Inflation and the Dispersion of Real Wages

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Abstract

The effects of inflation on real wage dispersion and welfare are studied in a cash-in-advance economy with a Walrasian goods market but a labor market with search friction in which firms enjoy monopsony power. In the labor market, firms post wages and both the employed and the unemployed workers search among the posted wages. In equilibrium, a higher inflation rate reduces the dispersion in real wages. This result is consistent with both the observed trends in wage dispersion and the inflation rate witnessed in the 1980's and the 1990's in the U.S. and the empirical literature linking reduced inflation to greater wage dispersion. While higher inflation always lowers consumption, output, and employment, the optimal inflation rate exceeds the Friedman rule.

Key Words: Inflation, Wage Posting, Search, Dispersion of Real Wages
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1. Introduction

In the eighties and the nineties, wage dispersion, including *residual* wage dispersion (*i.e.* differences in wages among workers with similar skills and job characteristics) increased dramatically in the U.S. (OECD (1997), Katz and Autor (1999)). Over this period the inflation rate also declined significantly relative to what it had been the 1970's. In this paper a general equilibrium monetary model is developed in which lower expected inflation increases the dispersion in real wages. This finding is consistent not only with the observed trends in wage dispersion and expected inflation for the U.S., but also with a sizeable empirical literature linking reduced inflation to greater wage dispersion for several different countries and time periods.

Lipsey and Swedenborg (1999) study the relationship between the price levels and wage dispersion for fifteen OECD countries for the period 1979-90. They find that the price level is negatively related to wage dispersion. Erikson and Ichino (1995) examine the effect of inflation on wage earnings differentials over the period 1976-90 in Italy using the wage data taken from metal-manufacturing firms. They find that a higher inflation rate significantly reduced changes in wage earnings differentials. Hammermesh (1986) analyzes the relationship between the inflation rate and the dispersion in the relative wage changes for the period 1955-81 in the United States using data from twenty two-digit manufacturing industries and finds that higher inflation, especially unexpected inflation, reduced the dispersion in the relative wage changes.[1]

These empirical findings are at variance with the predictions of the models which assume that firms face costs or other barriers to changing nominal wages. Inflation allows firms facing negative demand shocks to bring real wages in line with productivity (Tobin 1972). In the case of downward nominal rigidity, inflation increases real wage dispersion by allowing firms to provide real rewards to those whose productivity is increasing, while cutting rewards to those who are becoming less productive without reducing the nominal wages (Hammermesh 1986). Akerlof, Dickens, and Perry (1996) argue that inflation enables the firms facing downward nominal wage rigidity to change real wages in the case of negative demand shocks.[2]

[1] Allen (1987) studies the relative wage variability across industries in the U.S. and finds that both expected and unexpected inflation has negative though insignificant effect on the relative wage variability for the period 1947-83. Souza (2002) examines the changes in wage dispersion of male workers in metropolitan areas of Brazil for the period 1981-97 and finds that higher expected inflation rate reduced the standard deviation of log of wages, though increased the ratio of 90th to 10th percentile .

[2] Drazen and Hammermesh (1987) develop a model in which agents are confused between the aggregate and the relative shocks and the degree of indexation of wages depends on the uncertainty about inflation.

Many models with nominal rigidities imply that firms will follow a strategy of the [s,S] variety when setting wages (*e.g.* Sheshinski and Weiss (1977), Benabou (1988, 1992) , Diamond (1993)).[3] In such models, a reduction in the trend inflation rate reduces the bound of real wages within which wages are not changed. That is, lower inflation reduces the range in which nominal wages do not respond to price level changes, resulting in less dispersion of real wages rather than more.

In our model, the goods market is Walrasian with purchases subject to a standard cash-in-advance constraint. The labor market is characterized by search frictions, with workers and firms brought together by a matching function. Wages are determined in a general equilibrium variant of the model developed by Burdett and Mortensen (1998), in which firms post wages and both the employed and the unemployed workers search. In this framework, on-the-job search weakens the monopsony power of wage-posting firms, and thus wage dispersion is an equilibrium outcome even if firms and workers are both *ex ante* homogeneous. Variants of this model have been extensively used to explain wage dispersion (*e.g.* Bontemps, Robin, and van den Berg (2000), van den Berg and Ridder (1998), Vurren, van den Berg, and Ridder (2000)).

In the model studied, higher expected inflation, by eroding the expected future value of fiat money, reduces the profitability of firms. In addition, the real reservation wage of the unemployed workers rises, which further lowers the profitability of firms inducing them to post a smaller number of vacancies.

The higher real reservation wage of unemployed workers increases the lower support of the real wage earnings distribution. The effect of inflation on the upper support, however, is mitigated by two factors. Firstly, the firms posting the highest real wage do not face any competition from other firms to retain their workers, while the firms posting lower real wages do. The result is that the firms posting the highest real wage need not increase their real wage as much as the firms posting the lowest real wage. Secondly, the decline in the level of vacancies posted reduces the effectiveness of on-the-job search in eroding the monopsony power of firms by lowering the

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In such a set-up, if the degree of indexation is constant, higher unanticipated inflation increases the variance of relative wage changes. However, if the degree of indexation increases with unanticipated inflation, then the variance of relative wage changes may decline with unanticipated inflation. They find that unexpected inflation significantly reduced the relative wage dispersion in Israel during the period 1956-82. Expected inflation has negative though insignificant effect. In contrast to this model, in our model there is no confusion between the aggregate and the relative shocks and the degree of indexation is constant.

[3] In these models, firms post prices and incur menu cost in changing them. The results of these models remain the same if we assume that firms post wages and face menu cost in changing them.

matching rate of workers. Consequently, the support of the distribution of real wage earnings declines and the dispersion measured by the coefficient of variation and the ratio of 90th percentile to 10th percentile of the distribution of real wage earnings are reduced. In addition, a fall in the level of vacancies reduces employment and output.

The result that inflation affects the real reservation wage of unemployed workers has an interesting welfare implication, that the optimal rate of inflation exceeds the Friedman rule. The economy considered in this paper has two sources of inefficiency: i) the buyers' nominal money balance constraint and ii) the monopsony power of firms. The Friedman rule removes the first source of inefficiency. But, as the inflation rate approaches the Friedman rule, the real reservation wage of the unemployed workers falls for a given level of consumption, and firms create too many vacancies relative to the social optimum.

There is sizeable empirical literature linking lower real minimum wage with larger wage dispersion (DiNardo, Fortin, Lemieux 1996, Katz and Autor 1999, Lee 1999). For example, DiNardo et.al. (1996) and Lee (1999) find that the increased wage dispersion in the eighties in the U.S. is largely due to the decline in the federal minimum wage in real terms. Lee (1999) also suggests that the changes in the minimum wage has sizeable "spill-over" effect *i.e.*, it affects the distribution of wages above the minimum wage.

In this paper, we also consider the effect of changes in the binding real minimum wages (*i.e.* real minimum wage higher than the real reservation wage of the unemployed workers) on the distribution of real wage earnings. We find that lower real minimum wage increases the dispersion of real wages and does so by affecting the entire distribution. In addition, for a given real minimum wage higher inflation rate reduces the dispersion in real wages, though its effect on the dispersion is smaller compared to the case in which the real minimum wage is not binding.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, the optimal strategies of households are characterized. Section 4 defines a class of stationary monetary equilibrium and derives conditions for the existence of a unique stationary monetary equilibrium with non-degenerate real wage earnings distribution. Section 5 studies the effect of inflation on output and welfare in equilibrium. In Section 6, an example is constructed to illustrate the welfare cost of inflation. In Section 7, the effect of inflation rate on real wage dispersion is analyzed. Section 8 examines the robustness of the result that higher inflation reduces real wage dispersion. Section 9 examines the effect of real minimum wage on the dispersion of real wages. Section 10 concludes and summarizes. All the proofs are contained in Appendix 1.

2. The Economy

2.1 The Household Structure

Consider a cash-in-advance economy with no aggregate uncertainty comprised of a large number of infinitely-lived identical households with measure one. Each household, in turn, is comprised of three types of infinitely-lived members: a buyer, a firm, and a unit measure of identical workers.[4] Time is discrete.

Each type of member in the household plays a distinct role. The buyer buys the household's desired consumption goods in the goods market. The firm posts vacancies, hires workers in the labor market, produces, and sells goods in the goods market. The unemployed workers search for suitable jobs. The employed workers work and also search for better jobs. It is assumed that the firm uses a linear production technology, and each employee produces y units of goods.

Members of the household do not have independent preferences. Rather, the household prescribes the trading and production strategies for each member to maximize overall household utility. The members of the household share equally in the utility generated by the household consumption. The household maximizes the discounted sum of utilities from the sequence of consumption less the disutility arising from the workers' working and searching and posting of vacancies by the firm. The household's inter-temporal utility is represented by

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \Big[u(c_t) - (1+\phi)e_t - u_t - k(v_t) \Big] \tag{2.1}$$

where r is the rate of time-preference, and $u(c_t)$ and $k(v_t)$ are the utility derived from consumption and the disutility of posting vacancy respectively. ϕ is the disutility of working. For simplicity, it is assumed that the disutility of search is unity for both the employed and the unemployed workers.[5]

[4] The firm need not be the part of the household. One can assume that firms are owned by the households and firms take decisions in order to maximize the utility of the owners and rebate their profits equally to the owners. The construction of the household is similar to that in Fuerst (1992), Merz (1995), Shi (1999), and Head and Shi (2000). This construction makes the model highly tractable analytically.

[5] With different disutility of search the real reservation wage of unemployed workers depends not only on the disutility of work but also on the expected gain from search, which makes the model analytically complex (Kumar 2003). One can also endogenize the search-intensities of employed and unemployed workers. Endogenization of search-intensity will lead to considerable analytical and computational complexity. The reason is that the model generates a non-degenerate wage earnings distribution, which will induce non-degenerate distribution of search intensities, which is a high dimensional object on which the wage posting strategies of firms must depend.

Let $u'(c_t) > 0$ and $u''(c_t) \leq 0$. Also, let $k(v_t)$ be strictly increasing and convex and satisfy $\lim_{v_t \to 0} v_t k'(v_t) = 0$, and $\lim_{v_t \to 0} v_t k''(v_t) + k'(v_t) < \infty$.

Trade in this economy takes place in two separate markets – the goods market and the labor market. Since the focus of this paper is on the effects of inflation, it is assumed that buyers require cash in order to buy goods, and firms pay wages in cash to workers. In order to facilitate transactions, it is assumed that each household is endowed with $\hat{M}_0$ units of fiat money at time zero. At the beginning of each subsequent period, each household receives $(g-1)\hat{M}_{t-1}$ units of fiat money from the government as a lump-sum transfer, where $\hat{M}_t$ is the per-household aggregate holding of fiat money at time t in the economy. The government plays no role in the economy, other than making lump-sum transfers to the households. The households also acquire fiat money by selling goods and from wage receipts of the employed workers.

2.2 Goods Market

The goods market is assumed to be competitive. Let M_t be the post-transfer nominal money balances with the representative household at the beginning of period t. The household allocates the available money M_t to the buyer who goes to the goods market to acquire the consumption goods subject to the cash-in-advance constraint

$$p_t c_t \leq M_t \quad \forall\, t \tag{2.2}$$

where p_t is the price of the consumption goods and c_t is the amount purchased. The firm produces the consumption goods using the existing employees and sells the goods in the goods market. The total receipt of nominal money to the firm at time t is $y p_t J_t$, where J_t is the total measure of employees of the firm at time t.

2.3 Labor Market

The labor market is characterized by search frictions. Matches between workers and firms are not instantaneous. Rather, the firms who want to hire workers and the workers who either want to work or change their jobs have to search for suitable matches. The workers and the firms are brought together randomly through a matching function.

The process of wage determination is modeled using a version of the wage-posting model developed by Mortensen (2000), which extends the wage-posting model of Burdett and Mortensen

(1998) by endogenizing the job arrival rates which are exogenous in the Burdett-Mortensen model.[6] It is assumed that firms enjoy monopsony power in the labor market, and that they post wage offers. A *posted wage offer* is defined as a wage contract, which fully specifies the nominal wages payable to the workers for all time to come. The workers (both employed and unemployed) search among the posted wage offers.

Because of random matching, individual firms and workers face uncertainty in the matching outcomes. In particular, random matching induces a non-degenerate distribution of money holdings. As well, the model generates non-degenerate wage earnings distribution, which also induces a non-degenerate distribution of money holdings. The construct of a large household makes the distribution of money holding degenerate within the *households* and allows the analysis of a representative household, which makes the model highly tractable analytically.

The representative household chooses the level of vacancies, the distribution of wage offers to be posted (which can be degenerate), and the optimal job-acceptance strategies of both the employed and the unemployed workers. For simplicity, we abstract from nominal rigidities and assume that a wage offer promises to pay a constant real wage *i.e.*, a real wage offer [7]

$$w \equiv \left\{ \frac{w_t}{p_t}, \frac{w_{t+1}}{p_{t+1}}, ... \right\} \equiv \{w, w, ...\} \tag{2.3}$$

where w_t is the nominal wage at time t. In other words, the household offers fully-indexed wage contracts to the searching workers. In addition, we assume that there is no possibility of renegotiation as in Burdett and Mortensen (1998) and Mortensen (2000).[8]

In the model, the employed workers of the household, the employees of the firm, and the posted vacancies can be heterogeneous with respect to real wages. Table 1 lists the notations of the workers and the employees in the representative household and the vacancies posted by it.

[6] Rosholm and Svarer (2000) estimate the Mortensen (2000) model with firm-specific training expenditures using the Danish labor market data, and find that the model provides a good characterization of some empirical features of the labor market.

[7] One can assume that the households post infinite sequences of nominal wages to which they can credibly commit and then restrict attention to a stationary equilibrium, which supports constant real wage offers. However, the assumption that households post constant real wage offers simplifies the exposition a great deal without affecting the results.

[8] Section 8 below discusses extensions of the model, which allow for renegotiation similar to Coles (2001) and Postel-Vinay and Robin (2002). The qualitative results do not change.

Table 1

Notation: Posted Vacancies, Workers, and Employees in the Representative Household

	Distribution	Support	Total Measure
Vacancies	$F_t(w)$	$\mathcal{F}_t$	v_t
Employees of the Firm	$H_t(w)$	$\mathcal{H}_t$	J_t
Employed Workers	$G_t(w)$	$\mathcal{G}_t$	e_t
Unemployed Workers			u_t

As a convention, the aggregate variables and the decision variables of the other households, which are taken as given by the representative household, are denoted with superscript "ˆ".

After the decisions of the household, the firm and the workers go to the labor market. The firm posts the measure of vacancies, v_t, and the distribution of real wage offers, $F_t(w)$. In the case of a match, it pays the employed worker with the real wage earnings w, $p_t w$ at time t, $p_{t+1}w$ at time $t+1$, and so on as long as the match continues. It is assumed that the firm pays its workers wages from its nominal sales receipts.

Posting of vacancies is costly, and the household incurs disutility $k(v_t)$ for posting v_t vacancies. As is common in the search literature, it is also assumed that the firm treats each vacancy separately from other vacancies and hires only one worker per vacancy. Once a vacancy gets matched, the match starts producing from the next period and continues until the match is dissolved.

In the model, it is assumed that while the searching workers know the distribution of real wage offers posted. The actual real wage offers are revealed only after the search.

2.4 Matching and Job Separations

Workers and vacancies are matched through an aggregate matching function, which relates the flow of hiring, s, to the aggregate measure of the searching employed and unemployed workers and the aggregate measure of vacancies. The matching function $s(\hat{v}_t, \hat{e}_t, \hat{u}_t)$ is assumed to be concave, increasing, and is subject to constant returns to scale. It is also assumed that the searching employed and unemployed workers are perfect substitutes *i.e.*, only the aggregate measure of the workers searching matters and not their composition. The aggregate flow of matches then is given by

$$s(\hat{v}_t, \hat{e}_t, \hat{u}_t) = s(\hat{v}_t, 1) = s(\hat{v}_t) \tag{2.4}$$

Since the aggregate measure of the workers in the economy is unity, (2.4) also defines the aggregate matching rate of the workers. The aggregate matching rate of vacancies is given by

$$\frac{s(\hat{v}_t)}{\hat{v}_t} \tag{2.5}$$

Assume that $\lim_{\hat{v}_t \to 0} s(\hat{v}_t) \to 0$, $\lim_{\hat{v}_t \to 0} s'(\hat{v}_t) \to \infty$, and the aggregate matching rate of vacancies is decreasing in the level of vacancies, v_t.

The employed workers face the risk of unemployment. Each period fraction ρ of the household's existing matches are exogenously dissolved, with all such matches equally likely to fall in this group. Note that matches dissolve for two reasons: i) the employed worker in a match receives a better job offer and ii) the match is dissolved exogenously.

3. The Household's Choice Problem

3.1 Timing

The representative household at the beginning of period t enters with the measures of unemployed workers, u_t, employed workers, e_t, employees of the firm, J_t, and their distribution over real wage earnings. As previously mentioned, at the start of each period, the household receives a lump-sum money transfer, which is added to the nominal money balances carried from the previous period.

It is assumed that during any period t, trading takes place first in the goods market and then in the labor market.[9] The household gives the available money balance, M_t, to the buyer. The firm produces consumption goods using the existing employees. Buyers and firms trade in the goods market. After trading in the goods market, the buyer comes back to the household with the purchased goods and any residual nominal money balances and the firm with its nominal sales receipts. The firm pays wages to its employees and the employees return to their respective households with their nominal wage receipts. The profit of the firm, the wage receipts of the

[9] The results of the model do not hinge on whether the goods market opens first or the labor market. What is important is that there be time separation between the time when the households receive money and when they spend it. For the inflation rate (which is the focus here) to have impact, the time-separation between the acquisition of money and spending is essential.

employed workers, and any residual balance brought back by the buyer are added to the household nominal money balance for the next period.

After this, the labor market opens up. The household decides the measure of vacancies, v_t, and the distribution of real wage offers, $F_t(w)$, and prescribes the job-acceptance strategies to workers. Workers search among the posted real wage offers and accept or reject the offers received according to the prescribed job-acceptance strategies. Match dissolution takes place. Trading in the labor market and exogenous dissolution of matches determine the next period's measure of employed workers, e_{t+1}, and their distribution over real wage earnings, the measure of unemployed workers, u_{t+1}, and the measure of the employees of the firm, J_{t+1}, and their distribution over real wages. At the end of the labor market session, workers and firms go back to their respective households and consumption takes place. Time moves to the next period $t+1$.

3.2 The Optimal Job Acceptance Strategies of Workers

Before formally setting the household optimization problem, it is convenient to discuss the optimal job-acceptance strategies of workers prescribed by the household. In the current environment, the household knows that in any period t fraction ρ of the employed workers becomes unemployed and fraction $s(\hat{v}_t)$ receives new real wage offers, which they can accept or reject. Similarly, fraction $s(\hat{v}_t)$ of the unemployed workers realizes real wage offers, which they can accept or reject. The following lemma characterizes the real reservation wages of the employed and the unemployed workers.

Lemma 1:

The job acceptance strategies prescribed by the household have a reservation property.

1. The real reservation wage of an employed worker is the real wage he currently earns.
2. The real reservation wage of an unemployed worker, $\underline{w}$, satisfies

$$\omega_{Mt+1} p_{t+1} \underline{w} = \phi \tag{3.1}$$

where ω_{Mt+1} is the marginal value of nominal money balances to the household at time $t+1$.

Intuitively, the contribution of the employed workers earning higher real wages to the current utility of the household is higher compared to the employed workers earning lower real wages. At the same time, in the next period all the employed workers face the same aggregate matching rate, $s(\hat{v}_{t+1})$, as well as the aggregate distribution of real wage offers, $\hat{F}_{t+1}(w)$. Therefore, it is optimal for the household to instruct employed workers to accept any new real wage offer higher than their current real wage. (3.1) equates the utility of working at the real reservation wage of unemployed workers $\underline{w}$ (the left hand side) to the disutility of working (the right hand side). Thus it is optimal for the household to instruct unemployed workers to accept any real wage offer $w \geq \underline{w}$.

3.3 The Household Optimization Problem

Taking the aggregate distribution of real wage offers, $\hat{F}_t(w)$, the aggregate distribution of real wage earnings, $\hat{G}_t(w)$, the aggregate level of vacancies, $\hat{v}_t$, the price level in the goods market, p_t, the optimal choices of other households, the job-acceptance strategies of workers specified in Lemma 1, and the initial conditions $\{M_0,\ e_0,\ u_0,\ \ G_0(w)\}$ as given, the household chooses the sequence $\{c_t,\ M_{t+1},\ v_t,\ F_t(w)\}\ \forall\ t \geq 0$ to solve the following problem.

Household Problem (PH)

$$\max_{c_t, M_{t+1}, v_t, F_t(w)} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \Big[u(c_t) - (1+\phi)e_t - u_t - k(v_t) \Big] \tag{3.2}$$

subject to the buyer's nominal cash balance constraint given in (2.2) and the laws of motion:

$$M_{t+1} \leq M_t + (g-1)\hat{M}_t - p_t c_t + p_t J_t \big[y - \int_{\mathcal{H}_t} x dH_t(x) \big] + p_t e_t \int_{\mathcal{G}_t} x dG_t(x), \tag{3.3}$$

$$\begin{aligned} J_{t+1}H_{t+1}(w) \leq \big[1-\rho\big] J_t H_t(w) - s(\hat{v}_t) J_t \int_{\underline{\hat{w}}}^{w} (1 - \hat{F}_t(x)) dH_t(x) \\ + \frac{s(\hat{v}_t)}{\hat{v}_t} \hat{u}_t v_t [F_t(w) - F_t(\underline{\hat{w}})] + \frac{s(\hat{v}_t)}{\hat{v}_t} \hat{e}_t v_t \int_{\underline{\hat{w}}}^{w} \hat{G}_t(x) dF_t(x), \quad \forall w \in \mathcal{H}_t \end{aligned} \tag{3.4}$$

$$\begin{aligned} e_{t+1}G_{t+1}(w) \leq \Big[1 - \rho - s(\hat{v}_t)\big(1 - \hat{F}_t(w)\big) \Big] e_t G_t(w) \\ + s(\hat{v}_t) u_t [\hat{F}_t(w) - \hat{F}_t(\underline{w})] \hat{v}_t, \quad \forall w \in \mathcal{G}_t \end{aligned} \tag{3.5}$$

$$u_{t+1} \leq u_t + \rho e_t - s(\hat{v}_t)\big(1 - \hat{F}_t(\underline{w})\big) u_t, \tag{3.6}$$

$$\begin{aligned} \hat{e}_{t+1}\hat{G}_{t+1}(w) \leq \Big[1 - \rho - s(\hat{v}_t)\big(1 - \hat{F}_t(w)\big) \Big] \hat{e}_t \hat{G}_t(w) \\ + s(\hat{v}_t) \hat{u}_t [\hat{F}_t(w) - \hat{F}_t(\underline{\hat{w}})] \hat{v}_t, \quad \forall w \in \hat{\mathcal{G}}_t \end{aligned} \tag{3.7}$$

$$\hat{u}_{t+1} \leq \hat{u}_t + \rho \hat{e}_t - s(\hat{v}_t)\big(1 - \hat{F}_t(\underline{\hat{w}})\big) \hat{u}_t, \tag{3.8}$$

(3.3) describes the law of motion of the household's nominal money balances. The first term on the right hand side is the nominal money balances of the household at time t, the second term is the lump-sum monetary transfer at the beginning of period $t+1$, and the third term is the money spent by the buyer at time t. The fourth term is the nominal profit made by the firm at time t. The first integral is the average nominal wage payment made by the firm. The second integral is the average nominal wage payment received by the employed workers of the household.

(3.4) states the law of motion of the employees of the firm earning real wages w and less. The term on the left hand side is the measure of employees earning real wage w and less at the beginning of the period $t+1$. The first term on the right hand side is the measure of employees of the firm earning real wages w and less at the beginning of the period t who are not separated from their matches exogenously. The second term is the measure of employees who leave their matches because they receive higher real wage offers. The third and fourth terms together give the measure of new matches formed on the vacancies posted with real wage offers of w and less.

(3.5) specifies the law of motion of the employed workers of the household with real wage earnings of w and less. The term on the left hand side is the measure of employed workers with the real wage earnings of w and less at the beginning of period $t+1$. The first term on the right hand side is the measure of employed workers with real wage earnings of w and less at the beginning of the period t who remain in the same pool at the end of the period. An employed worker leaves this pool either due to exogenous dissolution or because he receives a real wage offer higher than w. The second term is the measure of unemployed workers who receive the real wage offers of w and less.

(3.6) describes the law of motion of the unemployed workers. The first term on the right hand side is the measure of unemployed workers in the household at the beginning of time t. The second term is the measure of employed workers who become unemployed due to exogenous dissolutions at time t. The third term is the measure of unemployed workers at time t who become employed.

(3.7) and (3.8) are the aggregate laws of motion of the employed workers earning real wages w and less and the unemployed workers respectively. They have similar interpretation to (3.5) and (3.6).

3.4 The Optimal Choice of c_t and M_{t+1}

Let ω_{ct} and ω_{Mt} be the Langrangian multipliers associated with the constraints (2.2)and (3.3) respectively. The first order conditions for the optimal choices of c_t and M_{t+1} are given by

$$c_t: \ \frac{u'(c_t)}{p_t} = \omega_{ct} + \omega_{Mt}, \tag{3.9}$$

$$M_{t+1}: \ \omega_{Mt} = \frac{1}{1+r}[\omega_{Mt+1} + \omega_{ct+1}]. \tag{3.10}$$

The slackness condition associated with the optimal choice of consumption is given by

$$\omega_{ct}[M_t - p_t c_t] = 0. \tag{3.11}$$

The sufficient condition for the buyer's nominal cash balance constraint to be binding is that the nominal interest rate be strictly positive

$$\frac{(1+r)u'(c_t)p_{t+1}}{u'(c_{t+1})p_t} - 1 > 0 \tag{3.12}$$

We will assume that the buyer's nominal cash balance constraint is binding for the rest of the paper. The first order conditions have the usual interpretations. For the optimal choice of consumption, the household equates the marginal benefit of spending one unit of money (the left hand side of (3.9)) with the marginal cost (the right hand side of (3.9)), which is the sum total of the Langrangian multipliers associated with the buyer's nominal cash balance constraint and the law of motion of nominal money holding.

(3.10) states that by not spending one unit of money in the current period, the household relaxes the buyer's nominal cash balance constraint and the constraint on the household nominal money balance next period. (3.9) and (3.10) together imply that the marginal value of nominal money balances, ω_{Mt}, is given by

$$\omega_{Mt} = \frac{1}{1+r}\frac{u'(c_{t+1})}{p_{t+1}}. \tag{3.13}$$

3.5 The Optimal Choice of the Level of Vacancies, v_t, and the Distribution of Real Wage Offers, $F_t(w)$

The household instructs the firm to post the level of vacancies, v_t, and the distribution of real wage offers, $F_t(w)$, which maximize the total net return on vacancies. Let us first consider the optimal choice of the distribution of real wage offers, $F_t(w)$.

Let $\underline{\hat{w}}$ be the household's belief about the aggregate real reservation wage of the unemployed workers. Then, the household will not post any real wage offer less than $\underline{\hat{w}}$, since it will not be able to attract any worker. Conditional on $w \geq \underline{\hat{w}}$, the expected gross return on a real wage offer w posted is

$$R_t(w) = \frac{s(\hat{v}_t)}{\hat{v}_t}\left[\hat{u}_t + \hat{e}_t \hat{G}_t(w)\right]\Omega_{Jt}(w). \tag{3.14}$$

where $\Omega_{Jt}(w)$ is the marginal value of a filled job at real wage w to the household defined below. The expected gross return on the posted real wage w is the product of the marginal value of the filled job at the real wage w, $\Omega_{Jt}(w)$, and the expected measure of workers who will receive and accept the offer. In turn, the expected measure of workers who will receive and accept the offer is equal to the product of the aggregate matching rate of vacancies and the aggregate measure of workers with real reservation wage less than w. It is immediately clear from (3.14) that by posting a higher real wage offer, the household can increase the expected measure of workers who will receive and accept the offer .

The marginal value of a filled job with the real wage w can be defined recursively as

$$\begin{aligned}\Omega_{Jt}(w) =& \frac{1}{1+r}\Big[p_{t+1}(y-w)\omega_{M_{t+1}} \\ & \Big[1-\rho-s(\hat{v}_{t+1})(1-\hat{F}_{t+1}(w))\Big]\Omega_{Jt+1}(w)\Big]\end{aligned} \tag{3.15}$$

The first term on the right hand side is the flow value of profit evaluated using the marginal value of nominal money balances at time $t+1$. The second term is the expected continuation value of the match. The term reflects the fact that the match can dissolve either exogenously or due to the employed worker leaving the match for a better offer. It also shows that the employed workers' turnover is lower at higher real wages.

The household will post real wage offer such that

$$w \in \underset{w}{\operatorname{argmax}}\ R_t(w) \equiv R_t^*. \tag{3.16}$$

Let $w*$ be an optimal real wage offer. Then, the household will post real wage offers other than $w*$ if and only if all the other posted real wage offers give return equal to R_t^*. Utilizing (3.16), one can express the total net return on posted vacancies as

$$TR_t \equiv -k(v_t) + R_t^* v_t. \tag{3.17}$$

Then, the optimal choice of the measure of vacancies, v_t, satisfies the following first order condition

$$k'(v_t) = \frac{s(\hat{v}_t)}{\hat{v}_t}\left[\hat{u}_t + \hat{e}_t\hat{G}_t(w*)\right]\Omega_{Jt}(w^*). \tag{3.18}$$

(3.18) equates the marginal cost of posting a vacancy to the expected marginal benefits. Also at the optimal choice of vacancies, it must be the case that $TR_t \geq 0$.

4. Stationary Monetary Equilibrium

This paper restricts its attention to an equilibrium in which consumption, unemployment, employment, vacancies, and the distributions of real wage offers and earnings are constant over time. Denote the real money balance, $M \equiv \frac{M_t}{p_t}$; the marginal value of real money balances $\Omega_M \equiv p_t \Omega_{Mt}$ and $\Omega_c \equiv p_t \omega_{ct}$. Also the subscript t is dropped in order to denote the endogenous variables in the stationary state.

Given the binding buyer's nominal cash balance constraint, we have $p_t = \frac{M_t}{c_t}, \forall\ t$ and in the stationary equilibrium the price level will grow at the rate equal to the money creation rate *i.e.*,

$$\frac{p_{t+1}}{p_t} = g \quad \forall t. \tag{4.1}$$

In the stationary state, (3.1) implies that the real reservation wage of unemployed workers, $\underline{w}$, satisfies

$$\Omega_M \underline{w} = \phi \tag{4.2}$$

where the marginal value of real money balances, Ω_M, is given by

$$\Omega_M = \frac{u'(c)}{(1+r)g}. \tag{4.3}$$

(4.2) is a key equation of the model. It implies that if the marginal value of real money balances falls, then the real reservation wage of unemployed workers must rise in order to induce them to work. Similarly, if the marginal value of real money balances rises, the real reservation wage of unemployed workers falls.

The stationary state also implies

$$e = J, \quad H(w) = G(w). \tag{4.4}$$

4.1 The Unemployment Rate and the Distribution of Real Wage Earnings

Given the optimal job-acceptance strategies of workers, one can easily derive the stationary state unemployment rate and the real wage earnings distribution of both the employed workers and the employees, $G(w)$, of the representative household. Since the households are identical, no household will post a real wage below the real reservation wage of unemployed workers, $\underline{w}$, which implies that $\hat{F}(\underline{w}) = 0$.

In the stationary state, the inflow to and the outflow from any employment status are equal. This implies that the measure of the unemployed workers in the household satisfies

$$(1-u)\rho = us(\hat{v}). \tag{4.5}$$

The left hand side of (4.5) is the total inflow to the unemployment pool, and the right hand side is the total outflow. Solving (4.5), we get the stationary state measure of unemployed workers in a household,

$$u = \frac{\rho}{\rho + s(\hat{v})}. \tag{4.6}$$

(4.6) implies that the measures of employed workers and the employees in the household are given by

$$e = J = \frac{s(\hat{v})}{\rho + s(\hat{v})}. \tag{4.7}$$

Similarly, the real wage earnings distribution for both the employed workers and the employees, $G(w)$, of a household satisfies

$$(1-u)G(w)[\rho + (1-\hat{F}(w))s(\hat{v})] = us(\hat{v})\hat{F}(w). \tag{4.8}$$

(4.8) states that in the stationary state, the outflow from the pool of workers earning real wage w and less should equal the inflow to the pool. The employed workers leave this pool either due exogenous match dissolution or because they receive real wage offers higher than w. The inflow to the pool comes from the unemployed workers who receive the real wage offer of w and less. (4.6) and (4.8) together imply that the household distribution of real wage earnings, $G(w)$, is given by

$$G(w) = \frac{\rho\hat{F}(w)}{\rho + (1-\hat{F}(w))s(\hat{v})}. \tag{4.9}$$

4.2 Existence and Uniqueness of an Equilibrium

Definition: A stationary monetary equilibrium (SME) with dispersed real wages is defined as a collection of variables $\{c, M, v, \hat{v}, u, \hat{u}, \underline{w}\}$ and distributions $\{F(w), G(w), \hat{F}(w), \hat{G}(w)\}$ such that

(i) Given $\hat{F}(w), \hat{G}(w)$, and $\hat{v}$, the household choice variables $\{c,\ M,\ v,\ F(w)\}$ solve (PH);

(ii) the real reservation wage of an unemployed worker $\underline{w}$ satisfies (4.2);

(iii) the unemployment rate, u, is given by (4.6);

(iv) the real wage earnings distribution, $G(w)$, is given by (4.9);

(v) the expected return on each posted real wage offer, defined in (3.14), $R(w) = R^* \ \forall\ w \in [\underline{w}, \overline{w}]$ where $\overline{w}$ is the highest real wage posted by the household;

(vi) aggregate variables are equal to the relevant household variables, $\hat{F}(w) = F(w),\ \hat{G}(w) = G(w),\ \hat{v} = v,\ \hat{u} = u$;

(vii) the goods market clears, $Jy = c$;

(viii) the marginal value of real money balances, Ω_M, is strictly positive and finite.

The optimal choices of the household and the equilibrium conditions induce the following equilibrium relations. Given that the buyer's nominal money balance constraint binds, the goods market clears, and in equilibrium, the measure of the employed workers, e, equals the measure of the employees of the firms, J we have

$$M = Jy \equiv \frac{s(v)}{\rho + s(v)} y = c. \tag{4.10}$$

(3.18) and the equilibrium condition that the expected gross return on all real wage offers posted be equal imply that the equilibrium measure of vacancies, v, implicitly solves

$$k'(v) = \frac{s(v)}{v} \frac{\rho}{\rho + s(v)} \Omega_J(\underline{w}) \tag{4.11}$$

where $\Omega_J(\underline{w})$ is the marginal value of a job at the lowest real wage offered.[10] (4.11) equates the marginal cost of posting a vacancy with the discounted expected marginal benefit from the job at

[10] From (3.15), the marginal value of a job at a real wage w in the stationary state is given by

$$\Omega_J(w) = \frac{(y - w)\Omega_M}{(r + \rho + s(v)(1 - F(w)))}.$$

the real reservation wage of the unemployed workers. After substituting for $\Omega_J(\underline{w})$ in (4.11), we get

$$k'(v) = \frac{s(v)}{v} \frac{\rho}{(\rho + s(v))} \frac{(y - \underline{w})\Omega_M}{(r + \rho + s(v))}. \tag{4.12}$$

The equilibrium condition for the measure of vacancies is similar to the equilibrium condition derived by Mortensen (2000). The difference is that the measure of vacancies in this model depends on the marginal value of real money balances, which is endogenously determined in the economy. This is the consequence of embedding the Mortensen model in a cash-in-advance economy. Using (4.2), (4.3), and (4.10), we can eliminate $\underline{w}$ and Ω_M from (4.12) and get

$$vk'(v) = s(v) \frac{\rho}{(\rho + s(v))(r + \rho + s(v))} \left[\frac{u'\left(\frac{s(v)y}{\rho + s(v)}\right)}{(1+r)g} y - \phi \right]. \tag{4.13}$$

It is clear from (4.13) that the solutions for the measure of vacancies, v, depend on the form of the household utility function, $u(c)$. Notice that in equilibrium the value of vacancies, v, is bounded from above by $\overline{v}$ satisfying the following equation

$$\frac{u'\left(\frac{s(v)y}{\rho + s(v)}\right)}{(1+r)g} y = \phi. \tag{4.14}$$

Let $\overline{c}$ be the consumption associated with $\overline{v}$. The lowest value the equilibrium measure of vacancies, v, can take is zero, and the associated consumption will also be zero. Now, define the coefficient of relative risk aversion as $\theta(c) \equiv -\frac{u''(c)}{u'(c)}c$. Then, the following lemma characterizes the possible solutions of v for different types of utility function.

Lemma 2:

(i) If the utility function is such that $\lim_{c \to 0} u'(c)c \to 0$, then (4.13) has a solution at $v = 0$. In addition, if $\lim_{c \to 0} \frac{u'(c)}{(1+r)g}(1 - \theta(c)) > \frac{\phi}{y}$, then (4.13) also has a solution at $v > 0$.

(ii) If the utility function is such that $\lim_{c \to 0} u'(c)c > 0$ and $\theta(c) \geq 1$, $\forall\ 0 \leq c \leq \overline{c}$, then (4.13) has a unique solution at $v > 0$.

For the rest of the paper, we will assume that there exists a unique equilibrium level of vacancies, $v > 0$. The equilibrium condition that the expected gross return on each posted real wage offer is equal implies that, for a given $v > 0$, the distribution of real wage offers, $F(w)$, implicitly solves

$$\frac{y - w}{(\rho + s(v)(1 - F(w)))(r + \rho + s(v)(1 - F(w)))} = \frac{y - \underline{w}}{(\rho + s(v))(r + \rho + s(v))}, \ \forall\, w \in \mathcal{F}. \tag{4.15}$$

(4.15) is a quadratic equation in the distribution of the real wage offers, $F(w)$. The equilibrium $F(w)$ is the positive root of this quadratic equation and is given by

$$F(w) = \frac{r + 2(\rho + s(v))}{2s(v)} \left[1 - \sqrt{\frac{r^2 + 4(\rho + s(v))(r + \rho + s(v))\frac{y-w}{y-\underline{w}}}{(r + 2(\rho + s(v)))^2}} \right]. \tag{4.16}$$

Putting $F(\overline{w}) = 1$ in (4.16), one can solve for the upper support of the real wage offer distribution, $\overline{w}$. $\overline{w}$ is given by

$$\overline{w} = \underline{w} + \left[1 - \frac{\rho(r + \rho)}{(\rho + s(v))(r + \rho + s(v))} \right] (y - \underline{w}). \tag{4.17a}$$

The ratio of the upper support to the lower support, $\overline{w}/\underline{w}$, is given by

$$\overline{w}/\underline{w} = 1 + \left[1 - \frac{\rho(r + \rho)}{(\rho + s(v))(r + \rho + s(v))} \right] \left(\frac{y}{\underline{w}} - 1 \right). \tag{4.17b}$$

The equations for the distribution of the real wage offers and its upper support are identical in form to the ones derived in the Mortensen (2000). However, the crucial difference here is that in the current model, unlike that of Mortensen (2000), the real reservation wage of unemployed workers, $\underline{w}$, and the measure of vacancies, v, depend on the endogenously determined marginal value of real money balances.

For a unique and finite equilibrium $v > 0$, the marginal value of real money balances, Ω_M, is strictly positive and finite. Hence, we have following proposition.

Proposition 1: For a unique, strictly positive, and finite equilibrium level of vacancies, there exists a unique SME with dispersed real wages characterized by equations 4.2, 4.3, 4.9, 4.16, and 4.17a.

To further characterize the SME with dispersed real wages, it is instructive to compare this equilibrium with the stationary competitive monetary equilibrium in which the labor market is frictionless. The results are summarized in the following proposition.

Proposition 2: In the SME with dispersed real wages, consumption, output, employment, and the real wages are lower than in the stationary competitive monetary equilibrium.[11]

Proposition 2 is the consequence of the fact that in the SME with dispersed real wages, firms enjoy monopsony power in the labor market. Because of that, firms make strictly positive profit on filled jobs. On the other hand, in the competitive equilibrium firms make zero profit on filled jobs. Because of this, employment and hence output and consumption in the SME with dispersed real wages is lower compared to the competitive equilibrium case. Also, in the competitive case the real wage is equal to the marginal product of the workers. But, in the SME with dispersed real wages, even the highest real wage paid is lower than the marginal product of the workers.

5. Inflation, Output, and Welfare

The effects of inflation on consumption, output, and employment are summarized in the following proposition.

Proposition 3: An increase in the inflation rate in the SME with dispersed real wages reduces vacancies, consumption, and output, and increases the unemployment rate.

An increase in the inflation rate erodes the value of fiat money, which reduces the return of firms on vacancies for a given level of consumption. In addition, higher inflation rate reduces the expected benefit from working and increases the real reservation wage of unemployed workers for a given consumption level, which further lowers the return on vacancies. This induces firms to reduce the equilibrium level of vacancies posted, which leads to lower output and consumption and a higher unemployment rate.[12]

[11] The competitive equilibrium program has two solutions depending on whether the work-force constraint binds or not. If the work-force constraint binds, optimal $e = 1$. In the case, it does not bind, the optimal $e < 1$ (see the proof of the Proposition 2).

[12] The result that consumption and output necessarily fall with inflation is because we do not allow match-specific investment. If we allow match-specific investment, consumption and output need not fall with inflation even though the level of vacancies decline. The reason is that a fall in the level of vacancies by reducing the turnover of employed workers encourages firms to incur higher match-specific investment, which by increasing workers' productivity may more than offset the decline in output due to higher unemployment. Kumar (2003) develops similar mechanism in a non-monetary setup in which higher unemployment benefits can increase output.

The effect of inflation on employment and consumption in the SME with dispersed real wages is similar to that in standard cash-in-advance economies (*e.g.* Cooley and Hansen 1989), where an increase in the inflation rate induces households to shift from consumption goods, which require cash, to leisure, which does not require cash.[13]

In the SME with dispersed real wages, however, a fall in consumption and output does not necessarily lead to a fall in social welfare. To see this, we compare the social optimal level of vacancies with the level of vacancies in the SME with dispersed real wages. The social planner maximizes

$$\max_{c_t, v_t, u_{t+1}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \Big[u(c_t) - (1+\phi)(1-u_t) - u_t - k(v_t) \Big]$$

subject to

$$u_{t+1} = \rho(1-u_t) + (1-s(v_t))u_t \quad \forall\, t \tag{5.1}$$

$$c_t = y(1-u_t)\ \forall\, t. \tag{5.2}$$

The first constraint is the labor matches constraint, which indicates that next period's level of unemployment equals the sum of the measure of the employed workers who become unemployed in the current period and the measure of the unemployed workers of the current period who are unable to find job. The second constraint is the feasibility constraint on consumption goods.

In the stationary state, the optimal level of vacancies is given by (see Appendix 2)

$$k'(v) = \frac{\rho s'(v)}{(\rho + s(v))(r + \rho + s(v))} \left[u'\Big(\frac{s(v)y}{\rho + s(v)}\Big) y - \phi \right]. \tag{5.3}$$

It can be easily shown that (5.3) has a unique solution. Let the elasticity of matching function with respect to vacancy be $\eta \equiv \frac{s'(v)v}{s(v)}$.[14] The following proposition compares the social optimal level of vacancies with the market equilibrium level of vacancies.

[13] In the competitive case with an interior solution, one can easily show that higher inflation reduces employment and consumption.

[14] Given the assumptions about the matching function, $0 < \eta < 1$.

Proposition 4: In the SME, the optimal inflation rate, g^*, exceeds the Friedman rule and is implicitly given by

$$g^* = \frac{1}{(1+r)\left[\eta + \frac{(1-\eta)\phi}{u'(c(g^*))y}\right]} > \frac{1}{1+r} \tag{5.4}$$

The optimal inflation rate exceeds the Friedman rule because here, the inflation rate affects the real reservation wage of unemployed workers, which, in turn distorts the level of vacancies posted by firms. The economy has two sources of inefficiency — the binding buyer's nominal cash balance constraint and the monopsony power of the firms in the labor market. The Friedman rule removes only the first source of inefficiency. As the inflation rate falls, the real reservation wage of the unemployed workers declines for a given consumption level increasing the monopsony power of the firms and the return on vacancies. Consequently, as the inflation rate approaches the Friedman rule, firms create too many vacancies and the unemployment rate falls too low relative to the social optimum.

6. A Numerical Example

We construct an example to illustrate the effects of inflation rate on output and welfare in the SME with dispersed real wages. We measure the welfare cost of inflation in terms of quantity of consumption, as a percentage of social optimum consumption, by which a representative household would have to be compensated to give it the same period utility in the SME as it would receive at the social optimum consumption. Let

$$U^* = u(c^*) - (1+\phi)e^* - u^* - k(v^*) \tag{6.1}$$

be the social optimum level of period utility, where c^*, e^*, u^*, and v^* are the social optimum levels of consumption, employment, unemployment, and vacancies respectively. Then, the welfare cost of inflation is given by $\frac{\Delta c}{c^*} \times 100$ where Δc solves

$$U^* = u(c + \Delta c) - (1+\phi)e - u - k(v) \tag{6.2}$$

Here c, e, u, and v are values in the SME with dispersed real wages for a given gross inflation rate, g.

In the example, we select functional forms and parameter values such that the economy generates the unemployment rate of 6 percent and the average unemployment duration of one quarter at the quarterly gross inflation rate of $g = 1.0075$, because this is roughly in line with the U.S. experience over the past twenty years. The functional forms and parameter values are given in Table 2.

Table 2

Functional Forms and Parameter Values

Utility Function $u(c)$	$\ln c$
Cost of Posting Vacancy $k(v)$	$0.058v^2$
Matching Function $s(v)$	$2.2v^{0.98}$
Productivity of Workers y	1
Exogenous Separation Rate ρ	0.07
Rate of Time Preference r	0.01
Disutility Cost of Working ϕ	0.5
Time Period	One Quarter

For these functional forms and parameter values, consider economies with quarterly gross inflation rates ranging from -1 percent to 10 percent (*i.e.* $g \in [0.991, 1.1]$). The effects of inflation rate on output and welfare are depicted in Figures 1 and 2 respectively.

Figure 1 shows that a higher inflation rate reduces output as expected, though the effect is quite small. The output declines by 0.42 percent (from 0.9401 to 0.9362) as the inflation rate rises from -1 percent to 10 percent per quarter. As discussed earlier (footnote 12), if we allow for match-specific investment, the impact of inflation on the output is likely to be even smaller. The output may even rise.

Figure 2 shows that the welfare cost is an 'U' shaped function of the inflation rate and is minimized at a strictly positive level of the inflation rate ($g = 1.0101$).[15] As discussed earlier, at low inflation rates the welfare cost of increased monopsony power exceeds the welfare gain from the lower inflation tax. However, as the inflation rate rises, the welfare gain from the decline in the monopsony power of firms is more than offset by the welfare cost from the higher inflation tax. Consequently, the social welfare declines at the higher inflation rate.

[15] The household utility in the competitive equilibrium (not depicted here) falls with inflation. The Friedman rule is optimal in this case.

The example shows that the welfare cost of deviating from the optimal inflation rate is quite small. The welfare cost of 10 percent inflation rate per quarter is just about 0.016 percent of the social optimum consumption. The small effect of inflation on welfare is due to the fact that our model is highly stylized, and several channels through which inflation could distort the economy have been closed down. For example, our assumption that all the workers in a household have identical consumption regardless of their real wages and employment status reduces the impact of inflation on welfare. As we will see in the next section, inflation has a significant effect on the dispersion of real wage earnings. If we allow for heterogenity in consumption and, in particular, wealth (which will make the model analytically intractable and numerically challenging), then inflation may have major effect on welfare (see also footnote 12).

7. Inflation and Real Wage Dispersion

In this section, we analyze the effects of inflation rate on the distributions of real wage offers and earnings. The results are summarized in Propositions 5 to 7.

A key issue determining inflation's effect on wage dispersion is its effect on the marginal value of real money balances $\left(\Omega_M = \frac{u'(c)}{(1+r)g}\right)$. In general, as long as $u''(c) < 0$, inflation may increase or reduce the marginal value of real money balances. Under certain restrictions, however, an increase in the rate of inflation unambiguously reduces the marginal value of real money balances.

Lemma 3: If in the SME $\rho(r + \rho) \leq s^2(v)$, then an increase in the inflation rate reduces the marginal value of real money balances, Ω_M.

Lemma 3 states that if the rate of time preference and the exogenous separation rate are small relative to the matching rate of workers, then the inflation rate reduces the marginal value of real money balances. The empirical evidence on the matching rate of workers and the exogenous separation rate suggests that in the real economies this condition is likely to be satisfied. For example, in the U.S. the average duration of unemployment is approximately one quarter. If we set the time-period to be a quarter, then this implies that $s(v) = 1$. The exogenous separation rate per quarter is variously estimated to be between 0.005 to 0.11. Also in the macro literature, the rate of time preference per quarter is commonly assumed to be 0.01. This suggests that the above condition is satisfied for the U.S. economy. Note that this condition is also satisfied in the example introduced in the Section 6.

If the condition in Lemma 3 is satisfied, the real reservation wage of the unemployed workers must rise as inflation increases. A reduction in the marginal value of real money balances reduces the expected benefit from working (4.2), and thus for a given disutility cost of working the real reservation wage $\underline{w}$ must rise in order to induce the unemployed workers to accept work.

Proposition 5: Let $\rho(r + \rho) \leq s^2(v)$ in the SME. Then, an increase in the inflation rate reduces the support of the distributions of real wage offers and earnings in the sense that $\overline{w}/\underline{w}$ falls.

Inflation affects the support of the distributions of real wage offers and earnings through its effect on the real reservation wage of the unemployed workers and the level of vacancies. An increase in the inflation rate increases the real reservation wage of unemployed workers and raises the lower support of the distributions of both real wage offers and earnings. An increase in the inflation rate may also raise the upper support. But this effect is mitigated by two factors.

Firstly, the firms posting the highest real wage do not face any competition from other firms to retain their workers. On the other hand, the firms posting lower real wages face competition from other firms to retain their workers. Consequently, the firms posting the highest real wage need not increase their wages as much as the firms posting the lowest real wage. Secondly, vacancies decline with an increase in the inflation rate reducing the matching rate of workers and weakening the effectiveness of on-the-job search in reducing the monopsony power of the firms. If the second factor is strong enough, then the upper support may in fact fall. These two factors imply that the support of the distributions of real wage offers and earnings falls with an increase in the inflation rate as long as the exogenous separation rate and the rate of time preference are small compared to the matching rate of workers.

If the condition $\rho(r + \rho) \leq s^2(v)$ is not satisfied, the real reservation wage of unemployed workers may rise or fall with the inflation rate. If the the real reservation wage of unemployed workers falls, then one can show that the higher inflation rate may increase or reduce the support of the distributions of real wage offers and earnings.

While analytically we are only able to show that higher inflation reduces the support of the distributions of real wage offers and earnings (under plausible conditions), through numerical computations it can be shown that a higher inflation rate reduces the dispersions of real wages.[16] Figure 3 depicts the lower and upper supports of the distributions of real wage offers and earnings generated using the numerical example introduced in the Section 6. The figure shows that higher

[16] It is well-known that the basic Mortensen (2000) model generates increasing wage offers and earnings densities, which are not supported empirically. However, this is the consequence of the assumption that the firms are of identical productivity. By introducing heterogeneous firms, one can match a variety of wage offers and earnings densities (*e.g.* Bontemps *et. al.* (2000)).

inflation rate increases the lower support (from .5001 to .5693), while it has little impact on the upper support reducing the range of real wage offers and earnings.

Figure 4 depicts the coefficients of variations of real wage offers and earnings. Figure 5 shows the ratios of 90th to 10th percentile, 90th to 50th percentile, and 50th to 10th percentile of the real wage offers distribution. Figure 6 plots the same ratios of the distribution of real wage earnings. All these figures show that a higher inflation rate reduces the dispersion of real wage offers and earnings. Quantitative experiments indicate that higher inflation reduces the dispersion in real wage offers and earnings for a wide range of parameter values and functional forms. The result of our model is consistent with empirical observation that inflation is associated with reduced wage dispersion.

Given that inflation affects the distributions of real wage offers and earnings, it is interesting to ask the question whether the household would prefer the distributions of real wage offers and earnings associated with higher inflation or lower inflation. The following two propositions address this issue.

Proposition 6: Let $\rho(r+\rho) \leq s^2(v)$ and r be small ($r^2 \approx 0$). Then, an increase in the inflation rate may or may not lead to a stochastic improvement in the distribution of real wage offers, $F(w)$.

An increase in the inflation rate has two conflicting effects on the monopsony power of firms. Firstly, an increase in the inflation rate, if it increases the real reservation wage of unemployed workers, reduces the monopsony power of the firms and leads to stochastic improvement in the distribution of the real wage offers. Secondly, an increase in the inflation rate reduces the level of vacancies increasing the monopsony power of the firms which may prevent any stochastic improvement in the distribution of the real wage offers. In this case, the effect of the inflation rate on the average real wage offer is ambiguous.

If the real reservation wage of unemployed workers falls, it can be shown that the proportion of low real wage offers rises. This happens because a fall in the real reservation wage of the unemployed workers increases the monopsony power of the firms unambiguously, and the firms post a greater proportion of low wage vacancies. In this case, the average real wage offer necessarily falls.

Proposition 7: If there is stochastic improvement in the distribution of real wage offers, $F(w)$, an increase in the inflation rate may or may not lead to stochastic improvement in the distribution of real wage earnings, $G(w)$. If there is no stochastic improvement in $F(w)$, then the proportion of employed workers with low real wage earnings increases with the rise in the inflation rate.

An increase in the inflation rate reduces the matching rate of workers and lowers the rate at which the employed workers move from low wage jobs to high wage jobs. *Ceteris paribus*, this shifts the mass of the employed workers from high wage jobs to low wage jobs. If there is a stochastic improvement in the distribution of the real wage offers, $F(w)$, then it may prevent the shift of mass of the employed workers from high wage jobs to low wage jobs caused by a fall in the matching rate of the workers. However, if there is no stochastic improvement in the distribution of the real wage offers, then the proportion of employed workers with low wage jobs rises unambiguously. In this case, the average real wage earnings falls as the inflation rate goes up.

Figure 7 depicts the effect of inflation on the average real wage offer and earnings. The figure shows that inflation increases both the average real wage offer and the average real wage earnings. Quantitative experiments show that while the average real wage offer increases with inflation for a wide range of parameter values and functional forms, this is not the case with the average real wage earnings.

8. Robustness

In this section, we discuss several extensions of our model. The Burdett and Mortensen (1998) model (on which Mortensen 2000 is based) puts severe restrictions on the wage setting behavior of firms. Firstly, it assumes that firms cannot change their posted wages. Secondly, they do not respond when their employees receive higher wage offers from other firms even though losing employees is costly. Thirdly, each firm is required to offer the same posted wage to all the workers it comes in contact with regardless of the reservation wages of the workers and thereby forgo a part of its rent. In this section, we discuss whether the result that inflation reduces real wage dispersion survives when firms are allowed more flexibility in setting their wages.

Coles (2001) relaxes the assumption that firms pre-commit to particular wage offers and allows firms to change their wages anytime they like. In other aspects, Coles is identical to the basic Burdett-Mortensen. Allowing firms to change their posted wages complicates the model a great deal. The optimal quit decisions of the workers depend on the expected future wages at their current employers, and those expectations must be consistent with the firms' wage setting strategies. Coles derives conditions under which firms do not wish to change the posted wages in the future period (*i.e.* pre-committed constant wage trajectory as in Burdett-Mortensen remains an equilibrium).[17]

[17] As the rate of time-preference $r \to 0$, the Coles equilibrium converges to Burdett-Mortensen equilibrium.

Given the workers' beliefs that firms will not change the posted wages ever in future, the marginal values of unemployed workers and employed workers at different wages, and thus the real reservation wage of unemployed workers, satisfy the same functional equations as in the Burdett-Mortensen (see Coles 2001, equation 9, page 168). Thus, with Coles's set-up in our monetary model, the real reservation wage of unemployed workers will still be given by (4.2) ($\phi = \underline{w}\Omega_M$).

The ratio of the upper and lower supports of the distributions of real wage offers and earnings is given by (see Coles 2001, Lemma 4, page 169-170)

$$\overline{w}/\underline{w} = 1 + \left(\frac{y}{\underline{w}} - 1\right)\left[\frac{\overline{s}(v)(2\rho + \overline{s}(v))}{(\rho + \overline{s}(v))(r + \rho + \overline{s}(v))}\right] \tag{8.1}$$

where $\overline{s}(v)$ indicates the fact that the job arrival rate is fixed in the model. One can immediately see that $\frac{d(\overline{w}/\underline{w})}{d\underline{w}} < 0$. Therefore, if the marginal value of real money balances falls, the support will decline.

One can endogenize the job arrival rate (level of vacancies) by equating the marginal value of vacancy with the real reservation wage of the unemployed workers to zero. Simple differentiation of (8.1) shows that $\frac{d(\overline{w}/\underline{w})}{d\underline{w}} < 0$ necessarily if $\frac{dv}{d\underline{w}} < 0$, which will be the case because an increase in $\underline{w}$ reduces the expected return on vacancies. Thus our findings with regard to the effect of inflation on wage dispersions are robust to relaxing the assumption that firms pre-commit to the posted wage offers.

Postel-Vinay and Robin (2002) modify the Burdett-Mortensen model by allowing firms to counter the offers received by their employees from competing firms and to vary their wage offers according to the characteristics of the particular workers with which they are matched (state-dependent offers — lower wage to low reservation wage workers and higher wage to high reservation wage workers).

For simplicity, consider their model with identical workers and identical firms. In this case, the equilibrium wage distribution will be degenerate at a mixture of two mass points — one at the real reservation wage of the unemployed workers , $\underline{w}$, and one at the productivity of the workers, y, because allowing the firms to counter alternative real wage offers made to their employees triggers a Bertrand competition between the firms. Then in the monetary set-up, the real reservation wage of the unemployed workers satisfies (see Appendix 3)

$$\left[\underline{w} + \frac{\overline{s}(v)}{r + \rho}y\right]\Omega_M = \frac{\overline{s}(v)}{r(r + \rho)} - \frac{r + \rho}{r} + (1 + \phi)\left[1 + \frac{\overline{s}(v)}{r + \rho}\right]. \tag{8.2}$$

Since the right hand of (8.2) is constant, if the marginal value of real money balances, Ω_M, falls, the real reservation wage of unemployed workers, $\underline{w}$, must rise, which will reduce the support of the distributions of real wage offers and earnings.

9. Minimum Wage

In this section, we consider the effect of changes in the real minimum wage on equilibrium variables. Let us assume that the mandated real minimum wage, w_{min}, is binding *i.e.* it is higher than the real reservation wage of the unemployed workers, $\underline{w}$, for a given set of parameters (also $w_{min} < y$). In this case, the real minimum wage will be the lower support of the distributions of real wage offers and earnings.

The equilibrium level of vacancies is given by

$$k'(v) = \frac{s(v)}{v} \frac{\rho}{(\rho + s(v))} \frac{(y - w_{min})\Omega_M}{(r + \rho + s(v))}. \tag{9.1}$$

The following lemma characterizes the solution of (9.1).

Lemma 4: Let $u(c) = \frac{c^{1-\theta}}{1-\theta}$ and $\theta \geq 1$, then (9.1) has a unique and finite solution $v > 0$.

For a unique and finite equilibrium level of vacancies, there will be a unique equilibrium and the associated distribution of real wage offers, $F(w)$, and the upper support $\overline{w}$ are continued to be given by (4.16) and (4.17) respectively with the real reservation wage $\underline{w}$ being replaced by the real minimum wage w_{min}.

The following proposition summarizes the effect of inflation and the real minimum wage on the equilibrium variables.

Proposition 8: Let the real minimum wage be binding:

(i) For a given real minimum wage, an increase in the inflation rate reduces the level of vacancies and the support of the distributions of real wage offers and earnings, $\overline{w}/\underline{w}$.

(ii) For a given inflation rate, an increase in the real minimum wage has the same effect.

For a given real minimum wage, higher inflation reduces the return on vacancies and the firms cut down the number of vacancies posted. The decline in the number of vacancies posted reduces the effectiveness of on-the-job search and the highest real wage posted falls lowering the support. In this case, the support falls solely due to the fall in the upper support of the wage distributions.

It is also easy to show that with the binding real minimum wage the effects of inflation on vacancies and $\overline{w}/\underline{w}$ are smaller than they would be when the real minimum wage is non-binding.

For a given inflation rate, an increase in the real minimum wage by lowering the return of firms on vacancies reduces the level of vacancies posted. This coupled with the increased lower support leads to compression in the support of the distributions of real wage offers and earnings. Mortensen (2000) derives this result in a non-monetary partial equilibrium set-up. We show that this result holds in a general equilibrium monetary set-up.

Quantitative experiments (not depicted) show that higher real minimum wage reduces other measures of dispersion. These results are consistent with empirical findings that the decline in the real minimum wage increases wage dispersion.

10. Conclusion

This paper has analyzed the effects of inflation on the dispersion of real wage earnings and welfare in a cash-in-advance economy with search friction in the labor market in which firms enjoy market power. The paper shows that in equilibrium an increase in the inflation rate can reduce real wage dispersion, a finding consistent with those of several empirical papers. This result is robust to several variations in the wage posting process. An increase in the inflation rate also reduces the level of vacancies and output and increases the unemployment rate. In addition, the Friedman rule is not optimal. Because of the monopsony power of the firms in the labor market, the optimal inflation rate exceeds the rate of discount. The paper also shows that the decline in the real minimum wages increases the dispersion of real wages by affecting the entire distribution, which is consistent with empirical findings.

The paper proposes a mechanism through which higher inflation can reduce wage dispersion. We would like to examine the quantitative significance of this mechanism. In future work, we would also like to examine the effects of inflation on the composition of vacancies (more or less skilled jobs) and inter-skill wage dispersion.

Appendix 1: Proofs

Lemma 1:

(i) The Optimal Job Acceptance Strategies of Employed Workers

The household prescribes the job-acceptance strategies to workers, which maximizes its expected return. Suppose that the household instructs the employed workers not to accept any new real wage offer and continue to work at their current real wages. The expected return at time t to the household on the measure of employed workers earning real wage w and less, $e_t G_t(w)$, is given by

$$R_{et}{}'(w) \equiv e_t G_t(w) \left[\omega_{Mt} \int_{\underline{w}}^{w} x dG_t(x) - (1+\phi) + E_t \sum_{i=t}^{\infty} \frac{l_{i+1}}{(1+r)^{i+1-t}} \right] \tag{A1.1}$$

where

$$\left. \begin{aligned} l_{i+1} &= -1, \text{with probability } \rho \\ &= \omega_{Mi+1} p_{i+1} \int_{\underline{w}}^{w} x dG_i(x) - (1+\phi), \text{ with probability } (1-\rho) \end{aligned} \right. \forall\ i. \tag{A1.2}$$

The future return l_{i+1} reflects the fact that the employed workers may become unemployed with probability ρ.

Suppose now that the household instructs the employed workers to accept any offer which gives them real wages higher than what they currently earn. In this case, the return to the household on the measure of employed workers earning real wages w and less is

$$R_{et}{}''(w) \equiv e_t G_t(w) \left[\omega_{Mt} \int_{\underline{w}}^{w} x dG_t(x) - (1+\phi) + E_t \sum_{i=t}^{\infty} \frac{l_{i+1}}{(1+r)^{i+1-t}} \right] \tag{A1.3}$$

where

$$l_{i+1} = -1, \text{with probability } \rho$$
$$=$$

$$\left\{ \begin{array}{l} \omega_{Mi+1} p_{i+1} \displaystyle\int_{\underline{w}}^{w} [[(1-F_i(x)) \int_{x}^{\overline{w}} z dF_i(z)] + F_i(x) x] dG_i(x) - (1+\phi), \\ \text{with probability } s(\hat{v}_i) \\ \omega_{Mi+1} p_{i+1} \displaystyle\int_{\underline{w}}^{w} x dG_i(x) - (1+\phi), \text{with probability} (1-\rho-s(\hat{v}_i)) \end{array} \right\} \forall i. \tag{A1.4}$$

Comparing A1.2 to A1.4, we can immediately see that ${R_{et}}''(w) > {R_{et}}'(w)$. By similar reasoning one can show that it is not optimal for the household to instruct employed workers to accept real wage offers lower than what they currently earn. Therefore, the household instructs employed workers to accept any real wage offer higher than what they currently earn. ▪

ii) The optimal real reservation wage of unemployed workers

Let

$S_t(w)$ = Set of workers receiving real wage offer of less than w

$S_t'(w)$ = The complement set of $S_t(w)$

w^i = Wage offer received by worker i

$R_{et}(w' \leq w \leq w'')$ = Expected return on the measure of employed workers receiving wage greater than or equal to w' but less than or equal to w''.

Suppose that the household prescribes unemployed workers to accept any real wage offer $x \geq w$. Then the expected return to the household on the measure of unemployed workers at time t, u_t, is

$$\begin{aligned} R_{ut}(w) = &- u_t + \frac{1}{1+r}\Big[\\ &- u_t(1 - s(\hat{v}_t)) - \int_{i \in S_t(w)} di \\ &+ \int_{i \in S_t'(w)} \left(x^i \omega_{Mt+1} p_{t+1} - (1+\phi)\right) di \\ &+ N_{t+1}(w)\Big] \end{aligned} \quad \forall\, t \tag{A1.5}$$

where $N_{t+1}(w)$ is the discounted future stream of utilities defined as

$N_{t+1}(w) =$

$$\left\{ \begin{array}{l} R_{u_t h_t}(w), \text{for the unemployed workers at time } t \\ \quad \text{who stay unemployed at time } t+1 \text{ where} \\ \quad h_t = u_t[1 - s_t(\hat{v}_t)(1 - \hat{F}_t(w))] \text{ and} \\ R_{et+1}(w \leq x \leq \overline{w}),\ \text{for the unemployed workers who accept the job offers} \end{array} \right\} \forall\, t. \tag{A1.6a}$$

where

$$R_{et+1}(w \leq w \leq \overline{w}) \equiv$$

$$u_t s_t(\hat{v}_t)(1 - \hat{F}_t(w)) \left[\omega_{Mt+2} \int_w^{\overline{w}} x dG_{t+2}(x) - (1+\phi) + E_t \sum_{i=t+2}^{\infty} \frac{l_{i+1}}{(1+r)^{i+1-t}} \right] \tag{A1.6b}$$

where

$$l_{i+1} = -1, \text{with probability } \rho$$
$$=$$

$$\left\{\begin{array}{l} \omega_{Mi+1}p_{i+1}\displaystyle\int_{w}^{\overline{w}}[[(1-F_i(x))\int_{x}^{\overline{w}} zdF_i(z)]+F_i(x)x]dG_i(x)-(1+\phi), \\ \text{with probability } s(\hat{v}_i) \\ \omega_{Mi+1}p_{i+1}\displaystyle\int_{w}^{\overline{w}} xdG_i(x)-(1+\phi), \text{with probability}(1-\rho-s_i(\hat{v}_i)) \end{array}\right\} \forall i. \quad \text{(A1.6c)}$$

Note that the measure of unemployed workers receiving real wage offers less than w is $u_t s_t(\hat{v}_t)\hat{F}_t(w)$, and the measure of unemployed workers receiving real wage offers of w and higher is $u_t s_t(\hat{v}_t)(1-\hat{F}_t(w))$.

Now suppose that $w < w*$ where $w*$ satisfies $\omega_{Mt+1}p_{t+1}w* = \phi$. In this case, one can immediately show that $R_{ut}(w*) > R_{ut}(w)$ since

$$\int_{i\in S_t'(w*)}\left(x^i\omega_{Mt+1}p_{t+1}-(1+\phi)\right)di - \int_{i\in S_t(w*)} di >$$

$$\int_{i\in S_t'(w)}\left(x^i\omega_{Mt+1}p_{t+1}-(1+\phi)\right)di - \int_{i\ \in S_t(w)} di \ \forall\ t. \quad \text{(A1.7)}$$

By similar logic we can show that $R_{ut}(w*) > R_{ut}(w)$ if $w > w*$. Therefore, it is optimal for the household to instruct unemployed workers to accept any real wage offer $w \geq w*$. ▪

Lemma 2:

The equilibrium value of vacancies, v, is given by

$$vk'(v) = \frac{\rho}{r+\rho+s(v)}\left[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\right] \equiv T(v). \quad \text{(A1.8)}$$

From the above, it is clear that when $u'(c)c = 0$ for $c = 0$, then (A1.8) has solution $v = 0$. But, when $u'(c)c > 0$ for $c = 0$, then (A1.8) does not have solution at $v = 0$ and $T(v) > vk'(v)$. From, (A1.8), it is also clear that the maximum value, which c or v can take is given by

$$\frac{u'(c)}{(1+r)g} = \frac{\phi}{y}. \quad \text{(A1.9)}$$

Denote the solution of (A1.9) by $\bar{c}$. It is also clear that at $\bar{c}$, $vk'(v) > T(v)$.

Differentiating $T(v)$ w.r.t. v, we get

$$T'(v) = \frac{\rho}{(r+\rho+s(v))^2}\Big[\frac{(r+\rho+s(v))y\rho}{(\rho+s(v))^2}\Big[\frac{u''(c)c+u'(c)}{(1+r)g} - \frac{\phi}{y}\Big] - \Big[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\Big]\Big]s'(v) \tag{A1.10}$$

.

The coefficient of relative risk aversion is given by $-\frac{u''(c)c}{u'(c)} \equiv \theta(c)$. Then, (A1.10) can be written as

$$T'(v) = \frac{\rho}{(r+\rho+s(v))^2}\Big[\frac{(r+\rho+s(v))y\rho}{(\rho+s(v))^2}\Big[\frac{u'(c)(1-\theta(c))}{(1+r)g} - \frac{\phi}{y}\Big] - \Big[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\Big]\Big]s'(v). \tag{A1.11}$$

From (A1.11), it is clear that in the case $\theta(c) \geq 1,\ \forall\ 0 \leq c \leq \bar{c}$, $T'(v) < 0$. Thus, in the case $u'(c)c > 0$ when $c = 0$, there exists a unique solution.

(A1.11) also shows that when $\lim_{c\to 0} \frac{u'(c)}{(1+r)g}(1-\theta(c)) > \frac{\phi}{y}$, then $T'(v) \to \infty$ when $c \to 0$ since $\lim_{v\to 0} s'(v) \to \infty$. Thus, in the case $\lim_{c\to 0} u'(c)c \to 0$, there exists at least one non-trivial solution. ▪

Proposition 2:

In the competitive case, the household maximization problem is

$$\max_{c_t, M_{t+1}, e_t, J_t} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t}\Big[u(c_t) - \phi e_t\Big]$$

subject to the cash-in-advance constraint given in (2.2), the law of motion of household nominal money balance,

$$M_{t+1} \leq M_t + (g-1)\hat{M}_t - p_t c_t + [p_t y - w_t]J_t + e_t w_t, \tag{A1.12}$$

and the work-force constraint

$$e_t \leq 1. \tag{A1.13}$$

Let μ_{ct}, μ_{Mt}, and μ_{et} be the Langrangian multipliers associated with the cash-in-advance constraint and the constraints (A1.12) and (A1.13) respectively. Then, the first order conditions associated with the optimal choices are given by

$$c_t : \frac{u'(c_t)}{p_t} = \mu_{ct} + \mu_{Mt}, \tag{A1.14}$$

$$M_{t+1} : \frac{1}{1+r}[\mu_{Mt+1} + \mu_{ct+1}] = \mu_{Mt}, \tag{A1.15}$$

$$e_t : \phi + \mu_{et} = w_t \mu_{Mt}, \tag{A1.16}$$

$$J_t : p_t y = w_t. \tag{A1.17}$$

The slackness condition associated with (A1.16) is given by

$$\mu_{et}[e_t - 1] = 0. \tag{A1.18}$$

If constraint (A1.13) is binding, then $e_t = 1$, and when it is not binding, there is a unique interior solution with optimal $e_t < 1$.

Steady State Equilibrium

In the steady state

$$\mu_M = p_t \mu_{Mt} = \frac{u'(c)}{(1+r)g}. \tag{A1.19}$$

Case (i): Constraint (A1.13) is binding.

In this case, $e = J = 1$, $c = y$, and $w = y$.

Case (ii): Constraint (A1.13) is not binding.

In this case, $e = J$, $c = ey$, $w = y$, and the equilibrium e is given by

$$\phi = \frac{yu'(ey)}{(1+r)g}. \tag{A1.20}$$

Notice that in the SME with dispersed real wages because of friction in the labor market

$$\phi < \frac{yu'(ey)}{(1+r)g}. \tag{A1.21}$$

Since the utility function is strictly increasing and concave, it immediately follows that in the SME with dispersed real wages employment and, hence, consumption and output levels are lower compared to the competitive case. Also, in the competitive equilibrium $w = y$, while in the SME with the dispersed real wages $\overline{w} < y$.

Notice also that in the competitive case with interior solution, the Friedman rule maximizes the social welfare. For the social optimal one requires that

$$yu'(c) = \phi. \tag{A1.22}$$

Comparing (A1.20) with (A1.22) we can immediately see that the Friedman rule maximizes the social welfare. ■

Proposition 3:

The equilibrium level of vacancies is given by

$$vk'(v) = \frac{\rho}{r + \rho + s(v)} \left[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y} c \right] \equiv T(v). \tag{A1.23}$$

We can immediately see that

$$\frac{dT(v)}{dg} < 0. \tag{A1.24}$$

From (A1.24), it follows that when equilibrium is unique, an increase in the inflation rate reduces the equilibrium level of vacancies and thus employment, consumption, and output.

Proposition 4:

The market equilibrium level of vacancies is given by

$$vk'(v) = s(v) \frac{\rho}{(\rho + s(v))(r + \rho + s(v))} \left[\frac{u'(c)}{(1+r)g} y - \phi \right]. \tag{A1.25}$$

The social optimum level of vacancies is given by

$$k'(v) = \frac{\rho s'(v)}{(\rho + s(v))(r + \rho + s(v)} [u'(c)y - \phi]. \tag{A1.26}$$

Now, define the elasticity of matching function w.r.t. vacancy $\eta \equiv \frac{s'(v)v}{s(v)}$. (A1.25) and (A1.26) imply that the market and the social levels of vacancies will be equal at the inflation rate g satisfying

$$\eta [u'(c)y - \phi] = \left[\frac{u'(c)y}{(1+r)g} - \phi \right]. \tag{A1.27}$$

Simplifying, we get the optimal rate of g^*

$$g^* = \frac{1}{(1+r)\left[\eta + \frac{(1-\eta)\phi}{u'(c)y}\right]}. \tag{A1.28}$$

From (A1.28), it is clear that the Friedman rule is optimal only when $\eta = 1$. But, given the restrictions on the matching function $0 < \eta < 1$. Also, in equilibrium, $u'(c)y > \phi$, which implies that

$$g^* > \frac{1}{1+r}. \blacksquare \tag{A1.29}$$

Lemma 3:

The equilibrium level of vacancies is given by

$$vk'(v) = \frac{\rho}{r+\rho+s(v)}\left[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\right] \equiv T(v,g) \tag{A1.30}$$

where $c = \frac{s(v)y}{\rho+s(v)}$. Now let

$$Z(v) \equiv \frac{s(v)}{(\rho+s(v))(r+\rho+s(v))}. \tag{A1.31}$$

Factoring out c from the RHS of (A1.30) and using the definition of $Z(v)$, we have

$$T(v,g) \equiv Z(v)\rho y\left[\Omega_M(g) - \frac{\phi}{y}\right]. \tag{A1.32}$$

.

Differentiating $Z(v)$ w.r.t. v we get

$$Z'(v) = \frac{s'(v)}{(\rho+s(v))^2(r+\rho+s(v))^2}[\rho(r+\rho) - s^2(v)]. \tag{A1.33}$$

From (A1.33), it is clear that $Z'(v) \leq 0$ if $\rho(r+\rho) \leq s^2(v)$ and $Z'(v) > 0$ otherwise.

Let the initial equilibrium satisfy $v^*k(v^*) = T(v^*, g^*)$. Now suppose that the gross inflation rises to g' ($g' > g^*$). From the Proposition 3, we know that at new equilibrium $v' < v^*$ and $T(v', g') < T(v^*, g^*)$.

Case I: $Z'(v) \leq 0$

In this case, $Z(v') \geq Z(v^*)$. Thus, in order to have $T(v', g') < T(v^*, g^*)$, from (A1.33) it must be the case that $\Omega_M(g') < \Omega_M(g^*)$. In this case, the marginal value of real money balances falls unambiguously with an increase in the inflation rate, g.

Case II: $Z'(v) > 0$

In this case, $Z(v') < Z(v^*)$ and $\Omega_M(g')$ can be higher or lower than $\Omega_M(g^*)$. Therefore, in this case the effect of inflation rate on the marginal value of real money balances is ambiguous. ▪

Proposition 5:

$$\overline{w}/\underline{w} = 1 + \left[1 - \frac{\rho(r+\rho)}{(\rho+s(v))(r+\rho+s(v))}\right]\left(\frac{y}{\underline{w}} - 1\right). \tag{A1.34}$$

Differentiating (A1.34) w.r.t. g we get

$$\begin{aligned} \frac{d\left(\overline{w}/\underline{w}\right)}{dg} = & -\left[1 - \frac{\rho(r+\rho)}{(\rho+s(v))(r+\rho+s(v))}\right]\left(\frac{y}{\underline{w}^2}\right)\frac{d\underline{w}}{dg} \\ & + \frac{\rho(r+\rho)}{(\rho+s(v))^2(r+\rho+s(v))^2}\left(\frac{y}{\underline{w}} - 1\right)(r+2\rho+2s(v))s'(v)\frac{dv}{dg} \end{aligned}. \tag{A1.35}$$

Given that $\frac{dv}{dg} < 0$, if $\frac{d\underline{w}}{dg} > 0$, then $\frac{d(\overline{w}/\underline{w})}{dg} < 0$. If $\frac{d\underline{w}}{dg} < 0$, then $\frac{d(\overline{w}/\underline{w})}{dg}$ is of ambiguous sign. ▪

Proposition 6:

The real wage offer distribution $F(w)$ is given by

$$F(w) = \frac{r+2(\rho+s(v))}{2s(v)}\left[1 - \sqrt{\frac{r^2 + 4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}}{(r+2(\rho+s(v)))^2}}\right]. \tag{A1.36}$$

Let $L \equiv (r+2(\rho+s(v)))^2$. Then,

$$\frac{dL}{dg} = 4(r+2\rho+2s(v))s'(v)\frac{dv}{dg}. \tag{A1.37}$$

Let $H \equiv r^2 + 4(r+\rho+s(v))(\rho+s(v))\frac{y-w}{y-\underline{w}}$. Then,

$$\frac{dH}{dg} = \frac{4(y-w)}{(y-\underline{w})^2}\left[(y-\underline{w})(r+2\rho+2s(v))s'(v)\frac{dv}{dg} + (\rho+s(v))(r+\rho+s(v))\frac{d\underline{w}}{dg}\right]. \tag{A1.38}$$

Differentiating (A1.36) w.r.t. g, we get

$$
\begin{aligned}
\frac{dF(w)}{dg} =& - \left[1 - \sqrt{\frac{H}{L}}\right] \frac{(r+2\rho)s'(v)}{2s(v)^2}\frac{dv}{dg} \\
&- 2\left(\frac{H}{L}\right)^{-1/2} \frac{r+2(\rho+s(v))}{s(v)L^2}\frac{2L(y-w)}{(y-\underline{w})^2}\Big[(y-\underline{w})(r+2\rho+2s(v))s'(v)\frac{dv}{dg} \\
&+ (r+\rho+s(v))(\rho+s(v))\frac{d\underline{w}}{dg}\Big] + 2\left(\frac{H}{L}\right)^{-1/2} \\
&\frac{r+2(\rho+s(v))}{s(v)L^2}\Big[r^2+4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}\Big](r+2\rho+2s(v))s'(v)\frac{dv}{dg}
\end{aligned}
\tag{A1.39}
$$

Rearranging and simplifying (A1.39), we get

$$
\begin{aligned}
\frac{dF(w)}{dg} =& - \left[1 - \sqrt{\frac{H}{L}}\right] \frac{(r+2\rho)s'(v)}{2s(v)^2}\frac{dv}{dg} \\
&- 2(y-w)(\rho+s(v))(r+\rho+s(v))\frac{r+2(\rho+s(v))}{Ls(v)(y-\underline{w})^2}\left(\frac{H}{L}\right)^{-1/2}\frac{d\underline{w}}{dg} - 2\left(\frac{H}{L}\right)^{-1/2} \\
&\frac{(r+2(\rho+s(v)))^2}{s(v)L^2}\Big[\frac{L(y-w)}{y-\underline{w}} - \big(r^2+4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}\big)\Big]s'(v)\frac{dv}{dg}
\end{aligned}
\tag{A1.40}
$$

Now $L \equiv (r+2(\rho+s(v)))^2 \equiv r^2+4(\rho+s(v))^2+4r(\rho+s(v))$. This implies that

$$
\frac{L(y-w)}{y-\underline{w}} - \big(r^2+4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}\big) = r^2\big(\frac{y-w}{y-\underline{w}} - 1\big). \tag{A1.41}
$$

Putting (A1.41) into (A1.40), we get

$$
\begin{aligned}
\frac{dF(w)}{dg} =& - \left[1 - \sqrt{\frac{H}{L}}\right] \frac{(r+2\rho)s'(v)}{2s(v)^2}\frac{dv}{dg} \\
&- 2(y-w)(\rho+s(v))(r+\rho+s(v))\frac{r+2(\rho+s(v))}{Ls(v)(y-\underline{w})^2}\left(\frac{H}{L}\right)^{-1/2}\frac{d\underline{w}}{dg}. \\
&- 2\left(\frac{H}{L}\right)^{-1/2}\frac{(r+2(\rho+s(v)))^2}{s(v)L^2}r^2\big(\frac{y-w}{y-\underline{w}} - 1\big)s'(v)\frac{dv}{dg}
\end{aligned}
\tag{A1.42}
$$

If we set $r^2 = 0$, the proposition follows. ∎

Proposition 7

The real wage earnings distribution $G(w)$ is given by

$$G(w) = \frac{\rho F(w)}{\rho + s(v)(1 - F(w))}. \tag{A1.43}$$

Differentiating (A1.43) w.r.t. g we get

$$\frac{dG(w)}{dg} = \frac{\rho}{(\rho + s(v)(1 - F(w)))^2}\Big[(\rho + s(v))\frac{dF(w)}{dg} - F(w)(1 - F(w))s'(v)\frac{dv}{dg}\Big]. \tag{A1.44}$$

The proposition follows from (A1.44). ∎

Lemma 4

The equilibrium level of vacancies solves

$$k'(v)v = \frac{\rho(y - w_{min})}{(1 + r)gy^{\theta}(r + \rho + s(v))}\left(\frac{\rho + s(v)}{s(v)}\right)^{\theta - 1} \equiv T(v) \tag{A1.45}$$

Since, $\lim_{v\to 0} T(v) > 0$, there is no solution at $v = 0$. Simple differentiation of $T(v)$ w.r.t v and rearrangement shows that $T'(v) < 0$. Therefore, given the properties of $vk'(v)$ there exists a unique, strictly positive, and finite solution to (A1.45). ∎

Proposition 8

Simple differentiation of $T(v)$ defined in (A1.45) shows that $\frac{dT(v)}{dg}$ & $\frac{dT(v)}{dw_{min}} < 0$. Thus, the level of vacancies falls with the higher inflation rate and the minimum real wage.

$\frac{\overline{w}}{w_{min}}$ is given by

$$\left(\frac{\overline{w}}{w_{min}}\right) = 1 + \left[1 - \frac{\rho(r + \rho)}{(\rho + s(v))(r + \rho + s(v))}\right]\left(\frac{y}{w_{min}} - 1\right). \tag{A1.46}$$

Simple differentiation of (A1.46) w.r.t g and w_{min} shows that the support declines with the higher inflation rate and the minimum real wage. ∎

Appendix 2: The Social Planner Problem

The social planner maximizes

$$\max_{c_t, v_t, u_{t+1}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \Big[u(c_t) - (1+\phi)(1-u_t) - u_t - k(v_t) \Big]$$

subject to

$$u_{t+1} = \rho(1-u_t) + (1-s(v_t))u_t \quad \forall\, t, \tag{A2.1}$$

$$c_t = y(1-u_t). \tag{A2.2}$$

Let λ_{ut} and λ_{ct} be the Langrangian multipliers associated with (A2.1) and (A2.2) respectively. Then, the first order conditions for the social optimum are given by

$$c_t: \; u'(c_t) = \lambda_{ct}, \tag{A2.3}$$

$$v_t: \; k'(v_t) = -\lambda_{ut} s'(v_t) u_t, \tag{A2.4}$$

$$u_{t+1}: \; \lambda_{ut} = \frac{1}{1+r} \Big[\phi - y\lambda_{ct+1} + [1-\rho-s(v_{t+1})]\lambda_{ut+1} \Big]. \tag{A2.5}$$

Combining (A2.3), (A2.4), and (A2.5), we get

$$\frac{k'(v_t)}{s'(v_t)u_t} = \frac{1}{1+r} \Big[u'(c_{t+1})y - \phi + \frac{k(1-\rho-s(v_{t+1}))}{s'(v_{t+1})u_{t+1}} \Big]. \tag{A2.6}$$

In the stationary state, (A2.6) reduces to

$$k'(v) = \frac{\rho s'(v)}{(\rho+s(v))(r+\rho+s(v))} \left[u'\left(\frac{s(v)y}{\rho+s(v)} \right) y - \phi \right]. \tag{A2.7}$$

It can be easily shown that (A2.7) has a unique solution. ■

Appendix 3: The Real Reservation Wage in the Postel-Vinay and Robin Model

The marginal value of unemployed workers satisfies

$$\Omega_u = \frac{1}{1+r}\left[-1 + \overline{s}(v)\Omega_e(\underline{w}) + (1-\overline{s}(v))\Omega_u\right]. \tag{A3.1}$$

The marginal value reflects the fact that unemployed workers receive only the real reservation wage in the case of a match (unlike in Burdett-Mortensen). Since by definition, $\Omega_u = \Omega_e(\underline{w})$, (A3.1) simplifies to

$$\Omega_u = -\frac{1}{r}. \tag{A3.2}$$

Similarly, the marginal value of an employed worker at the real reservation wage $\underline{w}$ is given by

$$\Omega_e(\underline{w}) = \frac{1}{1+r}\left[\underline{w}\Omega_M - (1+\phi) + \rho\Omega_u + \overline{s}(v)\Omega_e(y) + (1-\rho-\overline{s}(v))\Omega_e(\underline{w})\right] \tag{A3.3}$$

which reflects the fact that once the employed worker receives alternative wage offer, his real wage jumps to the worker's productivity y.

The marginal value of an employed worker at the labor productivity y is given by

$$\Omega_e(y) = \frac{1}{1+r}\left[y\Omega_M - (1+\phi) + \rho\Omega_u + (1-\rho)\Omega_e(y)\right]. \tag{A3.4}$$

Combining (A3.2)-(A3.4), we get the expression for the real reservation wage of unemployed workers, $\underline{w}$,

$$\left[\underline{w} + \frac{\overline{s}(v)}{r+\rho}y\right]\Omega_M = \frac{\overline{s}(v)}{r(r+\rho)} - \frac{r+\rho}{r} + (1+\phi)\left[1 + \frac{\overline{s}(v)}{r+\rho}\right]. \blacksquare \tag{A3.5}$$

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Figure 1
The Gross Inflation Rate and Output

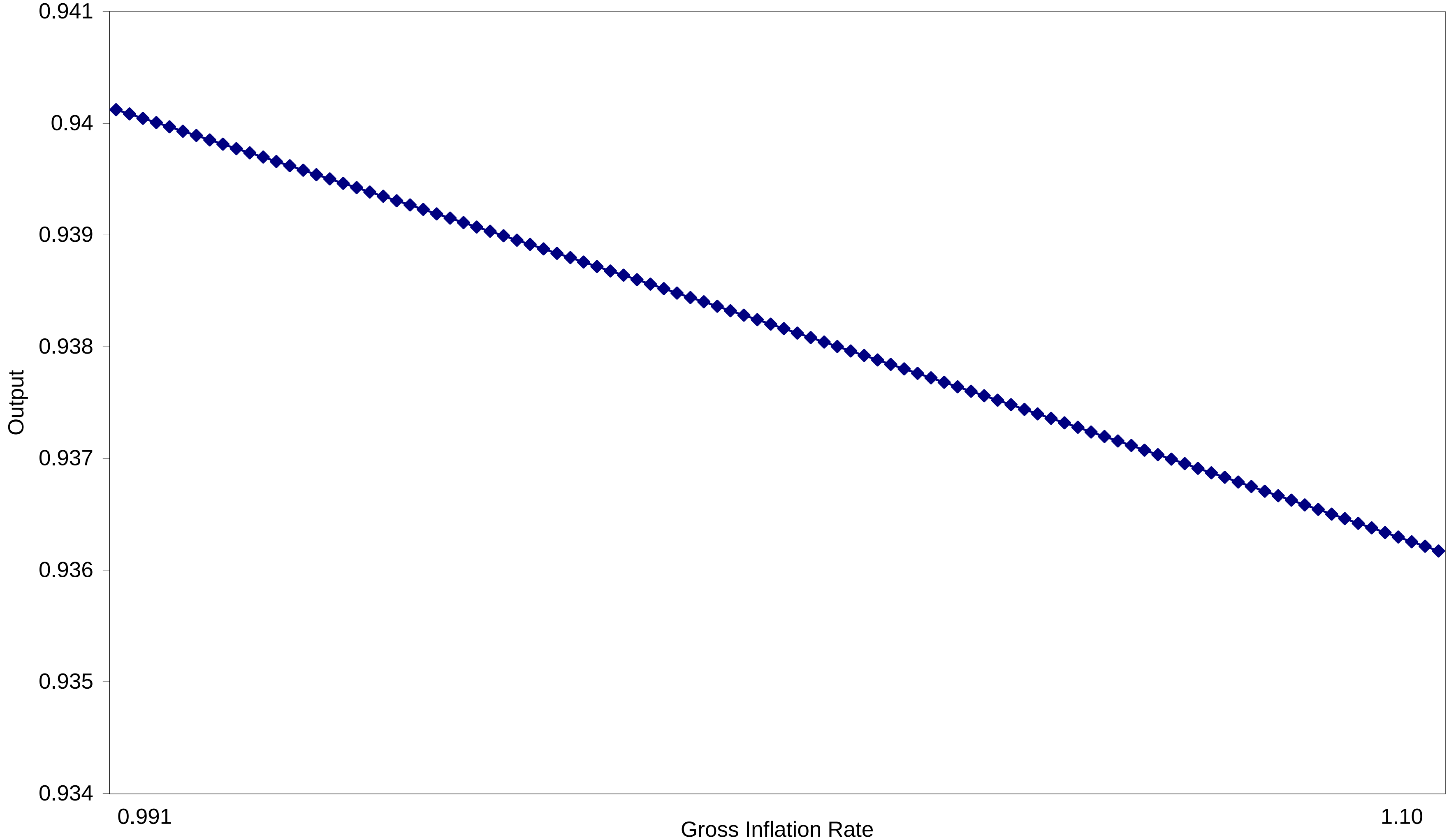

Figure 2
The Gross Inflation Rate and the Welfare Cost

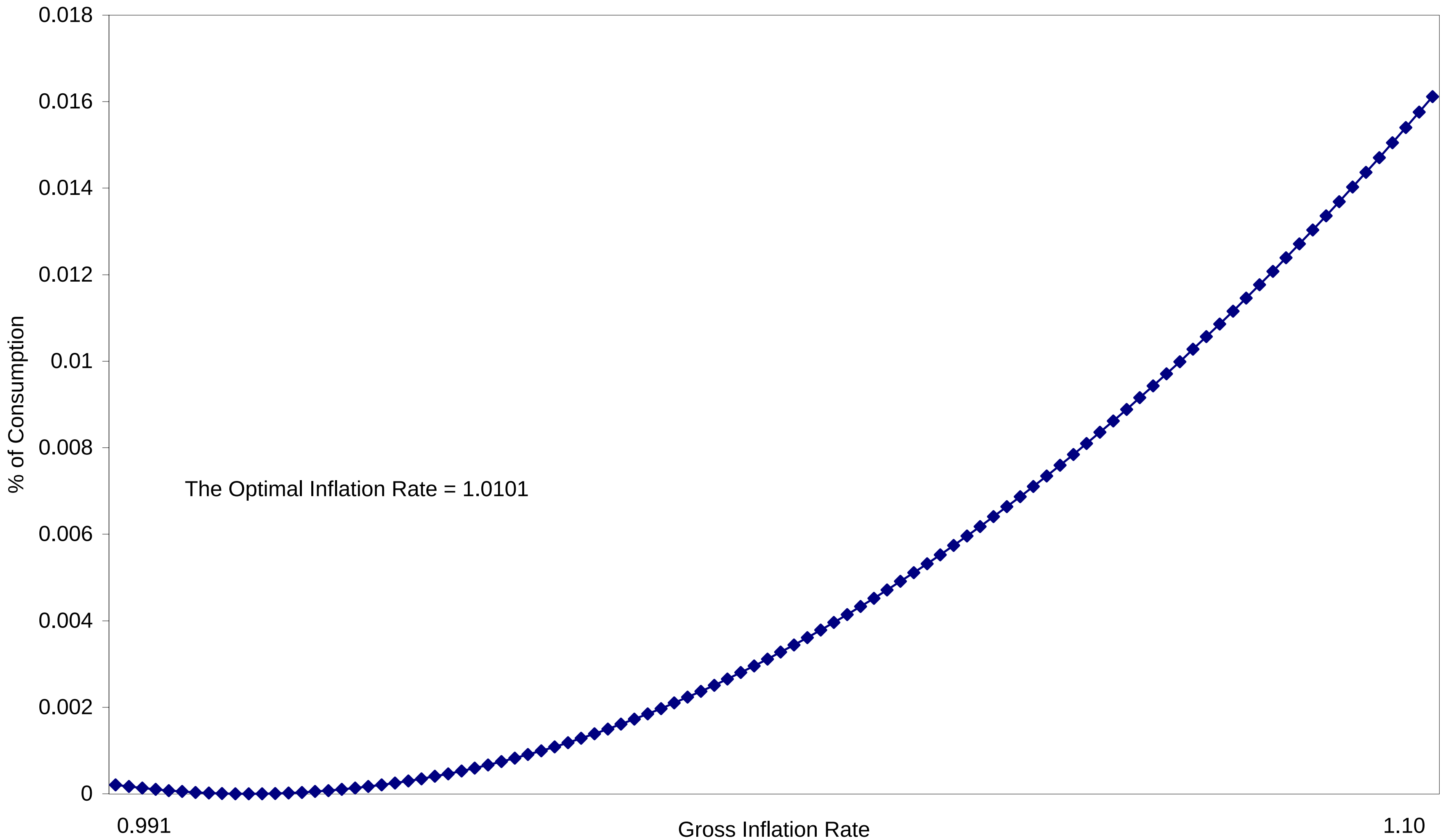

Figure 3
The Gross Inflation Rate and the Upper and the Lower Supports of
the Distributions of Real Wage Offers and Earnings

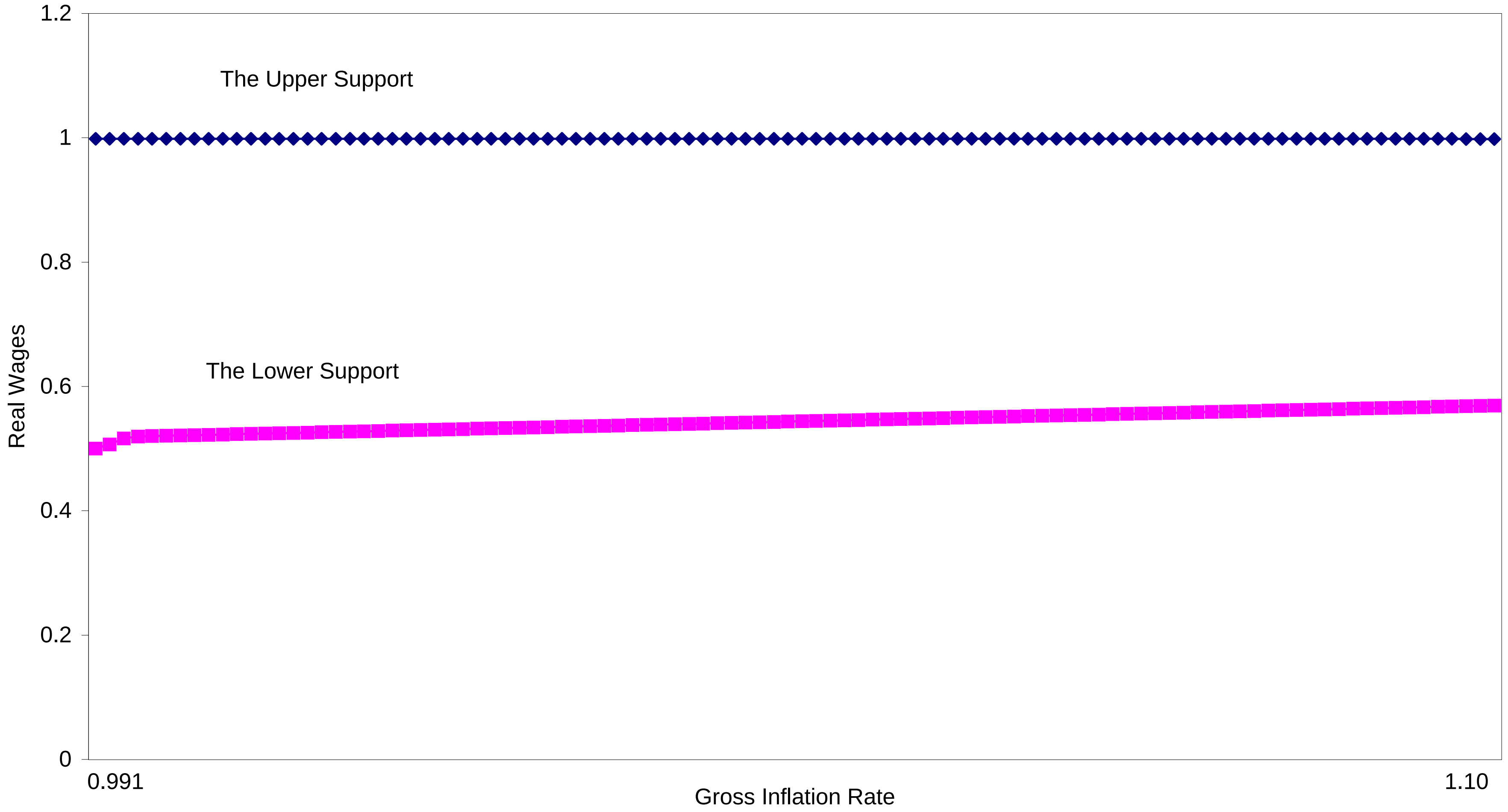

Figure 4
The Gross Inflation Rate and the Coefficient of Variations of Real Wage Offers and Earnings

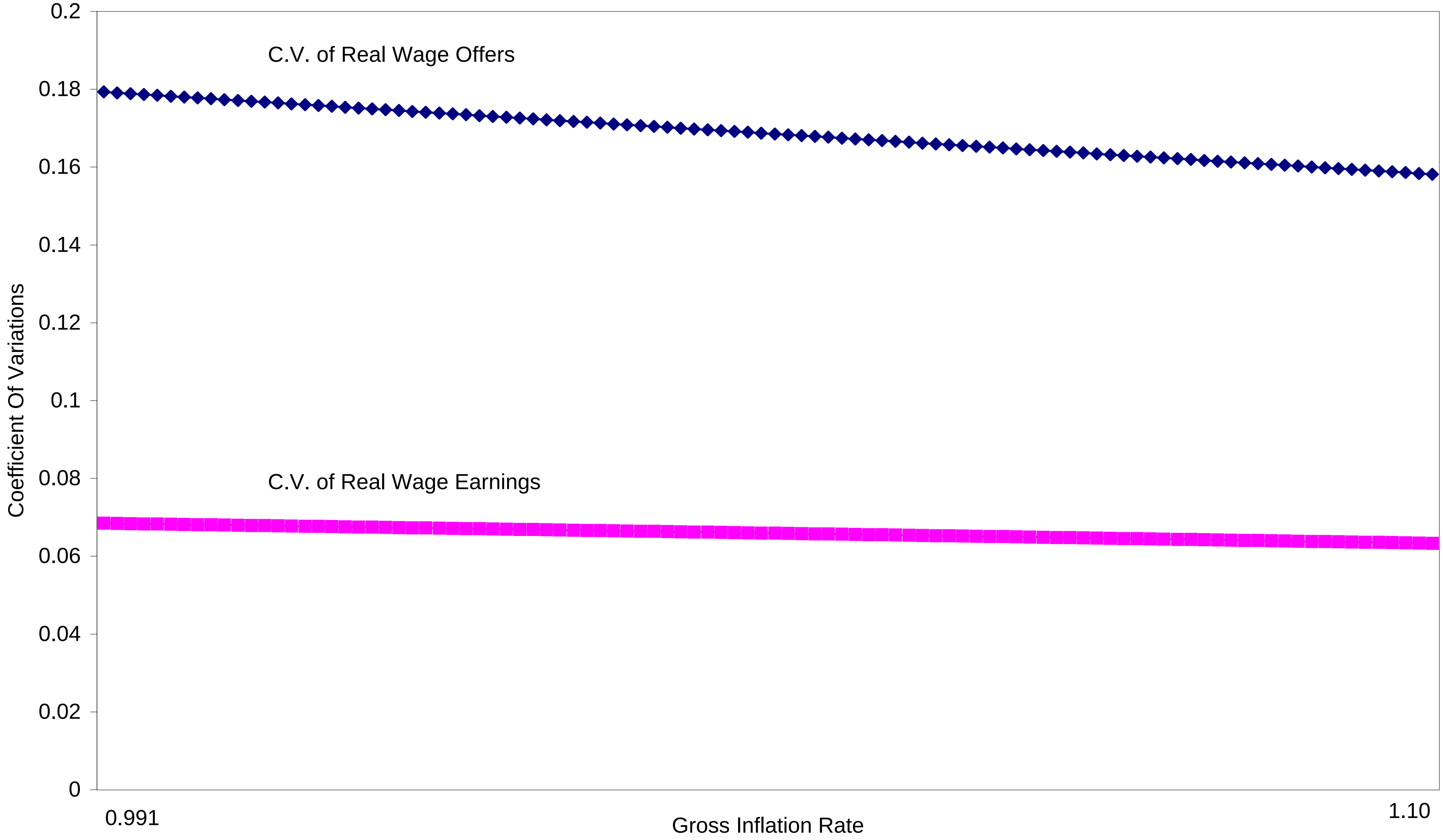

Figure 5
The Gross Inflation Rate and the 90th to 10th, 90th to 50th, and 50th to 10th Percentile Ratios of Real wage Offers

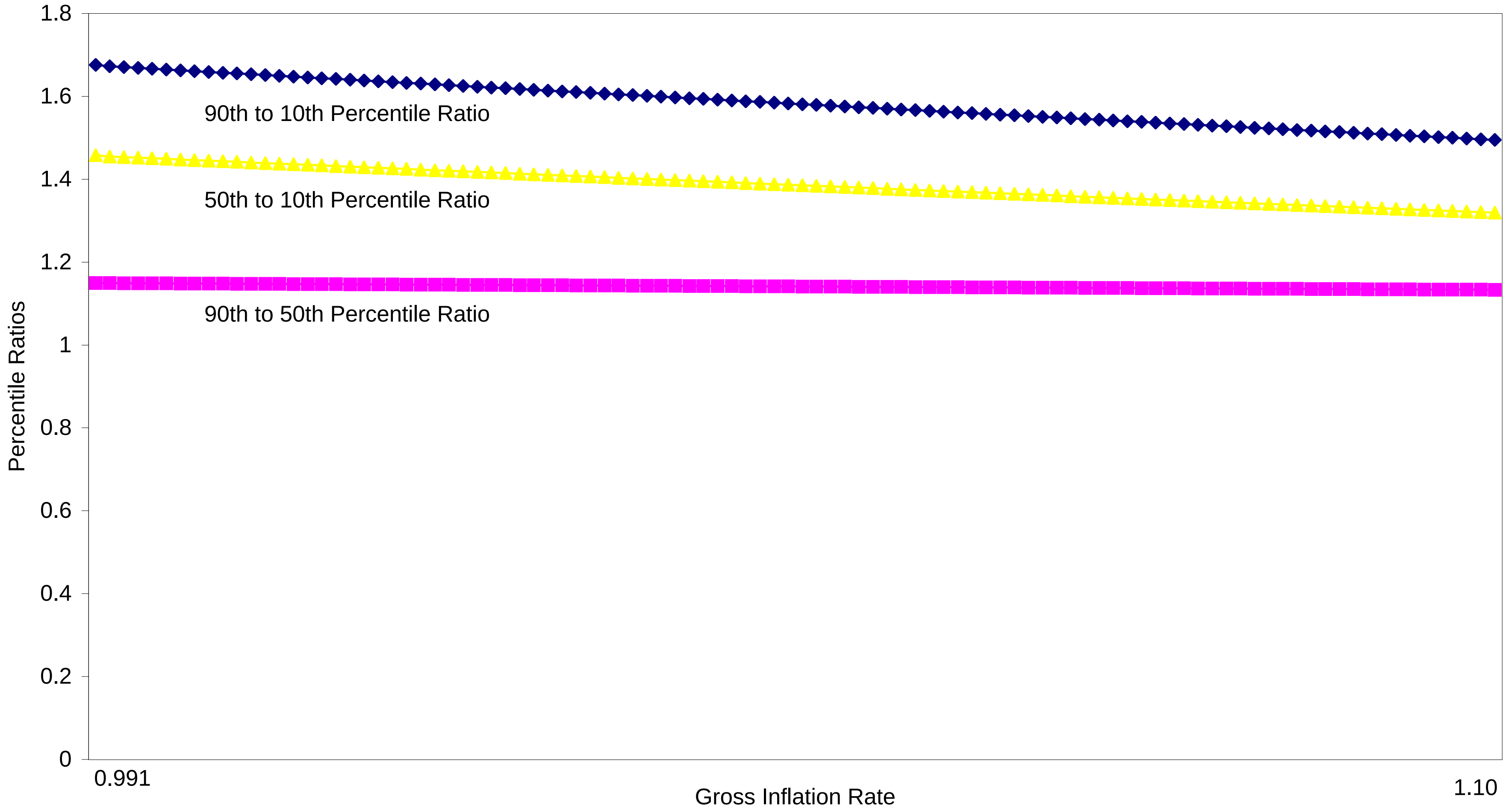

Figure 6
The Gross Inflation Rate and the 90th to 10th, 90th to 50th, and 50th to 10th Percentile Ratios of Real Wage Earnings

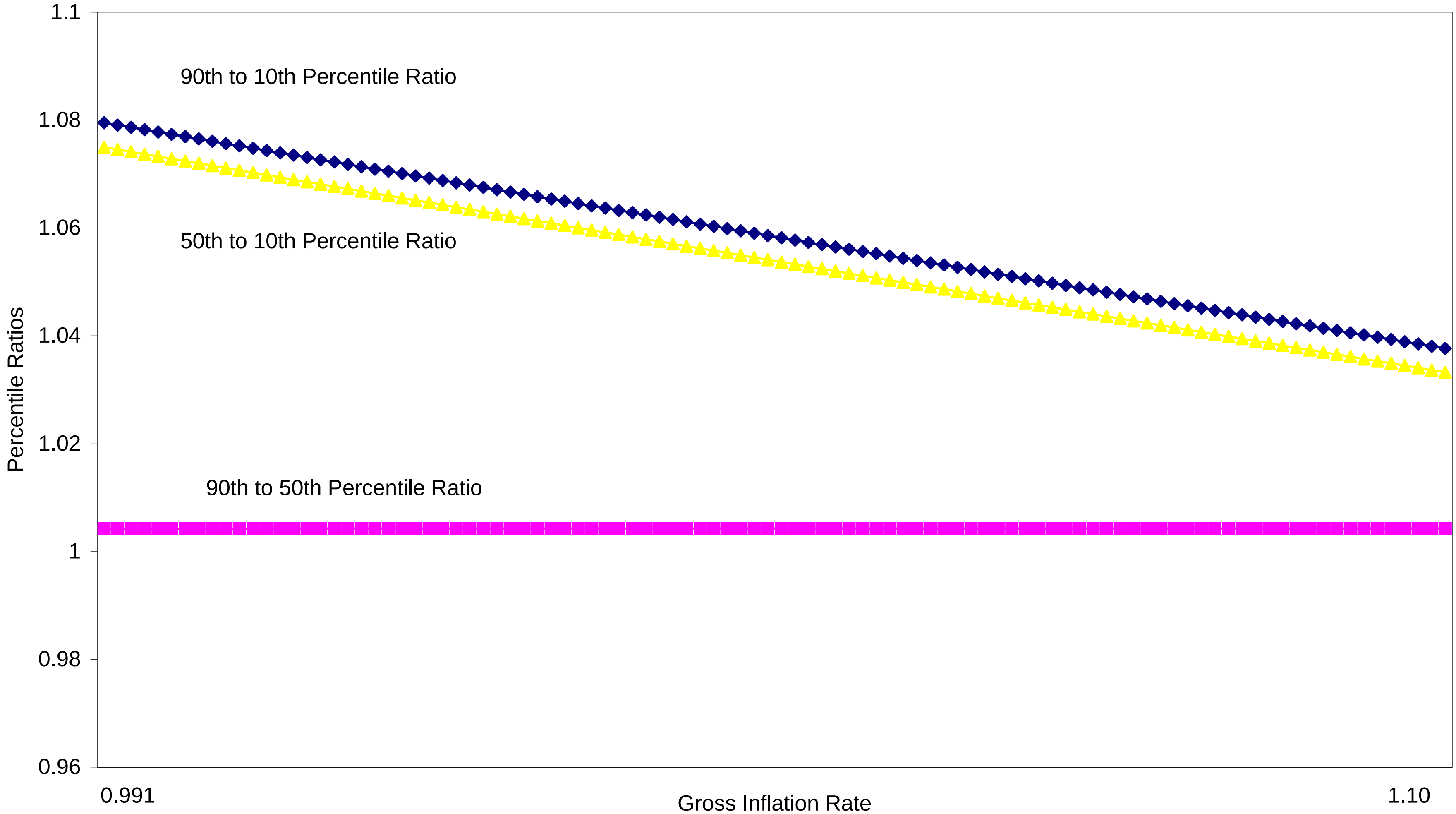

Figure 7
The Gross Inflation Rate and the Average Real Wage Offers and Earnings