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question paper

Student Number: ______________
Student Name:________________

Wilfrid Laurier UNIVERSITY

School of Business and Economics

EC 370 – Microeconomic Theory II

Suggested Solutions to Midterm Examination

June 18, 2008

Instructor: Sharif F. Khan

Time Limit: 1 Hour 30 Minutes

Instructions:

Important! Read the instructions carefully before you start your exam.

Write your answers in the answer booklets provided. Record your full name and student number on both the question and answer booklets.

Marking Scheme:

Part A [20 marks] Two of Three True/ False/Uncertain questions – 10 marks each

Part B [30 marks] Two problem solving / analytical questions – 15 marks each

Calculators:

Non-programmable calculators are permitted

Part A [20 marks]

Answer two of the following three questions. Each question is worth 10 marks.

Explain why the following statement is True, False, or Uncertain according to economic principles. Use diagrams and/or numerical examples where appropriate. Unsupported answers will receive no marks. It is the explanation that is important.

A1.

Imposition of a quantity tax on a monopolist will cause the market price to increase by the amount of the tax. [Note: Diagrams and mathematical explanation required]

False/ Uncertain

In the case of a monopolist facing a linear demand curve and constant marginal cost, a quantity tax increase the market price by half the amount of the tax. But in the case of a monopolist facing a constant-elasticity demand curve and constant marginal cost, a quantity tax increase the market price by more than the amount of the tax. See the example given in pages 427 to 429 and Figure 24.3 of the textbook for mathematical and graphical explanation.

A2.

In a market with a monopolist who exercises second-degree price discrimination the high-end consumers benefits from the presence of the low-end consumers. [Note: Diagrams required]

True

See pages 449 to 450 and Figure 25.3 of the textbook for a graphical explanation.

A3.

The quantity-leader can never get a lower profit in a Stackelberg equilibrium than it would get in the Cournot equilibrium. [Note: Diagrams required]

True

The quantity-leader could choose its Cournot-Nash output level, knowing that the follower would then also choose its C-N output level. The leader's profit would then be its C-N profit. But the leader does not have to do this, so its profit must be at least as large as its C-N profit.

See Figure 27.2 of the textbook, the 7th edition (page 486) for a graphical explanation.

Part B Problem Solving Questions [45 marks]

Answer three of the following four questions. Each question is worth 15 marks.

Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1. [15 Marks]

Suppose market demand is $P = 200 - y$ **.**

(a) If two firms compete in this market with constant marginal and average costs, $c = 20$, **find the Cournot equilibrium output and profit per firm. [5 marks]**

Suppose firm 1 takes firm 2's output choice y_2 as given. Then firm 1's problem is to maximize its profit by choosing its output level y_1. If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. Define $y \equiv y_1 + y_2$. The market price will be $P = 200 - y_1 - y_2$.

Firm 1's profit maximization problem:

$$\max_{y_1} \ \pi_1(y_1, y_2) = [200 - (y_1 + y_2)]y_1 - 20y_1$$

First order conditions:

$$\frac{\partial \pi_1}{\partial y_1} = 200 - (y_1 + y_2) + y_1(-1) - 20 = 0$$

$$\Rightarrow 180 - 2y_1 - y_2 = 0$$

$$\Rightarrow 2y_1 = 180 - y_2$$

$$\Rightarrow y_1 = \frac{180 - y_2}{2}$$

So, Firm 1's best response to y_2 or Firm 1's reaction function is

:

$$y_1 = R(y_2) = \frac{180 - y_2}{2} \qquad (1)$$

Since the profit- maximization problem faced by the two firms are symmetric in this case, Firm 2's best response to y_1 or Firm 2's reaction function will have the same functional form as (1):

$$y_2 = R(y_1) = \frac{180 - y_1}{2} \qquad (2)$$

The symmetry of the problem also implies that at the Cournot-Nash equilibrium both firms will produce the same level of output:

$$y_1^* = y_2^* = y^* \qquad (3)$$

Substituting $y_1 = y_2 = y^*$ into either (1) or (2) and solving for y^* we get:

$$\begin{aligned} y^* &= \frac{180 - y^*}{2} \\ \Rightarrow 2y^* &= 180 - y^* \\ \Rightarrow 3y^* &= 180 \\ \Rightarrow y^* &= 60 \end{aligned}$$

So, the Cournot-Nash equilibrium output is:

$$(y_1^*, y_2^*) = (60,60)$$

Firm 1's profit at the equilibrium: $\pi_1(y_1^*, y_2^*) = [200 - (y_1^* + y_2^*)]y_1^* - 20y_1^*$

$$\begin{aligned} &= [200 - (60 + 60)]60 - 20(60) \\ &= 4800 - 1200 \\ &= 3600 \end{aligned}$$

Because of the symmetry of the problem Firm 2's profit is same as Firm 1's profit at the equilibrium:

$$\pi_2(y_1^*, y_2^*) = \pi_1(y_1^*, y_2^*) = 3600 .$$

(b) Find the monopoly output and profit if there is only one firm with constant marginal and average costs, $c = 20$**. [4 marks]**

The monopolist's problem is to maximize profit by choosing its output level, *y*. The profit-maximization problem of the monopolist:

$$\max_{y} \ \pi_m(y) = (200 - y)y - 20y$$

First order conditions:

$$\frac{\partial \pi_m}{\partial y} = 200 - y + y(-1) - 20 = 0$$
$$\Rightarrow 180 - 2y = 0$$
$$\Rightarrow 2y = 180$$
$$\Rightarrow y = 90$$

So, the profit-maximizing equilibrium output of the monopolist is: $y_m^* = 90$.

The profit-maximizing equilibrium price of the monopolist is: $P_m^* = 200 - y_m^* = 200 - 90 = 110$.

Equilibrium profit of the monopolist is: $\pi_m^* = (P_m^* - c)y_m^* = (110 - 20)90 = 8100$.

(c) Using the information from parts (a) and (b), construct a 2×2 payoff matrix where the strategies available to each of two players are to produce the Cournot equilibrium quantity or half the monopoly quantity. [4 marks]

When both firms choose the Cournot equilibrium quantity, each earns the Cournot equilibrium profit which is calculated in part (a). Similarly, producing half the monopoly output garners each firm half the monopoly profit: $\pi_1 = \pi_2 = \frac{1}{2}\pi_m^* = \frac{1}{2}(8100) = 4050$.

When, for instance, firm 1 produces the Cournot output, $y_1 = 60$, while firm 2 produces half the monopoly output, $y_2 = \frac{1}{2}y_m^* = \frac{1}{2}(90) = 45$, the market price would be $P = 200 - (60 + 45) = 95$. Firm 1 earns $\pi_1 = (P - c)q_1 = (95 - 20)60 = 4500$ and firm 2 earns $\pi_2 = (P - c)y_2 = (95 - 20)45 = 3375$. When the roles are reversed, so are profits.

The payoff of matrix of a game where each of the two firms can choose between the half the monopoly quantity (45) or the Cournot equilibrium quantity (60) is as follows:

		Firm2 $y_2 = 45$	$y_2 = 60$
Firm 1	$y_1 = 45$	4050, 4050	3375, 4500
	$y_1 = 60$	4500, 3375	3600, 3600

(d) What is the Nash equilibrium (or equilibria) of the game you constructed in part (c)? Is there any mixed strategy Nash equilibrium in this game? If yes, what is the mixed strategy Nash equilibrium (or equilibria)? [2 marks]

The game in part (c) is a typical Prisoner's Dilemma game, where producing the Cournot equilibrium output, i.e., choosing $y_i = 60$ where $i = 1$ or 2, is the dominant strategy for each firm. If Firm 1 chooses $y_1 = 45$, the best response for Firm 2 is to choose $y_2 = 60$ because $4500 > 4050$. If Firm 1 chooses $y_1 = 60$, the best response for Firm 2 is to choose $y_2 = 60$ because $3600 > 3375$. Thus, $y_2 = 60$ is the dominant strategy for Firm 2. Since the game is symmetric, we can argue that $y_1 = 60$ is also the dominant strategy for Firm 1. This game has a unique Nash equilibrium where each firm plays its dominant strategy. That is, $(y_1 = 60, y_2 = 60)$ is the unique Nash equilibrium of this game. Note that this is a pure strategy Nash equilibrium.

In this game there is no mixed strategy Nash equilibrium because each firm has a dominant strategy which is to produce the Cournot equilibrium output. Given that firm 1 is choosing $y_1 = 60$ with a 100 percent probability, the best response of firm 2 is to choose $y_2 = 60$ with a 100 percent probability rather than to choose a randomized strategy over $y_2 = 45$ and $y_2 = 60$, vice versa.

B2. [15 Marks]

Consider the battle of the sexes game between Brenda and Peter. Figure 1 shows the payoff matrix of this game.

Figure 1: Payoff Matrix for the Battle of the Sexes Game

		Brenda	
		Hockey	**Ballet**
Peter	**Hockey**	4, 2	0, 0
	Ballet	0, 0	2, 4

(a) Find the best response strategies of each player. [6 marks]

		Brenda	
		Hockey *(p)*	**Ballet** *(1-p)*
Peter (*q*)	**Hockey**	4, 2	0, 0
(*1-q*)	**Ballet**	0, 0	2, 4

p is the probability Brenda chooses Hockey (H). $(1-p)$ is the probability Brenda chooses Ballet (B). q is the probability Peter chooses Hockey. $(1-q)$ is the probability Peter chooses Ballet.

Given that Brenda chooses Hockey with probability *p*,
Peter's expected payoff if he chooses Hockey, $EV^P(H)$:
$EV^P(H) = 4p + 0(1-p) = 4p$

Peter's expected payoff if he chooses Ballet, $EV^P(B)$: $EV^P(B) = 0(p) + 2(1-p) = 2(1-p)$.

$$
\begin{aligned}
EV^P(H) > EV^P(B) \text{ if } & 4p > 2(1-p) \\
& \Rightarrow 2p > 1-P \\
& \Rightarrow 3p > 1 \\
& \Rightarrow p > \frac{1}{3} \qquad (1)
\end{aligned}
$$

Similarly, $EV^P(H) < EV^P(B)$ if $p < \frac{1}{3}$ (2)

and $EV^P(H) = EV^P(B)$ if $p = \frac{1}{3}$. (3)

Using (1), (2) and (3), we can find Sarah's best response strategies:
Peter will choose Hockey (i.e. $q = 1$) if $p > \frac{1}{3}$.

Peter will choose Ballet (i.e. $q = 0$) if $p < \frac{1}{3}$.

Peter will be indifferent between Hockey or Ballet (i.e. $0 \le q \le 1$) if $p = \frac{1}{3}$.

Using Peter's best response strategies as outlined above, we plot his best response curve in Figure B4.

Now, given that Peter chooses Hockey with probability *q*,
Brenda's expected payoff if she chooses Hockey, $EV^B(H)$:
$EV^B(H) = 2(q) + 0(1-q) = 2q$
Brenda's expected payoff if she chooses Ballet, $EV^B(B)$:
$EV^B(B) = 0q + 4(1-q) = 4(1-q)$.

$$
\begin{aligned}
EV^B(H) > EV^B(B) \text{ if } & 2q > 4(1-q) \\
& \Rightarrow q > 2(1-q) \\
& \Rightarrow 3q > 2 \\
& \Rightarrow q > \frac{2}{3} \qquad (4)
\end{aligned}
$$

Similarly, $EV^B(H) < EV^B(B)$ if $q < \frac{2}{3}$ (5)

and $EV^B(H) = EV^B(B)$ if $q = \frac{2}{3}$. (6)

Using (4), (5) and (6), we can find Brenda's best response strategies:

Brenda will choose Hockey (i.e. $p = 1$) if $q > \frac{2}{3}$.

Brenda will choose Ballet (i.e. $p = 0$) if $q < \frac{2}{3}$.

Brenda will be indifferent between Hockey or Ballet (i.e. $0 \le p \le 1$) if $q = \frac{2}{3}$.

(b) Plot the best response curves of each player in a diagram. [4 marks]

Using Peter's best response strategies as outlined in the answer to part (a), we plot his best response curve in Figure B2. Using Brenda's best response strategies as outlined above, we plot her best response curve in Figure B2.

(c) Find all of the Nash equilibria (pure strategy equilibrium as well as mixed strategy equilibrium) of this game. Clearly identify all of the Nash equilibria on the diagram drawn for part (b). [5 marks]

The best response curves of Peter and Brenda are shown in Figure B2. The intersections of the best response curves are Nash equilibria. In this case they intersect at three points: (0, 0), (1/3, 2/3) and (1,1). So, there are two pure strategy Nash equilibria: (q^*, p^*) = (0,0) and (1,1) . There is also one mixed strategy Nash equilibrium in this game: (q^*, p^*) = (2/3, 1/3). All of these Nash equilibria are clearly identifies in Figure B4.

FIGURE B2: BEST RESPONSE CURVES AND NASH EQUILIBRIUM