

# Unemployment Insurance, Productivity, and Wage Dispersion

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## Abstract

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The effects of unemployment insurance on productivity, output, and within-skill wage dispersion are studied in a search environment in which firms enjoy monopsony power in the labor market. The model features wage posting by firms and on-the-job search by workers. The unemployed workers receive unemployment benefits financed by a proportional pay-roll tax. In equilibrium, unemployment insurance increases welfare not only by providing consumption-smoothing benefits but also by increasing output. Additionally, higher unemployment benefits lead to less within-skill wage dispersion, compression of the lower half of the wage earnings distribution, and a smaller incidence of low wages.

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# 1. Introduction

Models of perfectly competitive labor markets are not able to explain many empirical regularities in the labor market such as wage dispersion among similar workers (residual or within-skill wage dispersion) and co-existence of unemployed workers and vacancies. In order to explain such phenomena, models based on search frictions and imperfect information on the part of workers and firms are increasingly being used. The present paper employs one such model in order to study the effects of unemployment insurance.

This paper examines the effect of unemployment insurance on productivity, output, and within-skill wage dispersion in a general equilibrium search and matching framework. When firms have monopsony power in the labor market, unemployment insurance increases productivity and may increase output. This may increase welfare not only by providing consumption smoothing benefits but also by increasing output. In addition, unemployment insurance reduces within-skill wage dispersion.

Motivation for this study is provided by the literature on costs and benefits of unemployment insurance. Most analyses of unemployment insurance focus on the trade-off between its consumption smoothing benefits and the moral hazard induced by it.<sup>1</sup> In such models, unemployment insurance typically reduces output. Acemoglu and Shimer (1999a) posit a search model in which unemployment insurance increases output. The present paper is a contribution to this line of research.

Acemoglu and Shimer (1999a) show that in a search economy with risk-averse workers and firms, which make irreversible investment and post wages, unemployment insurance can increase output. However, their results depend crucially on the workers' risk aversion. In their model, in the case of risk-neutral workers, output is maximized when there are no unemployment benefits. In contrast to Acemoglu and Shimer (1999a), this paper shows that even with risk-neutral workers output is maximized at a positive level of unemployment benefits.

The analysis conducted in this paper differs from that of Acemoglu and Shimer (1999a) in three significant ways. Firstly, in the model studied here firms enjoy monopsony power in the labor market. Unemployment insurance removes the inefficiencies caused by the monopsony power of firms, which in theirs it does not, and may increase output even when workers are risk-neutral.

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<sup>1</sup> Hansen and Imrohorglu 1992, Andolfatto and Gomme 1996, Costain 1997, Fredriksson and Holmlund 1998 are some of the most recent studies in the general equilibrium framework.

Secondly, unemployment insurance is financed by a proportional pay-roll tax paid by firms while in their paper unemployment insurance is financed by a lump-sum tax. Finally, the model generates non-degenerate wage offers and earnings distributions, which allows us to make predictions about the effects of unemployment insurance on wage dispersion. In Acemoglu and Shimer (1999a) the distributions of wage offers and earnings are degenerate.

Other studies have examined the issue of the effect of unemployment insurance on productivity and output in a general equilibrium framework. Zhang (1996) finds that unemployment insurance leads to higher average productivity in a search economy. In his model, the job arrival rate and the distribution of job productivities are exogenous and not affected by unemployment insurance. In contrast, in our model the job arrival rate and the distribution of job productivities are endogenous. Acemoglu and Shimer (1999c) also find that in an economy with risk-averse workers, unemployment benefits raise the average productivity of workers and output is maximized at a positive level of unemployment benefits. However, in contrast to our model, in their model the job arrival rates and the wage offer distribution are exogenous.

Marimon and Zilibotti (1999) study a search economy with ex-ante heterogeneous firms and workers find that unemployment benefits can increase the growth rate of productivity by reducing the mismatch of skills. Also, if the mismatch of skills is severe, unemployment benefits may lead to higher output. In the model studied here, firms and workers are ex-ante homogeneous.

Our economy embeds a wage posting model with on-the-job search developed by Mortensen (2000) in a general equilibrium framework. Mortensen (2000) shows that in a model with ex-ante identical firms and identical workers, where firms post wages and both the employed and the unemployed workers search, non-degenerate wage offers and earnings distributions can be an equilibrium outcome.<sup>2</sup> In his model, the monopsony power of firms is partly eroded by on-the-job search. Firms are induced to post wages other than the reservation wage of unemployed workers, because posting of higher wages helps to attract the employed workers from low wage jobs and simultaneously reduces the workers' turnover.

The economy features wage posting by firms and sequential search by both the employed and the unemployed workers among the posted wages. Firms, while posting wage offers, also choose investment levels, which are incurred after the successful match. Each unemployed worker is paid

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<sup>2</sup> Rosholm and Svarer (2000) estimate the Mortensen (2000) model with firm-specific training expenditures using the Danish labor market data and find that the model provides a good characterization of some empirical features of the labor market.

a lump-sum unemployment benefit by the government, which finances unemployment insurance through a proportional pay-roll tax paid by firms.

In such an environment, we show that an equilibrium with non-degenerate wage offer distribution and wage earnings distribution exists. Quantitative experiments show that, in equilibrium, unemployment insurance increases workers' productivity and may increase output. Unemployment insurance may therefore increase welfare not only by providing consumption smoothing benefits but also by increasing output. The positive effect of unemployment insurance on output does not hinge on any specific assumption about the workers' attitude towards risk. Unemployment insurance can increase output both in cases where the workers are risk-neutral and where they are risk-averse.

Unemployment insurance affects the distributions of wage offers and earnings. We find that higher unemployment benefits, in equilibrium, lead to lower dispersion in wage offers and earnings, more compression in the lower part of the wage offer and earnings distributions, and a smaller incidence of low wage offers and earnings, where low wage offer and earnings are defined as two-third of the median wage offer and earnings respectively.

The mechanism generating these results may be described as follows. The level of capital investment depends on the workers' turnover, which depends on the matching rate of the employed workers due to on-the-job search. In the absence of unemployment insurance, the unemployed workers' reservation wage is low and firms create too many vacancies. This increases the matching rate of workers and the workers' turnover. Consequently, capital investment is lower than efficient.

Unemployment insurance increases the reservation wage and reduces the monopsony power and the profitability of firms. In addition, the pay-roll tax imposed on firms to finance unemployment insurance further reduces the profitability of firms. The decline in the profitability induces firms to reduce the level of vacancies posted, which reduces the matching rate of workers and the workers' turnover. A decline in the workers' turnover increases capital investment. The resultant increase in the productivity of workers may more than offset the decline in output due to higher unemployment rate, and output may rise.

Unemployment insurance also affects wage dispersion. An increase in the reservation wage of unemployed workers increases the lower supports of wage offer and earnings distributions. But the effect of unemployment insurance on the upper supports of wage offer and earnings distributions is mitigated by the fact that a fall in the matching rate of workers reduces the effectiveness of on-the-job search in eroding the monopsony power of firms. Consequently, the range of wage offers and earnings fall.

The rest of the paper is organized as follows. Section 2 describes the economy. In Section 3 stationary equilibrium is defined and characterized. In Section 4 the effects of unemployment insurance on the labor market flows and the distributions of wage offers, earnings, and productivity are analyzed in equilibrium. In Section 5, the effect of unemployment insurance on output and welfare is studied. Some numerical examples are also constructed to illustrate the effects of unemployment insurance. Section 6 contains concluding remarks. Proofs of all the propositions are in Appendix 2.

## 2. The Economy

Consider a search economy populated by continua of infinitely-lived identical firms and identical workers. Time is continuous, and there is no aggregate uncertainty. There is friction in the labor market: workers and firms have to search for suitable matches.

### 2.1 Preferences and Technology

All agents discount the future at the common rate  $r$ . Let the measure of workers in the economy be unity. Workers have preference over final consumption,  $c$ , given by  $U(c)$ , where  $U(c)$  is assumed to be strictly increasing.

Firms create job-sites which are identical except for wages and capital investment. Job-sites are either filled and producing or vacant and searching. For vacant job-sites, firms post wages and search for workers. For simplicity, the search-intensity of firms with vacant jobs is normalized to one. Posting of a vacancy costs a firm  $\delta$  per unit of time. In the case of a match, the searching firm hires at most one worker per job. After the match, the firm pays the worker the posted wage as long as the match continues. Let  $F(w)$  be the aggregate wage offer distribution and  $\underline{w}$  and  $\bar{w}$  be the lower and upper supports of the offer distribution respectively.

At the time it posts a vacancy, a firm also decides the level of capital investment,  $k$ , it will make in the case of a successful match. While the decision about the level of capital investment is made at the time of posting vacancy, actual investment is made only after the successful match. The level of capital investment determines the productivity of the job-site. In what follows, we will use terms productivity and the level of capital investment interchangeably.

For simplicity, it is assumed that capital investment is match-specific and does not depreciate as long as the match continues but depreciates completely in the case of match separation.<sup>3</sup> Once

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<sup>3</sup> Job-specific training expenditure can be one example of such an investment.

a firm with a vacant job gets matched, the firm incurs capital investment and production starts. The production function  $y(k)$  is assumed to be a strictly increasing and concave function of capital investment  $k$ . The timing of the capital investment implies that this economy, unlike that of Acemoglu and Shimer (1999b), does not feature “hold-up inefficiency” in which vacant firms first incur capital investment and then search for suitable workers.

## 2.2 Matching and Job Separation

In the economy, both the employed and the unemployed workers search among the posted job offers. The employed workers search for better jobs. The unemployed workers search for jobs which pay them at least their reservation wage. Search is assumed to be sequential. For simplicity, it is assumed that the employed and the unemployed workers search with fixed intensities  $s_e$  and  $s_u$  respectively.<sup>4</sup> Since the employed workers also search, the wage earnings distribution is distinct from the wage offer distribution. Let  $G(w)$  be the aggregate wage earnings distribution. As firms hire at most one worker per job, the distribution of filled jobs is identical to  $G(w)$ . In addition, the number of filled jobs is equal to the number of employed workers.

Workers and vacancies are matched through an aggregate matching function  $M(v, S)$  that relates the flow of hiring to the effective number of searching workers  $S$  and the number of vacancies  $v$ . The effective number of searching workers is given by

$$S = s_u u + s_e (1 - u) \tag{2.1}$$

where  $u$  is the measure of unemployed workers, and  $s_e$  and  $s_u$  are the fixed search intensities of employed and unemployed workers respectively.

Let the matching function  $M(v, S)$  be strictly increasing and concave in both its arguments. Also assume that the matching function is homogeneous of degree one. In addition, let  $M(0, S) = M(v, 0) = 0$ .

In this setting, the matching rate of searching workers is given by

$$m(q) \equiv \frac{M(v, s)}{S} \equiv M(q, 1) \tag{2.2}$$

where  $q \equiv v/S \equiv$  labor market tightness and  $m'(q) > 0$ . Let  $\lim_{q \rightarrow 0} m'(q) \rightarrow \infty$ , and

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<sup>4</sup> If search-intensity were endogenous, employed workers will search differently depending on their level of wages. Endogeneizing search intensity would lead to considerable computational complexity.

$\lim_{q \rightarrow \infty} m'(q) \rightarrow 0$ . The matching rate of a worker  $i$  for  $i = e, u$  is given by

$$\equiv m(q)s_i. \tag{2.3a}$$

The matching rate of a vacancy is given by

$$n(q) \equiv \frac{M(v, s)}{v} \equiv \frac{m(q)}{q} \tag{2.3b}$$

with  $\lim_{q \rightarrow 0} n(q) \rightarrow \infty$  and  $n'(q) < 0$ .

The employed workers face the risk of unemployment. More specifically, job matches are subject to exogenous idiosyncratic shocks, which arrive at Poisson rate  $\rho$ . Note that match separations occur for two reasons- (i) employed workers receive better job offers and (ii) job matches receive idiosyncratic shocks. Only in the second case do employed workers become unemployed.

### 2.3 Unemployment Insurance

All unemployed workers receive equal lump-sum unemployment benefit  $b$  per unit of time as long as they are unemployed. Benefits are paid by a government whose only function is to administer the unemployment insurance scheme. Note that the model generates non-degenerate distributions of wage offers and earnings. The assumption of lump-sum benefits implies that the implicit replacement ratio (the ratio of unemployment benefits to the average wage earnings) is declining in wage  $w$ . This can be considered as a continuous approximation to the actual unemployment insurance scheme where replacement ratio declines discretely. In the economy there is no financial market and thus workers cannot borrow or lend to smooth their consumption. Unemployment benefits are the only source of consumption smoothing, and this is the only form of insurance available.<sup>5</sup>

In order to finance the unemployment insurance expenditure, the government levies a proportional pay-roll tax  $t$  on filled jobs. It is assumed that the government cannot borrow or lend, and it has to finance current unemployment insurance expenditure through the current tax receipts. The budget constraint of the government satisfies

$$bu = (1 - u)t \int_{\underline{w}}^{\bar{w}} dG(w). \tag{2.4}$$

The term in the left hand side is the unemployment insurance expenditure. The term in the right hand side is the total tax receipt.

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<sup>5</sup> Workers do not own firms. Thus, the profits of firms do not directly affect the consumption of workers.



We now first describe the optimal strategies of workers, followed by the optimal strategies of firms.

## 2.4 Workers

Both the employed and the unemployed workers face the problem of choosing job-acceptance strategy, which maximizes their discounted utility. An individual worker takes the wage offer distribution,  $F(w)$ , and the matching rate of workers,  $m(q)$ , as given. In such a set-up, the value of a worker employed at wage  $w$ ,  $V_e(w)$ , and the value of an unemployed worker,  $V_u$ , satisfy following equations.

$$rV_e(w) = U(w) + m(q)s_e \left[ \int \max(V_e(w), V_e(x)) dF(x) - V_e(w) \right] - \rho(V_e(w) - V_u). \quad (2.5)$$

$$rV_u = U(b) + m(q)s_u \left[ \int \max(V_u, V_e(x)) dF(x) - V_u \right]. \quad (2.6)$$

In (2.5), the first term in the right hand side is the momentary utility flow to a worker employed at wage  $w$ .<sup>6</sup> The last two terms together give the expected capital gain from a job separation. An employed worker leaves the match either because he receives a better job offer or he receives idiosyncratic shocks.

In (2.6), the first term in the right hand side is the momentary utility flow to an unemployed worker. The second term is the expected capital gain from search.

Following Moretensen and Neumann (1988), one can characterize the value functions of workers and the optimal job-acceptance strategy.

**Proposition 1:** If the set of wage offers and the set of unemployment benefits are both bounded and the discount rate  $r$  strictly positive, then for a given continuous wage offer distribution,  $F(w)$ , and the matching rate,  $m(q)$ , unique, continuous and strictly increasing value functions  $V_e(w)$  and  $V_u$  exist.

The proof of the proposition is in Appendix 2. Since the value functions are unique, continuous, and strictly increasing, the optimal job-acceptance strategies of workers have a reservation property.

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<sup>6</sup> Since, we have assumed that workers do not save, consumption is equal to wages or unemployment benefits.

In the present environment, the optimal reservation wage of an employed worker is his current wage. The reservation wage of an unemployed worker  $R$  is given by the wage at which he is indifferent between accepting the realized wage offer and continuing the job search.

$$V_u = V_e(R). \quad (2.7)$$

Following Mortensen and Neumann (1988), one can show that the reservation wage of an unemployed worker,  $R$ , is a unique solution to the following equation (see Appendix 1).

$$U(R) = U(b) + m(q)(s_u - s_e) \int_R^{\bar{w}} \frac{(1 - F(x))U_x(x)}{r + \rho + m(q)s_e(1 - F(x))} dx. \quad (2.8)$$

The term in the left hand side of (2.8) is the momentary utility flow to an employed worker at wage  $R$ . The first term in the right hand side is the momentary utility flow to an unemployed worker. The second term is the difference between the expected capital gain from search when unemployed and the expected capital gain when employed at wage  $R$ . (2.8) shows that when the search-intensities of both the employed and the unemployed workers are identical, then the reservation wage depends only on unemployment benefits, as is the case in Mortensen (2000). Simple differentiation of (2.8) shows that the reservation wage  $R$  necessarily increases with unemployment benefit  $b$  as long as  $s_u \geq s_e$ , which is empirically realistic.

## 2.5 Firms

Firms with vacant jobs face the problem of choosing the level of wage,  $w$ , and capital investment,  $k$ , which maximize their discounted profits. The value of a vacant job,  $V_v$ , with wage  $w$  and capital investment  $k$  satisfies following equation

$$rV_v = -c + n(q) \left[ \frac{s_u u + (1 - u)s_e G(w)}{S} \right] [V_j(w, k) - k - V_v] \quad (2.9)$$

where  $V_j(w, k)$  is the value of a filled job with wage  $w$  and capital investment  $k$  and  $n(q)$  is the matching rate of vacancy. While posting a vacancy, a firm takes the wage offer distribution,  $F(w)$ , and the matching rate of vacancies,  $n(q)$ , as given. The first term in the right hand side is the momentary cost of posting a vacancy. The second term in the right hand side is the expected capital gain. The capital gain reflects the fact that the matching probability of a vacant firm posting wage  $w$  depends not only on the matching rate of vacancy but also on the pool of unemployed and employed workers receiving wages less than  $w$ . A firm posting a vacancy chooses wage  $w$  and

capital investment  $k$  to maximize the value of the vacancy. Because of on-the-job search, a vacant firm can affect its matching rate by varying the posted wage. By posting a higher wage, a vacant firm can increase the matching probability. In such an environment, no firm can profit by offering a wage below the lower support of the wage offer distribution. Thus, the lowest wage offered is  $\underline{w}$ .

Firms with vacant jobs choose capital investment to maximize the value of vacancy. The required first order condition is given by (see Appendix 1)

$$y'(k) = r + \rho + m(q)s_e(1 - F(w)). \quad (2.10)$$

(2.10) equates the marginal product of capital investment with the sum of the discount rate and the match separation rate at each  $w$ . (2.10) implies that the optimal capital investment is higher at higher wages:

$$\frac{\partial k}{\partial w} = -\frac{m(q)s_e F'(w)}{y''(k)} > 0. \quad (2.11)$$

At higher wages workers' turnover is less, encouraging firms to select a higher level of capital investment. Since at each wage, a unique level of capital investment is selected, corresponding to the wage offer distribution there will be a distribution of capital investment offered. Similarly, there will be a distribution of capital investment incurred corresponding to the distribution of filled jobs.

The value of a filled job with wage  $w$  and investment  $k$  satisfies

$$rV_j(w, k) = y(k) - w(1 + t) - \left[ \rho + m(q)s_e(1 - F(w)) \right] \left[ V_j(w, k) - V_v \right] \quad (2.12)$$

where  $t$  is the pay-roll tax rate. The first two terms on the right hand side of (2.12) give the momentary profit flow, and the last term is the expected capital gain in the event that the match is dissolved. A match is dissolved either due to idiosyncratic shocks or due to the employed worker receiving a better job offer. Because of on-the-job search, filled jobs paying higher wages have lower workers' turnover. In the model, the dependence of the matching probability of a vacancy and the match separation on the offered wage allows ex-ante homogeneous firms to post different wages in equilibrium.

### 3. Stationary Equilibrium

In a stationary environment, the wage offer distribution,  $F(w)$ , the labor market tightness,  $q$ , the tax rate,  $t$ , and the levels of capital investment,  $k(w)$ , are constant over time. Given the optimal job-acceptance strategies of workers, one can easily derive the stationary state unemployment rate and the wage earnings distribution.

#### 3.1 Stationary State Unemployment and the Wage Earnings Distribution

In the stationary state, flows in and out of any employment status are constant. Flows in and out of unemployment satisfy

$$u m(q) s_u = (1 - u)\rho. \quad (3.1)$$

The left hand side is the aggregate outflow from the unemployment pool and the right hand side the aggregate inflow. The aggregate outflow from the unemployment pool is equal to the number of unemployed workers being matched. The aggregate inflow to the unemployment pool is equal to the number of employed workers whose matches are destroyed due to idiosyncratic shocks. From (3.1) we have

$$u = \frac{\rho}{\rho + m(q)s_u}, \quad (3.2)$$

an expression for the steady state measure of unemployed workers. Since the total measure of workers in the economy is unity, (3.2) also gives the unemployment rate. It is clear that the unemployment rate is decreasing in the matching rate of workers,  $m(q)$ .

In the steady state, flows in and out of the pool of employed workers receiving wage  $w$  or less are equal.

$$\rho G(w)(1 - u) + m(q)s_e G(w)(1 - u)(1 - F(w)) = m(q)s_u F(w)u. \quad (3.3)$$

The left hand side is the outflow of employed workers who receive wage  $w$  or less. It consists of two terms. The first is the outflow of employed workers receiving wage  $w$  and less to the unemployment pool due to idiosyncratic shocks. The second is the outflow of employed workers to higher wage jobs. The right hand side gives the total inflow. The total inflow to the pool of employed workers receiving wage  $w$  and less is equal to the number of unemployed workers receiving

wage offer of  $w$  and less. The steady state relationship implies that  $G(w)$  satisfies the following equation:

$$G(w) = \frac{m(q)s_u F(w)u}{\left[\rho + m(q)s_e(1 - F(w))\right](1 - u)}. \quad (3.4)$$

Combining (3.2) and (3.4) we have

$$G(w) = \frac{\rho F(w)}{\left[\rho + m(q)s_e(1 - F(w))\right]}. \quad (3.5)$$

(3.5) gives the steady state distribution of workers as a function of the wage offer distribution and the matching rate of employed workers. The implication of (3.5) is that for a given wage offer distribution, an increase in the labor market tightness leads to stochastic improvement in the wage earnings distribution,  $G(w)$ . Similarly, a stochastic improvement in the wage offer distribution for a given labor market tightness leads to stochastic improvement in the wage earnings distribution. The intuition is as follows. An increase in the labor market tightness increases the matching rate of employed workers increasing the rate at which employed workers move from low wage jobs to high wage jobs. Similarly, if the proportion of high wage job offers increases, unemployed workers are more likely to find high wage jobs.

### 3.2 Equilibrium

**Definition:** Stationary equilibrium of the model is a reservation wage  $R$  of unemployed workers, a policy rule  $k(w)$  of vacant firms, a labor market tightness  $q$ , a wage offer distribution  $F(w)$ , a wage earnings distribution  $G(w)$ , and a tax rate  $t$  such that

- (i) the reservation wage  $R$  is optimal (2.8);
- (ii) the policy rule  $k(w)$  satisfies (2.10);
- (iii) the value of vacancy for each job  $V_v$ , defined in (2.9), is identical  $\forall w, k$  posted and equal to zero;
- (iv) the wage earnings distribution  $G(w)$  satisfies (3.5);
- (v) the government balances its budget each period (2.4).

Condition (iii) is a consequence of the fact that all firms are in a symmetric position. It ensures that a firm choosing pair  $(w, k(w))$  for a vacant job does not have incentive to deviate.

Since all unemployed workers are identical in the model, the lower support of the wage offer distribution  $\underline{w}$  equals the common reservation wage of unemployed workers  $R$  given by (2.8). This implies that in our model, unlike others in which the wage offer distribution is exogenous, the reservation wage effect is absent. In search models with an exogenous wage offer distribution, unemployment benefits increase the reservation wage of unemployed workers and thus reduce the probability of job-acceptance. In our model, in response to a change in unemployment benefits firms also change wage offers, and thus unemployed workers continue to accept job offers with probability one before or after the change in unemployment benefits.

The imposition of equilibrium conditions gives the following equilibrium relations (see Appendix 1 for details). Putting (2.12) in (2.9) and using (3.2) and (3.5) we get

$$q\delta = m(q)z_w \frac{y(k(w)) - w(1+t) - k(w)(r + \rho + m(q)s_e(1 - F(w)))}{r + \rho + m(q)s_e(1 - F(w))} \quad \forall w, \quad (3.6)$$

where

$$z_w \equiv \frac{s_u u + (1 - u)s_e G(w)}{S} \quad (3.7)$$

and  $k(w)$  is the optimal capital investment posted at wage  $w$ . (3.6) states that at any wage  $w$  the cost of creating a vacancy equals the present value of the expected future profit from filling the vacancy. Equilibrium condition (iii) requires that (3.6) holds for all wages in the support of the wage offer distribution,  $F(w)$ .

Setting  $F(R) = 0$  in (3.6), one can derive the equilibrium condition which the labor market tightness,  $q$ , must satisfy. The labor market tightness solves

$$q\delta = m(q) \frac{\rho}{(\rho + m(q)s_e)} \frac{y(k(R)) - R(1+t) - k(R)(r + \rho + m(q)s_e)}{(r + \rho + m(q)s_e)}. \quad (3.8)$$

Intuitively, firms post vacancies until the marginal cost of posting a vacancy (LHS) becomes equal to the expected marginal benefit (RHS). The first term in the right hand side is the matching rate of workers, and the second term is the measure of unemployed workers. The final term is the discounted flow of profitability on a vacancy with reservation wage  $R$  and the associated capital investment  $k(R)$ .

Since in equilibrium all vacant firms make an equal profit, (3.6) and (3.8) imply that the equilibrium wage offer distribution  $F(w)$  must satisfy

$$\begin{aligned}
& \frac{\rho}{(\rho + m(q)s_u)} \left[ \frac{y(k(R)) - R(1+t) - k(R)(r + \rho + m(q)s_e)}{(r + \rho + m(q)s_e)} \right] \\
& = z_w \frac{y(k(w)) - w(1+t) - k(w)(r + \rho + m(q)s_e(1 - F(w)))}{r + \rho + m(q)s_e(1 - F(w))} \forall w. \tag{3.9}
\end{aligned}$$

**Proposition 2:** For a given level of labor market tightness ( $q > 0$ ), reservation wage  $R$ , and tax rate  $t$ , there exists a unique wage offer distribution  $F(w)$ .

Since the left hand side is independent of  $w$  and  $F(w)$  and the right hand side is a strictly decreasing function of  $w$  for any  $q$ ,  $R$ , and  $t$ , if the above equilibrium condition is to be satisfied, then  $F(w)$  must be a strictly increasing function of  $w$ . Thus, for any given  $q$ ,  $R$ , and  $t$ , there exists a unique wage offer distribution  $F(w)$ , which satisfies (3.9).

Imposing the condition that  $F(\bar{w})=1$  in (3.9), one can derive the upper wage support  $\bar{w}$ . The upper wage support  $\bar{w}$  satisfies following equation

$$\begin{aligned}
\bar{w} = & \frac{(r + \rho)\rho}{T_R} R + \frac{1}{1+t} \left[ y(k(\bar{w})) - k(\bar{w})(r + \rho) \right. \\
& \left. - \frac{(r + \rho)\rho [y(k(R)) - k(R)(r + \rho + m(q)s_e)]}{T_R} \right] \tag{3.10}
\end{aligned}$$

where  $T_R = (\rho + m(q)s_e)(r + \rho + m(q)s_e)$ .

Regarding the existence of an equilibrium, if the production function  $y(k)$  satisfies constant returns to scale one can show that a non-trivial equilibrium exists for the appropriate choices of unemployment benefit  $b$  and the cost of posting vacancy  $\delta$ .

**Proposition 3:** Let the production function  $y(k)$  satisfy constant return to scale and  $s_u > s_e$ . Then, there exists a non-trivial equilibrium.

The proof of the proposition is in Appendix 2. In general, we cannot show that equilibrium is unique. It is possible that multiple equilibria *e.g.* equilibrium with high tax and unemployment rates may coexist with an equilibrium with low tax and unemployment rates exist. However, in the numerical examples computed below equilibrium is unique.

## 2.4. Effects of Unemployment Benefits in Equilibrium

In our model, given that the reservation wage,  $R$ , and the tax rate,  $t$ , depend on the distributions of wage offers,  $F(w)$ , and wage earnings,  $G(w)$ , it is not possible to compute an equilibrium analytically. In what follows, we delineate the likely mechanisms by which unemployment benefits affect the equilibrium variables in the model.

The effect of unemployment benefits on the labor market tightness depends on its effect on the reservation wage and the tax rate. An increase in the reservation wage and the tax rate reduces the return on vacancy, and thus firms reduce the number of vacancies posted. For the same reason, if the reservation wage and the tax rate fall, the number of vacancies posted increases.

In our model, unlike the model of Mortensen (2000) in which  $s_e = s_u$ , an increase in unemployment benefit  $b$  has an ambiguous effect on the reservation wage,  $R$ . In this model, the reservation wage,  $R$ , depends not only on the unemployment benefits but also on the expected capital gain from search (2.8). This, in turn, depends on the labor market tightness,  $q$ , and the wage offer distribution,  $F(w)$ . If unemployment benefits reduce the expected capital gain from search, the reservation wage need not rise. Similarly, the tax rate depends on the reservation wage, the labor market tightness, and the wage earnings distribution and may rise and fall with unemployment benefits.

Turning to the effects of unemployment benefits on the wage offer distribution,  $F(w)$ , an increase in unemployment benefits accompanied by an increase in the reservation wage leads to stochastic improvement in the wage offer distribution for a given tax rate and labor market tightness. This happens because an increase in the reservation wage reduces the monopsony power of firms, which induces them to create a larger proportion of high wage jobs.

A change in unemployment benefits, as discussed earlier, also affects the tax rate and the labor market tightness. If an increase in unemployment benefits reduces the labor market tightness, then it also reduces the effectiveness of on-the-job search in eroding the monopsony power of firms. Also, if an increase in unemployment benefits increases the tax rate, then it leads to a reduction in the profitability of firms inducing them to create low wage jobs in larger proportion. The latter two effects may prevent any stochastic improvement in the wage offer distribution even if the reservation wage rises.

Due to the ambiguous effect of unemployment benefits on the wage offer distribution and the matching rate of workers, unemployment benefits may or may not lead to stochastic improvement in the wage earnings distribution,  $G(w)$ , as well. A change in  $G(w)$  due to a change in  $b$  is positively



related to a change in  $F(w)$  and negatively related to a change in the matching rate of workers  $m(q)$  (3.5). If the mass of wage offers shifts from the lower wages to the higher wages, workers are more likely to find higher wage jobs and vice versa. On the other hand, if the matching rate of workers falls, then less employed workers will move to higher paid jobs, and the mass of earnings will shift towards lower wages.

Even where an increase in unemployment benefits leads to stochastic improvement in the wage offer distribution, there may not be stochastic improvement in the wage earnings distribution if the labor market tightness falls sufficiently. The ambiguous effect of unemployment benefits on the wage earnings distribution also implies that unemployment benefits have an ambiguous effect on the distribution of capital investment incurred.

In the model, the level of capital investment at any wage depends on employed workers' turnover, which, in turn, depends on the labor market tightness and the wage offer distribution. An increase in the labor market tightness by increasing workers' turnover reduces the level of capital investment. Similarly, a stochastic improvement in the wage offer distribution, which increases the probability of a worker finding higher wage jobs has the same effect. The overall effect of a change in unemployment benefits on capital investment depends on both its effect on the wage offer distribution and the labor market tightness. Since unemployment benefits have an ambiguous effect on both the variables, it makes the effect of unemployment benefits on capital investment ambiguous for all  $w \in (R, \bar{w})$ .

Unemployment benefits have no effect on the upper support of the distributions of capital investment offered and incurred because firms offering the highest wage do not face workers' turnover arising due to on-the-job search. Also an increase in unemployment benefits, if it reduces the labor market tightness, increase the level of capital investment posted and incurred at the lowest wage by reducing the workers' turnover.

Unemployment benefits through its effect on the reservation wage, tax rate, and labor market tightness also affects the dispersion of wage offers and earnings. An increase in the reservation wage for a given tax rate and labor market tightness reduces the range of wage offers and earnings. This happens because the firms posting the highest wage do not face any competition from other firms in retaining their workers, while the firms posting lower wages do. The result is that the firms posting the highest wage do not increase their wages to the same extent for a given increase in the reservation wage.

An increase in the tax rate reduces the profitability of firms, which induces them to lower the highest wage offered. This leads to a fall in the range of wage offers and earnings for a given

reservation wage and labor market tightness. Also a fall in the labor market tightness reduces the matching rate of employed workers. Consequently, as discussed earlier, on-the-job search becomes less effective in reducing the monopsony power of firms. Firms lower the highest wage offered, which reduces the range of wage offers and earnings.

## 2.5. Unemployment Insurance and Welfare

In the model, unemployment benefits may raise or lower the average productivity of employed workers. This ambiguity arises due to the ambiguous effect of unemployment benefits on the levels and the distribution of capital investment incurred. Production depends on the average productivity of workers as well as the number of workers employed. Since the effect of unemployment benefits on the average productivity of workers as well as the labor market tightness is ambiguous, its effect on production is ambiguous as well. In the model, an increase in unemployment benefits may increase the average productivity of employed workers, which may potentially offset the loss of output due to increased unemployment and total output may increase.

Define social welfare ( $Wel$ ) as the sum total of return on values of all the agents in the economy.

$$Wel = (1 - u) r \bar{V}_e(w) + u r V_u + (1 - u) r \bar{V}_j(w, k) + v r V_v \quad (5.1)$$

where  $\bar{V}_e(w)$  and  $\bar{V}_j(w, k)$  are the average values of employed workers and filled jobs respectively.

Two examples are now constructed in which unemployment benefits increase steady-state welfare compared to the situation when there are no unemployment benefits. Welfare gains are measured in terms of percentage increase in consumption, which workers must be given so that the economy with no unemployment benefits achieves the same level of steady-state social welfare (defined in 5.1) as one with unemployment benefits. In the first example, workers are risk-neutral and unemployment benefits do not provide consumption smoothing benefits. In the second example, workers are risk-averse. In this case, unemployment benefits provide consumption smoothing benefits in addition to their effect on productivity.

In the first example, we assume that workers are risk-neutral. More specifically, we assume that the utility function has the following form

$$U(c) = \mu c. \quad (5.2)$$

In the second example, we consider risk-averse workers. More specifically, we consider the logarithmic form of the utility function

$$u(c) = \ln c. \tag{5.3}$$

Apart from different preference, all other parameters and functions are identical in both examples. We assume that firms produce using the Cobb-Douglas production function

$$y(k) = hk^\alpha \tag{5.4}$$

where  $h$  is a constant. In addition, we assume that the matching function is of Cobb-Douglas form

$$M(v, S) = av^\eta S^{1-\eta} \tag{5.5}$$

In order to derive results for the hypothetical economy, specific values need to be assigned to the parameters  $a, b, c, h, r, s_e, s_u, \rho, \alpha, \mu$  and  $\eta$ . Some of the parameters are selected on the basis of empirical evidence. Other parameters are selected such that the baseline model with risk-neutral workers can match two key features of the U.S. labor market — the unemployment rate and the average duration of unemployment.

The unit of time period is assumed to be six weeks. The rate of discount  $r$  is set to be equal to 0.005 per period implying a 4 percent annual rate of discount. The elasticity of production with respect to capital  $\alpha$  is set equal to 0.3. These values of  $r$  and  $\alpha$  are commonly used in macroeconomics. Permanent separation rate  $\rho$  is taken to be 0.035 per period, which is in line with the value taken in Costain (1997). The remaining parameters are selected so that the benchmark model generates an unemployment rate of 6 percent and an average unemployment duration of about two periods. In the last thirty years, the average unemployment rate in the U.S. has been 6 percent and the average unemployment duration about a quarter. The match product at the reservation wage  $R$  is set equal to one, *i.e.*,  $hk(R)^\alpha = 1$ . Thus, unemployment benefit,  $b$ , the cost of posting a vacancy,  $\delta$ , and wages are expressed as fractions of the output per worker in the least productive job. The values of parameters are given in Table 1.

Table 1  
Parameter Values

Parameter Values	
Discount Rate ( $r$ )	0.005
Permanent Separation Rate ( $\rho$ )	0.035
Marginal Utility of Consumption ( $\mu$ )	1.0
Coefficient of Production Function ( $h$ )	1.4413
Elasticity of Production Function w.r.t Capital ( $\alpha$ )	0.30
Search Intensity of Employed Workers ( $s_e$ )	0.15
Search Intensity of Unemployed Workers ( $s_u$ )	0.30
Coefficient of Matching Function ( $a$ )	0.70
Elasticity of Matching Function w.r.t Vacancy ( $\eta$ )	0.50

### 5.1 Risk-Neutral Workers

In the benchmark model, unemployment benefit  $b$  is set equal to 0.3678. Once the model is calibrated, equilibrium values can be calculated numerically by an algorithm described in Appendix 3.

The numerical results are summarized in Tables 2, 3, and 4. Table 2 reports the values of selected equilibrium variables. In Table 3 the mean and the median wage offer, earnings, and match-specific capital investment incurred are reported. Table 4 reports the values of various measures of wage dispersion. Figures 1 and 2 depict the wage offer distribution and the wage earnings distribution respectively.

The upper panel of Table 2 shows that an increase in unemployment benefit  $b$  reduces the labor market tightness,  $q$ , and the upper wage support,  $\bar{w}$ . It increases the reservation wage,  $R$ , the tax rate,  $t$ , and the unemployment rate,  $u$ , and shows that the increase in the reservation wage,  $R$ , is less than the increase in unemployment benefits. This happens because of a decline in the labor market tightness,  $q$ . Also, because of a decline in the upper wage support and an increase in the reservation wage, the range of wage offers and earnings falls. In addition, the lower support of the distributions of capital investment offered and incurred rises, which is the result of a fall in the labor market tightness.

The lower panel of Table 2 shows that the average productivity of workers rises along with unemployment benefits. Also, total output and welfare gains are inverted ‘U’ shaped functions

of unemployment benefit  $b$ . Total output and welfare are maximized at strictly positive levels of unemployment benefits ( $b = 0.1604$  and  $b = 0.2857$  respectively). The welfare gain at the optimal level of unemployment benefits is equivalent to 0.033 percent of consumption in the economy with no unemployment benefits.

At moderate levels of unemployment benefits, a decline in output due to higher unemployment is more than offset by an increase in output due to increased average productivity of workers. This happens despite the presence of a pay-roll tax, which distorts the decisions of firms to post vacancies. In the current environment, the traditional trade-off between equity and efficiency is absent. This is the result of the presence of search friction, on-the-job search, and the dependence of match-specific capital investment on the employed workers' turnover.

Acemoglu and Shimer (1999a) consider a search economy in which unemployment benefits may increase output. In their model, firms make irreversible investment and post wages. Workers observe all the wage offers and then decide where to apply. Unemployment benefits increase the unemployment rate and the matching rate for vacancies. This induces firms to increase their desired capital-labor ratio. The resultant increase in the productivity more than offsets the decline in output due to higher unemployment.

Their results crucially depend on the assumption of risk-aversion. In the case of risk-neutral workers, output is maximized in a decentralized equilibrium with no unemployment benefits. This happens because workers observe all the wage offers before they decide where to apply. This leads to Bertrand type competition among firms. The result is that with risk-neutral workers, the efficient level of unemployment benefits is zero (see Acemoglu and Shimer 1999a,b).

However, in the case of risk-averse workers, the economy without unemployment benefits is not efficient. The risk-averse workers prefer to receive low wage jobs in order to reduce unemployment risk. Firms cater to these preferences by offering low wage jobs and higher level of vacancy. This increases the vacancy risk of firms, which induces them to incur inefficiently low levels of capital investment. The source of inefficiency in Acemoglu and Shimer (1999a) lies in the inability of risk-averse workers to completely insure against the unemployment risk. The introduction of unemployment benefits weakens this source of inefficiency.

In our model the efficient level of unemployment benefits is strictly positive, which is in contrast to Acemoglu and Shimer (1999a). This happens because in our model firms enjoy monopsony power, which causes inefficiency in the economy. With no unemployment benefits, firms post lower wages and create too many vacancies irrespective of workers' preferences, which increases the matching rate of workers. This leads to inefficiently low level of capital investment, and output is lower

despite lower unemployment rate. Output is thus maximized at a positive level of unemployment benefits even when workers are risk-neutral.

Table 3 shows that the average and the median wage offer are inverted ‘U’ shaped functions of unemployment benefits. Unemployment benefits shift the wage offer distribution to the right (Figure 1). However, the average and the median wage earnings fall as unemployment benefits rise. Despite an increase in the average and the median wage offer for moderate levels of unemployment benefits, the average and the median wage earnings decline due to a fall in the labor market tightness. This implies that an increase in the unemployment benefit,  $b$ , shifts the mass of employed workers from higher wages to lower wages. This is quite evident in Figure 2. An increase in unemployment benefit,  $b$ , makes the wage earnings distribution steeper. Table 3 also shows that an increase in the unemployment benefit,  $b$ , raises the mean and the median capital investment incurred. Higher average capital investment leads to higher average productivity of workers.

An increase in the unemployment benefits reduces the coefficient of variation of wage offers and earnings (Table 4). The decline in the coefficients of variation is mainly the result of a fall in the range of these distributions. Table 4 also shows that the decline in the inequality of wages (whether offer or earnings) comes mainly from the greater compression in the bottom half of the wage offer and earnings distributions. The ratio of the median wage earnings to the reservation wage declines from 4.0247 in the case of no unemployment benefits to 2.2208 in the case of baseline unemployment benefits. The ratio of the upper wage support and the median wage earnings falls relatively less. Also, an increase in unemployment benefits significantly reduces the incidence of low wage earnings where low wage earnings are defined as two-third of the median wage earnings. The effect of unemployment benefits on the inequality in the wage offer distribution is similar.

The predictions of the model are consistent with some of the salient features of the differences in the wage dispersion between the U.S. and Western European countries. Empirical studies find that residual wage dispersion is much higher in the U.S. than in Western European countries (OECD 1997, Devroye and Freeman 2001). Also, the lower part of the wage earnings distribution is much more compressed in Western European countries than in the United States (Blau and Kahn 1996, OECD 1997). In addition, the incidence of low wages is much higher in the U.S. than in Western European countries (OECD 1997). The results indicate that lower residual wage inequality in Western European countries compared to the U.S. may be due to a more generous unemployment insurance system in Western European countries.<sup>7</sup>

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<sup>7</sup> Hansen and Imrohroglu (1992) and Martin (1996) provide the evidence that unemployment insurance system is more generous in Western European countries compared to the United states.

## 5.2 Risk-averse Workers

The numerical results for risk-averse workers are summarized in Tables 5, 6, and 7. They show that an increase in unemployment benefits has similar qualitative as well as quantitative effects on the equilibrium variables as in the previous example. An increase in unemployment benefits reduces the labor market tightness and the upper wage support (Table 5). It increases the reservation wage, the tax rate, and the unemployment rate. The ranges of wage offers and earnings declines as unemployment benefits rise. In addition, the lowest capital investment offered and incurred rises with unemployment benefits.

In this example as well, the average productivity of workers increases with unemployment benefits. Also, output and welfare are inverted ‘U’ shaped functions of the unemployment benefits (Table 5). Output is maximized at a strictly positive level of the unemployment benefits ( $b=0.2343$ ). In this example, welfare is maximized at the level of unemployment benefits equal to 0.4664 and the welfare gain is equivalent to 0.038 percent of consumption in the economy with unemployment benefits.

As in the previous case, the mean and the median wage offer are inverted ‘U’ shaped functions of unemployment benefits (Table 6). The mean and the median wage earnings fall as unemployment benefits rise. With an increase in unemployment benefits, the mean and the median capital investment incurred rise.

As in the case of risk-neutral workers, an increase in unemployment benefits reduces the coefficients of variation of wage offer and earnings (Table 7). Also, the fall in the wage inequality comes mainly from the compression in the bottom half of the wage offer and earnings distributions. In addition, higher unemployment benefits reduce the incidence of low wage earnings.

## 6. Conclusion

This chapter has analyzed the effects of unemployment insurance on productivity, output, welfare, and the distributions of wage offers and earnings in a general equilibrium model with search in the labor market and wage posting by firms. Quantitative experiments show that unemployment benefits increase labor productivity and may increase output. This result is in contrast to standard models of unemployment insurance in which higher unemployment benefits typically reduce output. The positive effect of unemployment benefits on productivity and output does not require that workers be risk-averse. In the case where workers are risk-neutral, unemployment benefits increase

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welfare by increasing output. In the case of risk-averse workers, unemployment benefits increase welfare by increasing output as well as by providing consumption smoothing benefits. In addition, we find that higher unemployment benefits lead to a lower residual wage dispersion, a greater compression in the bottom half of the wage earnings distribution, and a smaller incidence of low wage earnings, where low wage earnings is defined as two-third of the median earnings.

The model can be extended in several directions. The unemployment insurance scheme analyzed in this chapter is quite simple. One can introduce various realistic features of unemployment insurance *e.g.* finite duration of unemployment benefit, insured and uninsured unemployed. One can also use this model to study various types of labor market policies and account for cross-country differences in the labor market flows and residual wage dispersion. The model can also be used to analyze the effects of unemployment insurance and other labor market policies on human capital formation (*e.g.* schooling decision). However, all these extensions are likely to be numerically challenging, so they are left for future research.



## Appendix 1

### The Optimal Choices of Workers and Firms

#### Determination of $R$ - The Optimal Reservation Wage:

The respective value functions of an employed worker at wage  $w$ ,  $V_e(w)$ , and an unemployed worker,  $V_u$ , can be written as

$$rV_e(w) = U(w) + m(q)s_e \left[ \int \max(V_e(w), V_e(x)) dF(x) - V_e(w) \right] - \rho(V_e(w) - V_u), \quad (\text{A1.1})$$

and

$$rV_u = U(b) + m(q)s_u \left[ \int \max(V_u, V_e(x)) dF(x) - V_u \right]. \quad (\text{A1.2})$$

$V_u$  is independent of  $w$  while  $V_e(w)$  is increasing function of  $w$ .

$$V_e'(w) = \frac{U_w(w)}{\left( r + \rho + m(q)s_e(1 - F(w)) \right)} > 0 \quad (\text{A1.3})$$

Thus there exists  $w = R$  such that  $V_u = V_e(R)$  and  $V_e(w) \geq V_u, \forall w \geq R$  and  $V_e(w) < V_u, \forall w < R$ .

Using this A1.2 can be written as

$$rV_u = U(b) + m(q)s_u \left[ \int_R^{\bar{w}} V_e(x) dF(x) - (1 - F(R))V_u \right]. \quad (\text{A1.4})$$

Also

$$rV_e(R) = U(R) + m(q)s_e \left[ \int_R^{\bar{w}} V_e(x) dF(x) - (1 - F(R))V_e(R) \right]. \quad (\text{A1.5})$$

Subtracting A1.4 from A1.5 and rearranging, we get

$$U(R) - U(b) = m(q)(s_u - s_e) \left[ \int_R^{\bar{w}} V_e(x) dF(x) - (1 - F(R))V_e(R) \right]. \quad (\text{A1.6})$$

Integration by part yields

$$U(R) - U(b) = m(q)(s_u - s_e) \left[ \int_R^{\bar{w}} \frac{U_x(x)(1 - F(x))}{r + \rho + m(q)s_e(1 - F(x))} dx \right]. \quad (\text{A1.7})$$

A1.7 implicitly determines the optimal reservation wage.

**Firm's Problem:**

The value of a vacant job with wage  $w$  and capital investment  $k$  is given by

$$rV_v = -\delta + n(q) \left[ \frac{s_u u + (1-u)s_e G(w)}{S} \right] (V_j(w, k) - k - V_v). \quad (\text{A1.8})$$

The value of a filled job with wage  $w$  and capital investment  $k$  is given by

$$rV_j(w, k) = y(k) - w(1+t) - \left[ \rho + m(q)s_e(1-F(w)) \right] (V_j(w, k) - V_v). \quad (\text{A1.9})$$

Free entry implies  $V_v = 0$ . Using this and A1.8 and A1.9, we get

$$q\delta = m(q)z_w \frac{y(k(w)) - w(1+t) - k(w)(r + \rho + m(q)s_e(1-F(w)))}{r + \rho + m(q)s_e(1-F(w))}, \quad (\text{A1.10})$$

where

$$z_w \equiv \frac{s_u u + (1-u)s_e G(w)}{S}.$$

A1.10 implicitly gives wage offer distribution. Setting  $F(w) = 0$ , we get the equilibrium level of labor market tightness

$$q\delta = \rho m(q) \frac{y(k(R)) - R(1+t) - k(R)(r + \rho + m(q))}{T_R}, \quad (\text{A1.11})$$

where  $T_R = (\rho + m(q)s_e)(r + \rho + m(q)s_e)$ . Combining A1.10 and A1.11, we get a equation for the wage offer distribution  $F(w)$ . By setting  $F(w)$  equal to one in A1.10 and combining with A1.11, we derive the upper support of the offer distribution.

$$\begin{aligned} \bar{w} = & \frac{(r + \rho)\rho}{T_R} R + \frac{1}{1+t} \left[ y(k(\bar{w})) - k(\bar{w})(r + \rho) \right. \\ & \left. - \frac{(r + \rho)\rho [y(k(R)) - k(R)(r + \rho + m(q)s_e)]}{T_R} \right] \end{aligned} \quad (\text{A1.12})$$

where  $k(\bar{w})$  is the optimal training expenditure posted at wage  $\bar{w}$ .

The optimal level of  $k$  posted solves

$$\max_k y(k) - w(1+t) - k \left( r + \rho + m(q)s_e(1-F(w)) \right)$$

for each  $w$ . The first order condition is given by

$$y'(k) = r + \rho + m(q)s_e(1 - F(w)). \blacksquare \tag{A1.13}$$

## Appendix 2

### Proofs of Propositions

**Proposition 1:**

Proof follows from the lemmas 1, 2, and 3 of Moretensen and Neumann (1988) pages 339-341 and thus omitted. ■

**Proposition 2:**

Proof follows from the arguments given in the text. ■

**Proposition 3:**

Note that for any  $R$  equations (2.4), (2.10), (3.5), (3.8), (3.9), and (3.10) together determine a vector  $(q; F(w))$ . Equation (2.8) generates a unique  $R \in z$  for any pair  $(q; F(w))$ . In other words, equilibrium conditions define a mapping of the reservation wage  $R$  and any fixed point of the map is an equilibrium.

The reservation wage  $R$  satisfies

$$U(R) = U(b) + m(q; R)(s_u - s_e) \left[ \int_R^{\bar{w}(R)} \frac{U_x(x)(1 - F(x; R))}{r + \rho + m(q; R)s_e(1 - F(x; R))} dx \right] \quad (\text{A2.1})$$

where  $q$ ,  $F(w)$ ,  $\bar{w}$  are functions of  $R$ .

Let

$$T(R) \equiv U(b) + m(q; R)(s_u - s_e) \left[ \int_R^{\bar{w}(R)} \frac{U_x(x)(1 - F(x; R))}{r + \rho + m(q; R)s_e(1 - F(x; R))} dx \right]. \quad (\text{A2.2})$$

$T(R)$  is clearly a continuous function of  $R$ . Let us consider a compact interval  $[b, y(\bar{k})]$  where  $\bar{k}$  satisfies  $y'(\bar{k}) = r + \rho$ .

When  $R = b$ , the LHS of (A2.1) =  $U(R)$ , but  $T(R) > U(b)$  as long as  $q > 0$ . Also when  $R = y(\bar{k})$ ,  $T(R) = U(b)$  since  $q = 0$ , because no firm will create any vacancy. Thus, at  $R = y(\bar{k})$ ,  $U(R) > T(R)$ . Given the continuity of  $T(R)$  over the compact interval  $[b, y(\bar{k})]$ , there exists a fixed point of the mapping (A2.1), provided  $q > 0$  when  $R = b$ .

In the case  $R = b$ ,  $q$  is given by

$$\frac{q\delta}{\rho} = m(q) \frac{y(k(R)) - b(1+t) - k(R)(r + \rho + m(q)s_e)}{(\rho + m(q)s_e)(r + \rho + m(q)s_e)} \quad (\text{A2.3})$$

where  $k(R)$  satisfies  $y'(k(R)) = r + \rho + m(q)s_e$ .

For balanced budget, the tax rate  $t$  satisfies

$$t = \frac{\rho b}{m(q)s_u \int_R^{\bar{w}} w dG(w)} \quad (\text{A2.4})$$

For a given  $q$ , the maximum value  $t$  can take satisfies

$$t_{max} = \frac{\rho}{m(q)s_u} \quad (\text{A2.5})$$

where we assume that the distribution of wage earnings is degenerate at  $b$ . Similarly, for a given  $q$ , the minimum value  $t$  can take satisfies

$$t_{min} = \frac{\rho b}{m(q)s_u y(\bar{k})} \quad (\text{A2.6})$$

where we assume that the distribution of wage earnings is degenerate at  $y(\bar{k})$ .

Putting (A2.6) in (A2.3), we can immediately see that at  $q = 0$ , the LHS of (A2.3) = 0 but the RHS < 0. Thus for any admissible tax rate  $t$ ,  $q = 0$  is not an equilibrium.

Now we derive the sufficient conditions for which the RHS of (A2.3) > 0 at  $q > 0$  when the tax rate is at the maximum or satisfies (A2.5). Then for the appropriate choice of the cost of posting vacancy  $\delta$ , the RHS of (A2.3) > the LHS and there will exist at least one  $q > 0$  satisfying (A2.3).

Essentially, we need to show that  $y(k(R)) - b(1+t) - k(R)y'(k(R)) > 0$ , since  $y'(k(R)) = r + \rho + m(q)s_e$ . Under the assumption that the production function is CRS,  $y(k(R)) - y'(k)k(R) > 0$ . Then for the appropriate choice of  $b$ ,  $y(k(R)) - b(1+t) - k(R)y'(k(R)) > 0$ . Thus there exists at least one  $q > 0$  satisfying (A2.3), and this proves the existence of a non-trivial equilibrium. ■

## Appendix 3

### Numerical Algorithm

- (i) Make initial guess of the reservation wage,  $R$ , and the tax rate,  $t$ ,  $(R_0, t_0)$ .
- (ii) Given  $R_0$  and  $t_0$ , solve for the labor market tightness  $q$  and the lower support of the distribution of the capital investment offered/incurred,  $k(R)$ , using equations (3.8) and (2.10) respectively.
- (iii) Given the values of  $q_0$ ,  $R_0$  and  $t_0$ , solve for the upper wage support,  $\bar{w}$ , using equation (3.10).
- (iv)  $R_0$  and  $\bar{w}_0$  give the lower and the upper wage support respectively. One needs to specify the values  $w$  will take within this range. It is assumed that  $w$  increases with the step of 0.01.
- (v) Given the values of  $w$ , solve for the wage offer distribution,  $F_0(w)$ , and the capital levels associated with each wage in the support of  $F_0(w)$ ,  $k_0(w)$ , using equations (3.9) and (2.10) respectively.
- (vi) Once  $F_0(w)$  is generated, generate the wage earnings distribution,  $G_0(w)$ , using (3.5).
- (vii) For given guesses and generated  $G_0(w)$ , solve for new equilibrium values of the reservation wage,  $R'$ , and the tax rate,  $t'$ , using (2.8) and (2.4) respectively.
- (viii) If  $R'$  and  $t'$  are close enough to  $R_0$  and  $t_0$  respectively, stop. Otherwise, take  $\frac{R'+R_0}{2}$  and  $\frac{t'+t_0}{2}$  as new guess and repeat these steps. ■

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Table 2  
The Effects of Unemployment Benefits on the Equilibrium Values  
(Risk-Neutral Workers)

	No UI	Maximum Output UI	Optimal UI	Baseline UI
Unemployment Benefit ( $b$ )	0	0.1604	0.2857	0.3678
Unemployment Rate ( $u$ )(%)	4.18	4.68	5.32	6.0
Mat Rate of Workers ( $m(v)$ )	0.8018	0.7126	0.6229	0.5482
Level of Vacancies ( $v$ )	14.5779	11.5133	8.7989	6.8141
Upper Wage Sup ( $\bar{w}$ )	1.9566	1.8590	1.7547	1.6618
Wage Range	1.5745	1.3783	1.2018	1.0660
Tax Rate ( $t$ )	0	0.0056	0.012	0.0183
Min. Cap. Inv. ( $k(b)$ )	0.9725	1.1327	1.3445	1.5788
Avg. Workers' Productivity	3.0396	3.0560	3.0752	3.0958
Total Output ( $Y$ )	2.9125	2.9129	2.9117	2.9100
Welfare Gain (%)	0.0	0.027	0.033	0.031

Note

- (1) Unemployment benefits and wages are expressed as fractions of the match-product at the reservation wage  $R$ ,  $y(k(R)) = 1$ .
- (2) Welfare gains are expressed in terms of percentage of equilibrium consumption for the economy with no unemployment benefits.

Table 3  
The Effects of Unemployment Benefits on the Mean and the Median  
Wage Offers, Earnings, and Capital Investment  
(Risk-Neutral Workers)

	No UI	Maximum Output UI	Optimal UI	Baseline UI
Unemployment Benefit ( $b$ )	0	0.1604	0.2857	0.3678
Avg. Wage Offer	0.9178	0.9314	0.9355	0.9317
Avg. Wage Earnings	1.4707	1.4082	1.3406	1.2801
Avg. Cap. Inv.	14.3420	14.4883	14.6511	14.8437
Median Wage Offer	0.8436	0.8547	0.8596	0.8589
Median Wage Earnings	1.5382	1.4667	1.3907	1.3231
Median Cap. Inv.	13.5358	13.6880	13.8810	14.0853

Note

- (1) Unemployment benefits and wages are expressed as fractions of the match-product at the reservation wage  $R$ ,  $y(k(R)) = 1$ .

Table 4  
The Effects of Unemployment Benefits on the Wage Dispersion  
(Risk-Neutral Workers)

	No UI	Maximum Output UI	Optimal UI	Baseline UI
Unemployment Benefit ( $b$ )	0	0.1604	0.2857	0.3678
Wage Offer Distribution				
Coeff. Var Wage Offer	0.5176	0.3829	0.2967	0.2628
$\bar{w}/\text{Median}$	1.6228	1.4537	1.2960	1.1705
Median/ $b$	2.2071	1.7781	1.5545	1.4416
Incidence of Low Wages	0.1767	0.1109	0.0321	0
Wage Earnings Distribution				
Coeff. Var Wage Earnings	0.2710	0.2477	0.2234	0.2099
$\bar{w}/\text{Median}$	0.8899	0.8471	0.8010	0.7599
Median/ $b$	4.0247	3.0514	2.5150	2.2208
Incidence of Low Wage	0.1438	0.1357	0.1256	0.1149

Note

- (1) Unemployment benefits and wages are expressed as fractions of the match-product at the reservation wage  $R$ ,  $y(k(R)) = 1$ .
- (2) Low wage offers and earnings are defined as two-third of the median wage offers and earnings respectively.

Table 5  
The Effects of Unemployment Benefits on the Equilibrium Values  
(Risk-Averse Workers)

	No UI	Maximum Output UI	Optimal UI
Unemployment Benefit ( $b$ )	0	0.2343	0.4664
Unemployment Rate ( $u$ )(%)	3.35	5.00	10.55
Mat Rate of Unp. Workers ( $m(v)$ )	1.0009	0.6646	0.2966
Level of Vacancies ( $v$ )	23.0901	10.0162	1.9951
Upper Wage Sup ( $\bar{w}$ )	2.1416	1.8039	1.2908
Wage Range	2.1163	1.2815	1.6393
Tax Rate ( $t$ )	0	0.0090	0.0531
Min. Cap. Inv. ( $k(b)$ )	0.7193	1.2383	3.2788
Avg. Workers' Productivity	3.0125	3.0664	3.2131
Total Output ( $Y$ )	2.9115	2.9130	2.8740
Welfare Gain (%)	0.0	0.033	0.038

Note

- (1) Unemployment benefits and wages are expressed as fractions of the match-product at the reservation wage  $R$ ,  $y(k(R)) = 1$ .
- (2) Welfare gains are expressed in terms of percentage of equilibrium consumption for the economy with no unemployment benefits.

Table 6  
The Effects of Unemployment Benefits on the Mean and the Median  
Wage Offer, Earnings, and Capital Investment  
(Risk-Averse Workers)

	No UI	Maximum Output UI	Optimal UI
Unemployment Benefit (b)	0	0.2343	0.4664
Avg. Wage Offer	0.8350	0.9347	0.8680
Avg. Wage Earnings	1.5844	1.3728	1.0359
Avg. Cap. Inv.	14.1307	14.5844	16.0357
Median Wage Offer	0.7825	0.8580	0.8203
Median Wage Earnings	1.6730	1.4265	1.0563
Median Cap. Inv.	13.2799	13.7854	15.3971

Note

- (1) Unemployment benefits and wages are expressed as fractions of the match-product at the reservation wage  $R$ ,  $y(k(R)) = 1$ .

Table 7  
The Effects of Unemployment Benefits on the Wage Dispersion  
(Risk-Averse Workers)

	No UI	Maximum Output UI	Optimal UI
Unemployment Benefit ( $b$ )	0	0.2343	0.4664
Wage Offer Distribution			
Coeff. Var Wage Offer	0.5710	0.3239	0.1929
$\bar{w}$ /Median	2.7368	2.1025	1.5735
Median/ $b$	30.9487	1.6426	1.2590
Incidence of Low Wages	0.2930	0.0705	0
Wage Earnings Distribution			
Coeff. Var Wage Earnings	0.2778	0.2326	0.1709
$\bar{w}$ /Median	1.2801	1.2646	1.2220
Median/ $b$	66.1661	2.7310	1.6212
Incidence of Low Wage	0.1577	0.1306	0.0428

Note

- (1) Unemployment benefits and wages are expressed as fractions of the match-product at the reservation wage  $R$ ,  $y(k(R)) = 1$ .
- (2) Low wage offers and earnings are defined as two-third of the median wage offers and earnings respectively.

Figure 1  
The Effects of Unemployment Benefits on the Wage Offer Distribution  
(Risk-Neutral Workers)

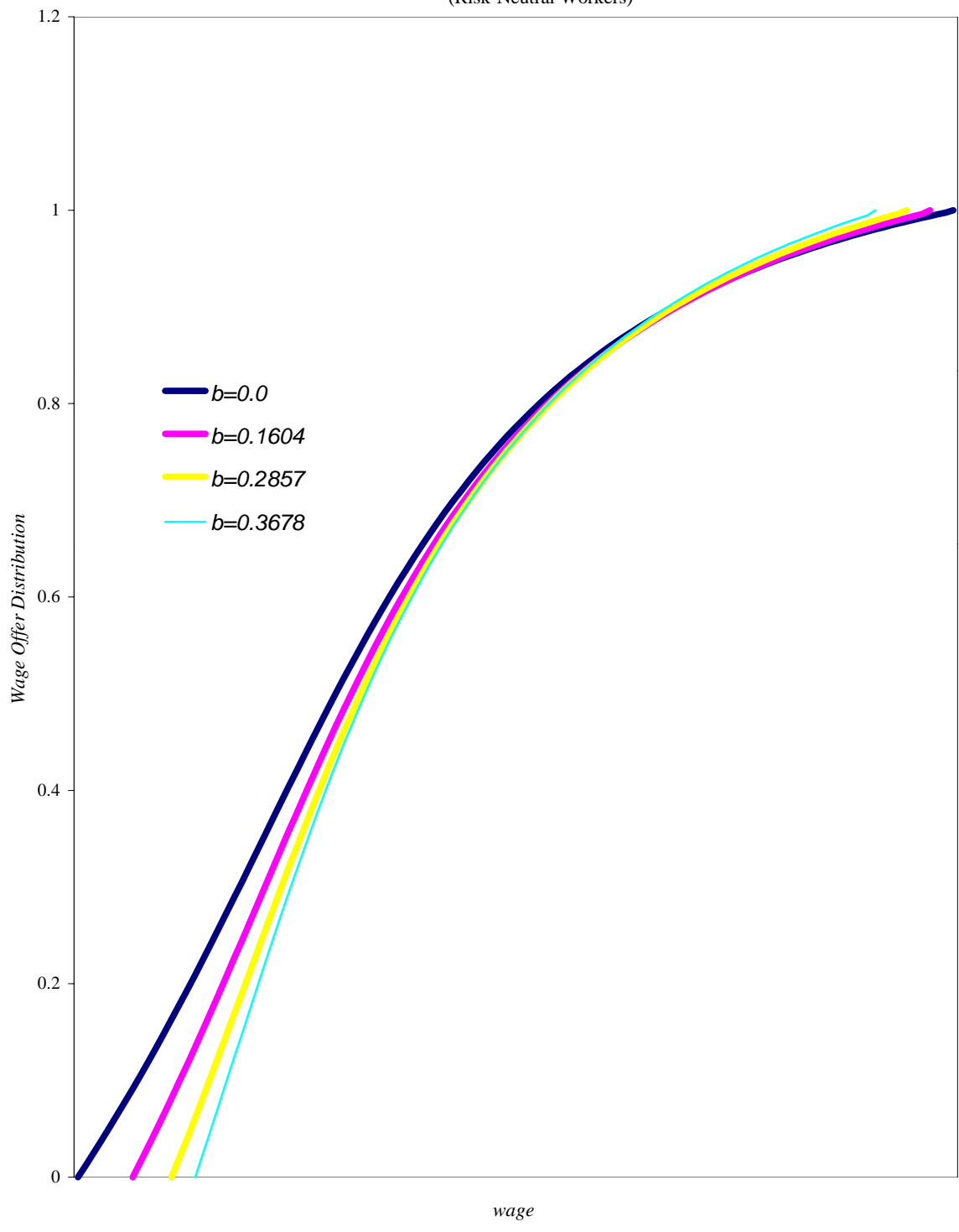


Figure 2  
The Effects of Unemployment Benefits on the Wage Earnings Distribution  
(Risk-Neutral Workers)

