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The Optimal Inflation Target in an Economy with Limited Enforcement

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Abstract

We formulate the central bank's problem of selecting an optimal long-run inflation rate as the choice of a distorting tax by a planner who wishes to maximize discounted utility for a heterogeneous population of infinitely-lived households in an economy with constant aggregate income. Households are divided into cash agents, who store value in currency alone, and credit agents who have access to both currency and loans. The planner's problem is equivalent to choosing inflation and nominal rates consistent with a resource constraint along with an incentive constraint that ensures credit agents prefer the superior consumption-smoothing power of loans to that of currency. We show that the optimum rate of inflation is positive, and the optimum nominal interest rate is higher than the inflation rate, if the social welfare function weighs credit agents no more than their population fraction.

Keywords: Deflation, debt constraints, limited participation, monetary policy, Friedman rule.

JEL codes: E31, E42, E58.

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1 Introduction

1.1 Overview

Central bankers have comfort zones for long-run inflation and nominal interest rates which deviate substantially from the prescriptions of economic theory. For example, Federal Reserve Chairman Bernanke has stated a preference for core inflation in the one-to-two percent annual range, in general agreement with the more explicit inflation targets of the European Central Bank, the Bank of England, the Reserve Bank of New Zealand and other institutions. This target is typically achieved with a nominal interest rate near five percent.

Economic theory, on the other hand, calls for an inflation target that is consistent with the Friedman rule of a zero nominal interest rate. That inflation target is minus the growth rate of real income in life cycle economies (Freeman (1993), Abel (1987)), or minus the sum of the rate of time preference plus an adjustment for income growth in representative household economies (Friedman (1969), Foley and Sidrauski (1969), Woodford (1990)).

Why do central banks prefer low inflation rates to outright deflation? One argument is that deflation subsidizes the holding of money at the expense of deposits and loans, causing disintermediation and a weakening of financial markets, as in Smith (2002). Another argument concerns the impact of the zero bound on nominal interest rates in environments where the central bank is committed to lower interest rates when economic activity weakens, as suggested by Summers (1991); for an analysis see Eggertsson and Woodford (2003).

In this paper we take the disintermediation argument seriously and use it in reverse: if a small deflation hurts asset markets, then a small inflation may help them. We explore an economy in which moderate inflation deepens financial markets and improves the ability of asset-trading households to smooth consumption. At the same time, inflation imposes a distortionary tax on money-trading households which works in the opposite direction. The central bank must choose the inflation rate to balance improvements in financial markets with deadweight losses from inflation.

1.2 What we do

We analyze an endowment economy with constant aggregate income, populated by a continuum of infinitely-lived households whose income shares fluctuate over time. There are two asset markets, for currency and consumption loans. Households are exogenously divided into two groups, called *cash* agents and *credit* agents. Members of the first group are anonymous and can store value only in currency of which they hold nonnegative amounts. Members of the second group can participate in either market subject to endogenous participation or debt constraints that successfully deter default: This group may hold assets in positive or negative amounts. Default is punished with perpetual exclusion from the loan market but still permits households to take long positions in currency.

In this environment, deflation raises the payoff from using money and makes default more attractive for borrowers. This, in turn, tightens the participation constraint (lowers debt limits) and weakens the loan market. Conversely, inflation raises debt limits and deepens the loan market up to the point where constraints cease to bind.

1.3 Main results

We formulate the central bank's problem of selecting an optimal long-run inflation rate as the choice of a distorting tax by a benevolent central planner who wishes to maximize a convex combination of discounted utilities for cash and credit agents, subject to a participation constraint that keeps credit agents from renouncing the loan market and switching to currency.

Deflation, as required by the Friedman rule, turns out to be an infeasible choice for any planner who assigns positive weight to credit households. Deflation subsidizes currency at the expense of consumption loans, and increases the payoff from cash-holding above the payoff to loan-trading, leading credit agents to default on their loans and forcing the credit market to shut down.

At the other end of possible inflation targets, an inflation rate higher than the minimum required to slacken debt constraints is equivalent to a distortionary income transfer from lower-welfare cash agents to higher-welfare credit agents. Planners who do not assign extraordinary weight to credit agents will reject inflation rates above the value needed to slacken debt constraints on credit agents.

If the relative weight of credit households in the social welfare function is above zero and less than or equal to their population weight, we show that the optimum rate of inflation is positive and the associated optimum nominal interest rate is larger than the inflation rate. We interpret these findings to be consistent with the comfort zones articulated by some of the world's leading central banks.

1.4 Recent related literature

Several recent papers in the monetary theory literature have themes related to the ones in this paper. Aiyagari and Williamson (2000), for instance, study an environment in which endowments are random and the outside option for defaulters is to use currency as the sole asset. These authors emphasize the role of financial intermediaries, which is not part of the framework in the present paper, and, unlike here, most of the analysis is computational. However, like us, Aiyagari and Williamson (2000) emphasize that an increase in inflation increases the penalty for default.

Berentsen, Camera, and Waller (forthcoming) analyze an alternating market model of money following Lagos and Wright (2005). Their analysis focuses on how credit can co-exist with money in an environment where money is essential for exchange, and on the role of financial intermediation in such an environment. They study cases in which financial intermediaries cannot force repayment and, instead, can only refuse future credit to defaulters. In this case, defaulting agents can only use money to facilitate exchange, and an increase in inflation increases the penalty for default.

Ragot (2006) studies a two-period life cycle economy with themes related to this paper. Money demand is introduced via a money-in-the-utility function specification, and credit constraints are introduced via an application of Holmstrom and Tirole (1997). In this economy, an increase in inflation relaxes credit constraints, and may lead the policy authority to select a positive inflation target.

Deviatov and Wallace (2007) study, computationally, an environment with features similar to the ones emphasized in the present paper. In particular, an exogenous fraction of agents are monitored and hence have known histories, while the remainder are anonymous; and in addition aggregate productivity has periodicity two, resembling the alternating endowment process in the present paper. Defaulters in credit arrangements become anonymous agents. The optimal monetary policy is relatively complicated and takes incentive constraints into account as in the present paper, but the analysis does not emphasize implications for the optimal inflation target as the present paper does.

The idea that an increase in inflation may deter activity in certain sectors of the economy, and through this effect produce desirable consequences in the economy as a whole, is a theme that has been analyzed from alternative points of view. One recent example is Huang, He, and Wright (2006). They study banking in an environment where money is essential for exchange, and in addition they include the possibility of theft, so that banks have an additional safekeeping role. An increase in inflation then taxes thieves and can be desirable.

Antinolfi, Azariadis, and Bullard (2006) is a precursor to the present paper. While the model structure is similar, it is simpler and the emphasis is on equilibrium selection and dynamics.

2 Environment

We describe the optimal rate of long-run inflation and analyze the associated optimal consumption plans in an endowment economy populated by four types of infinitely-lived household types, indexed by $i = 0, 1, 2, 3$. Household types 0 and 1 have mass $\lambda/2$ each, and households 2 and 3 have mass $1 - \lambda/2$ each, where $0 \leq \lambda \leq 1$. Individual income shares fluctuate deterministically and total income is constant. Time is discrete and is denoted by $t = 0, 1, 2, ...$ Agents have identical preferences given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right) \tag{1}$$

with $0 < \beta < 1$. Individual endowments and income shares are periodic,[1] given by,

$$\left(\omega_t^0, \omega_t^1\right) = \left(\omega_t^2, \omega_t^3\right) = \begin{cases} (1+\alpha, 1-\alpha) & \text{if } t = 0, 2, ... \\ (1-\alpha, 1+\alpha) & \text{if } t = 1, 3, ... \end{cases} \tag{2}$$

with $\alpha \in (0, 1)$. This endowment pattern means that type 0 and type 1 agents have negatively correlated income shares, as do agents 2 and 3. We introduce a critical difference between these two agent-pairs: We call agents 0 and 1 "Kehoe-Levine-type" or *credit* agents, and agents 2 and 3 "Bewley-type" or *cash* agents. Cash agents are anonymous households who may only use currency to smooth income fluctuations, as in Bewley (1980). No claims can be enforced on them or by them. Credit agents may enter into loan arrangements to smooth consumption subject to endogenous debt limits that give them proper incentives to repay, as in Kehoe and Levine (1993).

Incentives to repay loans are strongest, and debt limits are highest, when the payoff to default is lowest. We assume that credit agents who default are forever excluded from the loan market and must instead use money as a store of value. Clearly, the payoff to default at any point in time depends on future inflation rates.

The government acts as a benevolent central planner who chooses a constant inflation rate at which cash agents can trade currency across pe-

[1] In a growing economy, individual incomes need not be negatively correlated but income *shares* must be. This simple deterministic endowment process is the degenerate case of a stochastic economy with two Markovian states and a zero probability of remaining in the same state. Markovian endowments with two states are a straightforward extension. The assumption of two states or dates has obvious geometric advantages, but it is not innocuous where policy is concerned. We discuss this point further in the conclusion.

riods, and directly selects consumption vectors for credit agents who may either accept their allocations or behave like cash agents in perpetuity. The inflation target in this economy is similar to an optimal tax subject to an incentive constraint as understood by Mirrlees (1971). Positive rates of inflation impose a tax on cash agents and confer two benefits on credit agents: a transfer of resources from the cash sector as well as a reduction in the default payoff which brings about higher debt limits. Inflation, up to a point, deepens the credit market.

3 Inflation targeting as a planning problem

3.1 Overview

We now discuss a *modified* equal-treatment planning problem. A *pure* equal-treatment plan would amount to a choice of a periodic consumption sequence for each agent, which would treat similar households in similar fashion. In particular, a pure plan would select a consumption vector (c_H, c_L, x_H, x_L) for all households such that

$$\left(c_t^0, c_t^1, c_t^2, c_t^3\right) = \begin{cases} (c_H, c_L, x_H, x_L) & \text{if} \quad t = 0, 2, ... \\ (c_L, c_H, x_L, x_H) & \text{if} \quad t = 1, 3, ... \end{cases} \tag{3}$$

This plan assigns consumption (c_H, c_L) to high income and low income credit agents, and (x_H, x_L) to the corresponding cash agents. It is equivalent to lump-sum taxes on some agents and lump-sum subsidies to others. Because inflation is a distortionary tax, we will also define a slightly different or *modified* planning problem in three steps:

- The monetary authority sets a constant inflation factor π.
- Given π, high income cash agents choose a periodic consumption vector $(x_H, x_L) \geq 0$ to maximize stationary discounted utility

$$\frac{1}{1-\beta^2} \left[u\left(x_H\right) + \beta u\left(x_L\right)\right] \tag{4}$$

subject to

$$x_H \leq 1 + \alpha, \tag{5}$$

$$x_H + \pi x_L = 1 + \alpha + \pi (1 - \alpha), \tag{6}$$

and

$$u (x_H) + \beta u (x_L) \geq u (1 + \alpha) + \beta u (1 - \alpha). \tag{7}$$

The first inequality restricts excess demand for goods by high income cash agents to be nonpositive. This is equivalent to nonnegative demand for money balances. The second relation is a two-period budget constraint which assumes that money balances are completely used up to smooth consumption in low income dates. The third inequality allows households who dislike the announced inflation rate to renounce forever the use of money and consume their endowments in perpetuity.

- Let $x_H (\pi)$ and $x_L (\pi)$ solve the previous problem. Given π, the planner now chooses consumption values $(c_H, c_L) \geq 0$ for credit households to maximize the equal-treatment welfare function

  $$\frac{1}{1 - \beta^2} [u (c_H) + u (c_L)] \tag{8}$$

  of the credit community, subject to the resource constraint

  $$\lambda (c_H + c_L) + (1 - \lambda) [x_H (\pi) + x_L (\pi)] = 2, \tag{9}$$

  and the participation constraint

  $$u (c_H) + \beta u (c_L) \geq u [x_H (\pi)] + \beta u [x_L (\pi)]. \tag{10}$$

  Equal treatment of high income and low income households means that the discounted utilities are weighted equally. High income households are given the infinite periodic consumption vector $(c_H, c_L, ...)$ with payoff

  $$\frac{u (c_H) + \beta u (c_L)}{1 - \beta^2}.$$

Low income households consume the periodic vector $(c_L, c_H, ...)$ with discounted value

$$\frac{u(c_L) + \beta u(c_H)}{1-\beta^2}.$$

The welfare function in equation (8) is a linear combination of these two discounted utilities with each group's weight equal to $1/(1+\beta)$. In addition, note that the resource constraint equates aggregate consumption with aggregate income; the participation constraint ensures that high income credit agents prefer credit to money even if the planner forces them to consume less than their current income.

- If $c_H(\pi)$ and $c_L(\pi)$ solve the previous problem for a given $\pi > 0$, the planner selects the stationary inflation factor π to maximize the social welfare function

$$\mathcal{W}(\pi, \delta) = \delta \{u[c_H(\pi)] + u[c_L(\pi)]\} \\ + (1-\delta)\{u[x_H(\pi)] + u[x_L(\pi)]\}.$$

  This social welfare function assigns equal weights to members of the same group but potentially different weights to different groups. In particular, it weighs each credit community member by $\delta/(1+\beta)$, where $\delta \in (0,1)$, and cash community member by $(1-\delta)/(1+\beta)$. A strictly utilitarian welfare function would have equal weights for all, that is, $\delta = \lambda$.

3.2 Optimum inflation without incentive constraints

To build up intuition, we solve the planner's problem outlined in section 3.1, ignoring for the moment the incentive constraints laid out in equations (7) and (10). As a first step we allow lump-sum income transfers from cash agents to credit agents which permits us to also ignore the cash agents' budget constraints (5) and (6). All the planner has to do is maximize the social welfare function

$$\mathcal{W}(\pi, \delta) = \delta [u(c_H) + u(c_L)] + (1-\delta)[u(x_H) + u(x_L)] \tag{11}$$

subject to the economy's resource constraint

$$\lambda \left(c_H + c_L \right) + \left(1 - \lambda \right) \left(x_H + x_L \right) = 2. \tag{12}$$

The obvious solution is $\left(c_H, c_L, x_H, x_L \right) = \left(c^*, c^*, x^*, x^* \right)$ where c^* and x^* solve the following pair of equations:

$$\delta u' \left(c \right) = \left(1 - \delta \right) u' \left(x \right)$$

$$\lambda c + \left(1 - \lambda \right) x = 1.$$

We call this solution the *first best*. The implied optimal inflation and nominal interest rates can be inferred from the consumption Euler equation for the two household types, that is, from

$$\frac{\beta R^N}{\pi} = 1 \tag{13}$$

$$\frac{\beta}{\pi} = 1. \tag{14}$$

The first-best allocation is thus supported by Friedman's rule, $\left(\pi, R^N \right) = \left(\beta, 1 \right)$.

Suppose next that the planner cannot impose a lump-sum tax on any agent but must instead use inflation or deflation and redistribute the resulting seigniorage from one group to another. Inflation is a proportional tax on the excess supply of goods by high income cash agents; it transfers resources from cash to credit households. Deflation does the exact opposite. The planner must now choose $\left(\pi, c_H, c_L \right)$ to solve the problem outlined in section 3.1 subject to all constraints except (7) and (10). We call this outcome the *second best*.

To understand the optimum rate of inflation at the second best allocation, we examine the two polar cases $\delta = 1$ and $\delta = 0$. The first case, which assigns no welfare weight to the cash-using community, leads the planner to select that value of π which minimizes the consumption of that community. The maximum possible amount of seigniorage is transferred to the credit community, and the consumption of credit agents is smoothed completely.

Define the *maximal seigniorage inflation factor* from

$$\tilde{\pi} = \arg\min_{\pi \geq 1} \left[x_H(\pi) + x_L(\pi) \right] > 1. \tag{15}$$

Then the planner sets $(\pi, c_H, c_L) = (\tilde{\pi}, \tilde{c}, \tilde{c})$ where $\tilde{c}$ can be read from the resource constraint

$$2\lambda\tilde{c} + (1-\lambda)\left[x_H(\tilde{\pi}) + x_L(\tilde{\pi}) \right] = 2. \tag{16}$$

In addition, $c_H = c_L$ implies $\beta R^N = \tilde{\pi}$. The second best allocation turns out to be supported by high rates of inflation and nominal interest, that is

$$\left(\pi, R^N\right) = \left(\tilde{\pi}, \tilde{\pi}/\beta\right). \tag{17}$$

At the other extreme, $\delta = 0$ describes a society in which the planner cares about the cash community only. This planner will deflate the economy in order to reduce the aggregate consumption of credit households, pushing the inflation factor as close to zero as possible. That is obvious from Figure 1 below, which superimposes the budget constraint of the high income cash household against social indifference curves that turn out to be symmetric about the diagonal.

A utilitarian social welfare function with $\delta = \lambda$ represents a sensible compromise between the extremes just described. A planner endowed with a utilitarian social welfare function will choose a *modified Friedman rule* that combines mild deflation with a small positive interest rate to guarantee smooth consumption for credit agents. The following result is proved in the Appendix.

Theorem 1 *The second best optimum allocation under a utilitarian social welfare function satisfies* $(c_H, c_L, x_H, x_L) = (c^{**}, c^{**}, x_H(\pi^{**}), x_L(\pi^{**}))$. *It is supported by a modified Friedman rule for some inflation factor* $\pi^{**} \in (\beta, 1)$, *and a nominal yield such that* $R^N = \pi^{**}/\beta > 1$.

Proof. See Appendix. ■

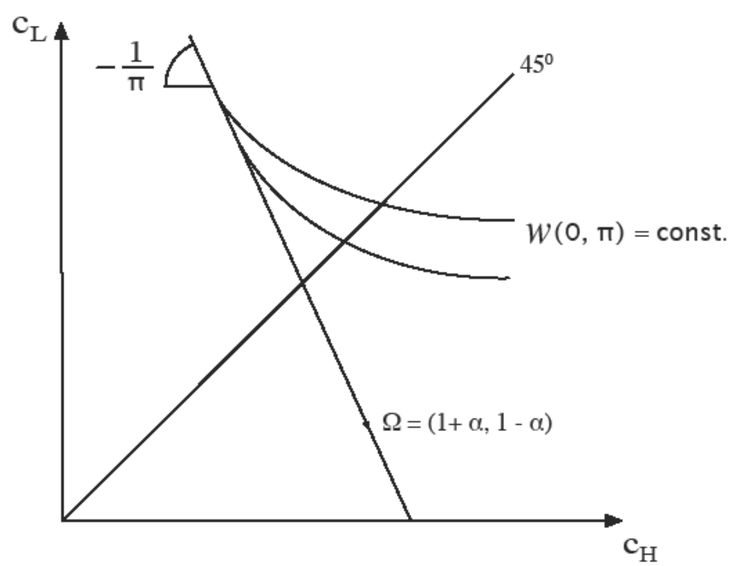

Figure 1: Social indifference curves for $\delta = 0$.

4 The role of incentive constraints

4.1 Basic assumptions

We suppose in what follows that the incentive constraints are restrictive enough to rule out both the first-best and the second-best allocations described in the previous section, and defeat the planner's desire to smooth completely the consumption profile for either the credit or the cash community. Define $y(\pi)$ to be the combined consumption of a pair of high income and low income credit agents. This consumption is maximal when the inflation factor π is equal to the maximal seigniorage inflation factor $\tilde{\pi}$. From the resource constraint we obtain

$$c_H + c_L = y(\pi) \equiv \frac{1}{\lambda}\left[2 - (1-\lambda)\left(x_H(\pi) + x_L(\pi)\right)\right].$$

Our key assumptions are these:

A1. $\bar{R} \equiv \frac{u'(1+\alpha)}{\beta u'(1-\alpha)} < 1$,

A2. $u(1+\alpha) + \beta u(1-\alpha) > (1+\beta)\, u(1)$, and

A3. $(1+\beta)\, u\left[\frac{y(\tilde{\pi})}{2}\right] > u\left[x_H(\tilde{\pi})\right] + \beta u\left[x_L(\tilde{\pi})\right]$.

Assumptions **A1** and **A2** state that individual income shares are neither very stable nor highly variable. In particular, **A1** asserts that autarky is an allocation with a low implied rate of interest $\bar{R}$ and therefore cannot be a constrained efficient allocation for the credit community.[2] Geometrically, we require the initial endowment point $\Omega = (1+\alpha, 1-\alpha)$ in Figure 2 to lie below the tangency point G on the budget line $c_H + c_L = 2$. This assertion is innocuous. It means that the income variability parameter α is large relative to the consumer's rate of time preference if α is the same for all households. If, however, α should vary across households, then autarky is a low interest rate equilibrium whenever the rate of time preference is small relative to the largest α in the population. Roughly speaking, **A1** amounts to asserting

[2] On this point, see Alvarez and Jermann (2000).

that there is at least one household in the economy whose income share fluctuates more than three or four percent per year.

The next assumption, **A2**, rules out plans that combine perfect consumption smoothing for credit agents with a zero rate of inflation, which would decentralize the golden rule allocation for cash agents. Zero inflation means no transfers of income between groups. Perfect consumption smoothing for credit agents is achieved by the allocation $c_H = c_L = 1$ whose payoff is below autarky by assumption **A2**. In Figure 2, the flat-consumption allocation point E lies below the indifference curve that goes through the initial endowment point Ω.

This assumption, too, is innocuous: It holds automatically for values of α near zero. If α were to vary across households, assumptions **A1** and **A2** would assert that income shares are nearly constant for some agents and quite variable for others. But, since we have only one endowment profile in the entire economy, we need to assume that income shares are neither too smooth nor too variable. That is what is embodied in assumptions **A1** and **A2**.

The last assumption is a bit more controversial. It claims that credit agents can achieve perfectly smooth consumption albeit at higher rates of inflation. **A3** asserts that it is within the power of the central planner, and of the central bank, to lower the rate of return facing users of cash to the point where the incentive constraint on credit users becomes slack. **A3** states that allocations with perfectly smooth consumption, $c_H = c_L = y(\pi)/2$, are feasible at the maximum seigniorage rate of inflation and also at lower rates. For all of these inflation rates the payoff from credit use exceeds the payoff from cash use. Figure 3 illustrates.

Let

$$v(\pi) \equiv u[x_H(\pi)] + \beta u[x_L(\pi)]$$

denote the two period payoff to any high income household using money. Then for any isoelastic utility function $u : \mathbb{R}_+ \to \mathbb{R}$ for which c_H and c_L are gross substitutes, the seigniorage function $y(\pi)$ is continuous, positive,

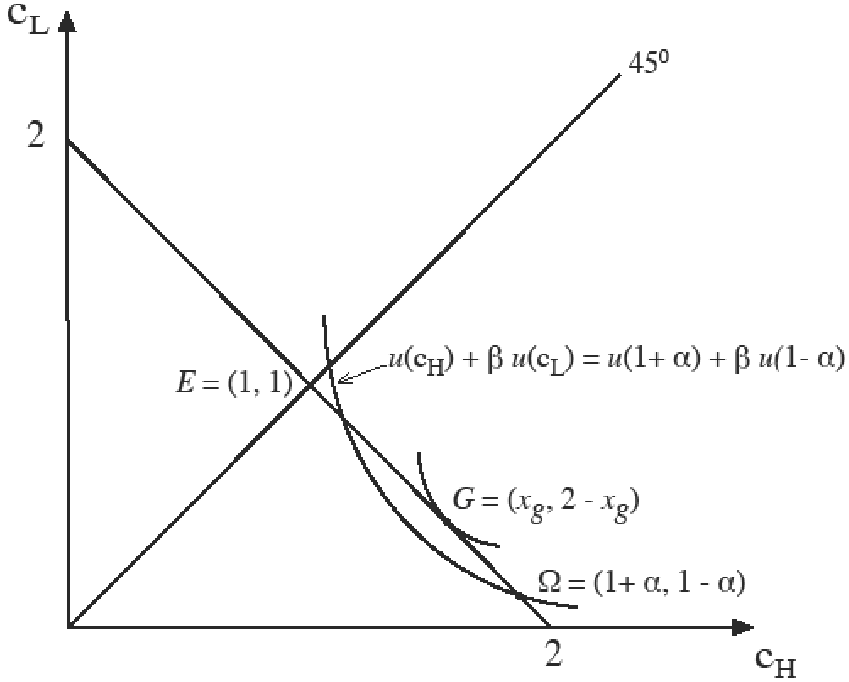

Figure 2: Assumptions $A1$ and $A2$.

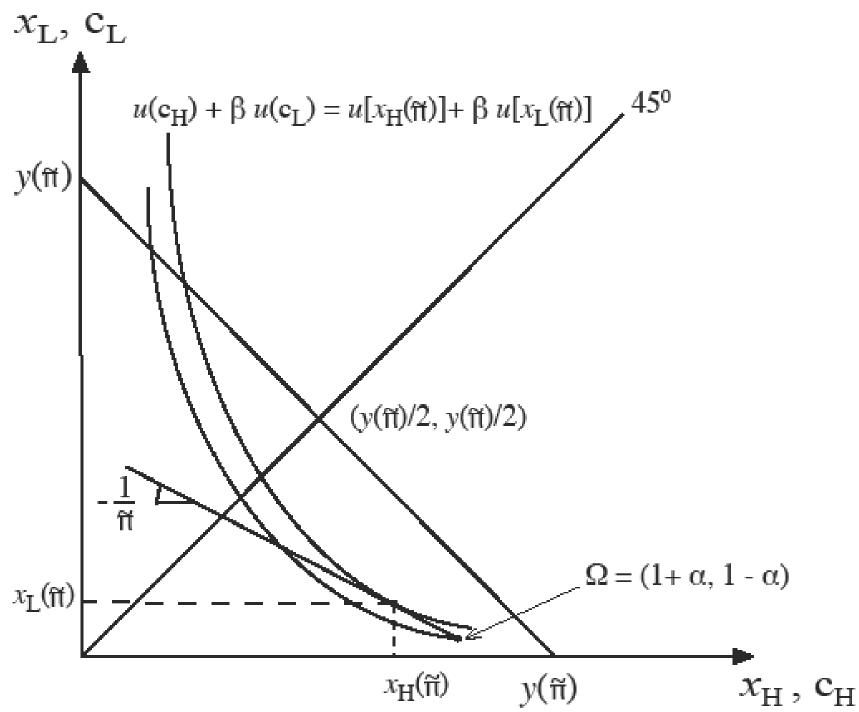

Figure 3: Assumption *A*3.

and increasing in π for all $\pi \in (1, \tilde{\pi})$; positive and decreasing in π for all $\pi \in (\tilde{\pi}, 1/\bar{R})$; and zero at $\pi = 1$ and $\pi = 1/\bar{R}$. The demand for money by cash agents vanishes at $\pi = 1/\bar{R}$ as households switch to autarky.

Assumption **A3**, together with the continuity of the function $y(\pi)$, guarantees the existence of an inflation factor $\bar{\pi}$ in the open interval $(1, \tilde{\pi})$ for which

$$(1 + \beta)\, u\, [y(\bar{\pi})\, /2] = v(\bar{\pi})\,. \tag{18}$$

High income credit households are indifferent between cash and credit at $\pi = \bar{\pi}$, and the participation constraint (10) becomes slack when inflation reaches that value. In a decentralized economy, debt constraints will cease to bind, and the loan market will smooth consumption perfectly, when inflation is in the closed interval $[\bar{\pi}, \tilde{\pi}]$.

Figure 4 illustrates the relationship between credit rationing and inflation by graphing the payoffs to credit and money users when the credit community enjoys constant consumption. These payoffs are exactly equal at $\pi = \bar{\pi}$ and again at some higher $\pi_m \in (\tilde{\pi}, 1/\bar{R})$. Discounted utility $v(\pi)$ from the use of money is a monotonically decreasing function of the inflation tax π for any π less than $1/\bar{R}$. When π reaches or exceeds $1/\bar{R}$, the rate of return on money falls below the implied yield on autarky, and the demand for money vanishes altogether.

Constant consumption for credit households rises as the inflation factor increases from 1 to $\tilde{\pi}$, then falls as π increases further from $\tilde{\pi}$ to $1/\bar{R}$. Seigniorage dries up at that point, and $c_H = c_L = 1$ for all $\pi \geq 1/\bar{R}$.

4.2 Inflation and social welfare

We are now ready to deal with the complete planning problem described in Section 3.1. Our strategy is to show that the social welfare function $\mathcal{W}(\pi, \delta)$:

- Is continuously differentiable for all $\pi \geq 1$;
- Is undefined for $\pi < 1$ because deflation contradicts the participation constraint (10);

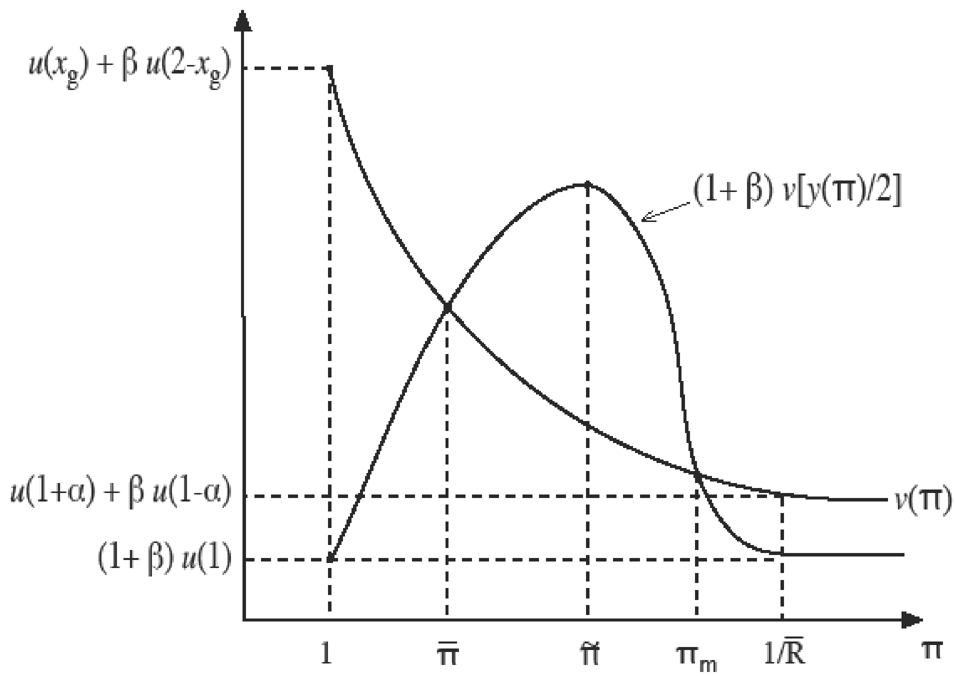

Figure 4: Inflation and credit rationing.

- Increases rapidly in π at $\pi = 1$;
- Decreases in π for all $\pi \in [\bar{\pi}, 1/\bar{R}]$ if $\delta \leq \lambda$;
- Is constant for π larger than $1/\bar{R}$.

These properties guarantee the existence of an optimum inflation factor

$$\pi^{\star}(\delta) = \arg \max_{\pi \in [1, 1/\bar{R}]} \mathcal{W}(\pi, \delta) \geq 1, \ > 1 \text{ if } \delta > 0.$$

The appendix contains a proof of the following result.

Lemma 2 *Define $\mathcal{W}_{\pi}(\pi, \delta) = \partial \mathcal{W} / \partial \pi$. Then (a) $\mathcal{W}_{\pi}(\pi, \delta) < 0 \ \forall (\pi, \delta) \in [\bar{\pi}, \tilde{\pi}] \times [0, \lambda]$, and (b) $lim_{\pi \to 1} \mathcal{W}_{\pi}(\pi, \delta) = +\infty$ when π converges from above.*

The intuition for part (*a*) is fairly simple. For any $\pi > \bar{\pi}$, assumption **A3** says that smoothing the consumption of credit households is consistent with the participation constraint. To raise π above $\bar{\pi}$ does not improve the ability of the planner to smooth the consumption of the credit community any further. Doing so merely transfers income from the cash community, who are consuming less than two units of total income, to the credit community who are consuming more. This transfer will reduce social welfare except in cases where the favored credit households are extraordinarily important to the central planner, that is, when $\delta > \lambda$.

Part (*b*) can be understood in a similar way. At very small positive rates of inflation, the aggregate consumption of each community is proportional to its population weight and, by assumption **A2**, c_H is substantially different from c_L. A tiny increase in the inflation tax transfers a tiny amount of resources between two groups with roughly the same marginal utility of income. This insignificant transfer would have essentially no impact on the social welfare function except that it lowers the discounted utility of money for the credit community, allowing the central planner to substantially smooth the consumption vector (c_H, c_L).

Next we prove, again in the appendix, Lemma 3.

Lemma 3 *$\mathcal{W}(\pi, \delta)$ is not defined for $\pi < 1$. It is decreasing in π for $\pi \in (\tilde{\pi}, 1/\bar{R})$ and constant for $\pi \geq 1/\bar{R}$.*

The key part of Lemma 3 is understanding why the reduced-form social welfare function $\mathcal{W}(\pi, \delta)$, defined at the end of Section 3.1, does not exist for $\pi < 1$ or, equivalently, why deflation violates the participation constraint for high income credit households. Deflation means that each high income cash household will consume a vector (x_H, x_L) such that $x_H + x_L > 2$, attaining a point above the budget line $x_H + x_L = 2$. The corresponding high income credit household will consume (c_H, c_L) such that $c_H + c_L < 2$, reaching a point below the previous budget line. The outcome of any deflation is that money has a higher payoff than credit.

The main result of this section, which follows directly Lemma 2 and Lemma 3, is stated below.

Theorem 4 *Suppose assumptions* **A1**, **A2**, *and* **A3** *hold, and $0 < \delta \leq \lambda$. Then the optimum inflation factor is $\pi^\star(\delta) > 1$ and the associated nominal interest yield, $R^N \in (\pi^\star(\delta), \pi^\star(\delta)/\beta)$, is even higher.*

Figure 5 uses Lemmas 2 and 3 to illustrate the planner's SWF for some welfare weight $\delta \in (0, \lambda)$ and any inflation factor $\pi \geq 1$. Assumption **A3** generates large improvements in the planner's consumption smoothing power from relatively small inflation rates. As inflation goes up, these improvements taper off, and after the optimum value $\pi^\star(\delta)$, they are negated by the deadweight loss of the inflation tax.

5 Extensions and conclusions

What factors should a benevolent, independent central bank consider when it sets a long run inflation target? Summers (1991) has expressed the view that zero bound on nominal interest rates dictates an inflation target above zero. This paper suggests that a very different mechanism may be at work. In particular, Theorem 4 shows that, for an economy with constant aggregate income and no collateral, the inflation target should strike a balance

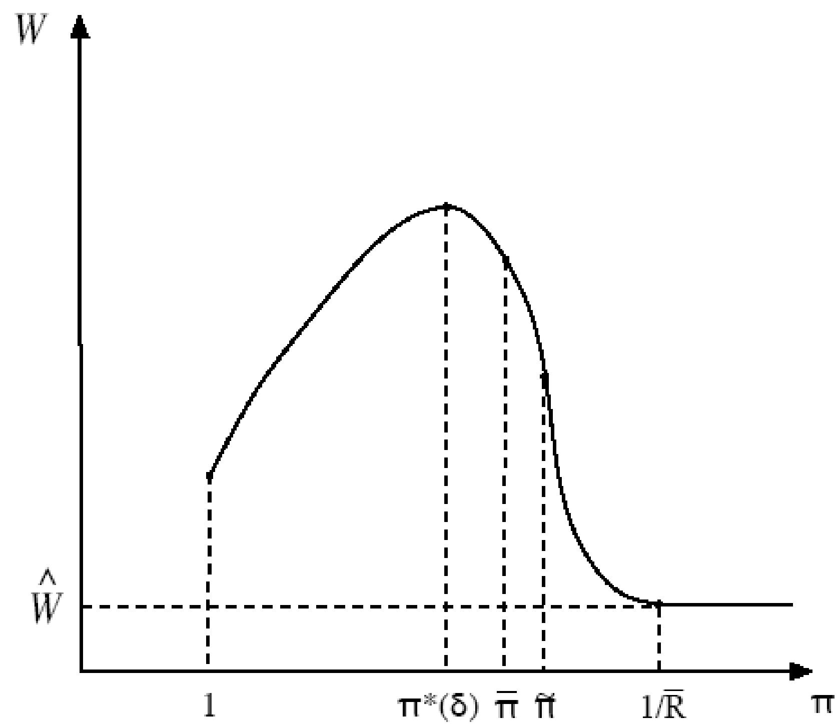

Figure 5: Inflation and social welfare.

between the deadweight loss from inflation and the potential improvement in credit market conditions.

How does economic growth affect inflation targets? Suppose, for example, that all the endowments described in equation (2) are multiplied by a growth factor $g \geq 1$, and that the utility function is isoelastic, that is,

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

where $\gamma \geq 0$, and $\beta_g \equiv \beta g^{1-\gamma} < 1$. In this growing economy the mathematical structure of the planning problem, defined in Section 3.1, remains the same if we replace the original discount factor β with a modified β_g and the original inflation factor π with the modified inflation factor $\pi_g \equiv g\pi$.

For any utility function with $\gamma \leq 1$ (which implies gross substitutability of intertemporal consumption goods), increases in g effectively raise the planner's patience and slacken the incentive constraints. We conjecture that this increase in effective patience will allow the planner to smooth consumption better at any given rate of inflation, and will lessen the need to subsidize the loan market at the expense of the currency market. The outcome should be a lower inflation target $\pi^\star$ for any given welfare weight δ. This conjecture is easily verified for the logarithmic utility function with $\gamma = 1$. In this case, the planner's effective discount rate remains at β and by Theorem 4 the optimum inflation rate should be $\pi_g = \pi^\star(\delta)$ or $\pi = \pi^\star(\delta)/g$. In other words, the sum of the inflation target plus the growth rate is a constant independent of the growth rate itself.

We also conjecture that collateral borrowing should have an effect on inflation targets similar to that of higher growth rates. Collateral improves the ability of credit agents to smooth consumption in a state of default by combining long positions in currency with short positions in collateralized loans. This will raise the payoff to default for cash agents and reduce the debt limits on non-collateral loans. Total borrowing, however, should improve as income becomes better collateral, and so will the planner's ability to smooth consumption without relying too much on the intermediating effect of higher inflation.

The main conclusion of this paper is that independent central banks will set low positive inflation targets in economies that possess highly developed financial markets. This finding seems to be broadly consistent with the comfort zones articulated by some of the world's leading central bankers. Less fortunate societies with relatively undeveloped asset markets will choose higher inflation targets to improve credit market performance. Slower growth tends to raise inflation targets, and the highest targets should be expected from stagnating economies with poorly developed financial institutions.

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A Proof of Theorem 1

The planner chooses (π, c_H, c_L) to maximize the utilitarian SWF

$$\mathcal{W}(\pi, c_H, c_L, \lambda) = \lambda \left[u(c_H) + u(c_L)\right] + (1-\lambda)\left[u(x_H(\pi)) + u(x_L(\pi))\right]$$

subject to the resource constraint (9) and the definitions of $x_H(\pi)$, $x_L(\pi)$ from equations (4), (5), and (6). The solution will clearly satisfy $c_H = c_L = c$. Using the resource constraint, we rewrite the SWF in the form

$$\mathcal{W}(\pi, \lambda) = 2\lambda u\left[\frac{2-(1-\lambda)(x_H + x_L)}{2\lambda}\right] + (1-\lambda)\left[u(x_H) + u(x_L)\right].$$

Denoting $\mathcal{W}_\pi = \partial\mathcal{W}/\partial\pi$, we differentiate the SWF with respect to π and obtain

$$\frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} = -u'(c)\left[x_H'(\pi) + x_L'(\pi)\right] + u'(x_H)\,x_H'(\pi) + u'(x_L)\,x_L'(\pi)$$

where $u'(x_H) = (\beta/\pi)\,u'(x_L)$ is the consumption Euler equation of the cash group. Therefore,

$$\frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} = -u'(c)\left(x_H' + x_L'\right) + u'(x_L)\left(\frac{\beta}{\pi}x_H' + x_L'\right). \tag{19}$$

Next we show that $\mathcal{W}$ is an increasing function of π at $\pi = \beta$, and a decreasing one for all $\pi \geq 1$. Since $\mathcal{W}$ is continuous in π, the intermediate value theorem implies that it attains a maximum in the interval $(\beta, 1)$. To check this, we note from (19) that

$$\frac{\mathcal{W}_\pi(\beta,\lambda)}{1-\lambda} = \left[x_H'(\beta) + x_L'(\beta)\right]\left[u'(x_L) - u'(c)\right]$$

where $x_H(\beta) = x_L(\beta) > 1 > c$ from the budget constraints, and $x_H'(\pi) + x_L'(\pi) < 0$ for all π, as shown by Figure 1. It follows that $\mathcal{W}$ is increasing in π at $\pi = \beta$.

Continuing along this line of argument, we observe that β/π is less than or equal to β for any $\pi \geq 1$, and $x_H'(\pi) > 0$ for all π if dated consumption goods are normal. Therefore, for any $\pi \geq 1$, we have

$$\frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} \leq -u'(c)\left(x_H' + x_L'\right) + u'(x_L)\left(\beta x_H' + x_L'\right). \tag{20}$$

Next, we differentiate the budget constraint in equation (6) and obtain

$$x_H' = 1 - \alpha - x_L - \pi x_L'. \tag{21}$$

Substituting (21) into (20) yields

$$\frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} \leq -u'(c)\left[1 - \alpha - x_L + (1-\pi)\,x_L'\right] + u'(x_L)\left[\beta\,(1 - \alpha - x_L) + (1 - \beta\pi)\,x_L'\right]. \tag{22}$$

Here, for any $\pi \geq 1$, the budget constraints and the consumption Euler equation for cash agents jointly imply $c > 1 > x_L$ and $1 - \alpha - x_L < 0$. Therefore, equation (22) leads to

$$\begin{aligned}\frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} &\leq [x_L-(1-\alpha)]\left[u'(c)-\beta u'(x_L)\right] \\ &\quad -x'_L(\pi)\left[(1-\pi)u'(c)-(1-\beta\pi)u'(x_L)\right] \\ &\leq [x_L-(1-\alpha)](1-\beta)u'(x_L) \\ &\quad -x'_L(\pi)u'(x_L)[1-\pi-1+\beta\pi]\end{aligned}$$

because $u'(c) < u'(x_L)$. Continuing,

$$\begin{aligned}\frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} &\leq (1-\beta)u'(x_L)\left[x_L-(1-\alpha)+\pi x'_L\right] \\ &= -(1-\beta)u'(x_L)x'_H(\pi)\end{aligned}$$

by equation (21). Since $x'_H(\pi)$ is positive for all π, $\mathcal{W}_\pi(\pi,\lambda) < 0$ for all $\pi \geq 1$. This completes the proof.

B Proof of Lemma 2

Part (*a*). The Lemma is trivial for large $\pi \in [\tilde{\pi}, 1/\bar{R}]$. We focus on $\pi \in [\bar{\pi}, \tilde{\pi}]$. Now note that the derivative

$$\mathcal{W}_\pi(\pi,\delta) = \delta\left[u'(c_H)c'_H + u'(c_L)c'_L\right] + \\ (1-\delta)\left[u'(x_H)x'_H + u'(x_L)x'_L\right]$$

can be written as

$$\mathcal{W}_\pi(\pi,\delta) = \delta u'[y(\pi)/2]\,y'(\pi) + (1-\delta)u'(x_L)\left[\frac{\beta}{\pi}x'_H + x'_L\right]$$

because $y(\pi)/2 = c_L = c_H$ and $u'(x_H) = (\beta/\pi)u'(x_L)$. Continuing, recall that $x'_H > 0$ by gross substitutes, $\beta/\pi < 1$ by assumption, and $x'_H + x'_L < 0$ because seigniorage is increasing in the interval $[1, \tilde{\pi}]$. Therefore,

$(\beta/\pi)\, x_H' < x_H'$ and

$$\begin{aligned}
\mathcal{W}_\pi(\pi,\delta) &< \delta u'(c_L)\, y'(\pi) + (1-\delta)\, u'(x_L)\left(x_H' + x_L'\right) \\
&= \delta u'(c_L)\left[-\frac{1-\lambda}{\lambda}\left(x_H' + x_L'\right)\right] + (1-\delta)\, u'(x_L)\left(x_H' + x_L'\right) \\
&= -\left(x_H' + x_L'\right)\left[\frac{\delta(1-\lambda)}{\lambda} u'(c_L) - (1-\delta)\, u'(x_L)\right] \\
&< -\left(x_H' + x_L'\right)\left[(1-\delta)\, u'(c_L) - (1-\delta)\, u'(x_L)\right]
\end{aligned}$$

since $\delta \leq \lambda$. Therefore,

$$\frac{-\mathcal{W}_\pi}{(x_H' + x_L')(1-\delta)} < u'(c_L) - u'(x_L)\,. \tag{23}$$

Note next that $\beta < \pi$ implies $x_L < x_H$, $c_L = c_H$ by assumption, and also $c_L + c_H > 2 > x_L + x_H$ for all $\pi \in (\bar{\pi}, \tilde{\pi})$. It follows that $c_L > x_L$ and therefore that the right hand side of inequality (23) is negative. From this and the fact that $x_H' + x_L' < 0$ we infer that $\mathcal{W}_\pi(\pi,\delta) < 0$ for all $\pi \in (\bar{\pi}, \tilde{\pi})$ and all $\delta \in (0, \lambda]$.

Part (*b*). Assumption **A3** asserts that the central planner cannot set $c_H = c_L$ for any $\pi \in (1, \bar{\pi})$ without violating the participation constraint (10). For any π in that interval, the planner will smooth consumption as much as the participation constraint allows, choosing $c_H(\pi)$ to be the smallest solution to the equation

$$\begin{aligned}
u(c_H) + \beta u\left[y(\pi) - c_H\right] &= v(\pi) \\
&\equiv u\left[x_H(\pi)\right] + \beta u\left[x_L(\pi)\right],
\end{aligned} \tag{24}$$

where $c_L(\pi) = y(\pi) - c_H(\pi)$. Differentiate (24) with respect to π and obtain

$$c_H'(\pi) = \frac{v'(\pi) - \beta u'(c_L)\, y'(\pi)}{u'(c_H) - \beta u'(c_L)}. \tag{25}$$

Note also that, at $\pi = 1$, we have

$$\begin{aligned}
c_H(1) &= x_H(1)\,, \\
c_L(1) &= x_L(1)\,, \\
c_H(1) + c_L(1) &= x_H(1) + x_L(1) = 2.
\end{aligned}$$

Next we compute

$$\mathcal{W}_{\pi}(1,\delta) = u'(x_H(1))\left[\delta c_H'(1) + (1-\delta)x_H'(1)\right] + u'(x_L(1))\left[\delta c_L'(1) + (1-\delta)x_L'(1)\right]$$

where

$$u'(x_H(1)) = \beta u'(x_L(1)).$$

Continuing we obtain

$$\begin{aligned}\frac{\mathcal{W}_{\pi}(1,\delta)}{u'(x_L(1))} &= \beta\left[\delta c_H'(1) + (1-\delta)x_H'(1)\right] + \\ &\quad \delta\left[y'(1) - c_H'(1)\right] + (1-\delta)x_L'(1) \\ &= Q + (\beta-1)\delta c_H'(1)\end{aligned}$$

where

$$Q \equiv \beta(1-\delta)x_H'(1) + \delta y'(1) + (1-\delta)x_L'(1).$$

Note now that

$$c_H'(1) = \lim_{\pi\searrow 1} c_H'(\pi) = -\infty$$

by equation (25) because

$$v'(1) - \beta u'(c_L(1))y'(1) < 0$$

as the sum of two negative terms, and

$$u'(c_H(1)) = \beta u'(c_L(1)).$$

Therefore $\lim_{\pi\searrow 1}\mathcal{W}_{\pi}(1,\delta) = +\infty$. This completes the proof.

C Proof of Lemma 3

The proof of this lemma is straightforward as shown in Figure 5. Note, however, that for $\pi > 1/\bar{R}$ the payoff to money is just autarky. Therefore, we have

$$\begin{aligned}\mathcal{W}(\pi,\delta) &= \delta\left[u(\hat{x}) + u(2-\hat{x})\right] + (1-\delta)\left[u(1+\alpha) + \beta u(1-\alpha)\right] \\ &\equiv \hat{W},\end{aligned}$$

where $\hat{x} \in (1, 1+\alpha)$ is the smallest solution to the equation

$$u\left(x\right) + \beta u\left(2 - x\right) = u\left(1+\alpha\right) + \beta u\left(1-\alpha\right).$$