

Leniency Programs and Quality of Information*

Mike Cloutier [†]

[**Job Market Paper**]

December 16, 2011

Abstract

In exchange for a lower fine, a leniency applicant shares its privileged information on the functioning of the cartel with the antitrust agency. This paper examines the gains obtainable should an antitrust agency condition the lenient fine on the quality of the evidence turned in. The analysis accounts for an important characteristic of leniency procedures in allowing firms to collect evidence of colluding practices (e.g., dates of meeting and pricing algorithm). Using repeated game theory, I show that the following three strategies are subgame perfect equilibria: firms choose (i) not to collude, (ii) to collude without the use of the leniency policy and (iii) to collude with the use of the leniency policy. My main result demonstrates the effect of the quality of the evidence on the sustainability of collusion: a more lenient antitrust agency induces firms to give away higher quality information, while at the same time creating a greater incentive for firms to use the leniency policy. I also solve for the optimal leniency policy set by the antitrust agency and show as a corollary that an antitrust agency conditioning the fine on the evidence welfare dominates the unconditional alternative.

JEL classification codes: K21; K42; L41; D83

Keywords: Collusion; Antitrust; Self-reporting; Amnesty; Information

*The financial support of Fonds de Recherche sur la Société et la Culture (FQRSC) is gratefully acknowledged. I wish to thank Jan Zabožnik for his time, patience and devotion to this project. I also thank Alexander Armstrong, Jean-Denis Garon, Joe Harrington, Susumu Imai, Jean-François Rouillard, Veikko Thiele, and Ruqu Wang for valuable comments.

[†]Author's affiliation: Ph.D. candidate at Queen's University, E-mail: ccloutier@econ.queensu.ca, Web-page: <http://qed.econ.queensu.ca/pub/students/phds/cloutier/>

1 Introduction

Leniency policies are effective tools for catching colluding firms. In exchange for a lower fine, a leniency applicant shares its privileged information on the functioning of the cartel¹ with the antitrust agency. The significant increase in the number of cases heard by competition tribunals and the concurrent increase in subsequent convictions spurred by leniency applicants giving away “smoking gun” evidence on the colluding practices point to the effectiveness of leniency policies (Hammond, 2009). Their recent success, starting with the 1993 revision of the Corporate Leniency Program in the United States, inspired a decade of research that still tries to understand the complex mechanisms behind the apparent simple idea of a leniency policy. Some key features of leniency policies remain without theoretical foundation.

This paper examines the gains obtainable should an antitrust agency condition the lenient fine to be paid on the evidence turned in by an applicant when granted leniency. I build on Motta and Polo (2003) and account for an important characteristic of leniency procedures in allowing firms to collect evidence of colluding practices. Using repeated game theory, I show that the following three strategies are subgame perfect equilibria: firms choose (i) not to collude, (ii) to collude without the use of the leniency policy and (iii) to collude with the use of the leniency policy. My main result demonstrates the effect of the quality of the evidence on the sustainability of collusion: a more lenient antitrust agency induces firms to give away higher quality information, while at the same time creating a greater incentive for firms to use the leniency policy. I also solve for the optimal leniency policy set by the antitrust agency and show that an antitrust agency conditioning the fine on the evidence welfare dominates the unconditional alternative.

This paper contributes to the economics literature on collusion, and more specifically to the leniency policy literature. I first contribute to these literatures in linking the quality of the evidence turned in with colluding firms’ behavior, and with welfare implications. Second, I relax current assumptions on strict leniency (100% fine discount) and on 100% rate of conviction by focusing on moderate leniency (less than 100% fine discount) and allowing for an uncertain conviction. Finally, I set the environment to correspond with the current European and Canadian setup, opposed to previous work (Feess and Walzl, 2010) focusing on the U.S. Before discussing in more depth the contribution, I introduce the relevant institutions to highlight the distinction with previous work.

¹For example: existence of a cartel, participation, dates, meetings and pricing algorithm

Divided into three sections, the Corporate Leniency Program lists conditions for granting leniency to the first firm reporting the illegal activity. Section A focuses on leniency before an investigation is undertaken, Section B is interested in alternative requirements for granting leniency when an investigation has already started and, finally, Section C discusses the fate of corporate directors, officers and employees when the firm has been granted leniency. This study focuses on leniency granted after an investigation has begun². During an undergoing investigation, leniency will be granted to the first applicant satisfying a set of predetermined criteria. Granting leniency to a corporation under Section B of the Corporate Leniency Program provides complete immunity from financial sanction and prosecution. This is also known as strict leniency.

Elsewhere in the world, antitrust agencies proceed somewhat differently. For example, the European Commission Leniency Policy³ stipulates that once an investigation has begun “companies ... may benefit from a reduction of fines if they provide evidence that represents significant added value”. This is also known as *moderate* or *partial leniency*. The main distinction between the E.U. and the U.S. comes from the plurality in the number of leniency beneficiaries and the conditionality on corporation “added value” to determine the extent to which leniency will be granted (expressed in percentage of the full fine otherwise charged). In Canada, the Competition Bureau - Leniency Program is comparable to the E.U. with regard to the treatment firms receive once an investigation has begun.⁴

The quality that the evidence (“added value”) turned in may possess is best illustrated by reviewing the famous 2001 Carbonless paper case (COMP/E-1/36.212). Carbonless paper, also known as self-copying paper, is used for the duplication of documents. The Official Journal of the European Union⁵ documents the European Commission decision on this case by giving precise details on the colluding agreements. Briefly, the case involves a continuing agreement resulting in price fixing between Sappi Limited and ten other companies⁶ over the period 1992 to 1995. As described in the Commission’s decision Sappi “submitted ... information about the cartel ... before the Commission had undertaken any investigation (p.69)”. Evidence provided by Sappi documents people at cartel meetings and the algorithm for price fixing. In the end, Sappi received total immunity from the fine it would have paid

²An alternative would have been to focus on the period “before an investigation has started”.

³<http://ec.europa.eu/competition/cartels/leniency/leniency.html>

⁴<http://www.competitionbureau.gc.ca/eic/site/cb-bc.nsf/eng/03288.html>

⁵<http://eur-lex.europa.eu/JOIndex.do>

⁶The companies are Arjo Wiggins Appelton Limited, Bolloré SA, Carrs Paper Ltd, Distribuidora Vizcaina de Papeles S.L., Mitsubishi HiTech Paper Bielefeld GmbH, Papelera Guipuzcoana de Zicunaga SA, Papeteries Mougeot SA, Papierfabrik August Koehler AG, Torraspapel SA and Zanders Feinpapier AG.

had it not cooperated with the Commission. Following the opening of the investigation, other corporations decided to take advantage of the Leniency Program. Depending on their level of cooperation with the European Commission and the quality of their evidence, these firms received moderate leniency somewhere between 10% to 50% in reduction of penalty for sharing details on meetings, on agreements, on participating companies and on executives responsibilities.

In the present framework, quality of evidence plays a dual role. It entitles firms to benefit from an accrued benefit, since they can, by having a fine set as a function of non-evidence based and *evidence based*, pay a lesser fraction of the full fine otherwise paid without applying for leniency. This new feature of the fine will lead them to apply more often for leniency. At the same time, a higher quality of evidence turned in implies that the antitrust agency is more likely to catch cartel members and this now happens at lower cost compared with low quality evidence.

The scholarly literature⁷ on the effectiveness of leniency policies is built on two assumptions.⁸ First, the incriminating evidence transferred from the leniency applicant to the antitrust agency leads to strict leniency (100% fine discount) regardless of the quality of the evidence handed in. Second, from the moment a firm applies for leniency, other cartel members are convicted for sure (100% rate of conviction). These two assumptions, however, raise many questions when looking at the applicability in the real life counterpart. For example, they may provoke strategic manipulation of evidence (e.g., falsification) since the fine is determined independently of the information accuracy. Leniency programs draw a clear line between the distinction of a leniency application happening “before” and “during” an investigation. Hence, by limiting the analysis to strict leniency granted before as well as during an undergoing investigation, these theories do not fit well the moderate leniency jurisdictions such as Canada and the European Union. Finally, a world in which cartel members get convicted for sure once a firm has applied for leniency fails to capture the possibility that the case may be dropped or may take longer than usual before conviction. This implies that the leniency applicant can, potentially, face retaliation⁹ from fellow cartel members or, worse, face higher scrutiny from the antitrust agency if the case is dropped, counter-acting the goal of the leniency policy in the first place. It is then important to understand the impact of leniency policies in an environment where these two assumptions (strict leniency and 100% conviction rate) are relaxed. Moderate leniency distinguishes the present study

⁷Two good surveys are OECD (2005) and Spagnolo (2008).

⁸An exception is Feess and Walzl (2010), discussed later.

⁹I do not model retaliation per se.

from others in restricting leniency to the period “once an investigation has started” and in considering discounted fines. As for the 100% rate of conviction assumption, the evidence turned in by the applicant will allow for an additional probability ($w(e)$) of catching cartel members. Only in a special case will a conviction happen for sure.¹⁰

Recent academic research on leniency policies, such as Motta and Polo (2003), Harrington (2008), Motchenkova (2009), Brenner (2009), and Miller (2009), have helped shape the understanding of the mechanisms at play from both positive and normative points of view. As discussed above, most of these earlier papers lack a systematic treatment of the quality of information. Only one paper touch this question. Feess and Walzl (2010) analyze the impact of strict leniency granted to the first applicant before an investigation has begun in a context identical to the E.U. program in the period pre-2002.¹¹ The authors discuss the pros and cons of allowing the antitrust agency to accept different amounts of evidence (low or high). They show the suboptimality of offering leniency to the low evidence provider and the optimality of granting leniency to the high evidence provider. High evidence provider makes leniency application necessary to obtain conviction, hence a justification for granting strict leniency to this evidence provider. The authors remain silent on the provenance of this evidence. I address this issue by endogenizing the amount of evidence kept by corporations. This allows, for example, to link the lenient fine with the cost of collecting the incriminating evidence. Also, my findings complement Feess and Walzl (2010) by extending the analysis to the E.U. in the post-2002 program (moderate leniency) and by setting the study within the “once an investigation has started” period rather than the period pre-investigation.

The rest of the paper is organized as follows. In Section 2, I describe the model. The antitrust agency is introduced in Section 3, followed by my theoretical findings in Section 4. In Section 5, I compare my results with Motta and Polo (2003) and in Section 6, I discuss policy implications. Section 7 concludes and presents future avenues of research.

2 Model

2.1 Environment

The game has two firms and an antitrust agency (AA). Firms compete in quantity *à la Cournot* an infinite number of times. Every period, a firm faces a stage game identical to

¹⁰This will be the case if the applicant has collected all available evidence.

¹¹Since 2002 in the European Union, firms are only granted strict leniency before an investigation has started.

the prisoners' dilemma, where $\pi_D > \pi_M > \pi_N > \pi_M^D$ denotes, respectively, the profit from deviating from collusion, half the monopoly profit¹², the Cournot Nash equilibrium, and the payoff if the other player has deviated from collusion.

	Collude	No Collusion
Collude	(π_M, π_M)	(π_M^D, π_D)
No Collusion	(π_D, π_M^D)	(π_N, π_N)

In addition to quantity, firms choose at a cost $C(e) = c\frac{e^2}{2}$ the amount of information¹³, e , revealing the level of their implication in the colluding agreement and, more importantly, their rivals' implication. Incriminating evidence is collected in the first period and will be used by firms in exchange for a lenient fine from the antitrust agency. A firm applying for leniency faces a fine $R(e) = sF - er$, where $r, F \geq 0$ and $s \in [0, 1]$. The parameter s denotes the level effect of leniency on the fine; that is, a more lenient agency has a lower s inducing firms to pay a smaller fraction of the full fine F independently of evidence e . The parameter r denotes the marginal effect of the quality of information (e); that is, a more lenient agency has a higher r and is more sensitive to the information revealed by the leniency applicant. I later investigate the effect of variations in s, r and F on firms behavior.

Note that a colluding firm can choose not to accumulate any information ($e = 0$). If so, the firm will face the probability p of being convicted by the AA once the investigation opens which happens with probability α . In that event, the firm faces the full fine F .

Firms punish a colluding deviator using a grim trigger strategy. As is common in applied work in the repeated game literature, my interest is in a symmetric equilibrium.

Even though I analyse a symmetric equilibrium, I still have to clarify what will happen in the event that a firm applies to leniency while the other does not. On the applicant side, nothing changes, it pays the lenient fine $R(e)$ once the investigation opens. From the point of view of the agency, if it is unable to catch the non-applicant on its own: it can now use the information e at hand and will catch the non-applicant with probability $w(e) = e$. Similarly, if two firms apply for leniency at the same time both will pay the lenient fine and the AA will save prosecution resources (since there are only two firms in the industry).

As for the antitrust agency, it chooses the following policy parameters: the probability of opening an investigation, α , the probability of leading a successful investigation, p , the full

¹²I limit the analysis to firms sustaining the monopoly outcome as it is a study about collusion. As we know, other quantities can be sustained using the appropriate discount factor (Folk Theorem).

¹³Following up on the Carbonless paper case, examples of information collected are time and frequency of the meetings, people at the meetings, pricing algorithm, etc.

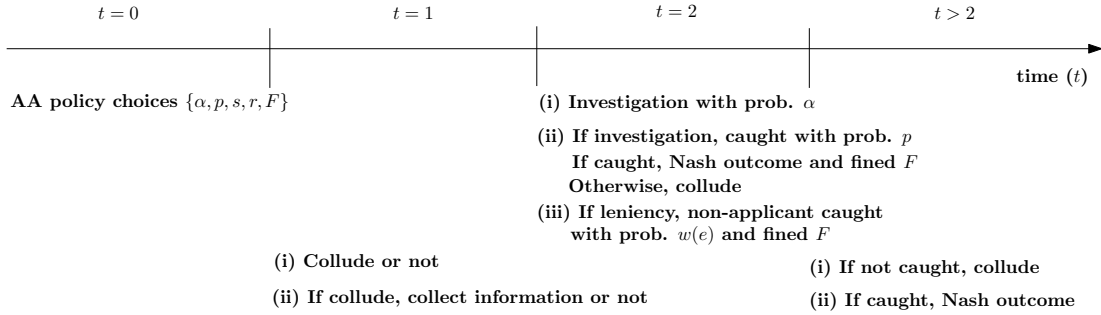


Figure 1: Timeline

fine faced by colluding firms, F , the level leniency parameter, s , and the marginal leniency parameter, r . The probabilities can be interpreted as resources to fight collusion at the agency’s disposition. From the antitrust agency point of view, the problem is to choose for $\{\alpha, p, s, r, F\}$ so as to maximize societal welfare.

The following subsection describes the timing of the game.

2.2 Timing

At time $t = 0$, the antitrust agency determines the policy parameters $\{\alpha, p, s, r, F\}$.¹⁴ In $t = 1$, firms select a strategy determining the extend of their colluding practices when the investigation opens in period $t = 2$. If colluding, a firm then chooses to collect (apply for leniency) or not (no application for leniency) information in $t = 1$ about colluding arrangements (meetings, people, etc). Period $t = 2$ has the following structure: an investigation opens with probability α and (i) if a firm chose to collect information in $t = 1$, it gives its incriminating evidence (e) to the AA, pays the lenient fine $R(e)$ and reverts to the Cournot Nash outcome forever, otherwise (ii) if a firm chose not to collect information in $t = 1$, with probability p (if the other firm has applied for leniency, this probability becomes $p + (1 - p)w(e)$), it will be fined F , and forced to the Cournot Nash outcome forever. On the other hand, with probability $1 - \alpha$, no investigation opens and firms continue with their previous arrangement (collusion or not). Figure 1 summarizes the timeline.

¹⁴The optimal policy choice $\{\alpha^*, p^*, s^*, r^*, F^*\}$ will be detailed in Section 4

2.3 Strategy

I introduce the concepts of action, payoff, history and strategy. The normal form stage game has two players ($i \in \{1, 2\}$), a set of actions ($A_i = \{C, NC\}$), and payoffs ($u_i(a_i) : A_i \rightarrow \mathbb{R}$) given by the prisoner's dilemma matrix depicted above. The set of period $t \in \{1, 2, \dots\}$ histories ($h^t = (a^0, a^1, \dots, a^{t-1})$, where $a^0 = \{\emptyset\}$ and $a^t = (a_1^t, a_2^t)$ denotes a vector of players' actions at t) is a sequence of realized actions from all periods before t .

Now, in the present framework, there is another layer to the simple game described above (and so far identical to a two persons prisoners' dilemma). If colluding, firms can collect information in $t = 1$ with repercussion in $t = 2$ if a firm chooses leniency. So the action space for a colluding firm applying for leniency is slightly different than in a prisoners' dilemma. A firm applying for leniency has an action in $t = 1$ of $a_i^1 = \{C, e\}$, where C stands for collusion and e for the collected information. In $t = 2$, the applicant will simply deliver the information in hand, that is $a_i^2 = \{NC\}$, where NC stands for no-collusion.

I study three possible symmetric equilibria: *no-collusion*, *collude and reveal information* and *collude and no leniency*. Each will be represented by a corresponding strategy, where a strategy is a mapping from the history set into an action ($\sigma_i(h^t) : H^t \rightarrow A_i$, where $h^t \in H^t$).

2.4 Subgame Perfect Equilibrium

The equilibrium concept I apply is subgame perfect equilibrium. The following definitions rely on Mailath and Samuelson (2006). The first building block is the concept of Nash equilibrium. Intuitively, a Nash equilibrium is a strategy profile in which each player is best responding to the other player strategies, formally:

Definition: Nash Equilibrium. *The strategy profile σ is a Nash equilibrium of the repeated game if for all players i and strategies σ'_i ,*

$$U_i(\sigma) \geq U_i(\sigma'_i, \sigma_{-i})$$

Then, if at each history the strategy profile is a Nash equilibrium we say that the strategy profile is a Subgame Perfect Nash Equilibrium:

Definition: Subgame Perfect Nash Equilibrium. *A strategy profile σ is a subgame-perfect equilibrium of the repeated game if for all histories h^t , $\sigma(h^t)$ is a Nash equilibrium of the repeated game.*

To solve for subgame perfectness in repeated games with perfect monitoring, I apply the one shot deviation principle and investigate the conditions under which an equilibrium exists and eventually fully characterize the equilibria. First, I define the one-shot deviation principle.

Definition: Profitable One-Shot Deviation. *Fix a profile of opponents' strategies σ_{-i} . A one-shot deviation $\hat{\sigma}_i$ from strategy σ_i is profitable if, at the history h^t for which $\hat{\sigma}_i(h^t) \neq \sigma_i(h^t)$,*

$$U_i(\hat{\sigma}_i(h^t), \sigma_{-i}(h^t)) > U_i(\sigma_i(h^t), \sigma_{-i}(h^t)) \equiv U_i(\sigma_i(h^t), \sigma_{-i}(h^t))$$

I will look for profitable one-shot deviations to find subgame perfect equilibria. A profitable one-shot deviation exists when a player has an incentive to deviate from its strategy and obtain a higher expected utility level. I can now introduce the one-shot deviation principle.

Definition: One-shot Deviation Principle. *A strategy profile is subgame perfect if and only if there are no profitable one-shot deviations.*

The one-shot deviation principle imposes structure on a Nash equilibrium by adding the additional requirement that every subgame is a Nash equilibrium. I can now look for subgame perfect equilibria.

2.4.1 No collusion

A firm playing the strategy *no-collusion* ($\sigma_i^{NC}(h^t)$) will play the Cournot-Nash outcome in every period, hence

$$\sigma_i^{NC}(h^t) = NC \text{ for all } t \in \{1, 2, \dots\}$$

The expected discounted profit for this strategy is given by $V_{NC} = \frac{\pi_N}{1-\delta}$. It is never a best response for a firm playing the Cournot-Nash outcome to deviate to collusion when the other firm is playing the Nash outcome since the Nash outcome is the only Nash equilibrium of the stage game. Lemma 1 formalizes the result.

Lemma 1. *A subgame perfect equilibrium exists in which firms do not collude.*

Proof. The proof is omitted since this is a well known result. □

2.4.2 Collude and Reveal Information

For this strategy, a firm colludes and collects information at time $t = 1$, and will apply for leniency if an investigation opens in time $t = 2$. If the investigation opens, the firm delivers every bit of information in its possession. If a deviation occurs in the marketplace¹⁵, firms use Nash punishment forever with profits π_N in every period. The rigorous formulation of the strategy collude and reveal information $\sigma_i^{CRI}(h^t)$ can be written as:

$$\sigma_i^{CRI}(h^t) = \begin{cases} C \text{ and } e & \text{if } t = 1 \text{ and will apply for leniency in } t = 2 \\ C & \text{if } a^\tau = (C, C), \tau \in \{1, 2, \dots, t - 1\} \text{ and no investigation (or unsuccessful)} \\ NC & \text{Otherwise} \end{cases}$$

The expected discounted profit, V_{CRI} , of this strategy is:

$$V_{CRI} = \pi_M - C(e) + \delta \left\{ \alpha \left[\frac{\pi_N}{1 - \delta} - R(e) \right] + (1 - \alpha) \frac{\pi_M}{1 - \delta} \right\}$$

From V_{CRI} , it is straightforward to determine the optimal amount of information (e^*) collected and delivered to the antitrust agency.

2.4.3 Choice of quality of information

Firms playing the strategy *collude and reveal information* have to choose the quality of information reported to the agency. This choice of quality is defined over the support¹⁶ $e \in [0, 1]$ and limited by a cost on the acquisition of information $C(e)$. The choice of quality is performed by selecting the highest return on “investment” in quality given by:

$$\max_{e \in [0, 1]} V_{CRI}(e)$$

The optimal information quality level, e^* , satisfies the following first-order condition:

$$-[C'(e) + \alpha \delta R'(e)] = 0$$

I formalize this result in Lemma 2.

¹⁵Marketplace can be defined as the competition arrangement implying that a deviation occurs when firms leave the cartel.

¹⁶I assume that c is big enough such that $0 \leq \frac{\alpha \delta r}{c} \leq 1$

Lemma 2. *Higher quality of information (higher e) follows from a lower cost of accumulating information (lower c), a higher value of the future (higher δ), a more lenient authority (higher r) and a higher probability of facing an investigation (higher α).*

Proof. Rewrite equation (2.4.3) using the functional form $C(e) = c\frac{e^2}{2}$ and $R(e) = sF - er$, implying the

$$e^* = \frac{\alpha\delta r}{c}$$

desired results¹⁷. □

Lemma 2 lists the factors driving the optimal choice of the quality of information e . A higher likelihood of an investigation by the Antitrust Agency, α , induces firms to collect better quality evidence. A higher cost, c , of acquiring information dissuades firms from collecting information. A higher discount factor, δ , induces firms to collect more information since the upcoming investigation weight more in firms' today decision. Finally, a more lenient authority, r , where, again, the fine paid is such that $R(e) = sF - er$, induces a firm to share a higher quality of information.

2.4.4 Collude and No Leniency

If firms are not benefiting from leniency because they evaluate the cost of acquiring information as too heavy or because they think they will not be successfully prosecuted, they can still collude by playing the strategy *collude and no leniency* ($\sigma_i^{CNL}(h^t)$).

$$\sigma_i^{CNL}(h^t) = \begin{cases} C & \text{if } t = 1 \\ C & \text{if } a^\tau = (C, C), \tau \in \{1, 2, \dots, t-1\} \text{ and no investigation (or unsuccessful)} \\ NC & \text{Otherwise} \end{cases}$$

The expected profit (V_{CNL}) from colluding and not using leniency is:

$$V_{CNL} = \pi_M + \delta \left\{ \alpha \left[p \left(\frac{\pi_N}{1-\delta} - F \right) + (1-p) \frac{\pi_M}{1-\delta} \right] + (1-\alpha) \frac{\pi_M}{1-\delta} \right\}$$

¹⁷Note that the second order derivative with respect to e is negative, implying a maximum:

$$\frac{d^2 V_{CRP}(e)}{de^2} = -C''(e) - \alpha\delta R''(e) < 0$$

Again, in $t = 1$, firms are colluding. In $t = 2$, either an investigation opens or not. If it opens, firms are caught with probability p and are forced to revert to the Cournot-Nash outcome, otherwise firms continue on colluding forever.

One may be tempted to conclude that the strategy *collude and no leniency* is in fact a subset of the strategy *collude and reveal information* where $e = 0$. A closer look at the strategy *collude and reveal information* reveals that a firm playing the strategy and choosing $e = 0$ will be giving itself to the AA without any evidence implying a fine to be paid of sF for sure. As for a firm playing the strategy *collude and no leniency*, it will pay F with probability p contrasting with the strategy *collude and reveal information*.

In what follows, I solve for subgame perfect Nash equilibria by applying the one-shot deviation principle to the three potential equilibria: *no-collusion*, *collude and reveal information* and, finally, *collude and no leniency*.

2.4.5 Collude and Reveal Information

In the strategy *collude and reveal information*, firms collude and collect incriminating evidence at a cost $C(e)$ and pay a fine $R(e)$ to the antitrust agency when the investigation opens.

Recall the ex-ante expected payoff from this strategy:

$$V_{CRI} = \pi_M - C(e) + \delta \left\{ \alpha \left[\frac{\pi_N}{1-\delta} - R(e) \right] + (1-\alpha) \frac{\pi_M}{1-\delta} \right\}$$

Since the game can be divided in three stages: (i) $t = 1$, (ii) $t = 2$ and (iii) $t > 2$; a profitable deviation can then happen in $t = 1$, or in $t = 2$ or somewhere in $t > 2$.

In the first case, in $t = 1$, I define V_D and V_{DNL} as respectively the payoff from a $t=1$ deviation from collusion and the payoff from a deviation from applying to leniency, where:

$$\begin{aligned} V_D &= \pi_D + \delta \frac{\pi_N}{1-\delta} \\ V_{DNL} &= \pi_M + \delta \left\{ \alpha \left[p \left(\frac{\pi_N}{1-\delta} - F \right) + (1-p)w(\hat{e}) \left(\frac{\pi_N}{1-\delta} - F \right) + \right. \right. \\ &\quad \left. \left. (1-p)(1-w(\hat{e})) \frac{\pi_M}{1-\delta} \right] + (1-\alpha) \frac{\pi_M}{1-\delta} \right\} \\ \Leftrightarrow V_{DNL} &= \pi_M + \delta \left\{ \alpha \left(p + (1-p)w(\hat{e}) \right) \left(\frac{\pi_N}{1-\delta} - F \right) + \alpha(1-p)(1-w(\hat{e})) \frac{\pi_M}{1-\delta} + (1-\alpha) \frac{\pi_M}{1-\delta} \right\} \end{aligned}$$

In V_D , the deviating firm earns the deviating profit in $t = 1$ (recall that $\pi_D > \pi_M$) and

faces the punishment for breaking the cartel agreement as the other firm reverts to the competitive outcome (π_N) in period $t > 1$. As for V_{DNL} , the colluding and deviating firm is not collecting any information in $t = 1$, implying no possibility for leniency in $t = 2$ if an investigation opens. In $t = 2$, the deviating firm faces the fine F if the investigation is successful (with probability $p + (1 - p)w(\hat{e})$), where $w(\hat{e})$ is the probability of conviction relying solely on the evidence (\hat{e}) handed over by the leniency applicant to the antitrust agency. Alternatively, if no investigation opens, the firm continues colluding forever, free of any investigation.

The inequalities $V_{CRI} \geq \max\{V_D; V_{DNL}\}$ ensure that in period 1 neither firm has an incentive to deviate from the proposed equilibrium.

In the second case, a firm may profitably deviate in $t = 2$. Similarly to V_D in $t = 1$, if no investigation opens in $t = 2$, a firm can deviate from collusion with expected payoff given by V_D ¹⁸. The resulting inequality is identical as the one obtained from the stage $t > 2$ application of the one-shot deviation principle. This is given by the standard argument based on the Folk Theorem, where $\delta \geq \frac{\pi_D - \pi_M}{\pi_D - \pi_N}$, in order to sustain the collusive outcome.

I can now describe the conditions for the existence of a subgame perfect equilibrium in which firms play the strategy *collude and reveal information* in Lemma 3.

Lemma 3. *For given policy values $\{\alpha, p, r, s, F\}$ a subgame perfect equilibrium in which firms play collude and reveal information¹⁹ exists if*

$$\alpha \leq \alpha_I \equiv \left\{ \alpha : -\frac{\alpha^2 \delta^2 r^2 (1 - \delta)}{2c} + \alpha \delta [\pi_M + sF(1 - \delta) - \pi_N] - [\pi_M - \pi_D(1 - \delta) - \delta \pi_N] = 0 \right\},$$

$$p \geq p_I(\alpha) \equiv 1 - \frac{Fc(1 - s) + \alpha \delta r^2 (1 - \delta)}{2(c - \alpha \delta r) [\pi_M - \pi_N + F(1 - \delta)]},$$

$$\delta \geq \delta_0 \equiv \frac{\pi_D - \pi_M}{\pi_D - \pi_N},$$

This parameter space is non-empty.

Proof. See Appendix. □

Figure 2 illustrates Lemma 3 where small x's show the region where a SPNE exists for CRI. Hence the existence of the strategy CRI being an equilibrium is conditional on α taking

¹⁸Note that a firm may be tempted to deviate by not revealing its collected evidence to the agency. This action is strictly dominated by revealing the evidence to the AA given the other firm will be revealing its information (symmetric equilibrium).

¹⁹Recall that firms playing CRI optimally pick $e^* = \frac{\alpha \delta r}{c}$. In order to make the passage to a graphical analysis, I set $e = e^*$ everywhere, hence under CRI, there will be no e showing in the equation.

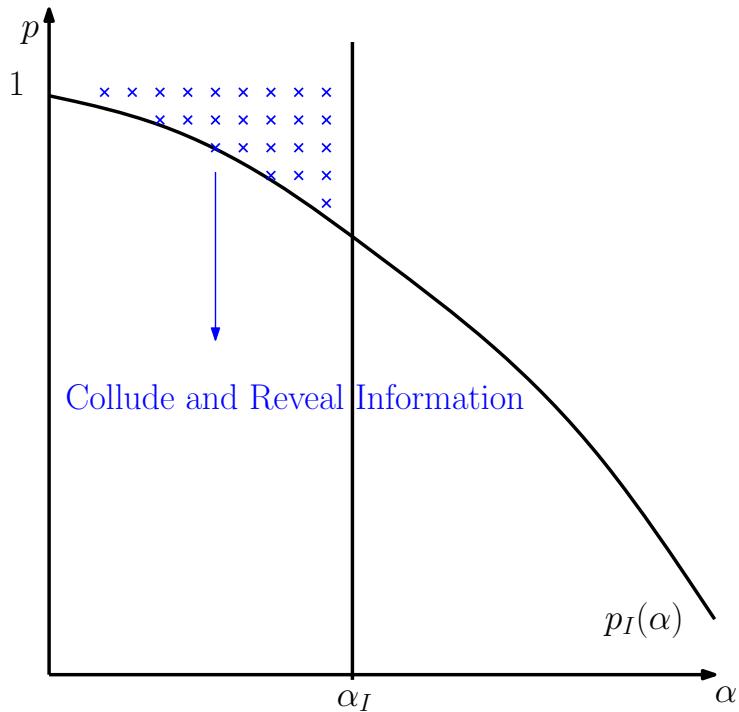


Figure 2: Existence CRI

small values and p taking high values. I now redo the exercise but this time for the strategy CNL.

2.4.6 Collude and No Leniency

In the strategy *collude and no leniency*, firms collude and don't apply for leniency (implying they collect no information).

Recall the ex-ante expected payoff for this strategy:

$$V_{CNL} = \pi_M + \delta \left\{ \alpha \left[p \left(\frac{\pi_N}{1-\delta} - F \right) + (1-p) \frac{\pi_M}{1-\delta} \right] + (1-\alpha) \frac{\pi_M}{1-\delta} \right\}$$

Similarly to the strategy *collude and reveal information*, a deviation may happen in one of the three stages: in $t = 1$, in $t = 2$, or in $t > 2$.

In the first case, in $t = 1$, I define V_D and V_{DRI} ²⁰ as respectively the payoff from a

²⁰Since the deviating firm is the sole applicant to leniency, the optimal collected level of evidence differs from the one calculated for strategy CRI. After optimizing the expected profit from the deviator, the optimal

deviation in $t = 1$ from collusion and the payoff from a deviation to applying for leniency (collecting information), where:

$$\begin{aligned}
V_D &= \pi_D + \delta \frac{\pi_N}{1 - \delta} \\
V_{DRI} &= \pi_M - C(e) + \delta \left\{ \alpha \left[p \left(\frac{\pi_N}{1 - \delta} - R(e) \right) + (1 - p)w(e) \left(\frac{\pi_N}{1 - \delta} - R(e) \right) \right. \right. \\
&\quad \left. \left. + (1 - p)(1 - w(e)) \left(\frac{\pi_M}{1 - \delta} - R(e) \right) \right] + (1 - \alpha) \frac{\pi_M}{1 - \delta} \right\} \\
\Leftrightarrow V_{DRI} &= \pi_M - C(e) + \delta \left\{ -\alpha R(e) + \alpha p \frac{\pi_N}{1 - \delta} + \alpha(1 - p) \frac{\pi_M}{1 - \delta} \right. \\
&\quad \left. - \alpha(1 - p)w(e) \frac{(\pi_M - \pi_N)}{1 - \delta} + (1 - \alpha) \frac{\pi_M}{1 - \delta} \right\}
\end{aligned}$$

V_D is identical as before. As for V_{DRI} , the colluding and deviating firm is now collecting information in $t = 1$, implying a lenient fine in $t = 2$ if an investigation opens. Even though the firm shares its incriminating evidence with the AA, the actual conviction of the rival is uncertain. In $t = 2$, since the other firm didn't apply for leniency, the deviating firm is the sole beneficiary of leniency, where with probability $p + (1 - p)w(e)$ the rival will be caught. Again, I impose that once the investigation opens, independently of the prosecution results, a firm applying to leniency pays the fine $R(e)$. Alternatively, if no investigation opens or if the investigation is unsuccessful (with probability $\alpha(1 - p)(1 - w(e)) + (1 - \alpha)$), the firms continue on colluding free of any investigation in the future.

The inequalities $V_{CNL} \geq \max\{V_D; V_{DRI}\}$ ensure that in period 1 neither firm has an incentive to deviate from the proposed equilibrium.

In the second case, a firm may deviate in $t = 2$. Similarly to V_D in $t = 1$, V_D denotes the expected profit from a deviation from collusion in $t = 2$. This can happen if an investigation is unsuccessful (with probability $\alpha(1 - p)$) or if no investigation has opened (with probability $1 - \alpha$). This inequality is identical to the one obtained when looking at profitable deviations after $t = 2$. This is again given by the standard argument based on the Folk Theorem, where

level of e becomes:

$$e_{DRI}^* = \frac{\alpha\delta[r(1 - \delta) - (1 - p)(\pi_M - \pi_N)]}{c(1 - \delta)}$$

Again, I impose that $e \in [0, 1]$. This implies the additional assumption that

$$r \geq \underline{r} \equiv \frac{\pi_M - \pi_N}{1 - \delta}$$

$\delta \geq \frac{\pi_D - \pi_M}{\pi_D - \pi_N}$, in order to sustain the collusive outcome.

I can now describe the conditions for the existence of a subgame perfect equilibrium in which firms play the strategy *collude and no leniency* in Lemma 4.

Lemma 4. *For given policy values $\{\alpha, p, r, s, F\}$ a subgame perfect equilibrium in which firms collude and use no leniency²¹ exists if*

$$\begin{aligned}
 p &\leq p_{NL}(\alpha) \equiv \frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\alpha\delta[\pi_M + F(1 - \delta) - \pi_N]}, \\
 \alpha &\leq \alpha_{NL}(p) \equiv \frac{2F(s - p)c(1 - \delta)^2}{\delta \left\{ r(1 - \delta) - (1 - p)(\pi_M - \pi_N) \right\}^2}, \\
 \delta &\geq \delta_0,
 \end{aligned}$$

This parameter space is non-empty.

Proof. See Appendix. □

Figure 3 illustrates Lemma 4 where small dots represent the solution region.

2.5 Uniqueness

The equilibrium regions for the different strategies (CRI, CNL and NC) may intersect for some values of α and p depending on the value of r, s, F and c . From Lemma 1, independently of the probabilities α and p , the strategy *no-collusion* is a subgame perfect equilibrium of this game. As Proposition 1 will show, firms can sustain a preferred strategy given the values of the policy parameters. Said otherwise, I will be looking at dominant equilibrium from the point of view of the firms. The underlying assumption is that a firm will always pick the equilibrium yielding the maximum expected profit.

To ensure no overlapping between the three existing equilibria, I restrict the analysis to regions where period $t = 1$ discounted payoff is strictly dominant. Recall period 1 discounted payoff for the three equilibria *no collusion*, *collude and reveal information*, and *collude and*

²¹As calculated, $e = e_{DRI}^*$ when a firm choose to deviate from CNL to CRI in period $t = 1$. In Lemma 4, there is no $e_{DRI}^* = \frac{\alpha\delta[r(1-\delta) - (1-p)(\pi_M - \pi_N)]}{c(1-\delta)}$ showing, as it has already been replaced.

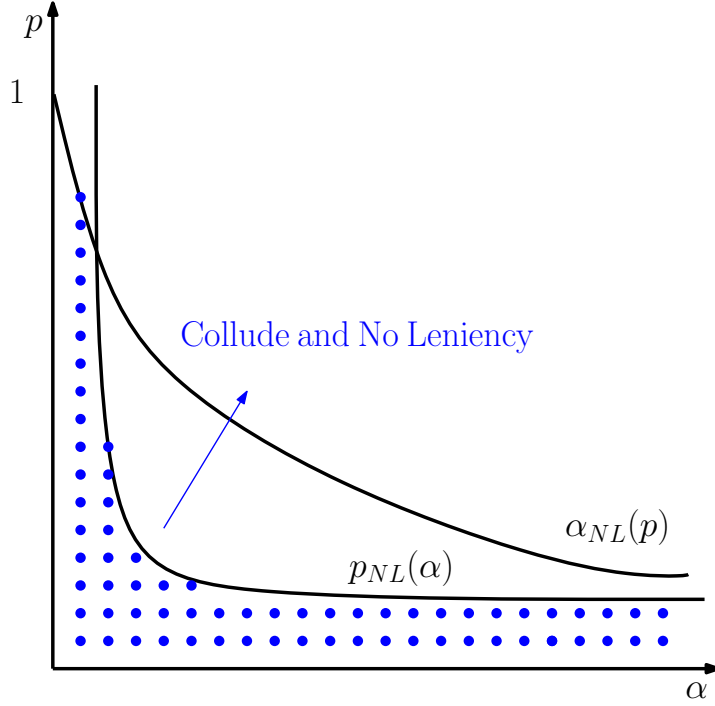


Figure 3: Existence CNL

no leniency.

$$\begin{aligned}
 V_{NC} &= \frac{\pi_N}{1-\delta} \\
 V_{CRI} &= \pi_M - C(e) + \delta \left\{ \alpha \left[\frac{\pi_N}{1-\delta} - R(e) \right] + (1-\alpha) \frac{\pi_M}{1-\delta} \right\} \\
 V_{CNL} &= \pi_M + \delta \left\{ \alpha \left[p \left(\frac{\pi_N}{1-\delta} - F \right) + (1-p) \frac{\pi_M}{1-\delta} \right] + (1-\alpha) \frac{\pi_M}{1-\delta} \right\}
 \end{aligned}$$

I then obtain uniqueness by comparing these three equilibria. With these additional inequalities resulting from such a comparison and from Lemmas 1, 3 and 4, I obtain subgame perfectness as profitable deviations are constrained not to be profitable. I describe subgame perfect equilibria in Proposition 1.

Proposition 1. For given policy values $\{\alpha, p, r \geq \underline{r}, s, F \geq \underline{F}\}$, the dominant Subgame Perfect Nash Equilibria of the repeated game are given by:

$$\begin{aligned}
\text{Collude and Reveal Information:} & \left\{ \begin{array}{l} \alpha \leq \min \left\{ \alpha_I; \alpha_{IN} \right\}, \\ p \geq \max \left\{ p_I(\alpha); p_{INL}(\alpha) \right\}, \text{ and} \\ \delta \geq \delta_0 \end{array} \right. \\
\text{Collude and No Leniency:} & \left\{ \begin{array}{l} p \leq \min \left\{ p_{NL}(\alpha); p_{INL}(\alpha); p_{NLN}(\alpha) \right\}, \\ \alpha \leq \alpha_{NL}(p), \text{ and} \\ \delta \geq \delta_0 \end{array} \right. \\
\text{No Collusion:} & \left\{ \begin{array}{l} \alpha \geq \alpha_{IN}, \text{ and} \\ p \geq p_{NLN}(\alpha) \end{array} \right. \\
\text{Otherwise:} & \left\{ \text{The unique SPNE is NC} \right.
\end{aligned}$$

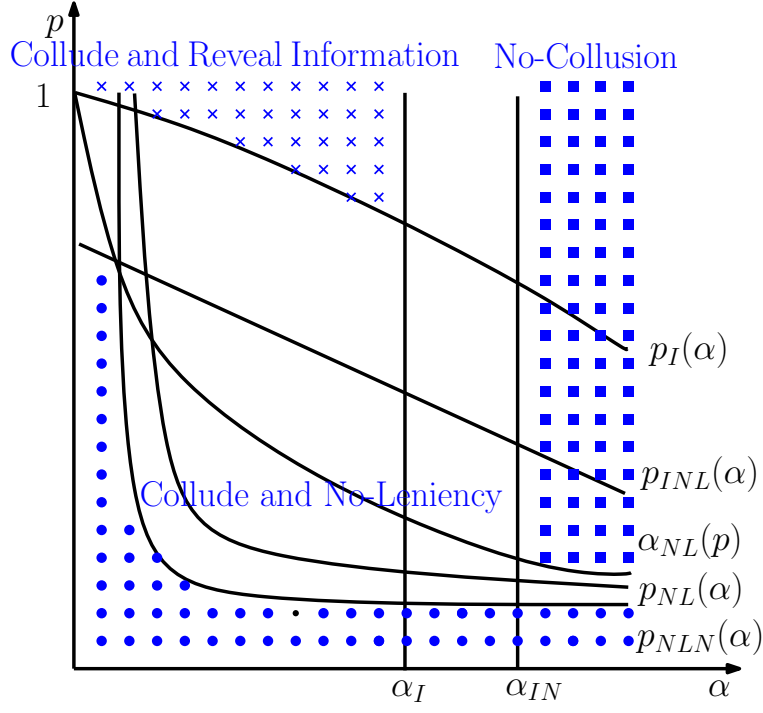


Figure 4: SPNE

Where:

$$\alpha_{IN} \equiv \left\{ \alpha : -\frac{\alpha^2 \delta^2 r^2 (1 - \delta)}{2c} + \alpha \delta [\pi_M + F(1 - \delta) - \pi_N] - (\pi_M - \pi_N) = 0 \right\}$$

$$p_{NLN}(\alpha) \equiv \frac{\pi_M - \pi_N}{\alpha \delta [\pi_M + F(1 - \delta) - \pi_N]}$$

$$p_{INL}(\alpha) \equiv \frac{\pi_M - \pi_N + (1 - \delta) \left[sF - \frac{\alpha \delta r^2}{2c} \right]}{\pi_M + F(1 - \delta) - \pi_N}$$

$$F_{\Delta 1} \equiv \frac{2r^2(1 - \delta) [\pi_M - \pi_D(1 - \delta) - \delta \pi_N]}{cs^2} - \frac{\pi_M - \pi_N}{s}$$

$$F_{\Delta 2} \equiv \sqrt{\frac{2r^2(\pi_M - \pi_N)}{cs^2(1 - \delta)}} - \frac{\pi_M - \pi_N}{s(1 - \delta)}$$

$$F_{R1} \equiv \frac{\frac{r^2 \delta^2}{2c} - (\pi_D - \pi_M)}{\delta s}$$

$$F_{R2} \equiv \frac{2c(\pi_M - \pi_N) - \delta^2 r^2}{2cs\delta}$$

$$F_{INL} \equiv \frac{r^2 \delta (1 - \delta) - 2c(\pi_M - \pi_N)}{2cs(1 - \delta)}$$

$$F_{NLN} \equiv \frac{\pi_M - \pi_N}{\delta}$$

$$F \equiv \max\{F_{\Delta 1}, F_{\Delta 2}, F_{R1}, F_{R2}, F_{INL}, F_{NLN}\}$$

$$r \equiv \frac{\pi_M - \pi_N}{1 - \delta}$$

Proof. See Appendix. □

Figure 4 illustrates the equilibria.²² To obtain this figure, I fix the values of $\{s, r, F, c\}$; the reader should take notice that the curves level and slope depend on this assumption. Independently of the parameters variation, Proposition 1 holds as long as $F \geq \underline{F}$. Aside from the dominant equilibria, figure 4 depicts regions where no equilibrium can be sustained, that is, from a firm's point of view it is impossible to choose which equilibrium is preferred or that the only possibility is a non-symmetric equilibrium. Also, from Lemma 1, independently of the probabilities (α, p) , the non-collusive equilibrium exists. This existence does not necessarily mean dominance. As figure 4 shows, only when (α, p) are both sufficiently high that the equilibrium *no-collusion* is dominant.

Figure 4 results make sense. Whenever the gain from collusion is high and the probability of being successfully prosecuted is low, firms will collude. Whenever the likelihood of a successful prosecution is too high, firms prefer applying for leniency. Finally, again, whenever both probabilities are too high, firms prefer not colluding at all and revert to the Cournot-Nash outcome forever.

So far, the only limitation imposed on parameters is $(F \geq \underline{F})$. Previous studies using similar framework (e.g., Motta and Polo (2003) and Motchenkova (2009)) all needed at some point to restrict fine values for feasibility and a representation of the solution in the first quadrant determined by the probabilities (α, p) . This requirement is essential to give rise to a credible and harmful punishment as otherwise the expected fine for collusive behavior will be too low and equilibria CRI and CNL will always exist.

In the next section, I investigate variations in parameters, $\{s, r, F, c\}$, determining the frontlines solution region and extract intuitions on the way. The main purpose of the next section is to build the basics for the optimal policy derivation. In what follows, I focus on dominant equilibrium since the treatment is most straightforward.

3 Results

In this section, I evaluate the impact of an infinitesimal change in the model parameters: $\{s, r, F, c\}$. To first set the ideas, I look at the effect of s and F on each equilibrium in Lemmas 5 and 6. Again, s represents the level gain from a firm applying for leniency ($R(e) = sF - er$) where F affects both the fine under leniency and the fine under no leniency.

²²In the figure, only dominant equilibria are depicted.

Lemma 5. *An increase in F creates a larger (smaller, smaller) region where the equilibrium NC (CRI, CNL) is a dominant SPNE.*

Proof. See Appendix. □

Such result occurs from the dual impact of F . It enters the strategy *collude and no leniency* as the fine to be paid if an investigation is successful. It also affects the strategy *collude and reveal information* via the fine to be paid. It comes as no surprise that an increase in the fine to be paid will widen the range of parameters (α, p) such that *no-collusion* is a dominant equilibrium.

Lemma 6 considers a similar scenario, but this time restricting the change to s which only enters the fine to be paid by firms applying to leniency.

Lemma 6. *An increase in s creates a larger (smaller, larger) region where the equilibrium no-collusion (CRI, CNL) is a dominant SPNE.*

Proof. See Appendix. □

For the expansion of the region where the strategy *no-collusion* is an equilibrium, again, this comes as no surprise as this is a special case of Lemma 5. The next lemmas look at variations in parameters affecting the optimal quality of information: the marginal cost of acquiring information c and the marginal effect of r .

Recall Lemma 2, the quality of information is influenced by four variables: the marginal cost c of acquiring information, the discount factor δ , the probability of an investigation opening α and the leniency parameter r . The quality of information is negatively correlated with c , while positively correlated with α , δ , and r . Unfortunately, the effects of α and δ on the equilibria are impossible to sign leaving c and r as the single representatives of the quality of information.

In Lemma 7, I show that a decrease in the cost of acquiring information, c , induces a wider range of parameters (α, p) such that the strategy *collude and reveal information* is an equilibrium. Again, the direct effect of a lower cost, c , is the collection of an higher quality of information passed to the AA.

Lemma 7. *A decrease in c creates a larger region where the strategy collude and reveal information is a dominant SPNE.*

Proof. See Appendix. □

Another channel influencing the quality of information is to vary the AA's receptiveness (r) to information. Lemma 8 presents the effect of increasing r .

Lemma 8. *An increase in r creates a larger region where the strategy collude and reveal information is a dominant SPNE.*

Proof. Identical to Lemma 7. □

Lemma 8 is interesting on many regards. First, a policy maker with a strong propensity for the implementation of leniency policies will force r to tend to infinity to extend the range of applicability of leniency. On one side, the firms prefer leniency for a wider range of (α, p) while at the same time providing higher quality information to the antitrust agency. Similarly, Lemma 9 shows the effect on the solution region NC and CNL

Lemma 9. *An increase in r , or a decrease in c , creates a smaller(smaller) region where the strategy no-collusion(CNL) is a SPNE.*

Proof. See Appendix. □

This implies that two of the tools (r and s) at the AA's disposition yield similar result in terms of leniency applications. Hence, if NC equilibrium is not reachable²³ because for example of a budget restriction, both tools should be used together: r will control for the quality of information and s will induce firms to use leniency independently of the quality of information in hand. For example, if F has to be bounded above (as it is the case in most jurisdiction around the world), it is still possible to force firms to apply for leniency with high quality information by setting r to a high value while at the same time allowing a sufficiently high discount on the fine to be paid by setting s sufficient low.

In the next section, I solve for the optimal policy parameters. By doing so, I move to time $t = 0$ and solve for the whole game since the AA is the first player to move in the game²⁴.

4 Antitrust Agency

Until now, firms were taking the policy parameters $\{\alpha, p, s, r, F\}$ as given. In this section, I investigate the choice of those parameters by the antitrust agency. Similarly to Motta

²³More on this point in the next Section

²⁴See Figure 1.

and Polo (2003) and Motchenkova (2009), I argue that an AA always acts under a budget constraint. Its budget constraint is given by $Ap + B\alpha \leq M$, where $A, B > 0$. The coefficients A and B illustrate different impact of respectively leading a successful investigation and opening an investigation, while M denotes the AA's total budget for fighting cartels. By expressing the budget constrain in terms of α , I define $p_{BC}(\alpha)$. The antitrust agency also has a second, independent, "political" fund²⁵ to compensate firms that comes forward (for courageous leniency). The maximum amount of this courageous fund is set to M_R .

The agency's goal is to maximize welfare under a budget constraint. For example, if the cost of opening an investigation (proxied by α) is relatively high compared to the cost of prosecution (proxied by p), it may be preferable to the AA to settle on inducing firms to apply for leniency instead of the heavy burden of both opening an investigation and leading a successful prosecution.

The welfare function of each equilibrium differs and mainly depends on the probability of successfully avoiding collusion. As it is mathematically simpler to express things in terms of savings in terms of deadweight loss rather than deadweight loss itself, the welfare according to the Nash-Cournot outcome (π_N) will be denoted by $SDWL$. Hence, the equilibrium where firms are not colluding is given by $W_{NC} = \frac{SDWL}{1-\delta} \equiv K$, where the agency saves a deadweight loss in every periods.

If firms preferred the symmetric equilibrium *collude and no-leniency*, society incurs savings in deadweight loss only when the cartel is successfully prosecuted (with probability αp) in which case the welfare is given by $W_{CNL} = \frac{\alpha p \delta SDWL}{1-\delta}$. Similarly for the equilibrium *collude and reveal information*, now, the opening of an investigation (with probability α) is sufficient to catch colluding firms and the welfare function is $W_{CRI} = -2C(e^*) + \frac{\alpha \delta SDWL}{1-\delta}$. The welfare comparison of the equilibrium is such that $W_{NC} \geq W_{CRI}$ and $W_{NC} \geq W_{CNL}$. The relation between W_{CRI} and W_{CNL} is for now ambiguous and will be discussed shortly.

The antitrust agency maximizes the welfare function with respect to the budget constraint. Since *no-collusion* is an equilibrium independently of the policy parameters, the question for the AA is simply a matter of budget and domination by firms. Recall that firms with colluding possibilities will prefer the Nash outcome only if α and p are sufficiently high.

²⁵This assumption requires justification. First, it simplifies the algebra, and, most importantly, allows for courageous leniency (money in exchange of information). I could have simply shut down the possibility of courageous leniency by forcing $R(e) \geq 0$. By restraining from doing so, I get an extra result in terms of comparability with the lenient fine non-conditional on evidence. Only Spagnolo (2004) addresses the issue of courageous leniency and, so far, no jurisdictions have adopted this practice even in the situation before an investigation opening. For practical reason, this fund allows for compensation, but it is assumed to be sufficiently small. Political reasons justify the size of the fund.

I break the analysis in four parts. First, in Proposition 2, I derive the optimal fines and Propositions 3 - 5 derive the optimal policies from the point of view of the antitrust agency.

Proposition 2. *The optimal policies $\{s^*, r^*, F^*\}$ that implement the equilibria are given by no-collusion ($F^* \rightarrow \infty, s^* = 1, r^* = \underline{r}$), collude and no leniency ($F^* = \underline{F}, s^* = 1, r^* = \underline{r}$) and collude and reveal information ($F^* = \underline{F}, s^* = 0, r^* = \bar{r}$) where:*

$$\begin{aligned}\underline{F} &\equiv \max\{F_{\Delta 1}, F_{\Delta 2}, F_{R1}, F_{R2}, F_{INL}, F_{NLN}\} \\ \underline{r} &\equiv \frac{\pi_M - \pi_N}{1 - \delta} \\ \bar{r} &\equiv \sqrt{\frac{cM_R}{\alpha^* \delta}}\end{aligned}$$

Proof. See Appendix. □

Proposition 2 formalizes the intuition developed so far. If feasible, the AA will seek to implement the NC equilibrium as the welfare extracted from its application is higher than the two alternatives. The optimal policy sets the full fine to the highest possible level. The equilibrium CNL calls for low values of F and r , while a high value for s . On the other hand, the equilibrium CRI requires high values for r , and low values for s and F . As expected, F is the key parameter to address collusive Vs. non-collusive behavior while r and s serves the purpose of creating an incentive for CNL Vs. CRI.

Proposition 3 gives the optimal policies that implement the equilibrium *no-collusion* to be optimal.

Proposition 3. *The antitrust agency chooses to implement the dominant equilibrium no-collusion whenever the relative cost, A and B and the budget constraint M are such that $p_{BC}(\alpha_{IN}) \geq p_{NLN}(\alpha_{IN})$. The optimal policy chosen is then*

$$\alpha^* \geq \alpha_{IN} \text{ and } p^* \in [p_{NLN}(\alpha^*), p_{BC}(\alpha^*)]$$

Proof. See appendix. □

This should come as no surprise since the equilibrium *no-collusion* yield the highest welfare. The only condition for its optimality is feasibility under the budget constraint M

and the relative cost of opening an investigation, B , and of leading a successful investigation, A .

Similarly, proposition 4 presents the optimal policy when the equilibrium collude and reveal information is Pareto optimal.

Proposition 4. *The antitrust agency chooses to implement the dominant equilibrium collude and reveal information whenever A, B and M are such that $W_{CRI} \geq W_{CNL}$ and $p_{BC}(\alpha_{IN}) \leq p_{NLN}(\alpha_{IN})$. The optimal policy chosen by the AA is then*

$$\alpha^* = \left\{ \alpha : p_{BC}(\alpha) = \max\{p_I(\alpha); p_{INL}(\alpha)\} \leq p_W, \alpha \leq \min\{\alpha_I; \alpha_{IN}\} \right\} \text{ and } p^* = p_{BC}(\alpha^*)$$

where:

$$p_W \equiv 1 - \frac{M_R}{2K}$$

Proof. See Appendix. □

Figure 5 illustrates Proposition 4. Again, the crosses represent the region where the equilibrium *collude and reveal information* dominates the two alternatives. For the equilibrium CRI to be implemented by the agency means that, first, NC is not feasible and, second, that CRI induces a higher welfare than CNL. In the figure, the budget constraint do not cross the dominant *no-collusion* equilibrium under the specific values of A, B and M ($p_{BC}(\alpha_{IN}) < p_{NLN}(\alpha_{IN})$). At the same time, the solution region where CRI is an equilibrium is bounded by four curves: incentive compatibility constraints ($\max\{p_I(\alpha); p_{INL}(\alpha)\}$, and $\min\{\alpha_I; \alpha_{IN}\}$), the budget constraint $p_{BC}(\alpha)$ and p_W , where $p \leq p_W$ delimitates the region where CRI induces an higher welfare than CNL ($W_{CRI} \geq W_{CNL}$). Once this region has been identified, the antitrust agency selects the optimal policy (α, p) that yields the highest possible welfare. The iso-welfare curves are drawn in green and the resulting optimal policy is given by the star point.

I conduct similar analysis in Proposition 5, but this time for the equilibrium *collude and no leniency*.

Proposition 5. *The antitrust agency chooses to implement the dominant equilibrium collude and no leniency whenever A, B and M are such that $W_{CNL} \geq W_{CRI}$ and $p_{BC}(\alpha_{IN}) \leq$*

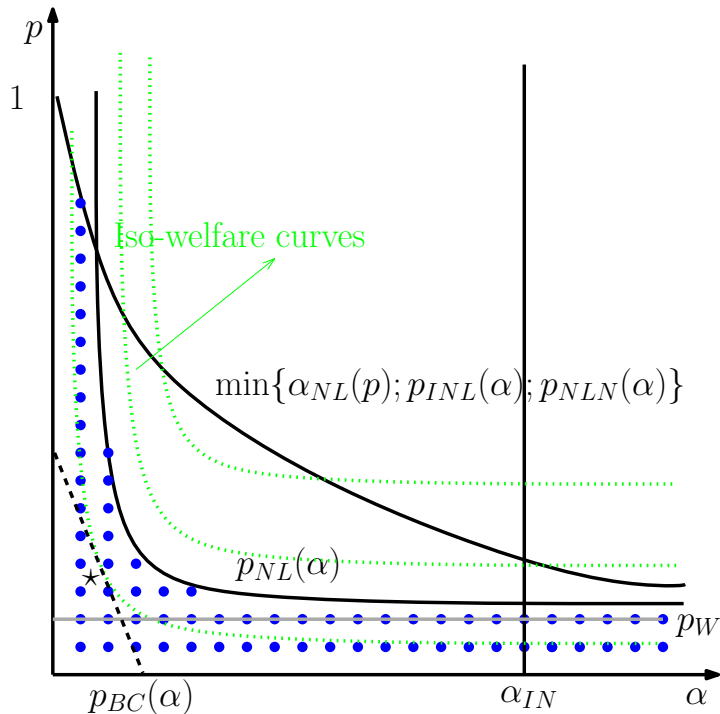


Figure 7: Proposition 5 (ii). The AA implements CNL whenever firms play the dominant equilibrium CNL and the welfare obtained from this equilibrium is higher than the alternatives. It then selects the optimal policy given these restrictions yielding (α^*, p^*) . The figure depicts an interior solution.

5 Comparison with Motta and Polo (2003)

As mentioned before, the present framework builds on Motta and Polo (2003). The main point of departure is that I account for an important characteristic of leniency procedures in allowing firms to collect incriminating evidence. Evidence, e , helps the AA in two ways: it creates an incentive for higher quality information while at the same time allows for new evidence that complements its own prosecution resources (p). The model in Motta and Polo (2003) is a special case of my framework. To obtain the authors' model, set $e = 0$ and replace sF by R , where R is now the leniency fine (unconditional case). To resume, in the present framework $R(e) = R - er$ and in Motta and Polo (2003) the lenient fine is simply R .

In this section, I investigate the optimality of conditioning the fine with the evidence. Results show that both firms and AA gain from this addition. Proposition 6 describes the conditions under which firms will prefer applying for leniency for a wider ranger of parameters

when the AA conditions on the evidence turned in. As a reminder, the “political” fund, is an independent fund usable by the AA, if needed, to pay firms that come forward.

Proposition 6. *Conditioning the fine to be paid under leniency induces firms to apply more often for leniency whenever the “political” fund is big enough.*

Proof. See Appendix. □

In this simple framework, as shown in the Proposition 6, the additional tool of conditioning on the quality of information once an investigation opens widens the region where the equilibrium *collude and reveal information* is preferred by firms. This feature is explainable by principally one reason. The additional tool is favored by firms since it decreases the amount to be paid under leniency.

In Propositions 7 and 8, I perform a similar analysis but this time for the other two equilibria.

Proposition 7. *Conditioning the fine to be paid under leniency induces firms to prefer less to collude and not use leniency.*

Proof. See Appendix. □

Proposition 8. *Conditioning the fine to be paid under leniency induces firms to prefer less no-collusion.*

Proof. Follows from the analysis of α_{IN} in Proposition 6. □

Proposition 8 is an undesirable feature of the model where, again, by decreasing the lenient fine, firms will prefer to collude and reveal information to not colluding simply because it pays more to collude and reveal information while facing a lower fine.

On a slightly different topic, one can notice that for the equilibrium CRI in Motta and Polo (2003) the optimal policy is to set $R = 0$, while in my framework $R(e^*) < 0$, hence we might be tempted to inquiry about the optimality of setting $R < 0$ (called courageous leniency). Proposition 9 discusses such a possibility. Courageous leniency was first introduced by Spagnolo (2004) in a related context. The author defines the concept in an environment where the leniency program is self-financed by the cartel members caught because of the leniency applicants. Optimally, only the first applicant receives courageous leniency and the (negative) fine equals the sum of the collectively received fines. To my knowledge, Spagnolo (2004) and the present study, in a different settings, constitute the only two papers obtaining as an equilibrium that the antitrust agency should remunerate leniency applicants.

Proposition 9. *Courageous leniency is almost never optimal under unconditional leniency ($R \leq -R_1$), while always optimal under conditional leniency.*

where:

$$R_1 \equiv \frac{\pi_M - \pi_N}{1 - \delta}$$

Proof. See Appendix. □

Proposition 9 exposes a mathematical result that simply occurs because of the additional component in the fine. This conditionality on the evidence renders possible the usage of the “political fund”. No jurisdictions around the world use courageous leniency. As one can imagine, in practice, it is far from trivial to justify its usage as it is already so costly to lead a successful investigation even with direct insider evidence.

After discussing the additional benefit on the firms side and some induced differences between Motta and Polo (2003) and the present framework, I address the most relevant question: “Should the antitrust agency conditions on the quality of the evidence?”

Leniency policies were designed to deter and force desistance of colluding practices. Deterrence in the present framework implies an accrued welfare from sustaining the *non-collusive* equilibrium when conditioning on the evidence. As a corollary of Proposition 8, the additional variable “evidence” of the fine shrinks the region where the equilibrium no-collusion is preferred by firms. Hence, for the same budget constraint the antitrust agency has lower possibility to choose from to sustain the equilibrium. Corollary 1 formally states this result.

Corollary 1. *If the goal of the antitrust agency is to deter colluding practices conditioning the fine on the quality of the evidence destroys possibilities of reaching this goal, hence decreases welfare.*

Proof. Follows from the above discussion. □

Fortunately, the situation is different in terms of desistance. Forcing desistance of the colluding practices in the present framework implies an accrued welfare from sustaining the *leniency* equilibrium when conditioning on the evidence. Proposition 6 showed that the leniency equilibrium region is wider when conditioning on the evidence. The AA has then a wider range of policy choices, in terms of α and p , to maximize societal welfare W_{CRI} . At the same time, the AA, when conditioning, faces the additional societal cost coming from the collection of the evidence $C(e)$. Hence, with a sufficiently low marginal cost of collecting the evidence, society ultimately benefits from the conditionality on the evidence. Corollary 2 resumes the reasoning.

Corollary 2. *If the goal of the antitrust agency is to force desistance of colluding practices conditioning the fine on the quality of the evidence increases welfare when the marginal cost of collecting the evidence tends to zero.*

Proof. Follows from the above discussion. □

I discuss the model policy implication in the next section.

6 Policy Implication

Several lessons can be learnt from the above discussions. Most importantly, I provide justification for the usage of leniency fines based on the quality of evidence turned in. An antitrust agency providing the right incentive for high quality evidence will benefit from it. Compared with setting the lenient fine unconditionally on this information, it favors the application for leniency and induced alternatives to be less profitable while, at the same, saving the agency the cost of successfully prosecuting colluding firms.

Under reasonable assumptions, I also reinforce the benefit of courageous leniency (money in exchange for information). Again, this result is striking and relies on the disentanglement of the two parts of the lenient fine: a level effect (sF) and a marginal effect (r). This allows the agency to optimally shut down the level effect and solely rely on the marginal effect (the quality of the information) when sustaining the dominant SPNE *collude and reveal information* equilibrium.

Going back to the definitions of those parameters allows for alternate interpretation of the results. Recall that the level effect represents all non-evidence based factors taken into consideration to set the optimal fine. In the European Commission Leniency Policy, the leniency percentage discount applicable depends mainly on three components: the evidence (“significant added value”), the timing and the level of cooperation. I have made the implicit assumption in the present model that those three components are uncorrelated. Hence, by optimally shutting down the level effect (non-direct-evidence based factors, such as the timing and the level cooperation of an applicant) and by setting the marginal effect to the highest possible value, I am assigning a weight to those three components. This weight will be different than one and definitely bigger than zero for all three components.

As a corollary to this last point, I address an interesting puzzle of competition policy: “Why colluding firms keep incriminating evidence ?” (Brenner, 2009) . According to the reality of the model, firms benefit more from collecting evidence in terms of fine reduction than the alternatives (e.g. of being the first applicant or the level of cooperation). Similarly to a contract literature, when antitrust agency increases their sensitivity to the evidence handed in, it brings forth firms to collect higher quality information.

7 Conclusion

I study the impact of the quality of evidence given by firms to the antitrust agency when taking advantage of leniency programs. The discussion is set in an environment corresponding with the current European Commission (E.U.) and the Competition Bureau (Canada) in allowing firms to benefit from leniency based on the quality (quantity) of their evidence once an investigation has opened. Analytically, I use repeated game theory in a game similar to the prisoner's dilemma where colluding firms can be detected and may collect information about the illegal activities to alleviate the full fine paid otherwise.

Although simple, my framework sheds light on the main motivation for a firm handing in higher quality information in the form of a higher reward. The results show that by having an antitrust agency more sensitive to the evidence, firms will use more often the leniency program, while at the same time turn in higher quality information. As a corollary, I show that by conditioning the fine to be paid on the quality of the evidence, firms will apply more often for leniency than they would in the unconditional case. I also give a justification to the usage of courageous leniency (i.e., paying firms to come forward).

The present paper also raises questions about the usage of moderate leniency. Future work should aim at addressing the issues of the timing of the applicant, the heterogeneity of firms and should empirically investigate the effect of the information revealed on firms' deterrence.

The model is a rather static one even given the plurality of periods. It is static in the sense that the information given away to the antitrust agency is completely independent of the rivals collected information. As is typically the case, once a *Race to the Courthouse* is underway, all firms want to benefit from leniency. Cooperation, timing and the information in hand ultimately dictate how leniency will be the agency. In the present paper, I have addressed the latter leaving for future work the degree of cooperation and the timing of applicant. I think, the most promising avenue is the one of timing issues, since firms have a new incentive to move according to the information held by the other firms.

On firms' asymmetry, an interesting idea will be to consider the effect of a leading firm (big fish) holding more information than smaller colluding firms (small fishes) holding less information. Now, the big firm can either manipulate the information or use its effect on colluding firms. At the same time, the leading firm can react using its more accurate information by using side payments to its smaller rivals to sustain the colluding agreement more efficiently. This information will act as a device to counterbalance the effect of the leniency policy.

Finally, it will be nice to estimate empirically the impact of incriminating evidence on firms' behavior and on the leniency of the fine granted. Brenner (2009) addresses a similar issue. He tests and finds support for the hypothesis that under leniency the total amount of fine paid by prosecuted firms is higher than the total amount in the pre-leniency era. The total amount of fine acts as a proxy for the information revealed since better documented cases (with the help of leniency) lead to higher financial penalty from colluding firms (an average difference of €31 million). Of course it can be argued that this proxy is inadequate as firms having more information in hand may more efficiently "game" the system by stabilizing the cartel agreement. More work on moderate leniency remains to be done while at the same time more empirical work in general on the topic should be a priority, if we are to understand the impact of leniency policy²⁶.

²⁶I document the effectiveness of the Corporate Leniency Program in Cloutier (2011).

8 Appendix

Proof: Lemma 3

Proof. The proof is straightforward from the above discussion. The condition $\alpha \leq \alpha_I$ comes from eliminating the first possible profitable deviation in $t = 1$, that is $V_{CRI} \geq V_D$. Similarly, the condition $p \geq p_I(\alpha)$, for $t = 1$, is obtained from $V_{CRI} \geq V_{DNL}$. Other possible profitable deviations are constrained by imposing $\delta \geq \delta_0$. The rest of the argument shows that the parameter space is non-empty. I start with α_I . The polynomial describing α_I is of the form:

$$g(x) = -Ax^2 + Bx - C = 0, \text{ where } A, B, C > 0$$

I investigate the property of $g(x)$. The main concern is that the roots are not in $x \in [0, 1]$. First, $g(x)$ is a concave function ($g''(x) < 0$) with its maximum value in $x > 0$ (at $x^* = \frac{2A}{B} > 0$ precisely). Since $g(0) = -C < 0$, I conclude that the roots are in the range $x > 0$. I then impose a positive determinant ($\Delta = B^2 - 4AC > 0$) for obtaining two real roots. The condition is given by:

$$F > F_{\Delta 1} \equiv \frac{2r^2(1-\delta)[\pi_M - \pi_D(1-\delta) - \delta\pi_N]}{cs^2} - \frac{\pi_M - \pi_N}{s}$$

Finally, since this polynomial has two roots in the range $x > 0$, one needs to check if can be the case that both roots be in $x \in [0, 1]$. Remember that the quadratic formula for computing the roots of $g(x)$ is:

$$x = \frac{-B \pm \sqrt{\Delta}}{-2A}$$

It is straightforward exercise to check that only a single root can be such that $x < 1$ with a sufficiently high fine F . So the small root will be this single root if the following inequality hold: $A + C < B$, rewritten most precisely as

$$F > F_{R1} \equiv \frac{\frac{r^2\delta^2}{2c} - (\pi_D - \pi_M)}{\delta s}$$

I can then conclude that $g(x) = 0$ yield a single root in the range $[0, 1]$ with the minimal requirement that $F > \max\{F_{\Delta}, F_R\}$. The second check for non-emptiness is for $p \geq p_I(\alpha)$. To have an equilibrium, there is no floor value for $p_I(\alpha)$, but only a ceiling value of one: $p_I(\alpha) < 1, \forall \alpha \in [0, 1]$. Finally, it is a straightforward exercise to show that $p_I(0) = 1, \frac{\partial p_I(\alpha)}{\partial \alpha} < 0$ and $\frac{\partial^2 p_I(\alpha)}{\partial \alpha^2} < 0$, since the inequality $1 - \frac{\alpha \delta r}{c} > 0$ always hold ($e \in [0, 1]$). Hence, $p \geq p_I(\alpha)$ is non-empty. \square

Proof: Lemma 4

Proof. The proof is straightforward from the above discussion. The condition $p \leq p_{NL}(\alpha)$ comes from eliminating the first possible profitable deviation in $t = 1$, that is $V_{CRI} \geq V_D$. Similarly, the condition $\alpha \leq \alpha_{NL}(p)$, for $t = 1$, is obtained from $V_{CNL} \geq V_{DRI}$. Other possible profitable deviations are constrained by imposing $\delta \geq \delta_0$. The rest of the argument shows that the parameter space is non-empty. I start with $p_{NL}(\alpha)$. The only requirement here is that $p_{NL}(\alpha) \geq 0$. This condition is always satisfied once imposed $\delta \geq \delta_0$ implying a non-negative numerator. Similarly for $\alpha_{NL}(p)$, I only need to check that $\alpha_{NL}(p) \geq 0$. This is always satisfied for $p \leq s$ and won't be an issue as the optimal policy will be to set $s^* = 1$ (to come later). Hence, the solution region is non-empty. \square

Proof: Proposition 1

Proof. Follows from Lemmas 1, 3 and 4 with the additional conditions: $V_{NC} \geq V_{CRI}$, $V_{NC} \geq V_{CNL}$, and $V_{CRI} \geq V_{CNL}$ yielding $\alpha \geq \alpha_{IN}$, $p \geq p_{NLN}(\alpha)$, and $p \geq p_{INL}(\alpha)$. The rest of the argument shows that the parameter space is non-empty. For those three curves, conditions have to be imposed for characterizing the dominant equilibria from the point of view of the firms and ensuring that the solution fits in $[0, 1]$. I start with $p_{INL}(\alpha)$. Imposing that $p_{INL}(\alpha) \geq 0$ boils down to the following condition:

$$F \geq F_{INL} \equiv \frac{r^2\delta(1-\delta) - 2c(\pi_M - \pi_N)}{2cs(1-\delta)}$$

On the other side, the condition $p_{INL}(\alpha) \leq 1$ is always satisfied by straightforward algebraic manipulation. The case of α_{IN} follows the analysis performed for α_I . The polynomial describing α_{IN} is of the form:

$$f(x) = -Ax^2 + Bx - C = 0, \quad , \text{ where } A, B, C > 0$$

I investigate the property of $f(x)$. First, note that $f''(x) < 0$ and $f(0) < 0$. I conclude that the roots are in the range $x > 0$. I then impose a positive determinant ($\Delta = B^2 - 4AC > 0$) for obtaining two real roots. The condition is given by:

$$F > F_{\Delta 2} \equiv \sqrt{\frac{2r^2(\pi_M - \pi_N)}{cs^2(1-\delta)}} - \frac{\pi_M - \pi_N}{s(1-\delta)}$$

Finally, since this polynomial has two roots in the range $x > 0$, one needs to check if can be

the case that both roots be in $x \in [0, 1]$. Remember that the quadratic formula for computing the roots of $f(x)$ is:

$$x = \frac{-B \pm \sqrt{\Delta}}{-2A}$$

It is straightforward exercise to check that only a single root can be such that $x < 1$ with a sufficiently high fine F . So the small root will be this single root if the following inequality hold: $A + C < B$, rewritten most precisely as

$$F > F_{R2} \equiv \frac{r^2 \delta^2}{2c} + \frac{(\pi_M - \pi_N)}{\delta s}$$

I can then conclude that $f(x) = 0$ yield a single root in the range $[0, 1]$ with the minimal requirement that $F > \max\{F_{\Delta 2}, F_{R2}\}$. Finally, I study the non-emptiness of $p_{NLN}(\alpha)$. The existence of a solution in $[0, 1]$ is given by

$$F \geq F_{NLN} \equiv \frac{\pi_M - \pi_N}{\delta}$$

This condition comes from imposing an intersection between $p = 1$ and $p_{NLN}(\alpha)$, where the correspondent α fits in $[0, 1]$. From the above discussion, the proposition only holds if

$$F > \underline{F} \equiv \max\{F_{\Delta 1}, F_{\Delta 2}, F_{R1}, F_{R2}, F_{INL}, F_{NLN}\}$$

□

Proof: Lemma 5

Proof. To conclude on a larger region where the strategy no-collusion is a SPNE, I look at curves delimiting this equilibrium. Two curves are delimiting that region: α_{IN} and $p_{NLN}(\alpha)$

Starting by α_{IN} , I first write down the simplyfied polynomial $f(x) = -Ax^2 + Bx - C$, where $A, B, C > 0$. This polynomial function has two zeros: one where $x < 0$ and one where $x > 0$. Since the interest is on the region where the polynomial is negative and such that $x \leq 1$, I look at the property of the function $x = \frac{B - \sqrt{B^2 - 4AC}}{2A}$. From the parametrization of the polynomial, an increase in F is equivalent to an increase in B , implying a root closer to the y-axis, hence α_{IN} is lower.

$$\frac{\partial x}{\partial B} = \frac{\sqrt{B^2 - 4AC} - B}{2A} < 0$$

For $p_{NLN}(\alpha)$, the computation is straightforward: an increase in F implies a lower area under the curve.

$$\frac{\partial p_{NLN}(\alpha)}{\partial F} = -\frac{(1-\delta)[\pi_M - \delta\pi_N]}{\alpha\delta[\pi_M + F(1-\delta) - \pi_N]^2} < 0$$

As for the CRI part, F enters the three following curves delimitating the equilibrium: $p_I(\alpha)$, p_{INL} and α_I . I start with α_I . Recall $g(x)$ from Lemma 3's proof. The root of interest (discriminant given by: $\Delta = B^2 - 4AC$) can be written as:

$$x = \frac{B - \sqrt{\Delta}}{2A}$$

The partial derivative of x is negative implying a root closer to the y-axis with higher F .

$$\frac{\partial x}{\partial B} = \frac{\sqrt{B^2 - 4AC} - B}{2A} < 0$$

As for $p_I(\alpha)$ and p_{INL} , it is straightforward to see that $\frac{\partial p_I}{\partial F} > 0$ and $\frac{\partial p_{INL}}{\partial F} > 0$. Hence, an increase in F shrinks the region where the strategy CRI is an equilibrium.

As for the CNL part, F enters three following curves delimitating the equilibrium: $p_{NL}(\alpha)$, $\alpha_{NL}(p)$ and $p_{INL}(\alpha)$. The analysis for the first curve is straightforward and it follows that an increase in F will move downward the curve. As for $\alpha_{NL}(p)$, an increase in F move the curve upward. This is a counter-intuitive result. From the formulation of $p_{INL}(\alpha)$, an increase in F moves the curve downward. By increasing F the curve $\alpha_{NL}(p)$ will move upward leaving binding the two other incentive constraints. As we will later see in the antitrust agency section, this problem vanishes by nothing that $p_{NL}(\alpha)$ is an iso-welfare curve. As long as the curves ($\alpha_{NL}(p), p_{INL}(\alpha)$ and $p_{NL}(\alpha)$) intersect in $[0, 1]^2$, no problem will arise. \square

Proof: Lemma 6

Proof. For the NC part of the proposition, s only enters the curve α_{IN} . The proof is identical to Lemma 5. Similarly for CRI, it follows from Lemma 5, where the effect is identical. Finally, for the equilibrium CNL, it follows from the analysis on $\alpha_{NL}(p)$ in Lemma 5. \square

Proof: Lemma 7

Proof. Four curves delimitating the equilibrium CRI are affected by a change in c : $p_I(\alpha)$,

p_{INL} , α_{IN} , and α_I . I start with α_I . Again, the root of interest is given by:

$$x = \frac{B - \sqrt{\Delta}}{2A}$$

The partial derivative with respect to A is positive.

$$\frac{\partial x}{\partial A} = \frac{\overbrace{B[B - \sqrt{\Delta}] - 2AC}^{>0}}{4A^2} > 0$$

Similarly for α_{IN} where, Again, the root of interest is given by:

$$x = \frac{B - \sqrt{\Delta}}{2A}$$

From straightforward calculation and since $A, B, C > 0$, I get that the partial derivative with respect to A is positive.

$$\frac{\partial x}{\partial A} = \frac{B[B - \sqrt{\Delta}] - 2AC}{4A^2\sqrt{\Delta}} > 0$$

As for the curve $p_I(\alpha)$, I get the desire result from straightforward calculation:

$$\frac{\partial p_I(\alpha)}{\partial c} = -2\alpha r F \delta \overbrace{[2c - \alpha \delta r]}^{>0} - 4\alpha c F^2 \delta (1 - s) < 0$$

Finally, it is straightforward to see that a decrease in c negatively increase the slope of the curve p_{INL} . A decrease in c creates a larger region where CRI is an equilibrium. \square

Proof: Lemma 9

Proof. The only curve determining NC equilibrium and affected by a change in r (or c) is α_{IN} . From Lemma 8, it implies that the NC equilibrium region shrink as r increases (c decreases).

As for the CNL part, two curves are affected by such a change: $\alpha_{NL}(p)$ and $p_{INL}(\alpha)$. The analysis is similar to Lemma 5. \square

Proof: Proposition 2

Proof. I start with the NC. The optimal value of sF and r follow from Lemmas 5, 6 and 9. As for CNL, it follows from Lemmas 5, 6, and 9. Finally, for CRI, from Lemmas 5, 6,

7, and 8, I set values for $\{s, r, F\}$, where r is bounded above by $R(e^*) \geq -M_R$. Hence, $r^* = \bar{r} \equiv \sqrt{\frac{cM_R}{\alpha^*\delta}}$ \square

Proof: Proposition 3

Proof. The budget constraint (BC) of the AA is given by: $Ap + B\alpha \leq M$. Rewriting the BC in terms of α , I get $p \leq p_{BC}(\alpha) \equiv \frac{M-B\alpha}{A}$. I start with NC. Recall that the welfare under that equilibrium is $W_{NC} = \frac{SDWL}{1-\delta}$, the budget constraint is $p \leq p_{BC}(\alpha)$ and the solution region established in Proposition 1. The key fact here is the BC. Since the welfare attached to the equilibrium NC for the AA is the highest of the three strategies, the only requirement is feasibility. Hence, if the BC crosses the solution region for NC, $p_{BC}(\alpha_{IN}) \geq p_{NLN}(\alpha_{IN})$, any feasible point is an optimal policy. So, any point such that $\alpha^* \geq \alpha_{IN}$ and $p^* \in [p_{NLN}(\alpha^*), p_{BC}(\alpha^*)]$ is an optimal policy. \square

Proof: Proposition 4

Proof. For the the AA to prefer the equilibrium CRI over NC, it must the case that NC is not feasible: $p_{BC}(\alpha_{IN}) < p_{NLN}(\alpha_{IN})$. Similarly, it must the case that CRI pareto dominates the equilibrium CNL. This is satisfied when $W_{CRI} > W_{CNL}$. As for the optimal policy, this is a little trickier. To do so, I first build the iso-welfare curves. I write $W = f(\alpha)$, where

$$f(\alpha) \equiv -C(e^*(\alpha)) + \alpha\delta K$$

This implies that $f(0) = 0$, $f''(\alpha) = 0$ and the first derivative $f'(\alpha) > 0$ when $M_R < 2K$. So, whenever $M_R < 2K$, iso-welfare curves are straight lines increasing with α . It turns out that to have CRI Pareto dominating CNL, the following inequality has to be satisfied:

$$W_{CRI}(\alpha) > W_{CNL}(\alpha, p) \Leftrightarrow p < p_W \equiv 1 - \frac{M_R}{2K}$$

Hence, if a solution exists, it will satisfy

$$\alpha^* = \left\{ \alpha : p_{BC}(\alpha) = \max \{p_I(\alpha); p_{INL}(\alpha)\} \leq p_W, \alpha \leq \min\{\alpha_I; \alpha_{IN}\} \right\} \text{ and } p^* = p_{BC}(\alpha^*)$$

This solution assumes that p_W intersects with $p_{BC}(\alpha)$ at an higher value in terms of α than $\max\{p_I(\alpha); p_{INL}(\alpha)\}$ intersects with $p_{BC}(\alpha)$. Whenever the intesection of $\max\{p_I(\alpha); p_{INL}(\alpha)\}$ and $p_{BC}(\alpha)$ is such that $\alpha \leq \min\{\alpha_I; \alpha_{IN}\}$, it must be the case that CRI Pareto dominantes CNL. \square

Proof: Proposition 5

Proof. For the the AA to prefer the equilibrium CRI over NC, it must the case that NC is not feasible: $p_{BC}(\alpha_{IN}) < p_{NLN}(\alpha_{IN})$. Similarly, it must the case that CNL pareto dominates the equilibrium CNL. This is satisfied when $W_{CNL} > W_{CRI}$. As for the optimal policy, I first build the iso-welfare curves: $p = p_{WCNL}(\alpha) \equiv \frac{W}{\alpha\delta K}$. The iso-welfare curves has as an element the curve $p_{NL}(\alpha)$. From the proof of Proposition 4, I have established that CNL is dominated by CRI if $p \leq p_W$, so in the case of interest, I simply inverse the relation to get CRI dominated by CNL when $p \geq p_W$. So two cases might happen, either the budget constraint crosses the uppercountour delimited by $p_{NL}(\alpha)$ or there is a tangent point between an iso-welfare curve and the budget constraint represented the two solutions:

- (i) $\alpha^* = \{\alpha : p_{BC}(\alpha) = p_{NL}(\alpha) \geq p_W\}$ and $p^* = p_{BC}(\alpha^*)$
- (ii) $\alpha^* = \{\alpha : p'_{BC}(\alpha) = p'_{WCNL}(\alpha), p_{BC}(\alpha) \geq p_W\}$ and $p^* = p_{BC}(\alpha^*)$

In (i), the budget constraint crosses the uppercounter $p_{NL}(\alpha)$ and the intersection happens above $p_W(\alpha)$. Finally, in (ii), there is a tangency between the budget constraint and the iso-welfare and this happens above $p_W(\alpha)$. □

Proof: Proposition 6

Proof. The proof is essentially a comparison of the solution region at optimality under the two regimes: $R(e)$ and R . To do so, I compare Motta and Polo (2003)'s result and mine.

The equivalent to α_I is α_I^M . It is straightforward to show that $\alpha_I \geq \alpha_I^M$.

$$\alpha \leq \alpha_I^M \equiv \frac{\pi_M - \pi_D(1 - \delta) - \pi_N}{\delta[\pi_M + R(1 - \delta) - \pi_N]}$$

Rewritting the inequality describing α_I , I get that:

$$\alpha \leq \alpha_I^M + \left\{ \frac{\alpha^2 \delta^2 r^2 (1 - \delta)}{2c\delta[\pi_M + R(1 - \delta) - \pi_N]} \right\}$$

Hence, α_I is necessarily bigger or equal than α_I^M .

The equivalent to $p_I(\alpha)$ is the inequality $R \leq F$. It is a straightforward exercise to show that whenever M_R is big enough, the constraint $p \geq p_I(\alpha)$ is identical to $p \geq 0$, which means

that the constraint never binds.

The equivalent to $p_{INL}(\alpha)$ is $p_{INL}^M(\alpha)$, where:

$$p \geq p_{INL}^M(\alpha) \equiv \frac{\pi_M + R(1 - \delta) - \pi_N}{\pi_M - \pi_N + F(1 - \delta)}$$

Rewriting $p_{INL}(\alpha)$, I get that:

$$p_{INL}(\alpha) = p_{INL}^M(\alpha) - \left\{ \frac{\alpha \delta r^2 (1 - \delta)}{2[\pi_M - \pi_N + F(1 - \delta)]} \right\}$$

Hence, $p_{INL}(\alpha) \leq p_{INL}^M(\alpha)$.

Finally, the equivalent α_{IN} is α_{IN}^M , where:

$$\alpha \leq \alpha_{IN}^M \equiv \frac{\pi_M - \pi_N}{\delta[\pi_M - \pi_N + R(1 - \delta)]}$$

Rewriting the inequality for α_{IN} , I get that:

$$\alpha \leq \alpha_{IN}^M + \left\{ \frac{\alpha^2 \delta^2 r^2 (1 - \delta)}{2c} \right\}$$

Hence, α_{IN} is necessarily bigger or equal than α_{IN}^M .

□

Proof: Proposition 7

Proof. Two defining CNL solution frontlines are determined by $R(e)$ and subject to the change of regime. The case of $p_{INL}(\alpha)$ has been dealt with in the proof of Corollary 1. The case of interest here is what happen to the profitable deviation to collude and reveal information. In Lemma 4, it has been establish that the solution region is bounded by:

$$\alpha \leq \alpha_{NL}(p) \equiv \frac{2F(s - p)c(1 - \delta)^2}{\delta \left\{ r(1 - \delta) - (1 - p)(\pi_M - \pi_N) \right\}^2}$$

The parallel in Motta and Polo (2003)'s paper is:

$$p \leq p_{NL}^M(\alpha) \equiv \frac{\pi_M - \pi_N + (1 - \delta)R}{\pi_M - \pi_N + (1 - \delta)F}$$

The optimal policy as described in Motta and Polo (2003) is to set $R = F$ implying that $p_{NL}^M(\alpha) = 1$. Since $\alpha_{NL}(p)$ is decreasing in p and either cross or not the line $p = 1$ at a point $\alpha \leq 1$. I then obtain that either the solution region shrink or is identical for that curve. Adding to this the effect of $p_{INL}(\alpha)$, I get the desired result. \square

Proof: Proposition 9

Proof. For the non-conditioning case, recall $\alpha_I^M(R)$. The global maximum of this function with respect to R happens whenever the denominator is as close to zero as possible. The denominator tends to zero as R tends to $R_1 \equiv \frac{\pi_M - \pi_N}{1 - \delta}$.

For the conditioning case, since at optimality $\{s^*, r^*, F^*\} = \{0, \bar{r}, \underline{F}\}$, it implies that $R(e^*) < 0$. \square

References

- Brenner, S. (2009). An Empirical Study of the European Corporate Leniency Program. *International Journal of Industrial Organization* 27.
- Cloutier, M. (2011). An Empirical Investigation of the U.S. Corporate Leniency Program. Working Paper.
- Feess, E. and M. Walzl (2010). Evidence Dependence of Fine Reductions in Corporate Leniency Programs. *Journal of Institutional and Theoretical Economics* 166.
- Hammond, S. D. (2009). Recent Developments Relating to the Antitrust Divisions's Corporate Leniency Program. *National Institute on White Collar Crime, San Francisco CA, March 5*.
- Harrington, J. (2008). Optimal Corporate Leniency Programs. *Journal of Industrial Economics* LVI(2).
- Mailath, J. and L. Samuelson (2006). *Repeated Games and Reputations: Long-Run Relationships*. Oxford University Press.
- Miller, H. N. (2009). Strategic Leniency and Cartel Enforcement. *American Economic Review* 99.
- Motchenkova, E. (2009). Effects of leniency programs on cartel stability: A timing game approach. *CentER Discussion paper 2004-98*.
- Motta, M. and M. Polo (2003). Leniency Programs and Cartel Prosecution. *International Journal of Industrial Organization* 21.
- OECD (2005). Hard Core Cartels: Third report on the implementation of the 1998 Council Recommendation.
- Spagnolo, G. (2004). Divide et Impera: Optimal Leniency Programs. *CEPR Discussion Paper No.4840*.
- Spagnolo, G. (2008). Leniency and Whistleblowers in Antitrust. *Handbook of Antitrust Economics (The MIT Press, Cambridge, Massachusetts, U.S.A.)*.