $\theta$ . As the figure makes clear, the costs basically converge when  $\tau$  reaches 150 percent since decentralized trade has all but shut down by this point. Hence, the difference between models with  $\theta = 1$  and  $\theta < 1$  is especially relevant for small to moderate inflation rates.

Finally, we want to contrast our method with the traditional way of measuring the cost of inflation, which is to compute the area under the “money demand” curve (see the discussion and references in Lucas [2000]). Our results show that this procedure does not work in general. If we start with a value for  $\theta$  and fit parameters to “money demand” and then change  $\theta$  and refit the parameters, we match the data equally well but get very different values for the welfare cost. Knowing the empirical “money demand” curve is not enough: one really needs to understand the micro foundations, and especially how the terms of trade are determined, in order to correctly estimate the welfare cost of inflation.

## VI. Conclusion

We have presented a new framework for monetary economics, explicitly based on the frictions used in search theory, but without the restrictions

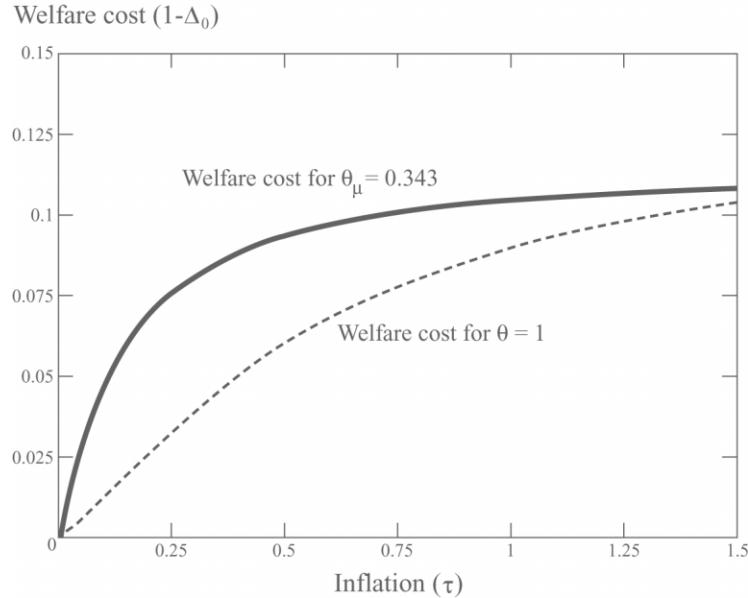


FIG. 3.—Welfare cost of higher inflation

on money holdings usually made in those models. A key innovation is to allow agents to interact periodically in centralized as well as decentralized markets. Given that agents have quasi-linear preferences, the distribution of money is degenerate in equilibrium, and this keeps the model tractable. We characterized equilibria and discussed some policy issues. The Friedman rule is optimal but does not achieve the first-best for  $\theta < 1$ . We found that this has sizable implications for the cost of inflation: going from 10 percent to 0 percent inflation here is worth between 3 percent and 5 percent of consumption—much larger than most previous estimates. This indicates that building monetary theories with explicit foundations matters for quantitative analysis. We think that all of this constitutes progress in terms of bringing micro and macro models of money closer together.

We also think that we have only scratched the surface, and much more can be done. In Lagos and Wright (2004), we report several extensions. For example, we add real shocks, either match-specific or aggregate, and either independently and identically distributed or persistent. Although in that model the constraint  $d \leq m$  may not bind with probability one, we show that  $F$  is still degenerate. We also discuss the effects of uncertainty in  $M$ . One experiment is to keep total  $M$  constant and randomly transfer  $m$  across agents. Again  $d \leq m$  may not bind with probability one, and we show that a mean-preserving spread in  $m$  always

reduces welfare, even though it may increase  $\phi$  and  $q$  if  $u'' \geq 0$  because of a “precautionary demand” effect. We also consider transfers  $\tau$  that are the same for all agents but are random over time. This version delivers natural results and remains fairly tractable: we show that if  $\tau$  is independently and identically distributed, then  $\phi$  and  $q$  are constant; and if  $\tau$  is persistent,  $\phi$  and  $q$  are smaller in periods of high  $\tau$  because this implies forecasts of higher future inflation.

Other extensions include the paper by Aruoba and Wright (2003), who add neoclassical firms and capital—that is, they make general goods storable—and integrate the framework with a standard real business cycle model. However, they assume that capital is not used in the decentralized market, which implies some very special results; Waller (2003) and Aruoba, Waller, and Wright (2004) generalize this. Lagos and Rocheteau (2004) make the general good storable and allow it to compete with money as a medium of exchange. Rocheteau and Wright (2005) add heterogeneity and free entry, which allows one to analyze effects on the extensive margin (number of trades), in addition to the intensive margin (quantity per trade). Lagos and Rocheteau (2005) endogenize search intensity and study the effects of inflation on velocity, output, and welfare. Some of these papers also consider alternative pricing mechanisms, such as price taking or posting, instead of bargaining. See also Ennis (2004), Faig and Huangfu (2004), and Rocheteau and Waller (2004). Ennis (2004) and Rocheteau and Wright (forthcoming) redo the quantitative experiments in this paper under some of these alternative pricing mechanisms.

Faig (2004) asks when credit and insurance markets can replace quasi-linear utility. Williamson (forthcoming) studies policy in a version with seasonal and other fluctuations in the demand for liquidity. Lagos (2005) extends the basic environment to study liquidity and asset prices. Bhattacharya, Haslag, and Martin (2005) study policy with heterogeneous agents. Reed and Waller (2004) discuss risk sharing. Berentsen, Camera, and Waller (2004) and He, Huang, and Wright (2005) introduce roles for banks. Rocheteau and Craig (2004) consider “sticky” prices. Berentsen et al. (2005) assume that agents may be in the decentralized market for more than one round of trade, which makes the distribution no longer degenerate but still tractable. Kahn, Thomas, and Wright (2004) assume that utility is not quasi-linear; this model can be solved numerically to show that when wealth effects are not too big, the results are close to those derived here. While not an exhaustive list, this gives a sense of a few of the applications and extensions that are possible.

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