Department of Economics
Wilfrid Laurier University
Summer 2008

Suggested Solutions to Assignment 1 (OPTIONAL)

Total Marks: 50

Part A True/ False/ Uncertain Questions

Explain why the following statement is True, False, or Uncertain according to economic principles. Use diagrams and / or numerical examples where appropriate. Unsupported answers will receive no marks. It is the explanation that is important. Each question is worth 10 marks.

A1.

A monopoly produces a Pareto inefficient level of output at the equilibrium. [Diagrams Required]

Uncertain

Without perfect price discrimination, a monopoly produces a Pareto inefficient level of output at the equilibrium. See Section 24.4 and Figure 24.4 of Varian's textbook (7th ed.) for a graphical explanation.

With perfect price discrimination or first degree price discrimination, a monopoly produces a Pareto efficient level of output at the equilibrium. See Section 25.2, Figure 25.1 and Figure 25.2 of Varian's textbook (7th ed.) for a graphical explanation.

A2.

In a location model of product differentiation with only two firms, the equilibrium location pattern is Pareto efficient. [Diagrams Required]

False

In a location model of product differentiation with only two firms, the equilibrium location pattern is Pareto inefficient. See Section 25.8 and Figure 25.7 of Varian's textbook (7th ed.) for a graphical explanation. Alternatively, see pages 62 to 76 of Lecture Slides for Chapter 25, which are available on the course website, for a graphical explanation.

Part B Problem Solving Questions

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1. [15 Marks]

The monopolist faces a demand curve given by $D(p) = 100 - p$ **. Its total cost function is** $c(y) = 700 + 20y$.

> **(a) Calculate the monopoly output level, the monopoly price, and the profits of the firm. Calculate the deadweight loss from this monopoly. [4 marks]**

The given market demand function, $D(p) = 100 - p$ **,** can be also be written as,

$$p(y) = 100 - y. \qquad (1)$$

Total revenue function of the monopolist: $TR(y) = p(y) * y = (100 - y) * y = 100y - y^2$.

Marginal revenue function of the monopolist: $MR \equiv \frac{dTR(y)}{dy} = 100 - 2y$.

Marginal cost function of the monopolist: $MC \equiv \frac{dc(y)}{dy} = 20$.

Profit maximization requires that the monopolist produces where $MR = MC$. Solving for the profit maximizing quantity:

$$\begin{aligned}
&MR = MC \\
&\Rightarrow 100 - 2y = 20 \\
&\Rightarrow 80 = 2y \\
&\Rightarrow y = 40.
\end{aligned}$$

Monopoly price is determined by the demand curve. So, substituting $y = 40$ into the demand function, equation (1), we get:

$$p(y) = 100 - y = 100 - 40 = 60.$$

So, the equilibrium levels of output and price of the monopolist are 40 and 60, respectively.

The monopolist's profits at the equilibrium $= \text{Total revenue} - \text{Total cost}$

$$= p(y) * y - (700 + 20y)$$
$$= 60 * 40 - 700 - 20 * 40$$
$$= 2400 - 700 - 800$$
$$= 900.$$

To find the deadweight loss we have to first find the competitive equilibrium price and output by setting price equals to marginal cost.

$$p(y) = MC$$
$$\Rightarrow 100 - y = 20$$
$$\Rightarrow y = 80$$

So, the competitive equilibrium price and output are 20 and 80, respectively. The deadweight loss due to the monopoly is given by the area of the triangle Δbfd in Figure B1.

Deadweight loss = area of Δbfd

$$= \frac{1}{2}(80 - 40)(60 - 20)$$
$$= \frac{1}{2}(40)(40)$$
$$= 800.$$

So, the deadweight loss due to the monopoly is 800.

(b) Draw a diagram to illustrate your answers to part (a). [2 marks]

Point "b" in Figure B1 shows the monopoly equilibrium price and quantity combination. The areas of the deadweight loss and the profit are identified clearly in Figure B1.

(c) Explain why this market might be considered to be a "natural" monopoly. [2 marks]

The market can be considered to be a natural monopoly because average costs are declining over the whole range of the market. The average cost function of the monopolist is:

$$AC(y) \equiv \frac{c(y)}{y} = \frac{700 + 20y}{y} = \frac{700}{y} + 20.$$

The ever-decreasing average cost function shows that it is cheaper for one firm to serve this market than it would be for more than one firm to do so.

(d) If the government regulates the monopolist by imposing average cost pricing, what will be the monopolist's price and quantity? Draw a diagram to illustrate your answers. Explain the problem that exists with this average cost pricing. [4 marks]

The monopolist's price and quantity with average cost pricing can be calculated by setting price equals to average cost.

$$p(y) = AC$$
$$\Rightarrow 100 - y = \frac{700}{y} + 20$$
$$\Rightarrow 100 - y = \frac{700 + 20y}{y}$$
$$\Rightarrow 100y - y^2 = 700 + 20y$$
$$\Rightarrow 80y - y^2 - 700 = 0$$
$$\Rightarrow y^2 - 80y + 700 = 0$$
$$\Rightarrow y^2 - 70y - 10y + 700 = 0$$
$$\Rightarrow y(y-70) - 10(y-70) = 0$$
$$\Rightarrow (y-70)(y-10) = 0$$

So, either $y - 70 = 0$ or $y - 10 = 0$. Therefore, $y = 70$ or $y = 10$. However, $y = 10$ is not a reasonable value of the monopolist's quantity with average cost pricing because without government regulations the monopolist will choose to produce 40 units of output. So, the reasonable value of the monopolist's quantity with average cost pricing is $y = 70$, which close to the competitive equilibrium quantity.

The monopolist's price with average cost pricing is:

$$p(y) = 100 - y = 100 - 70 = 30.$$

So, the monopolist's price and quantity with average cost pricing are 30 and 70, respectively. This is shown as point "c" in Figure B1.

The problem with this average cost pricing equilibrium is that it is still not efficient. Some deadweight loss remains. It is shown in Figure B1 as area of the triangle Δcde.

Deadweight loss with average cost pricing $= \text{area of } \Delta cde$

$$= \frac{1}{2}(80-70)(30-20)$$
$$= \frac{1}{2}(10)(10)$$
$$= 50.$$

So, the deadweight loss due to the monopoly with average cost pricing is 50.

(e) Illustrate and explain the problem that emerges when the government regulates the monopolist by imposing marginal cost pricing. What level of subsidy should the government provide to the monopolist if the government wants to overcome this problem? [3 marks]

The monopolist's price and quantity with marginal cost pricing can be calculated by setting price equals to marginal cost.

$$p(y) = MC$$
$$\Rightarrow 100 - y = 20$$
$$\Rightarrow y = 80$$

So, the monopolist's price and quantity with marginal cost pricing are 20 and 80, respectively. This price and quantity combination, which is shown as point "*d*" in Figure B1, is same as the price and quantity combination at the competitive equilibrium. This solution is Pareto efficient. But at this solution price is less than average cost, which is clearly evident from Figure B1. This means the monopolist is making a loss and would require a subsidy to stay in business.

The monopolist's profits with marginal cost pricing = Total revenue – Total costs

$$= p(y) * y - (700 + 20y)$$
$$= (100 - y)y - 700 - 20y$$
$$= (100 - 80)80 - 700 - 20(80)$$
$$= -700$$

Thus, the government should provide a subsidy of 700 to the monopolist.

B2. [15 Marks]

Suppose the market demand can be separated into two distinct markets, where $p_1 = 80 - 5y_1$, $p_2 = 180 - 20y_2$, and the common total cost function is $C = 50 + 40(y_1 + y_2)$.

(a) Determine the equilibrium price and quantity in each market and the overall profit that result from the actions of a price discriminating monopolist. Draw a diagram to illustrate your answers. [8 marks]

In this scenario the monopolist can exercise third-degree price discrimination. We can find the third-degree price discriminating price and quantity by setting marginal revenue in each market equals to marginal cost in each market. That is, the optimal solution must have

$$MR_1(y_1) = MC_1(y_1 + y_2)$$
$$MR_2(y_2) = MC_2(y_1 + y_2)$$

Total revenue function in market 1: $TR_1(y_1) = p_1(y_1)y_1 = (80 - 5y_1)y_1 = 80y_1 - 5y_1^2$

Marginal revenue function in market 1: $MR_1(y_1) \equiv \dfrac{dTR_1(y_1)}{dy_1} = 80 - 10y_1$

Common total cost function: $C(y_1 + y_2) = 50 + 40(y_1 + y_2)$

Marginal cost function in market 1: $MC_1(y_1 + y_2) \equiv \dfrac{dC(y_1 + y_2)}{dy_1} = 40$

Total revenue function in market 2:
$TR_2(y_2) = p_2(y_2)y_2 = (180 - 20y_2)y_2 = 180y_2 - 20y_2^2$

Marginal revenue function in market 2: $MR_2(y_2) \equiv \dfrac{dTR_2(y_2)}{dy_2} = 180 - 40y_2$

Marginal cost function in market 2: $MC_2(y_1 + y_2) \equiv \dfrac{dC(y_1 + y_2)}{dy_2} = 40$

At the equilibrium in market 1,

$$MR_1(y_1) = MC_1(y_1 + y_2)$$
$$\Rightarrow 80 - 10y_1 = 40$$
$$\Rightarrow 10y_1 = 40$$
$$\Rightarrow y_1 = 4$$

The equilibrium price in market 1: $p_1(y_1) = 80 - 5y_1 = 80 - 5(4) = 80 - 20 = 60$

So, the equilibrium price and quantity in market 1 are 60 and 4, respectively.

At the equilibrium in market 2,

$$MR_2(y_2) = MC_2(y_1 + y_2)$$
$$\Rightarrow 180 - 40y_2 = 40$$
$$\Rightarrow 40y_2 = 140$$
$$\Rightarrow y_2 = 3.5$$

The equilibrium price in market 2: $p_2(y_2) = 180 - 20y_2 = 180 - 20(3.5) = 180 - 70 = 110$

So, the equilibrium price and quantity in market 2 are 110 and 3.5, respectively.

The overall profit at the equilibrium: $\pi = p(y_1)y_1 + p(y_2)y_2 - (50 + 40(y_1 + y_2))$

$$= 60 * 4 + 110 * 3.5 - (50 + 40(4 + 3.5))$$
$$= 240 + 385 - 350$$
$$= 275$$

I have illustrated the results in Figure B2. Point "a'' in Panel A of Figure B2 shows the equilibrium price and quantity combination in market 1 and Point "b'' in Panel B of Figure B2 shows the equilibrium price and quantity combination in market 2. I have clearly identified the areas of profits in each market in Figure B2.

(b) Determine the price elasticity of demand in each market, evaluated at the equilibrium price and quantity. [3 marks]

The demand function in market 1: $p_1 = 80 - 5y_1$

$$\Rightarrow 5y_1 = 80 - p_1$$
$$\Rightarrow y_1 = 16 - \left(\frac{1}{5}\right)p_1$$

So, $\dfrac{dy_1}{dp_1} = -\dfrac{1}{5}$.

The price elasticity of demand at the equilibrium in market 1:

$$\varepsilon_{y_1, p_1} = \frac{\dfrac{dy_1}{y_1}}{\dfrac{dp_1}{p_1}} = \frac{dy_1}{dp_1}\frac{p_1}{y_1} = \left(-\frac{1}{5}\right)\left(\frac{60}{4}\right) = -3$$

The demand function in market 2:

$$p_2 = 180 - 20y_2$$
$$\Rightarrow 20y_2 = 180 - p_2$$
$$\Rightarrow y_2 = 9 - \left(\frac{1}{20}\right)p_2$$

So, $\dfrac{dy_2}{dp_2} = -\dfrac{1}{20}$.

The price elasticity of demand at the equilibrium in market 2:

$$\varepsilon_{y_2,p_2} = \frac{\frac{dy_2}{y_2}}{\frac{dp_2}{p_2}} = \frac{dy_2}{dp_2}\frac{p_2}{y_2} = \left(-\frac{1}{20}\right)\left(\frac{110}{3.5}\right) = -1.57$$

(c) What is the relationship between the price elasticity of demand in each market and the price prevailing in each market? [2 marks]

Market 2 is relatively less elastic than market 1. The equilibrium price is higher in market 2 than in market 1. Therefore, we can conclude that there is a negative relationship between the price elasticity of demand in each market and the price prevailing in each market.

(d) Suppose now that the government imposes a 10% tax on the overall profit of the monopolist. Explain what will be the monopolist's equilibrium price and quantity in each market under this form of taxation? [2 marks]

The maximization problem the monopolist faces with 10% tax on the overall profit, $\pi(y_1, y_2)$, is

$$\max_{y_1, y_2} \quad (1-0.10)\pi(y_1, y_2) = (1-0.10)[p(y_1)y_1 + p(y_2)y_2 - (50 + 40(y_1 + y_2))].$$

But the value of y_1 and y_2 that maximize $\pi(y_1, y_2)$ will also maximize $(1-0.10)\pi(y_1, y_2)$. Thus a pure profits tax will have no effect on a monopolist's choice of output. This means that the monopolist's equilibrium price and quantity in each market under 10% tax on the overall profit will be same as the ones we found in part **(a)**.