Monetary Transmission Mechanism in a Small Open Economy: A Bayesian Structural VAR Approach

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Abstract

This paper develops an open-economy Bayesian structural VAR model for Canada in order to estimate the effects of monetary policy shocks, using the overnight target rate as the policy instrument. I allow the policy variable and the financial variables of the model to interact simultaneously with each other and with a number of other home and foreign variables. When I estimate this over-identified VAR model, I find that the policy shock transmits to real output through both the interest rate and exchange rate channels, and the shock does not induce a departure from uncovered interest rate parity. I also find that the impulse response of the monetary aggregate, $M_1$, does not exactly follow the impulse response of the target rate. Finally, I find that Canadian variables significantly responds to the US federal funds rate shock, and external shocks are an important source of Canadian output fluctuations.

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Keywords: Monetary policy, structural VAR, block exogeneity, impulse response
1. Introduction

This paper identifies monetary policy shocks employing a Bayesian structural VAR model for a small open economy, using Canada as a case study. To identify the monetary policy function, I follow the general procedure of Cushman and Zha (1997) and Kim and Roubini (2000), but change it in a number of respects. First, unlike these authors, I do not use money in the basic model and therefore do not define a money demand function or a money supply function. Instead, I use the overnight target rate as the policy instrument, which is what the Bank of Canada actually uses to conduct monetary policy. The estimation of the basic model without money also gives me the opportunity to check the robustness of the impulse responses and to study the role of money, after estimating an extended model that incorporates money. Second, in order to increase the precision of the model identification, I allow the policy variable and the financial variables of the model to interact simultaneously with each other and with a number of other home and foreign variables within the month. Finally, since the over-identified structural VAR model developed in this paper entails simultaneous interactions, in order to obtain accurate statistical inference I employ a Bayesian Gibbs sampling method to estimate the posterior distribution of the model parameters.

I argue that it is difficult to measure changes in monetary policy by shocks to money supply or market interest rates, since these shocks might reflect some other shocks in the economy. In contrast, unlike money and market interest rates, the overnight target rate is under the sole control of the Bank of Canada. Therefore, shocks to the target rate cannot be influenced by private-sector behaviour, except through the channel of an endogenous policy response of the central bank to changing economic conditions. Since the structural VAR model developed in this paper explicitly identifies the policy reaction function as a function of all key home and foreign variables, I am able to estimate policy innovations to the overnight target rate that are orthogonal to these variables.

I assume that the Bank of Canada contemporaneously reacts to some foreign variables, such as the federal funds rate and the world export commodity price index, in addition to the domestic nominal market interest rate and the exchange rate. I use the world export commodity price index as a proxy for inflationary expectations. I allow the nominal interest rate to react contemporaneously to the financial variables of the model, assuming that financial variables interact instantaneously with each other. I also assume that an efficient foreign-exchange market has all the relevant information within the month and let the exchange rate react to all variables in the model contemporaneously. The use of the structural VAR ap-
proach, as opposed to the recursive VAR approach, enables me to capture these economically meaningful cross-directional relationships among variables. Following Cushman and Zha (1997) and Zha (1999), I treat the foreign variables as exogenous from Canada’s point of view in order to maintain the small-open-economy assumption strictly.

This identification approach involves a great deal of simultaneity among the contemporaneous variables. Therefore, the shape of the posterior density of the model parameters tends to be so non-Gaussian that the widely used importance sampling technique of obtaining finite-sample inferences from the posterior distribution becomes inefficient. In order to obtain accurate statistical inferences from the parameter estimates and subsequent forecasts, I estimate the model employing the Waggoner and Zha’s (2003) Bayesian Gibbs sampling method that incorporates prior information into the VAR following Sims and Zha (1998). The other advantage of this method is that it can be efficiently used for this over-identified structural VAR model that has a restricted covariance matrix of the reduced-from residuals as well as restrictions on the lagged coefficients.

When I apply this model to the Canadian data, I find that the monetary policy shock transmits to real output through both the interest rate and exchange rate channels, as opposed to Cushman and Zha (1997), who found that the transmission operates through the exchange rate only. I also find that the higher return in Canadian currency, induced by the contractionary policy shock, is offset by the gradual depreciation of the Canadian dollar after an impact appreciation. This result differs from that of Eichenbaum and Evans (1995), who, in a recursive VAR model, found that the higher return in home currency is further magnified by a persistent appreciation of the exchange rate. This paper finds that the Bank of Canada responds to any home and foreign variables that embody information about future inflation. This reaction of the Bank is consistent with its inflation-targeting policy. I also find that US federal funds rate shocks have significant effects on Canadian macroeconomic variables, and external shocks are an important source of Canadian output fluctuations.

Since some previous studies, such as those by Gordon and Leeper (1994) and Bhuiyan and Lucas (2007), used money as the policy instrument, I investigate the role of money by incorporating the monetary aggregate, $M_1$, identified as a money demand function, into the basic model. When I re-estimate this model, I find that the impulse responses of all variables in the extended model remain unchanged from the basic model. The key finding of this extended model, however, is that the impulse response of $M_1$ does not exactly follow the
impulse response of the target rate. In particular, I find that the contractionary policy shock of increasing the target rate peaks almost immediately, while, following the same shock, the money stock keeps declining, the highest impact of which is not realized until the end of the second year. This imprecise response of the money stock casts further doubt on the rationalization of using money as the policy instrument and justifies the use of the overnight target rate.

The remainder of the paper is organized as follows: section 2 presents the context of the research, section 3 provides the data sources, section 4 describes the structural VAR model that identifies the exogenous monetary policy shock, section 5 presents the results, and section 6 draws conclusions.

2. Research Context

Sims (1980) suggested the use of impulse responses from the VAR model for policy analysis. Subsequently, a great deal of VAR literature has been developed to estimate the impulse responses of various macroeconomic variables due to monetary policy shocks (see Christiano, Eichenbaum and Evans (1999) for details). Bernanke and Blinder (1992) argued that innovations in the federal funds rate, identified in a recursive approach, are in some respects better measures of monetary policy shocks than are innovations in monetary aggregates for the US. This argument was challenged by Gordon and Leeper (1994). Using innovations both in the federal funds rate and monetary aggregates in a recursive approach, they found dynamic responses that are at odds with what we expect from monetary policy shocks. Identifying contractionary policy shocks with innovations in the ratio of non-borrowed reserves to total reserves in a recursive VAR model, Eichenbaum and Evans (1995) reported a persistent appreciation of the US dollar for a prolonged period of time. Using the same policy instrument in a recursive VAR approach for the US, Strongin (1995) found a strong liquidity effect but an insignificant effect on the price level.

Sims (1992) pointed out that innovations in any type of monetary aggregates may not correctly represent changes in monetary policy in the presence of money demand shocks and therefore suggested using innovations in short-term interest rates as policy shocks. Using short-term interest rates as policy instruments in a recursive approach for G-7 countries, Grilli and Roubini (1995) found that home currencies depreciate in response to innovations in home interest rates for every country except the US. For Canada, identifying contractionary policy
shocks with innovations in both the monetary aggregate, $M_1$, and the overnight target rate in a recursive VAR model, Bhuiyan and Lucas (2007) also found that the exchange rate depreciates due to these shocks.

The recursive approach of monetary policy identification might make some sense for the US, since it is a large and relatively closed economy and the movement of US monetary policy due to foreign shocks is relatively small. In addition, in closed economy models, such as those used by Christiano and Eichenbaum (1992) and Kim (1999), the monetary policy transmission mechanism operates primarily through the interest rate, not the exchange rate. Therefore, the conditions of recursive identification that are somewhat valid for the US are very unlikely to be valid for smaller and more open economies, since central banks of small open economies contemporaneously respond to movements in exchange rates and other foreign variables.

In an attempt to identify monetary policy more realistically, Sims and Zha (1995) proposed a structural VAR model for the relatively closed US economy. Cushman and Zha (1997) and Kim and Roubini (2000), among others, extended this structural model for more open economies. The gist of the structural approach is that, rather than relying solely on the recursive Choleski technique, it allows simultaneous interactions among the policy variable and other macroeconomic variables of the model within the month. Faust and Rogers (2003) also incorporated these standard assumptions of the structural VAR model using an inference procedure of identification. On the other hand, Bernanke, Boivin, and Eliasz (2004) used a factor-augmented VAR model that allows the monetary policy variable to interact with a large set of variables simultaneously.

Both Cushman and Zha (1997) and Kim and Roubini (2000) identified a money demand function and a money supply function in their models and treated the money supply equation as the reaction function of the monetary authority. While both studies developed structural models in an open-economy context, Cushman and Zha (1997)’s model incorporated the small-open-economy assumption strictly by treating the foreign block of variables as exogenous in the model. In addition, Cushman and Zha (1997) realistically allowed the policy reaction function to react to the Fed policy contemporaneously and conditioned the reaction function to more foreign variables.

Although Cushman and Zha’s (1997) model is more realistic than other existing struc-
tural models for a small open economy, there is room to build a better model for Canada under evolving circumstances. In 1994, the Bank of Canada adopted a target band and a target rate for the overnight rate on loans among banks and other financial institutions, which the Bank calls its main monetary policy instrument. The target band is of 50 basis points and is designed to allow for small and presumably temporary adjustments of the overnight rate to market conditions, while adjustments in the target rate are reserved for the implementation of changes in the monetary policy. Since the Bank of Canada controls the overnight target rate, innovations in this rate should be a more precise measure of monetary policy shocks.

Therefore, I build a structural VAR model using the target rate as the policy instrument. In addition, since some previous studies used different measures of monetary aggregates as policy instruments, it would be a useful exercise to build the basic model without money, so that, by extending the model to incorporate money, we can examine the robustness of the impulse responses and study the role of money. Another important dimension of improving existing structural VAR models is allowing more simultaneous interactions among the policy variable and the financial variables of the model. Since the information about financial variables is available to the monetary authority instantaneously, and since financial variables react to the policy variable and also react to each other contemporaneously, allowing simultaneous interactions among them will increase the precision of the model identification.

Having identified a monetary policy by incorporating these economically meaningful identifying assumptions into the structural VAR model, how much can we rely on its impulse response functions as measures of the dynamic responses of the macroeconomic variables? In response to the skepticism expressed by Chari, Kehoe, and McGrattan (2005) regarding the ability of the structural VAR model to document empirical phenomena, Christiano, Eichenbaum, and Vigfusson (2006) demonstrated that, if the relevant short-run identifying restrictions are justified, the structural VAR procedures reliably recover and identify the dynamic effects of shocks to the economy. In a recent paper, Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007) also demonstrated that, if the variables chosen by the econometricians are accurate and if the identifying restrictions are precise, then the impulse responses of the VAR model do a good job of portraying the dynamic behaviour of the macroeconomic variables due to shocks.

3. Canadian Monthly Data

The data runs monthly from 1994 to 2007. Over the years, the Bank of Canada has
shifted the way it conducts monetary policy. Since 1994, the Bank has been using the target for the overnight rate as its key monetary policy instrument. Therefore, I choose to run the sample from 1994 to 2007. All the data is collected from Statistics Canada’s CANSIM database and the International Monetary Fund’s *International Financial Statistics* (IFS). The variables are: \(i_0\), the overnight target rate (Cansim, V122514); \(i\), the three-month Treasury bills rate (Cansim V122529); \(s\), the logarithm of the nominal exchange rate in units of Canadian currency for one unit of US dollar (Cansim, B3400); \(\pi\), the annualized monthly inflation rate calculated from the consumer price index (Cansim, V737311); \(y\), the logarithm of the gross domestic product (GDP) (Cansim, V41881478); \(m\), the logarithm of the monetary aggregate, \(M_1\) (Cansim, V37199); \(i^*\), the US federal funds rate (IFS, 11164B.ZF..); \(y^*\), the logarithm of the US gross domestic product (GDP) (IFS, 11166..CZF..); \(\pi^*\), the annualized monthly US inflation rate calculated from the US consumer price index (IFS, 11164..ZF..); and \(wxp^*\), the logarithm of the world total export commodity price index (IFS, 06174..DZF..).

4. A Structural VAR Model with Block Exogeneity

In the first subsection, I develop the structural VAR model to identify the monetary policy of the Bank of Canada, and in the second subsection, I describe the Bayesian Gibbs sampling method of estimating the model.

4.1 Identification of Monetary Policy

Omitting constant terms, the standard structural system can be written in the following linear and stochastic dynamic form:

\[
Ax_t = \sum_{l=1}^{p} B_l x_{t-l} + \varepsilon_t, \tag{1}
\]

where \(x_t\) is an \(n \times 1\) column vector of endogenous variables at time \(t\), \(A\) and \(B_l\) are \(n \times n\) parameter matrices, \(\varepsilon_t\) is an \(n \times 1\) column vector of structural disturbances, \(p\) is the lag length, and \(t = 1, \ldots, T\), where \(T\) is the sample size. The parameters of the individual equations in the structural VAR model (1) correspond to the rows of \(A\) and \(B_l\). I assume that the structural disturbances have a Gaussian distribution with \(E(\varepsilon_t \mid x_1, \ldots, x_{t-1}) = 0\) and \(E(\varepsilon_t \varepsilon_t' \mid x_1, \ldots, x_{t-1}) = I\).

In the model, \(x\) comprises two blocks of variables—the Canadian block, \(x_1: [i_0, i, s, y, \pi]\) and the non-Canadian block, \(x_2: [y^*, \pi^*, i^*, wxp^*]\), where the variables in each block have
been defined in the previous section. For the sake of clarity, I rewrite the structural system (1) in the following matrix notation:

\[ Ax_t = F z_t + \varepsilon_t, \quad (2) \]

where \( z_t = [x_{t-1}, \ldots, x_{t-p}]' \) and \( F = [B_1 \ldots B_p] \). Here, \( z_t \) is the \( np \times 1 \) column vector of all lagged variables and \( F \) is the \( n \times np \) matrix of all lagged coefficients.

For the precision of the model identification, it is important to treat the relationship between the Canadian and the non-Canadian blocks of variables appropriately. Canada’s economy is about one-tenth of the size of the US economy, and about 75 percent of Canada’s exports go to the US, while only about 20 percent of US exports come to Canada. Therefore, it seems economically more appealing to treat the foreign block of variables as exogenous. Zha (1999) demonstrated that failing to impose such exogeneity restrictions is not only unappealing but also results in misleading conclusions. Incorporating the exogeneity assumption, the structural model (2) can be rewritten as follows:

\[
\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \quad (3)
\]

The restriction that \( A_{21} = 0 \) follows from the assumption that the Canadian block of variables does not enter into the non-Canadian block contemporaneously, and the restriction that \( F_{21} = 0 \) follows from the assumption that it does not enter into the non-Canadian block in lag. It is worth noting that this concept of block exogeneity is similar to Granger causal priority defined by Sims (1980) in the context of the reduced-form VAR. The reduced-form version of the structural model (2) can be written as follows:

\[ x_t = Ez_t + e_t, \quad (4) \]

where \( E = A^{-1}F \) and \( e_t = A^{-1}\varepsilon_t \). The block-exogeneity restrictions in the reduced-form model (4) is \( E_{22} = 0 \), which implies that \( z_{2t} \) is Granger causally prior to \( x_{1t} \) in the sense of Sims (1980).

I perform the likelihood ratio test to examine if the non-Canadian block is Granger causally prior to the Canadian block. With a lag length of eight, which is determined on the basis of the likelihood ratio test and the Akaike information criterion, the Chi-squared statistic is \( \chi^2(160) = 160.612 \), where 160 is the total number of restrictions on the non-Canadian block. This value of the Chi-squared statistic implies that the null hypothesis is...
not retained at a standard significance level. Therefore, any structural identification for small open economies that treats both the home and the foreign blocks of variables as endogenous, such as that of Kim and Roubini (2000), is likely to produce imprecise estimates resulting in misleading forecasts.

Let $\Sigma$ be the variance-covariance matrix of the reduced-form residuals, $e_t$. Since the structural disturbances, $\varepsilon_t$, and the regression residuals, $e_t$, are related by $\varepsilon_t = A e_t$, we can derive that $\Sigma = (AA')^{-1}$. To reveal the identifying restrictions on the contemporaneous-coefficients matrix $A$, I display the relationship between the reduced-form residuals and the structural shocks in the system of equations (5). These restrictions do not merely describe the relationships between the residuals and the structural shocks, but they also describe the contemporaneous relationships among the levels of these variables. I do not impose any restrictions on the lagged coefficients except the block-exogeneity restrictions on the foreign block of variables, as shown in the structural model (3).

\[
\begin{pmatrix}
\varepsilon_{i0} \\
\varepsilon_i \\
\varepsilon_s \\
\varepsilon_y \\
\varepsilon_{\pi} \\
\varepsilon_{\pi^*} \\
\varepsilon_{i^*} \\
\varepsilon_{wxp^*}
\end{pmatrix}
= 
\begin{pmatrix}
a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & a_{18} & a_{19} \\
a_{21} & a_{22} & a_{23} & 0 & 0 & 0 & a_{28} & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} \\
0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{54} & a_{55} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{66} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{76} & a_{77} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{86} & a_{87} & a_{88} & 0 \\
0 & 0 & 0 & 0 & 0 & a_{96} & a_{97} & a_{98} & a_{99}
\end{pmatrix}
\begin{pmatrix}
e_{i0} \\
e_i \\
e_s \\
e_y \\
e_{\pi} \\
e_{\pi^*} \\
e_{i^*} \\
e_{wxp^*}
\end{pmatrix}
\]

As mentioned before, there are two special features of the above identification scheme. First, there is no money in the model, and I use the overnight target rate as the monetary policy instrument. The contemporaneous identification of this policy equation is given by the first equation of the system of equations (5), where I condition the overnight target rate as a function of the nominal market interest rate ($i$), the exchange rate ($s$), the federal funds rate ($i^*$), and the world export commodity price index ($wxp^*$). I assume that the Bank of Canada certainly has access to the information on these variables within the month. I use the

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1 Before 1999, the target rate could be anywhere within the band, but since 1999 it has been set at the midpoint of the band. Before 2001, the target rate could be changed on any day, but since 2001 there have been eight fixed dates of the year on which the target rate can be changed. For the four months of the year when the Bank is not scheduled to meet to decide the target rate, I replace the target rate with the overnight rate.
world export commodity price index as a proxy for inflationary expectations. Since a key objective of the Bank is to maintain a stable inflation rate, I assume that the Bank looks at some measures of inflationary expectations when it determines the monetary policy. On the other hand, the Bank of Canada would be unable to observe data on output and the general price level of both domestic and foreign countries within the month.

The second feature of this structural identification is the simultaneous interactions among the policy variable and the financial variables within the month. The second equation of the system of equations (5) is the nominal market interest rate equation, which I assume to be contemporaneously affected by the overnight target rate, the exchange rate, and the federal funds rate. The third equation is the exchange-rate equation. Since the exchange rate is a forward-looking asset price, following the information equation of Cushman and Zha (1997) and Kim and Roubini (2000), I assume that an efficient foreign exchange market is able to respond to all macroeconomic variables in the model within the month. As the data on the exchange rate may reflect other sources of domestic and foreign information, which may not be contemporaneously available to the monetary authority, identification of the exchange-rate equation in this way is important for the monetary policy identification.

The structural model identified this way allows the policy variable, the market interest rate, and the exchange rate to interact simultaneously with each other and with other important home and foreign variables within the month. Since the recursive approach, with any ordering of the variables, cannot capture this simultaneity, it produces flawed monetary policy shocks, resulting in unreliable dynamic responses. Recursive identification approaches, such as those used by Eichenbaum and Evans (1995), Kahn, Kandel, and Sarig (2002), and Bhuiyan and Lucas (2007), assume that monetary policy does not react to the exchange rate contemporaneously, which is inconsistent with what the central bank actually does. These recursive VAR studies, as well as some structural VAR studies such as those by Kim and Roubini (2000) and Kim (2005), also assume that non-US central banks do not respond to the Fed policy move until a month later. This assumption is particularly inappropriate for the Bank of Canada, which always seems to adjust the target rate on the fixed action date following any change in the federal funds rate.

Finally, I specify the production sector of the Canadian block, which comprises two variables: output ($y$) and the inflation rate ($\pi$). I assume that the financial variables of both Canadian and non-Canadian blocks do not affect real activities contemporaneously but with
lag. Although the exchange rate will eventually feed through to the domestic price level, evidence suggests that this pass-through effect is not instantaneous. Also, firms do not change their output and price in response to changes in signals of financial variables or monetary policy within the month due to inertia, adjustment cost, and planning delays. Therefore, I normalize this subsystem in the lower-triangularized order of $y$ and $\pi$. The estimated results, however, are robust to the reverse order of $\pi$ and $y$. As shown in the system of equations (5), I also do not impose any structure on the foreign block of variables but follow Cushman and Zha (1997) to keep them in the lower-triangularized fashion of the order $y^*, \pi^*, i^*, wxp^*$.

4.2 A Bayesian Approach of Imposing Restrictions and Estimation

Two circumstances unfold from the identification scheme in the previous subsection. First, while a total of 45 zero restrictions on the contemporaneous-coefficient matrix, $A$, would exactly identify the model, I have imposed a total of 50 zero restrictions, which makes the covariance matrix of the reduced-form residuals, $\Sigma$, restricted. Second, the identifying restrictions involve simultaneous interactions among the target rate, the market interest rate, and the exchange rate. Due to this high degree of simultaneity, the shape of the posterior density for the model parameters tends to be non-Gaussian, making the widely used importance sampling method of obtaining finite-sample inferences inefficient, as noted by Leeper, Sims, and Zha (1996) and Zha (1999). Waggoner and Zha (2003) demonstrated how the use of the importance sampling method in a simultaneously interacted over-identified model results in misleading inferences. Therefore, I cannot use the existing importance sampling technique as did Cushman and Zha (1997) and Kim and Roubini (2000), although their identification approaches also had simultaneous interactions but to a lesser extent than my approach.

To circumvent the problem incurred due to the simultaneity involved in this over-identified structural VAR model, I estimate the model following the Bayesian Gibbs sampling method of Waggoner and Zha (2003), who incorporated prior information into the VAR as suggested by Sims and Zha (1998). The advantage of this approach is that it delivers accurate statistical inferences for models with a high degree of simultaneity among the contemporaneous variables, as well as for models with restricted variance-covariance matrices of the residuals and for models with restrictions on lagged coefficients.

To explain how the Gibbs sampling method can be applied to this structural VAR model, let $a_i$ be the $i$th row of the contemporaneous-coefficient matrix, $A$, and $f_i$ be the
ith row of the lagged-coefficient matrix, $F$, defined in the structural equation (2), where $1 \leq i \leq n$. Let $Q_i$ be any $n \times n$ matrix of rank $q_i$, and $R_i$ be any $k \times k$ matrix of rank $r_i$. Therefore, the linear restrictions on the contemporaneous-coefficient matrix, $A$, and on the lagged-coefficient matrix, $F$, can be summarized, respectively, as follows:

\[ Q_i a_i = 0, \quad i = 1, ..., n, \]  
\[ R_i f_i = 0, \quad i = 1, ..., n. \]

(6)  
(7)

Assuming that there exist non-degenerate solutions to the above problems, I can define a $n \times q_i$ matrix $U_i$ whose columns form an orthonormal basis for the null space of $Q_i$, and a $k \times r_i$ matrix $V_i$ whose columns form an orthonormal basis for the null space of $R_i$. Therefore, $a_i$ and $f_i$, which, respectively, are the rows of $A$ and $F$, will satisfy the identifying restrictions (6) and (7) if and only if there exists a $q_i \times 1$ vector $b_i$ and a $r_i \times 1$ vector $g_i$ such that

\[ a_i = U_i b_i, \]  
\[ f_i = V_i g_i. \]

(8)  
(9)

The model then becomes much easier to handle by forming priors on the elements of $b_i$ and $g_i$, since the original parameters of $a_i$ and $f_i$ can be easily recovered via the linear transformations through $U_i$ and $V_i$. Waggoner and Zha (2003) demonstrated that using this approach, simulations can be carried out on an equation-by-equation basis, which vastly reduces the computational burden of the problem. To obtain the finite-sample inferences of $b_i$ and $g_i$, and their functions, that is, impulse responses, it is necessary to simulate the joint posterior distribution of $b_i$ and $g_i$. To do this simulation, I follow Waggoner and Zha’s (2003) two-step Gibbs sampling procedure. First, I simulate draws of $b_i$ from its marginal posterior distribution, and then, given each draw of $b_i$, I simulate $g_i$ from the conditional posterior distribution of $g_i$. The second step is straightforward, since it requires draws from multivariate normal distributions. The first step, however, is less straightforward as the over-identifying restrictions on the contemporaneous-coefficient matrix, $A$, makes reduced-form covariance matrix, $\Sigma$, restricted.

5. Empirical Evidence of the Effects of Monetary Policy Shocks

The first step of estimation is to test the over-identifying restrictions imposed on the contemporaneous and the lagged coefficients. Following Cushman and Zha (1997), I perform

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2For a detailed explanation of the algebra and the algorithm, see Waggoner and Zha (2003).

---
a joint test of the contemporaneous and the lagged identifying restrictions. As long as all restrictions are treated as a restricted subset of the complete unrestricted parameter space, the likelihood ratio test can be applied to test the overall identifying restrictions. In the model, the contemporaneous-coefficient matrix, $A$, has 5 over-identifying restrictions, and with a lag-length of eight, the number of lagged restrictions on the non-Canadian block is 160. Therefore, with a total of 165 restrictions, the estimated Chi-squared statistic $\chi^2(165) = 176.543$ implies that the null is not retained at a standard significance level.

As mentioned in subsection 4.1, a greater degree of simultaneous interactions among the variables makes the structural approach developed in this paper different from the existing approaches in the literature. Therefore, the estimated contemporaneous coefficients will be informative about the effectiveness of this approach. The estimated contemporaneous coefficients of the first three equations of the model are reported in table 1. Since the production sector and the foreign block of variables do not have any structural interpretations, those contemporaneous coefficients are not produced here. The significance of most of the contemporaneous coefficients and, in particular, the strong significance of the simultaneously interacted coefficients—$a_{12}$, $a_{21}$, $a_{13}$, $a_{31}$, $a_{23}$, $a_{32}$—indicates that both a recursive identification and a structural identification that do not allow the financial variables to interact with each other simultaneously would be erroneous.

<table>
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<th>Parameter</th>
<th>$a_{11}$</th>
<th>$a_{12}$</th>
<th>$a_{13}$</th>
<th>$a_{14}$</th>
<th>$a_{15}$</th>
<th>$a_{16}$</th>
<th>$a_{17}$</th>
<th>$a_{18}$</th>
<th>$a_{19}$</th>
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<td>Estimate</td>
<td>1.177</td>
<td>0.871</td>
<td>-9.328</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>(SE)</td>
<td>(0.512)</td>
<td>(0.379)</td>
<td>(4.405)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(0.071)</td>
<td>(0.915)</td>
</tr>
<tr>
<td>Parameter</td>
<td>$a_{21}$</td>
<td>$a_{22}$</td>
<td>$a_{23}$</td>
<td>$a_{24}$</td>
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<tr>
<td>Estimate</td>
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<td>-7.423</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.214</td>
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</tr>
<tr>
<td>(SE)</td>
<td>(3.594)</td>
<td>(3.489)</td>
<td>(3.467)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(0.104)</td>
<td>-</td>
</tr>
<tr>
<td>Parameter</td>
<td>$a_{31}$</td>
<td>$a_{32}$</td>
<td>$a_{33}$</td>
<td>$a_{34}$</td>
<td>$a_{35}$</td>
<td>$a_{36}$</td>
<td>$a_{37}$</td>
<td>$a_{38}$</td>
<td>$a_{39}$</td>
</tr>
<tr>
<td>Estimate</td>
<td>4.851</td>
<td>4.693</td>
<td>53.881</td>
<td>0.848</td>
<td>-53.164</td>
<td>75.719</td>
<td>0.013</td>
<td>-1.333</td>
<td>57.568</td>
</tr>
<tr>
<td>(SE)</td>
<td>(1.646)</td>
<td>(1.542)</td>
<td>(18.025)</td>
<td>(0.318)</td>
<td>(29.748)</td>
<td>(12.240)</td>
<td>(0.121)</td>
<td>(0.582)</td>
<td>(19.178)</td>
</tr>
</tbody>
</table>

Note: Entries correspond to rows 1 through 3 of the contemporaneous-coefficient matrix, $A$, defined in equation (5), and apply to shocks to $i_0$, $i$, and $s$ respectively.

In the monetary policy equation, the positive and significant coefficient of the market interest rate implies that the Bank of Canada tightens monetary policy if the current nominal
market interest rate is low. The negative and significant coefficient of the exchange rate shows that, as a measure of leaning today against tomorrow’s wind, the Bank increases the overnight target rate to offset currency depreciation. These findings are consistent with the inflation-targeting monetary policy of the Bank of Canada. Since both the lower market interest rate and the depreciation of the Canadian dollar are indications of future inflation, the Bank would want to tighten the monetary policy by raising the overnight target rate in order to reduce future inflation by influencing these variables. The negative and significant coefficient of the federal funds rate confirms the traditional belief that the Fed is the leader and the Bank of Canada is the follower. Although the coefficient of the world export commodity price index is not significant, its negative sign implies that the Bank undertakes a contractionary monetary policy when seeing a higher world export price.

All the contemporaneous coefficients of the market interest rate equation and the exchange-rate equation are statistically significant at less than the 0.05 level, except the coefficient of the US inflation rate on the exchange-rate equation. The significance of these coefficients validates the structural identification and the simultaneity I assumed among the target rate, the market interest rate, and the exchange rate. On the other hand, the significance of the coefficients of the exchange-rate equation with the non-financial variables, except the US inflation rate, justifies the assumption that an efficient exchange-rate market can contemporaneously respond to these variables within the month. Although this identification scheme does not allow the market interest rate to react directly to the non-financial variables within the month, it allows this rate to react indirectly to these variables via reacting to the exchange rate, which in turn reacts to all the variables. When I test whether the market interest rate directly responds to the non-financial variables, I find that the coefficients of these variables are both individually and jointly insignificant.

Before presenting impulse responses, it is worth discussing the theory underlying the open-economy monetary transmission mechanism. Following the pioneering work by Obstfeld and Rogoff (1995), a number of open-economy monetary transmission models, such as those by Chari, Kehoe, and McGrattan (2002), Corsetti and Pesenti (2001), Galí and Monacelli (2004), and Kollmann (2001), made contributions to the New Open Economy Macroeconomics literature. These models with stickiness in prices and wages imply that monetary policy operates through the interest rate and exchange rate channels. For example, following a contractionary monetary policy shock, the market interest rate rises, which causes an inflow of capital into the country from around the world, leading to an appreciation of domestic
currency. The rise in the interest rate also increases the cost of borrowing and thus tends to dampen the demand for interest-sensitive consumption and investment expenditures. On the other hand, the appreciation of domestic currency increases prices of home products relative to foreign ones, leading to a decline in net exports. Therefore, the contractionary policy shock leads to a reduction in aggregate demand.

Over short periods of time, since output is determined by aggregate demand, the fall in aggregate demand causes a fall in aggregate output. With a given underlying growth rate of potential output, this reduction in actual output implies a negative output gap. While this negative output gap might continue for a while, eventually the economic slack leads to a fall in wages and prices of other inputs. Finally, this reduction in firms’ costs of production leads to a reduction in the price of output, that is, to a low inflation rate in the economy. Therefore, theoretically, while the effects of the policy shock on interest rates and exchange rates are realized immediately, this effect on the level of output is realized with lag, and on the price level with further lag.

The estimated impulse responses of the macroeconomic variables are displayed in figure 1. The response horizon, in months, is given in the horizontal axis. The solid lines are the estimated impulse responses computed from the values of $a_i$ and $f_i$, defined in subsection 4.2, at the peak of their posterior distributions. The upper and lower dashed lines are one-standard-error bands, derived using the Bayesian Gibbs sampling method of Waggoner and Zha (2003).[^3]

We observe from the figure that a one-standard-deviation contractionary monetary policy shock of increasing the overnight target rate by 25 basis points increases the nominal market interest rate by 20 basis points, which remains statistically significant for about a year. Following the same shock, the exchange rate appreciates on impact and gradually depreciates to its terminal value. Therefore, this reaction of the exchange rate is consistent with Dornbusch’s prediction that following a policy shock the exchange rate overshoots its long-run level on impact, followed by a gradual adjustment to the initial value. We also observe that due to this contractionary policy shock the output level falls with a lag of over half a year. Finally, this policy shock lowers the inflation rate by 15 basis points, the highest impact of which is realized with a lag of about one year and remains significant up to a year and a half.

[^3]: The error bands are computed from a set of 10000 draws. I gratefully acknowledge Tao Zha for helping me with the Matlab codes.
after the shock was introduced. Therefore, while the market interest rate and the exchange rate respond to the policy shock immediately, output responds with a lag and the response of the inflation rate is further delayed. These impulse responses are consistent with the theoretical predictions of the dynamic responses of a contractionary monetary policy shock, both in terms of the direction and the timing of the responses.

At this point it is interesting to compare the findings of this paper with other findings in the literature for Canada. Both Cushman and Zha (1997) and Kim and Roubini (2000) found that the contractionary monetary policy shock appreciates the exchange rate on impact, which then gradually depreciates to the terminal value. However, Cushman and Zha (1997) did not find any significant effect on the nominal interest rate and concluded that the monetary policy transmission mechanism operates through the exchange rate, not the interest rate. They also found that the effect on the price level started to be significant after about two years, which remained significant up to the end of the third year.

The contractionary monetary policy shock in Kim and Roubini’s (2000) approach pro-
duced the liquidity puzzle—a decrease in the nominal interest rate following a contractionary policy shock—for Canada and a few other G-7 countries. They also found that this policy shock had a statistically significant effect on the Canadian price level for more than four years. On the other hand, the contractionary monetary policy shock of raising the overnight target rate in a recursive identification approach by Bhuiyan and Lucas (2007) increased the real interest rate and lowered inflationary expectations. However, following this contractionary policy shock, the exchange rate depreciated, and there was no significant effect on output.

I believe that the superiority of the impulse responses generated in this paper, in terms of matching with the theoretical predictions, is due to a more accurate identification of the structural model, which estimates a more precise measure of the exogenous monetary policy shock. In the structural model I realistically allow the target rate, the nominal interest rate, and the exchange rate to react to each other and to a number of other home and foreign variables within the month, which increases the precision of the model identification. On the other hand, since the overnight target rate cannot be influenced by other shocks in the economy, except through an endogenous policy response of the Bank to changes in the variables captured in the policy equation, innovations to this equation truly estimate exogenous policy shocks. In a recent paper, Carpenter and Demiralp (2008) made a similar argument to demonstrate the liquidity effect for the US. They showed that the liquidity puzzle or the time-insensitive liquidity effect is the result of the misspecification of the VAR model arising from the use of wrong variables.

I check the robustness of the results estimating the model using some other alternative identifications. I find that the recursive approach with various orderings of the variables generates some puzzles similar to those found in previous studies. For example, a recursive identification that does not allow the overnight target rate to contemporaneously respond to the exchange rate causes a depreciation of the Canadian dollar after a contractionary policy shock. On the other hand, when I use the market interest rate instead of the overnight target rate as the policy instrument, I find less significant impulse responses of all variables and an insignificant impulse response of the exchange rate. If I impose zero restrictions on output and inflation of both home and foreign countries, the exchange-rate effect becomes almost insignificant. Finally, although the omission of the world export price index does not generate a price puzzle, it causes an insignificant effect of the price level.

Since uncovered interest rate parity (UIP) is directly related to the interest rate and
the exchange rate, I also investigate whether the monetary policy shock induces a systematic departure from this parity. To explore this issue, I follow Eichenbaum and Evans (1995) to define $\psi$ as the *ex post* difference in the return between investing in one-period Canadian assets and one-period US assets, that is, $\psi_t = i_t - i^*_t + 12(s_{t+1} - s_t)$. For a direct comparison with interest rates, which are already in annual terms, I multiply the exchange rate change by twelve. If the UIP condition holds and expectations are rational, the conditional expectation of this excess return should be zero. From the estimated impulse responses of the interest rate and the exchange rate, I compute the impulse response of this excess return due to the monetary policy shock as shown in the lower right block of figure 1. Figure 1 shows that there is no systematic evidence of excess returns due to the contractionary monetary policy shock; the excess returns are highly noisy and statistically insignificant.

This finding that the gradual depreciation of the exchange rate offsets the excess return from home country assets differs from that of Eichenbaum and Evans (1995). These authors, identifying contractionary monetary policy shocks with innovations in non-borrowed reserves in a recursive V AR model, found that the higher return in home currency is further magnified by a persistent appreciation of the exchange rate for a prolonged period of time. This result similar to that of Cushman and Zha (1997), who also reported an insignificant impulse response of the UIP deviation due to the policy shock. This empirical confirmation of the UIP condition found in this paper as well as in Cushman and Zha’s (1997) study reflects the importance of identifying the monetary policy function in a small-open-economy context for Canada.

Next I extend the basic model by incorporating the monetary aggregate, $M1$. In the extended model, I still keep the overnight target rate as the policy instrument but include an informal money demand function, where money holding is a contemporaneous function of the nominal market interest rate, the inflation rate, and output. I also let the nominal interest rate and the exchange rate be contemporaneously affected by money. No other variables either affect money or are affected by money contemporaneously. There are no restrictions in the lagged coefficients of the money demand function, and the rest of the identification scheme of this extended model is the same as the basic model.

The impulse responses of the variables due to an overnight target rate shock in the extended model are reported in figure 2. We observe from the figure that there is no marginal contribution of the inclusion of money into the model: the pattern of dynamic responses of
the variables due to the policy shock remain unchanged in the extended model. While the impulse responses of the other macroeconomic variables are robust to the incorporation of money into the model, the dynamic response of the money stock itself is not an exact mirror image of the dynamic response of the overnight target rate. Figure 2 shows that the contractionary monetary policy shock of increasing the overnight target rate peaks in the second month, followed by a gradual decline which becomes insignificant after about one year. In contrast, following the same shock, the money stock keeps declining, the highest impact of which is not realized until the end of the second year, and the effect remains statistically significant for about three years. This imprecise dynamic response of the money stock might be due to the fact that, in addition to the monetary policy decision, monetary aggregates are influenced by other factors in the economy, such as private-sector behavior. Therefore, this impulse response of $M1$ casts further doubt on the justification of using money as the policy instrument and rationalizes the use of the target rate as the policy instrument.

**Figure 2: Impulse Responses in the Extended Model**

Given the relative size of the US economy and the trade relationship between the US
and Canada, we assume that shocks to the US variables might play an important role in the movement of the Canadian variables. Therefore, I also investigate the dynamic responses of the Canadian macroeconomic variables due to shocks to the US federal funds rate. The impulse responses are presented in figure 3. We observe from the figure that the contractionary policy shock of increasing the federal funds rate is accompanied by an increase in the overnight target rate and the market interest rate. This result is consistent from the viewpoint that the Bank of Canada would want to increase the interest rate to avoid the inflationary effect of its currency depreciation caused by the rise in the federal funds rate. Figure 3 also shows that the contractionary US monetary policy shock causes a short-lived depreciation of the Canadian dollar, increases the inflation rate, and lowers the GDP. The increase in the inflation rate is a consequence of the stimulated aggregate demand caused by the depreciation of the Canadian dollar. On the other hand, two opposite directional forces originating from the increase in the federal funds rate influence Canadian output. First, the depreciation of the Canadian dollar raises the aggregate demand for Canadian goods and hence Canadian output increases. Second, the increase in the Canadian interest rate following the increase in the
federal funds rate dampens aggregate demand and reduces output. Since the Bank of Canada
strongly reacts to the Federal Reserve System by adjusting its interest rate, the second effect
dominates the first effect. Therefore, the exchange rate does not depreciate much and the
level of output decreases.

Table 2: Forecast error variance decomposition of output

<table>
<thead>
<tr>
<th>Months</th>
<th>Overnight Target Rate</th>
<th>Market Interest Rate</th>
<th>Exchange Rate</th>
<th>Output</th>
<th>Inflation</th>
<th>Foreign Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.75</td>
<td>0</td>
<td>2.48</td>
<td>90.41</td>
<td>1.33</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>11.32</td>
<td>5.13</td>
<td>13.54</td>
<td>37.01</td>
<td>1.99</td>
<td>31.97</td>
</tr>
<tr>
<td>12</td>
<td>6.03</td>
<td>3.90</td>
<td>16.81</td>
<td>27.89</td>
<td>1.68</td>
<td>44.16</td>
</tr>
<tr>
<td>24</td>
<td>3.57</td>
<td>3.92</td>
<td>15.12</td>
<td>20.81</td>
<td>2.20</td>
<td>54.47</td>
</tr>
<tr>
<td>48</td>
<td>3.20</td>
<td>6.72</td>
<td>12.59</td>
<td>16.92</td>
<td>4.11</td>
<td>59.47</td>
</tr>
</tbody>
</table>

Note: Entries are in percentage points. Foreign variables include the federal funds rate, the US GDP,
the US inflation rate, and the world export price index.

Finally, I report the sources of Canadian output fluctuations. Table 2 presents
the variance decomposition of the Canadian GDP due to shocks to the home and foreign macroe-
conomic variables over different horizons. We see from the table that the monetary policy
shock is not the dominant sources of output fluctuations in Canada at any horizons. Among
domestic shocks, those from the exchange rate and from the domestic GDP itself are the
primary source of output fluctuations. On the other hand, external shocks, which are shocks
from the foreign block of variables, become the dominant source of output fluctuations after
twelve months. These results are similar to the findings of Cushman and Zha (1997) and Kim

6. Conclusion

This paper develops an open-economy Bayesian structural VAR model for Canada in
order to estimate the effects of a monetary policy shock, using the overnight target rate as
the policy instrument. The structural model developed here allows the financial variables
of the model to interact contemporaneously with each other and with a number of other
home and foreign variables. Since the identification involves simultaneous interactions in the
contemporaneous relationships of the financial variables in the model, in order to increase
the precision of the parameter estimates, I use a Bayesian Gibbs sampling method to estimate
the model. This paper finds that the liquidity effect and the exchange-rate effect of the policy
shock are realized immediately, while output responds with a lag of over half a year, and the inflation rate responds with a lag of about one year.

The results of this paper differ from those of other studies in the literature in a number of important respects. While Cushman and Zha (1997) found that the transmission of the monetary policy shock to real output operates through the exchange rate channel only, I find that this transmission operates through both the interest rate and the exchange rate. I also find that, due to the contractionary policy shock, the exchange rate depreciates gradually after an impact appreciation, which offsets the higher return in home currency. This result differs from that of Eichenbaum and Evans (1995), who, in a recursive VAR model, found that the higher return in home currency is further magnified by a persistent appreciation of home currency for a prolonged period of time after the policy shock was introduced. On the other hand, in the extended model, the impulse response of M1 confirms that shocks to monetary aggregates reflect some other shocks in the economy, and hence, cannot be a good measure of exogenous monetary policy shocks. Finally, I find that the Canadian macroeconomic variables including the overnight target rate significantly respond to the federal funds rate, and external shocks are an important source of Canadian output fluctuations.
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