The Effects of Monetary Policy Shocks in a Small Open Economy: A Structural VAR Approach

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Abstract. This paper develops an open-economy structural VAR model for Canada in order to estimate the effects of monetary policy shock, using the overnight target rate as the policy instrument. To increase the precision of parameter estimates of my over-identified structural VAR model, where some contemporaneous variables simultaneously interact with each other, I use a Bayesian Gibbs sampling method to estimate the model. I find that the policy shock transmits to real output through both the interest rate and the exchange rate. I also find that the contractionary policy shock induces a departure from UIP in spite of the deprecation of the exchange rate after an impact appreciation. On the other hand, the contractionary monetary policy of raising the target rate is not exactly followed by a decrease in the monetary aggregate.

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1. Introduction

This paper identifies monetary policy shocks in a structural VAR approach for a small open economy, using Canada as a case study. I follow the general procedure of Cushman and Zha (1997) and Kim and Rouibini (2000), but change it in a number of respects. Unlike these authors, I do not use money in my basic model. Instead, I use the overnight target rate as the policy instrument, which the Bank of Canada actually uses to conduct monetary policy. I assume that the Bank contemporaneously reacts to some foreign variables, in addition to some key domestic macroeconomic variables. Likewise, these variables also respond to the policy variable within the month, except the foreign variables, which I assume to follow an exogenous process in order to maintain the small-open-economy assumption strictly. This identification approach, therefore, entails simultaneous interactions among the policy variable and some other variables in the model. In order to address this simultaneity in my over-identified structural VAR model, I employ a Bayesian Gibbs sampling method to estimate the model, as opposed to the importance sampling technique, widely used in literature, including by Cuhsman and Zha (1997) and Kim and Rouibini (2000).

In the identification scheme, I define the monetary policy feedback rule as a function of the contemporaneous values of the nominal market interest rate, the exchange rate, the federal funds rate, and the world export commodity price index, as well as of the lagged values of all variables in the model. I use the world export commodity price index as a proxy for inflationary expectations. I allow the nominal interest rate to react contemporaneously to the financial variables of the model, assuming that financial variables interact instantaneously with each other. I also assume that an efficient foreign-exchange market has all the relevant information within the month and let the exchange rate react to all the variables in the model contemporaneously. The use of the structural VAR approach, as opposed to the recursive VAR approach, enables me to capture these economically meaningful cross-directional relationships among these variables. Since shocks from Canada have little effect on the rest of the world, I treat the foreign block of variables as exogenous from Canada's point of view.

I argue that it is difficult to measure changes in monetary policy by shocks to money supply, since these shocks might reflect other shocks in the economy. In contrast, unlike money, the overnight target rate cannot be influenced by private-sector behaviour, except through the channel of an endogenous policy response of the central bank to changing economic conditions. Since my structural VAR approach explicitly identifies the policy reaction function as a function of all key home and foreign variables, I am able to estimate policy innovations that are orthogonal to these variables.

This identification approach involves a great deal of simultaneity among the contemporaneous variables, so the shape of the posterior density for the model parameters tends to be non-Gaussian. It makes the importance sampling technique of obtaining finite-sample inferences from the posterior distribution prohibitively inefficient. Therefore, in order to obtain accurate statistical inferences from my over-identified structural VAR model, I employ the Bayesian Gibbs sampling method of Waggoner and Zha (2003) to estimate the model.

When I apply my model to the Canadian data, I find that the monetary policy shock transmits real output through both the interest rate and the exchange rate, as opposed to Cushman and Zha (1997), who found that the transmission operates through the exchange rate only. I also find that the higher return in Canadian currency, induced by the contractionary policy shock, is only slightly reduced by the gradual depreciation of the Canadian dollar after an impact appreciation. This result differs from that of Eichenbaum and Evans (1995), who, in a recursive VAR model, found that the larger return in home currency is further magnified by a persistent appreciation of home currency due to a contractionary policy shock.

I also investigate the role of money by incorporating a monetary aggregate (M1), identified as a money demand function, into the structural VAR model. When I re-estimate this extended model, I find that the impulse responses of the variables remain unchanged from the basic model, but the contractionary monetary policy of raising the target rate is not exactly followed by a decrease in the monetary aggregate. This imprecise impulse response of the money stock casts further doubt on the rationalization of using money as the policy instrument and justifies the use of the overnight rate.

The remainder of the paper is organized as follows: Section 2 presents the context of my research, and Section 3 gives the data sources. Section 4 describes the structural VAR model that identifies the exogenous monetary policy shock and explains the estimation procedure. Section 5 presents the results, while Section 6 draws conclusions.

2. Research Context

In his classic paper, Sims (1980) suggested the use of impulse responses from the VAR model for policy analysis. Since then, a great deal of VAR literature, adopting both recursive and structural approaches, has been developed to identify monetary policy shocks and to estimate impulse responses of various macroeconomic variables due to these shocks. In order to have meaningful inference from the impulse responses, however, it is important to condition the VAR model to an information set that the central bank and the private sector actually observe when they make decisions. If the information that the central bank and the private sector have is not included in the model, then the measurement of policy shocks is likely to be contaminated, which will lead to misleading impulse responses. The choice of the appropriate monetary policy variable is also crucial for correctly identifying monetary policy shocks. If shocks to the policy variable reflect some other shocks in the economy – for example, shocks to monetary aggregates might reflect shocks to private-sector behaviour – then the measurement of the policy shocks is bound to be imprecise.

The identification of monetary policy requires even more care if the economy under consideration is a small open economy. In addition to the domestic macroeconomic variables, which impact all economies, for a small open economy, some external variables, such as exchange rates and foreign monetary policy variables, are crucial considerations for the monetary authority. Since these variables are known to the monetary authority when they make decisions, it is natural that the monetary policy would contemporaneously respond to these home and foreign variables. Another important aspect of the small open economy is that the decisions of its monetary authority and private sector are unlikely to have any effects on the rest of the world. Therefore, it might be more plausible to let the foreign block of variables follow an exogenous process from the small open economy's point of view. On the other hand, since the policy shock transmits to the financial sector immediately, the financial variables of the home country would also react to the policy shock within the month.

To identify monetary policy shocks, we now have many models, adopting both recursive and structural approaches, some using the short-term interest rate and others monetary aggregates as monetary policy instruments. Bernanke and Blinder (1992) argued that the federal funds rate innovations, identified in a recursive approach, are in some respects better measures of monetary policy shocks than are the innovations in monetary aggregates for the US. This argument was challenged by Gordon and Leeper (1994), who, using innovations in monetary aggregates in a recursive approach, however, found dynamic responses that are at odds with the theoretical predictions. Identifying contractionary monetary policy shocks with negative innovations in the narrow monetary aggregates in a recursive VAR model, Eichenbaum and Evans (1995) also reported persistent appreciation of the US dollar for a prolonged period of time.

The recursive approach of monetary policy identification might make some sense for the US, since it is a large and relatively closed economy, and the movement of US monetary policy due to foreign shocks is relatively small. In addition, in closed economy models, such as those used by Christiano and Eichenbaum (1992) and Kim (1999), the monetary policy transmission mechanism operates primarily through the interest rate, not the exchange rate. Therefore, the conditions of recursive identification that are somewhat valid for the US are very unlikely to be valid for smaller and more open economies. Using the recursive approach, Sims (1992) reported that positive interest rate innovations are associated with a persistent increase in home price levels and depreciation of home currencies, for several non-US countries. Some other studies, such as those of Grilli and Roubini (1995) and Bhuiyan and Lucas (2007), also found dynamic responses of various macroeconomic variables due to monetary policy shocks that are inconsistent with economic theory using the recursive approach for some non-US countries.

In an attempt to identify monetary policy more realistically, Sims and Zha (1995) proposed a structural VAR model for the relatively closed US economy. Cushman and Zha (1997) and Kim and Roubini (2000), among others, extended this structural model for more open economies. The gist of the structural approach is that, rather than relying solely on the recursive Choleski technique, it allows simultaneous interactions between the monetary policy variable and other macroeconomic variables of the model within the month. Faust and Rogers (2003), using an inference procedure that allows them to relax dubious identifying assumptions of the recursive approach, also incorporated these standard assumptions of the structural VAR model in order to identify monetary policy shocks. The idea of conditioning monetary policy simultaneously to a large set of home and foreign variables was approached in a different way by Bernanke, Boivin, and Eliasz (2004), by using a factor-augmented VAR model, exploiting the indexes or factors of the dynamic factor model.

Having identified a monetary policy by incorporating these economically meaningful identifying assumptions into the structural VAR model, how much can we rely on its impulse response functions as measures of the dynamic responses of the macroeconomic variables? In response to the skepticism expressed by Chari, Kehoe, and McGrattan (2005) regarding the ability of the structural VAR model to document empirical phenomenon, Christiano, Eichenbaum, and Vigfusson (2006) demonstrated that, if the relevant short-run identifying restrictions are justified, the structural VAR procedures reliably recover and identify the dynamic effects of shocks to the economy. In a recent paper, Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007) also demonstrated that, if the variables chosen by the econometricians are accurate and if the identifying restrictions are precise, then the impulse responses of the VAR model do a good job of portraying the dynamic behaviour of the macroeconomic variables due to shocks. Therefore, it would be a useful exercise to test how monetary policy shocks in a structural VAR model influence a small open economy after factoring in all of its features into the model.

3. Canadian Monthly Data

My data runs monthly from 1994 to 2007. Over the years, the Bank of Canada has shifted the way it conducts monetary policy. Since 1994, the Bank has been using the target for the overnight rate as its key monetary policy instrument. Therefore, I choose to run the sample from 1994 to 2007. All the data is collected from Statistics Canada's CANSIM database and the International Monetary Fund's *International Financial Statistics* (IFS). The variables are: m, the logarithm of the Canadian monetary aggregate $(M1)$ (Cansim, V37199), i_0, the overnight target rate (Cansim, V122514), i, the three-month Treasury bills rate (Cansim V122529), s, the logarithm of the nominal exchange rate in units of Canadian currency for one unit of US dollar (Cansim, B3400), π, the annualized monthly inflation rate calculated from the consumer price index (Cansim, V737311), i^*, the US federal funds rate (IFS, 11164B..ZF..), y^*, the logarithm of US industrial production (IFS, 11166..CZF..), π^*, the annualized monthly inflation rate calculated from the US consumer price index (IFS, 11164..ZF..), wxp^*, the logarithm of the world total export commodity price index (IFS, 06174..DZF...).

4. An Identified VAR Model with Block Exogeneity

In this section, I develop a structural VAR model to identify monetary policy for Canada. Considering Canada seriously as a small open economy, I follow Cushman and Zha (1997) and Zha (1999) to treat the non-Canadian block of variables as exogenous from Canada's point of view, both contemporaneously and for the lagged values. In the first subsection, I identify the structural VAR model, and in the second subsection, I describe how I estimate the model in a Bayeisian approach.

4.1 Identification of Monetary Policy

To begin, I start with the general specification, where, omitting constant terms, the structural system can be written in the following linear and stochastic dynamic form:

$$Ax_t = \sum_{l=1}^{p} B_l x_{t-l} + \varepsilon_t, \tag{1}$$

where x_t is an $n \times 1$ column vector of endogenous variables at time t, A and B_l are $n \times n$ parameter matrices, ε_t is an $n \times 1$ column vector at structural disturbances, p is the lag length, and $t = 1, ..., T$, where T is the sample size. The parameters of the individual equations in the structural VAR model (1) correspond to the rows of A and B_l. I assume that the structural disturbances have a Gaussian distribution with $E(\varepsilon_t \mid x_1, ..., x_{t-1}) = 0$ and $E(\varepsilon_t \varepsilon_t' \mid x_1, ..., x_{t-1}) = I$.

In my model, x comprises two blocks of variables – the Canadian block, x_1:$[i_0, i, s, y, \pi]$ and the non-Canadian block, x_2:$[y^*, \pi^*, i^*, wxp^*]$, where i_0 is the overnight target rate, i is the nominal interest rate, s is the logarithm of the exchange rate, y is the logarithm of output, π is the inflation rate, y^* is the logarithm of US output, π^* is the US inflation rate, and wxp^* is the logarithm of the world export commodity price index. For convenience of explanation, I rewrite the structural system (1) in the following matrix notation:

$$Ax_t = Fz_t + \varepsilon_t, \tag{2}$$

where $z_t = [x_{t-1}....x_{t-p}]'$ and $F = [B_1....B_p]$ with z_t being the $np \times 1$ column vector of all lagged variables and F being the $n \times np$ matrix of all lagged coefficients.

For the accuracy of parameter estimates and subsequent forecasts, it is important to treat appropriately the relationship between the Canadian and the non-Canadian blocks of variables. Canada's economy is about one-tenth of the size of the US economy, and about 75 percent of Canada's exports go to the US, while only about 20 percent of US exports come to Canada. Therefore, it seems economically more appealing to treat the foreign block of variables, including the world commodity price index (wxp^*), as exogenous. Zha (1999) demonstrated that failing to impose such exogeneity restrictions is not only unappealing but also results in misleading conclusions. Adding the exogeneity assumption into the structural model (2), it can be rewritten as follows:

$$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}. \tag{3}$$

The restriction that $A_{21} = 0$ follows from the assumption that the Canadian block of variables does not enter into the non-Canadian block contemporaneously, and the restriction that $F_{21} = 0$ follows from the assumption that it does not enter into the non-Canadian block in lag. It is worth noting that this concept of block exogeneity is similar to Granger causal priority defined by Sims (1980) in the context of the reduced-form VAR. The reduced-form version of the structural model (2) can be written as follows:

$$x_t = Ez_t + e_t, \tag{4}$$

where $E = A^{-1}F$ and $e_t = A^{-1}\varepsilon_t$. If I translate the block exogeneity restrictions that are imposed on the structural model (3) into the reduced-form model (4), they will imply that $E_{22} = 0$. This property will then mean that z_{2t} is Granger causally prior to x_{1t} in the sense of Sims (1980). I perform the likelihood ratio test to examine if the non-Canadian block Granger causally prior to the Canadian block. With a lag length of eight, which was determined on the basis of the likelihood ratio test and the Akaike information criterion, the Chi-squared statistic $\chi^2(160) = 160.612$, where 160 is the total number of restrictions on the non-Canadian block. This result implies that the null hypothesis is acceptable at the significance level of 0.471. Therefore, any structural identification for small open economies that treats both the home and the foreign blocks of variables as endogenous, such as that of Kim and Roubini (2000), is likely to produce imprecise estimates, resulting in misleading forecasts.

Let Σ be the variance-covariance matrix of the reduced-form residuals (e_t). Since the structural disturbances (ε_t) and the regression residuals (e_t) are related by $\varepsilon_t = Ae_t$, we can derive that:

$$\Sigma = (AA')^{-1}. \tag{5}$$

The right-hand side of equation (5) has $n \times (n+1)$ free parameters to be estimated, while the estimated variance-covariance matrix of the residuals, Σ, contains $n \times (n+1)/2$ estimated parameters. After normalizing the n diagonal elements of A to 1's, an exact identification still requires $n \times (n-1)/2$ more restrictions on A.

To reveal the restrictions I impose on the contemporaneous-coefficients matrix A, I display the relationship between the reduced-form residuals and the structural shocks in the system of equations (6). It is important to note that these contemporaneous restrictions do not merely describe the relationships between the residuals and the structural shocks, but they also describe the contemporaneous relationships among the levels of variables. Therefore, each equation of the system of equations (6) shows the contemporaneous relationships of a variable with other variables in the model. I do not impose any restrictions on the lagged coefficients except the block exogeneity restrictions on the foreign block of variables, as shown in the structural model (3).

There are two special features of my identification scheme. First, there is no money in the model, and I use the overnight target rate as the monetary policy instrument. In 1994, when my sample starts, the Bank of Canada adopted a target band and a target rate for the overnight rate on loans among banks and other financial institutions, which the Bank calls its main monetary policy instrument. The target band is of 50 basis points and is designed to allow for small and presumably temporary adjustments of the overnight rate to market conditions, while adjustments in the target rate are reserved for the implementation of changes in the monetary policy. However, if the overnight rate threatens to break through the upper or lower band, the Bank intervenes in the overnight market.[1] Since the overnight target rate is under the sole control of the Bank of Canada, innovations in this rate should be a more precise measure of monetary policy shocks than innovations in monetary aggregates.

Therefore, unlike previous structural VAR studies, such as those by Cushman and Zha (1997) and Kim and Roubini (2000), I use the overnight target rate as the monetary policy instrument of the Bank of Canada. The contemporaneous identification of this policy equation is given by the first equation of the system of equations (6), where I condition the overnight target rate as a function of the nominal interest rate (i), the exchange rate (s), the federal funds rate (i^*), and the world export commodity price index (wxp^*). I assume that the Bank of Canada certainly has access to the information on these variables within the month. I use the world export commodity price index as a proxy for inflationary expectations. Since a key objective of the Bank is to maintain a stable inflation rate, I assume that the Bank looks at some measures of inflationary expectations when it determines the monetary policy. On the other hand, the Bank of Canada would be unable to observe data on output and the general price level of both domestic and foreign countries within the month.

[1]Before 1999, the target rate could be anywhere within the band, but since 1999 it has been set at the midpoint of the band. Before 2001, the target rate could be changed on any day, but since 2001 there have been eight fixed dates of the year on which the target rate can be changed. For the four months of the year, when the Bank is not scheduled to meet to decide the target rate, I replace the target rate with the overnight rate in my data.

$$
\begin{pmatrix} \varepsilon_{i_0} \\ \varepsilon_i \\ \varepsilon_s \\ \varepsilon_y \\ \varepsilon_\pi \\ \varepsilon_{y^*} \\ \varepsilon_{\pi^*} \\ \varepsilon_{i^*} \\ \varepsilon_{wxp^*} \end{pmatrix} = \begin{pmatrix} 1 & a_{12} & a_{13} & 0 & 0 & 0 & 0 & a_{18} & a_{19} \\ a_{21} & 1 & a_{23} & 0 & 0 & 0 & 0 & a_{28} & 0 \\ a_{31} & a_{32} & 1 & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{76} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{86} & a_{87} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{96} & a_{97} & a_{98} & 1 \end{pmatrix} \begin{pmatrix} e_{i_0} \\ e_i \\ e_s \\ e_y \\ e_\pi \\ e_{y^*} \\ e_{\pi^*} \\ e_{i^*} \\ e_{wxp^*} \end{pmatrix} \tag{6}
$$

The second feature of my structural identification is the simultaneous interactions of the financial variables within the month. The second equation of the system of equations (6) is the nominal interest rate equation, which I assume to be contemporaneously affected by the overnight target rate, the exchange rate, and the federal funds rate. The third equation is the exchange-rate equation. Since the exchange rate is a forward-looking asset price, following the information equation of Cushman and Zha (1997) and Kim and Roubini (2000), I assume that an efficient foreign exchange market is able to respond to all macroeconomic variables in the model within the month. As the data on the exchange rate may reflect other sources of domestic and foreign information, which may not be contemporaneously available to the monetary authority, identification of the exchange-rate equation in this way is important for the monetary policy identification.

The structural model identified this way allows the policy variable, the nominal interest rate, and the exchange rate to interact simultaneously with each other and with other important home and foreign variables within the month. Such identification is important for properly addressing the interrelationships among the monetary policy variable and the other financial variables. While the theoretical transmission mechanism suggests that the policy shock transmits to the real sector through its immediate effects on the nominal interest rate and the exchange rate, it is also a reality that the current values of these variables affect the monetary policy decision. Since the recursive approach, with any ordering of the variables, cannot capture this simultaneity, it produces flawed monetary policy shocks, resulting in unreliable dynamic responses of variables.

The recursive identification, such as by Eichenbaum and Evans (1995), Kahn, Kandel, and Sarig (2002), and Bhuiyan and Lucas (2007), assumes that monetary policy does not react to the exchange rate contemporaneously, which is inconsistent with what the central bank actually does. These recursive VAR studies, as well as some structural VAR studies such as the one by Kim and Roubini (2000), also assume that non-US central banks do not respond to the Fed policy move until a month later. This assumption is inconsistent with the striking movements in the non-US rates that regularly follow within minutes of the Fed policy announcements. Particularly for Canada, evidence suggests that any change in the federal funds rate is followed by a similar adjustment in the target rate in the fixed action date.

Finally, I specify the production sector of the Canadian block, which comprises two variables: output (y) and the inflation rate (π). I assume that the financial variables of both Canadian and non-Canadian blocks do not affect real activities contemporaneously, but with lag. Although the exchange rate will eventually feed through to the domestic price level, evidence suggests that this pass-through effect is not instantaneous. Also, firms do not change their output and price in response to changes in signals of financial variables or monetary policy within the month due to inertia, adjustment cost, and planning delays. Therefore, I normalize this subsystem in the lower-triangularized order of y and π. My estimated results, however, are robust to the reverse order of π and y. As shown in the system of equations (6), I also do not impose any structure on the foreign block of variables, but follow Cushman and Zha (1997) to keep them in the lower-triangularized fashion of the order y^*, π^*, i^*, wxp^*.

4.2 A Bayesian Approach of Imposing Restrictions and Estimation

Two circumstances unfold from the identification scheme in the previous subsection. First, the number of contemporaneous restrictions imposed is greater than the number of restrictions necessary to exactly identify the model: while we need a total of 36 zero restrictions for an exact identification, we have imposed a total of 50 zero restrictions. Therefore, the imposition of 14 over-identifying restrictions on the contemporaneous-coefficient matrix A restricts the variance-covariance matrix of the reduced-form residuals (Σ). Second, the identifying restrictions involve simultaneous interactions among the target rate, the nominal interest rate, and the exchange rate. Because of this high degree of simultaneity, the shape of the posterior density for the model parameters tends to be non-Gaussian, which makes the importance sampling method of obtaining finite-sample inferences inefficient, as also noted by the original developers of this technique (Leeper, Sims, and Zha (1996) and Zha (1999)). Therefore, we cannot use the existing importance sampling technique as did Cushaman and Zha (1997) and Kim and Roubini (2000), although their identification approaches also had simultaneous interactions among the contemporaneous variables, but to a lesser extent than my approach.

To circumvent the problem incurred due to this simultaneity in my over-identified structural VAR model, I use the Gibbs sampling method, developed by Waggoner and Zha (2003), in order to obtain accurate statistical inference from the parameter estimates and the impulse responses. The advantage of this approach is that it delivers accurate statistical inferences for models with a high degree of simultaneity among the contemporaneous variables, as well as for models with restricted variance-covariance matrices of the residuals and for models with restrictions on lagged coefficients. To explain how the Gibbs sampling method can be applied to my over-identified structural VAR model, let a_i be the ith row of the contemporaneous-coefficient matrix A, and f_i be the ith row of the lagged-coefficient matrix F, defined in the structural equation (2), where $1 \leqslant i \leqslant n$. Let Q_i be any $n \times n$ matrix of rank q_i, and R_i be any $k \times k$ matrix of rank r_i. Therefore, the linear restrictions on the contemporaneous-coefficient matrix A and on the lagged-coefficient matrix F can be summarized, respectively, as follows:

$$Q_i a_i = 0, \quad i = 1, ...n, \tag{7}$$

$$R_i f_i = 0, \quad i = 1, ...n. \tag{8}$$

Assuming that there exist non-degenerate solutions to the above problems, I can define a $n \times q_i$ matrix U_i whose columns form an orthonormal basis for the null space of Q_i, and a $k \times r_i$ matrix V_i whose columns form an orthonormal basis for the null space of R_i. Therefore, a_i and f_i, which, respectively, are the rows of A and F, will satisfy the identifying restrictions (7) and (8) if and only if there exists a $q_i \times 1$ vector b_i and a $r_i \times 1$ vector g_i such that

$$a_i = U_i b_i, \tag{9}$$

$$f_i = V_i g_i. \tag{10}$$

The model then becomes much easier to handle by forming priors on the elements of b_i and g_i, since the original parameters of a_i and f_i can be easily recovered via the linear transformations through U_i and V_i. Waggoner and Zha (2003) demonstrated that using this approach, simulations can be carried out on an equation-by-equation basis, which vastly reduces the computational burden of the problem. To obtain the finite-sample inferences of b_i and g_i, and their functions, that is, impulse responses, it is necessary to simulate the joint posterior distribution of b_i and g_i. To do this simulation, I follow Waggoner and Zha's (2003) two-step Gibbs sampling procedure.[2] First, I simulate draws of b_i from its marginal posterior distribution, and then, given each draw of b_i, I simulate g_i from the conditional posterior distribution of g_i. The second step is straightforward, since it requires draws from multivariate normal distributions. The first step, however, is less straightforward, since my structural identification of the contemporaneous-coefficient matrix A makes a restricted reduced-form covariance matrix.

5. Empirical Evidence of the Effects of Monetary Policy Shocks

This section presents estimated results. First, I report the results from the basic model, and then I present the results of the extended model, where I incorporated money, identified with a money demand function, into the model.

The first step of estimation is to test the over-identifying restrictions imposed on the contemporaneous and the lagged coefficients. Following Cushman and Zha (1997), I perform a joint test of the contemporaneous and the lagged identifying restrictions. As long as all restrictions are treated as a restricted subset of the complete unrestricted parameter space, the likelihood ratio test can be applied to test the overall identifying restrictions. In my model, the contemporaneous-coefficient matrix A has 14 over-identifying restrictions, and, with a lag-length of eight, the number of lagged restrictions on the non-Canadian block is 160. Therefore, with a total of 174 restrictions, the estimated Chi-squared statistic $\chi^2(174) = 176.543$ implies that the null is acceptable at the significance level of 0.432.

As mentioned in section 4.1, because of a greater degree of simultaneous interactions among the variables, my structural approach differs from the existing approaches in the literature. Therefore, the estimated contemporaneous coefficients will be informative about the effectiveness of my approach. The estimated contemporaneous coefficients of the first three equations of the model are reported in Table 1. Since the production sector and the foreign block of variables do not have any structural interpretations – because they are estimated in

[2]For a detailed explanation of the algebra and the algorithm, see Waggoner and Zha (2003).

a triangularized fashion – those contemporaneous coefficients are not produced here. The significance of most of the contemporaneous coefficients, and in particular the strong significance of the simultaneously interacted coefficients – a_{12}, a_{21}, a_{13}, a_{31}, a_{23}, a_{32} – indicates that both the recursive identification and the structural identification that do not allow the financial variables to interact with each other simultaneously would be erroneous.

Table 1: Estimated contemporaneous coefficients

Parameter	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}
Estimate	-0.740	7.923	0	0	0	0	0.133	1.384
(SE)	(0.115)	(3.420)	–	–	–	–	(0.092)	(3.765)
Parameter	a_{21}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}
Estimate	0.928	-9.822	0	0	0	0	-0.214	0
(SE)	(0.540)	(3.467)	–	–	–	–	(0.134)	–
Parameter	a_{31}	a_{32}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}
Estimate	-0.090	0.087	0.015	-0.986	-1.405	-0.0002	-0.024	1.068
(SE)	(0.036)	(0.034)	(0.004)	(0.520)	(0.720)	(0.0043)	(0.011)	(0.273)

Note: Entries correspond to rows 1 through 3 of the contemporaneous-coefficient matrix A defined in equation (6), and apply to shocks to i_0, i, and s respectively.

In the monetary policy equation, the negative and significant coefficient of the nominal interest rate implies that the Bank of Canada tightens monetary policy if the current nominal market interest rate is low. The positive and significant coefficient of the exchange rate shows that, as a measure of leaning today against tomorrow's wind, the Bank increases the overnight target rate to offset currency depreciation. These results are also consistent with the inflation-targeting monetary policy of the Bank of Canada. Since both the lower market interest rate and the depreciation of the Canadian dollar are indication of future inflation, the Bank would want to tighten the monetary policy by raising the overnight target rate to reduce future inflation by influencing these variables. The positive and significant coefficient of the federal funds rate confirms the traditional belief of the Fed being the leader and the Bank of Canada being the follower. Although the coefficient of the world export commodity price index is not significant, its positive sign implies that the Bank undertakes contractionary monetary policy, seeing the higher world export price.

All the contemporaneous coefficients of the nominal interest rate equation and the exchange-rate equation are statistically significant at less than the 0.05 level, except the coefficient of the US inflation rate on the exchange-rate equation. The significance of these coefficients validates my structural identification and the simultaneity I assumed among the target rate, the nominal interest rate, and the exchange rate. On the other hand, the significance of the coefficients of the exchange-rate equation with the non-financial variables, except the US inflation rate, justifies the assumption that an efficient exchange-rate market can contemporaneously respond to these variables within the month. It is important to note that although my identification scheme does not allow the market interest rate to react to the non-financial variables directly within the month, it allows this rate to react to these variables indirectly via reacting to the exchange rate, which, in turn, reacts to all the variables. When I test whether the nominal interest rate directly responds to the non-financial variables, I find the coefficients of these variables both individually and jointly insignificant.

Next I report the estimated impulse responses due to the monetary policy shock identified in my structural VAR model. Before presenting the impulse responses, it would be worth discussing what the Bank of Canada says about how the monetary policy transmission mechanism operates. According to the Bank, since Canada is a small open economy, the monetary policy operates through the interest rate and the exchange rate channels.[3] Following a contractionary monetary policy shock, for example, the nominal market interest rises, which causes an inflow of capital into the country from around the world. This capital inflow then appreciates the domestic currency. The rise in interest rates also increases the cost of borrowing, and thus tends to dampen the demand for interest-sensitive consumption and investment expenditures. On the other hand, the appreciation of the domestic currency increases prices of home products relative to foreign ones, leading to a decline in net exports. Taken together, the effects of the rise in interest rates and the appreciation of the currency cause a reduction in aggregate demand.

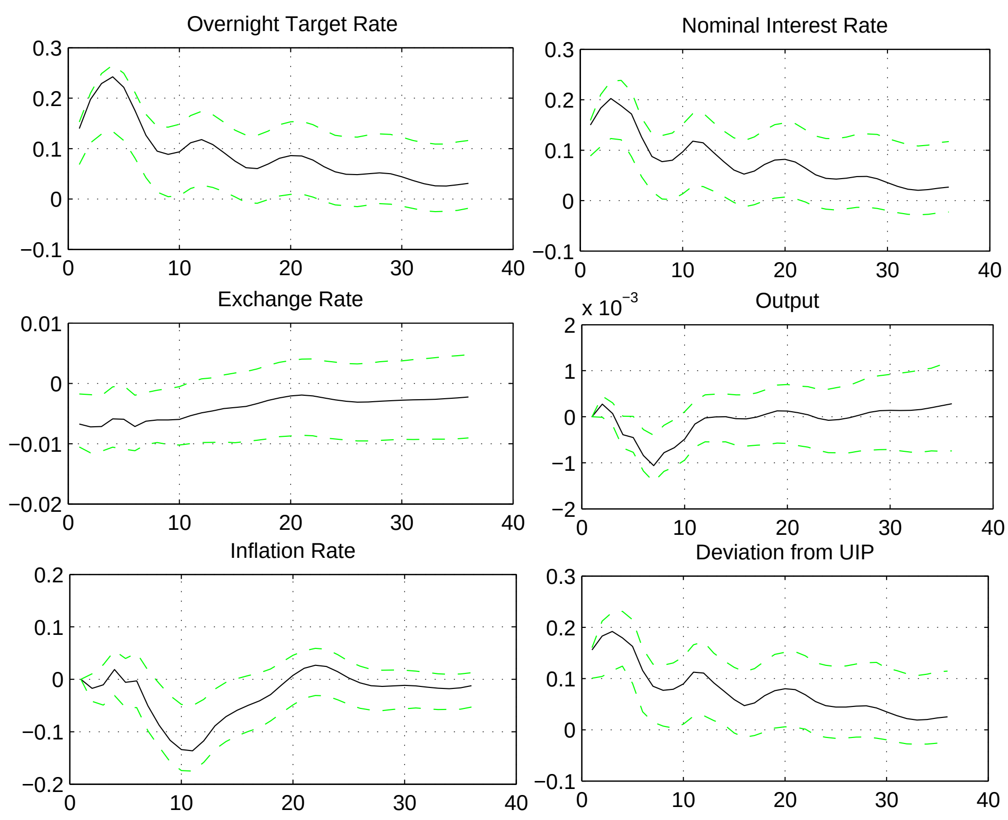
Over short periods of time, since output is determined by aggregate demand, the fall in aggregate demand causes a fall in aggregate output. With a given underlying growth rate of potential output, this reduction in actual output implies a negative output gap. The final step of the monetary policy transmission mechanism is the link from this output gap to the inflation rate in the economy. While this negative output gap might continue for a while, but eventually the economic slack leads to a fall in wages and prices of other inputs. Finally, this reduction in firms' costs of production leads to a reduction in the price of output, that

[3]These information is available at the Bank of Canada's web site: $bankofcanada.ca/en/ragan\_paper/monetary.html$

is, to a low inflation rate in the economy. Therefore, according to this mechanism, while the effects of the policy shock on interest rates and the exchange rate are realized immediately, this effect on the level of output is realized with lag and on the price level with further lag.

The estimated impulse responses of the macroeconomic variables are displayed in Figure 1. The response horizon, in months, is given in the horizontal axis. The solid lines are the estimated impulse responses computed from the values of a_i and f_i, defined in subsection 4.2, at the peak of their posterior distributions, while the upper and lower dashed lines are one-standard-error bands, derived using the Bayesian Gibbs sampling method of Waggoner and Zha (2003).[4]

Figure 1: Impulse Responses Due to Monetary Policy Shock



We observe from the figure that a one-standard-deviation contractionary monetary policy shock of increasing the overnight target rate by 25 basis points increases the nominal market interest rate by 20 basis points, which remains statistically significant for about a year.

[4]The error bands are computed from a set of 10000 draws, which hardly changes with the change in the number of draws. I gratefully acknowledge Tao Zha for helping me with the Matlab codes.

Following the same shock, the exchange rate appreciates on impact, and gradually depreciates to the terminal value. This reaction of the exchange rate is, therefore, consistent with Dornbusch's prediction that, following a policy shock, the exchange rate overshoots its long-run level on impact, followed by a gradual adjustment to the initial value. We also observe that due to this contractionary policy shock, the output level falls, which is realized with a lag of about half a year. Finally, this policy shock lowers the inflation rate by 15 basis points, which starts to be significant with a lag of more than half a year and remains significant up to a year and a half after the shock was introduced. We, therefore, see that the financial variables respond to the policy shock immediately, and the non-financial variables respond with lags. Therefore, these impulse responses are consistent with the Bank of Canada's predictions of the dynamic responses of a contractionary monetary policy shock, both in terms of the direction of the responses and the timing of the responses.

At this point, it would be interesting to compare my findings with other findings in the literature. Both Cushman and Zha (1997) and Kim and Roubini (2000) found that the contractionary monetary policy shock appreciates the currency rate on impact, which then gradually depreciates to the terminal value. However, Cushman and Zha (1997) did not find any significant effect on the nominal interest rate and concluded that the monetary policy transmission mechanism operates through the exchange rate, not the interest rate. About the effect on the price level, they found that this effect started to be significant after about two years, which remained significant up to the end of the third year.

The contractionary monetary policy shock in Kim and Roubini's (2000) approach produced the liquidity puzzle – a decrease in the nominal interest rate following a contractionary policy shock – for Canada and a few other G-7 countries. They also found that this policy shock had a statistically significant effect on the Canadian price level for more than four years. On the other hand, the contractionary monetary policy of raising the overnight target rate in a recursive identification approach by Bhuiyan and Lucas (2007) increased the real interest rate and lowered inflationary expectations. However, the impulse response of the exchange rate was puzzling, and there was no significant effect on output.

I believe that the superiority of the impulse responses generated in my model, in terms of matching with the theoretical prediction, is due the more accurate identification of the exogenous monetary policy shock. Since the overnight target rate cannot be influenced by other shocks in the economy, except through an endogenous policy response of the Bank to changes in the variables captured in the policy equation, innovations to this equation will truly estimate exogenous policy shocks. In addition, in my structural model, I realistically allow the financial variables – the target rate, the nominal interest rate, and the exchange rate – to react to each other and to a number of other home and foreign variables within the month, which increases the precision of the model identification. On the other hand, the Gibbs sampling technique of estimation, in the current context of increased simultaneous interactions among the contemporaneous variables, produces more reliable parameter estimates and subsequent impulse responses.

Since uncovered interest rate parity (UIP) is directly related with the interest rate and the exchange rate, I also investigate if the monetary policy shock induces a systematic departure from this parity. To explore this issue, I follow Eichenbaum and Evans (1995) to define ψ as the *ex post* difference in the return between investing in one-period Canadian assets and one-period US assets, that is, $\psi_t = i_t^* - i_t + 12(s_{t+1} - s_t)$. For a direct comparison with interest rates, which are already in annual terms, I multiply the exchange rate change by twelve. If the UIP condition holds and expectations are rational, the conditional expectation of this excess return should be zero. From the estimated impulse responses of the interest rate and the exchange rate, I compute the impulse response of this excess return due to the monetary policy shock as shown in the lower right block of figure 1. We see from the figure that the contractioanry monetary policy shock induces a significant excess return in Canadian currency for about one year.

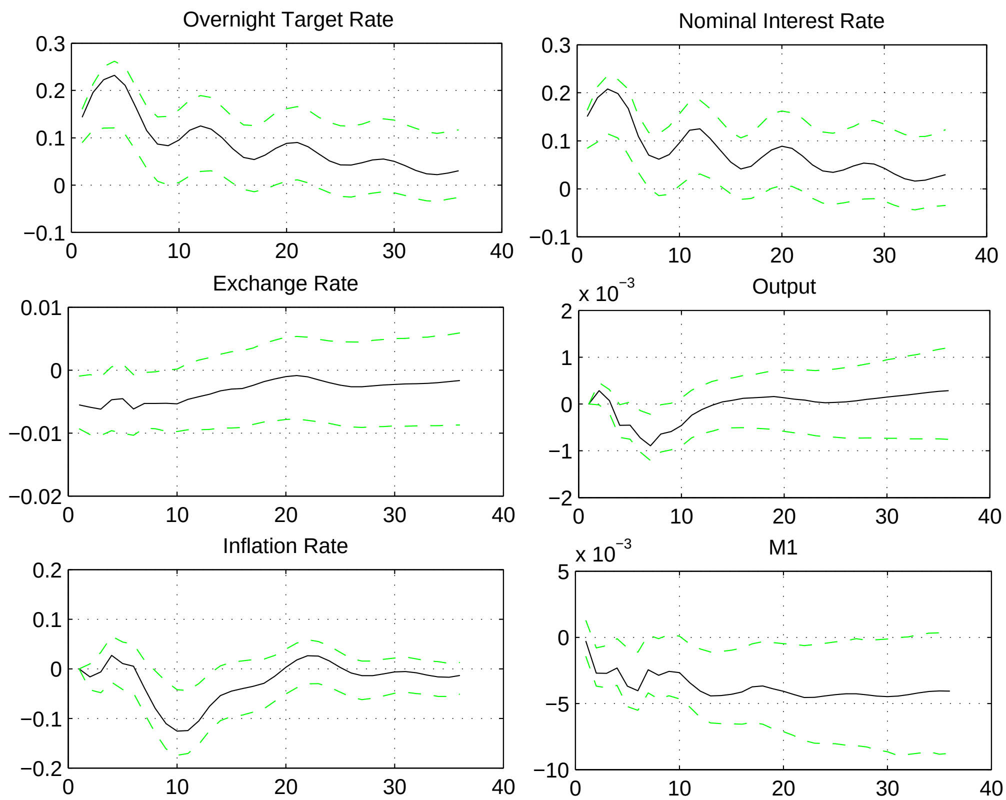
It is interesting to further explore the implication of the increase in the deviation from the parity condition, since the effect of the policy shock on this deviation embodies the effects on the nominal interest rate and the exchange rate. We observe from figure 1 that the nominal market interest rate and the deviation from the UIP condition follow a similar pattern of impulse response, except that the interest rate increases slightly more than the UIP. On the other hand, following the same shock, the exchange rate gradually depreciates toward the terminal value after an impact appreciation. Since the effect of the policy shock on the US interest rate is zero, the only factor that can minimize the excess return in Canadian assets is the depreciation of the Canadian dollar. Taken these results together, we can conclude that although the exchange rate depreciates over time, the magnitude of depreciation is not high enough to offset the excess return from Canadian bonds, which causes this large departure from the parity condition.

This finding differs from that of Eichenbaum and Evans (1995), who, identifying contractionary monetary policy shocks with innovations in the narrow monetary aggregates in a recursive VAR model, found that the higher return in home currency is further magnified by the persistent appreciation of the exchange rate for a prolonged period of time. My result also differs from Cushaman and Zha's (1997), who, using a similar type of model as mine, reported somewhat insignificant impulse response of the UIP deviation due to the policy shock. As mentioned before, one important difference between my findings and Cushman and Zha's (1997) is that I find strong and significant effect of the policy shock on both the market interest rate and the exchange rate, whereas they found strong and significant effect on the exchange rate only and insignificant effect on the interest rate. Therefore, given the relationship of the UIP deviation with the interest rate and the exchange rate, we can conclude that the significant effect of the policy shock on the UIP deviation in my model is due to the domination of the interest rate effect of the policy shock over the exchange rate effect, while the reverse is true for the insignificant effect of the policy shock on the UIP deviation in Cushman and Zha's (1997) model. Finally, the immediate overshooting of the exchange rate coupled with the large deviation from the parity condition in my model imply that the immediate or delayed overshooting is not driven by uncovered interest rate parity as in Dornbusch (1976).

Some of the previous studies used money as the monetary policy instrument, so it is interesting to see how the impulse responses change due to the incorporation of money into the model. Therefore, still keeping the overnight target rate as the policy instrument, I add an informal the money demand function into the structural VAR model, where I allow money holding to respond contemporaneously to the nominal interest rate, the inflation rate, and output. In the contemporaneous identification, I also let the nominal interest rate and the exchange rate be contemporaneously affected by money. No other variables either affect money or are affected by money contemporaneously. There are no restrictions in the lagged coefficients of the money demand function, and the rest of the identification scheme of this extended model is the same as the basic model.

The impulse responses of the variables due to an overnight target rate shock in the extended model are reported in figure 2. We observe from the figure that there is no marginal contribution of the inclusion of money into the model: the pattern of dynamic responses of the variables due to the policy shock remain unchanged in the extended model. In the extended model also, the contractionary monetary policy shock increases the nominal interest rate and appreciates the exchange rate on impact. This policy shock then transmits to the real sector, lowering the level of output with a lag of more than half a year and decreasing the inflation rate with a lag of about a year.

Figure 2: Impulse Responses in the Extended Model



While the impulse responses of the other macroeconomic variables are robust to the incorporation of money into the model, the dynamic response of the money stock itself is not an exact mirror image of the dynamic response of the overnight target rate. Figure 2 shows that the contractionary monetary policy shock of increasing the overnight target rate peaks in the second month, followed by a gradual decline, which becomes insignificant after about one year. On the other hand, following to the shock, the money stock keeps declining, the highest impact of which is not realized until the end of the second year, and the effect remains statistically significant for about three years. This imprecise dynamic response of the money stock might be due to the fact that monetary aggregates are influenced by other factors in the economy, such as private-sector behavior, in addition to the monetary policy decision. This impulse response of M1, therefore, casts further doubt on the justification of using money as the policy instrument and rationalizes the use of the target rate as the policy instrument.

6. Conclusion

This paper develops an open-economy structural VAR model for Canada in order to estimate the effects of a monetary policy shock, using the overnight target rate as the policy instrument. The structural model developed here allows the financial variables of the model to interact contemporaneously with each other and with a number of other home and foreign variables. Since my identification involves simultaneous interactions in the contemporaneous relationships of the financial variables in the model, in order to increase the precision of the parameter estimates, I used a Bayesian Gibbs sampling method to estimate the model. This paper finds that the liquidity effect and the exchange-rate effect of the policy shock are realized immediately, while output responds with a lag of six months, and the inflation rate responds with a lag of about one year.

The results of this paper differ from those of other studies in the literature in a number of important respects. I find that the transmission of the monetary policy shock to real output operates through both the interest rate and the exchange rate, as opposed to Cushman and Zha (1997), who found that the transmission operates through the exchange rate only. I also find that, due to the contractionary policy shock, the exchange rate depreciates gradually after the impact appreciation, which helps only slightly to shrink the larger return in home currency. This result differs from that of Eichenbaum and Evans (1995), who, in a recursive VAR model, found that the larger return in home currency is further magnified by the persistent appreciation of home currency for a prolonged period of time after the policy shock. On the other hand, in the extended model, the impulse response of M1 confirms that shocks to monetary aggregates reflect some other shocks in the economy, and hence cannot be a good measure of exogenous monetary policy shocks.

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