**MAPLE V Tutorial - Economics 421**

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**Tutorial Notes:**

Economics 421 website has several resources to help you with MAPLE. Also, the online help can be used instead of a manual.

**Raison d’Maple**

1) Can run in batch or interactive mode  
2) Manipulate symbolically and graphically (instead of numerically)  
3) Programmable (build your own functions)  
4) Worksheet environment

**A Short Introduction to the Maple Language**

- Expressions, Names, Statements, and Assignments  
- Functions (mention its usefulness in assignments)  
- Using the Packages that come with Maple

    ```maple
    with(plots);
    with(linalg);
    ```

- Trouble Shooting Notes

- Four methods of solving equations

    i) Guess and check (avoid)
    ii) By Hand, as with pencil and paper (too slow)
    iii) Graphical solution.
    iv) Using fsolve.

**Using the Linear Algebra routines**

To use the linalg library we have to invoke:

```maple
>with(linalg);
```

Solving a system of equations
From the online help in MAPLE V

linalg[linsolve] - Solution of linear equations

Calling Sequence:

linsolve(A, b, 'r', v)
linsolve(A, B, 'r', v)

Parameters:

A - a matrix
b - a vector
B - a matrix
r - (optional) a name
v - (optional) a name

Description:

The function linsolve(A, b) finds the vector x which satisfies the matrix equation
A x = b. If A has n rows and m columns, then vectdim(b) must be n and
vectdim(x) will be m, if a solution exists. If A x = b has no solution, then the null
sequence NULL is returned. If A x = b has many solutions, then the result will
use global names (see below) to describe the family of solutions parametrically.
The call linsolve(A, B) finds the matrix X which solves the matrix equation A X =
B where each column of X satisfies A col(X,i) = col(B,i) . If A X = B has does not
have a unique solution, then NULL is returned. The optional third argument is a
name which will be assigned the rank of A. The optional fourth argument allows
you to specify the seed for the global names used as parameters in a parametric
solution. If there is no fourth argument, the default, then the global names _t[1],
_t[2], _t[3], ... will be used in the vector case, _t[1][1], _t[1][2], _t[2][1], ... in the
matrix case (where _t[1][i] is used for the first column, _t[2][i] for the second,
etc). This is particularly useful when programming with linsolve. If you declare v
as a local variable and then call linsolve with fourth argument v, the resulting
parameters (v[1], v[2], ...) will be local to the procedure. An inert linear solver,
Linsolve, is known to the mod function and can be used to solve systems of
linear equations (matrix equations) modulo an integer m. The command
with(linalg,linsolve) allows the use of the abbreviated form of this command.

e.g.

Ax=b

x=inv(A)*b

or

Ax=B

x=inv(A)*B
more concretely if we want to solve 2 equations for 2 unknowns such as;

\[0.2x_1 + 0.3x_2 = 0.5\]
\[0.4x_1 + 0.7x_3 = 1\]

\[A = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}\]
\[b = \begin{bmatrix} 0.5 \\ 1.0 \end{bmatrix}\]

\[x = \text{vector of unknowns} \rightarrow \text{we want to solve this!}\]

Translate into maple:

> A := matrix([[0.2, 0.3], [0.4, 0.7]]);
> b := vector([0.5, 1.0]);

*Note you can check to see if there is 
A is nonsingular using determinants in Maple:

> det(A);

*Everyone knows that if the determinant is not zero then A is a nonsingular matrix.

To solve for x explicitly use linsolve function:

> linsolve(A, b);

Supposed now you were given the following problem:

\[z x_1 + 0.3 x_2 = 5\]
\[0.4 x_1 + 0.7 x_2 = 10\]

Instead of 0.2 you had z in the equation. In this case you would not get a specific solution but a range of solutions. To solve re-write it in Maple as:

> C := matrix([[z, 0.3], [0.4, 0.7]]);

then using linsolve(C, b); we get:

vector([2.500000000*1/(-6.+35.*z), 10.*(1.+5.*z)/(-6.+35.*z)])

Thus we get a solution that:

\[x_1 = 2.500000000*1/(-6.+35.*z)\]
and
\[ x_2 = 10 \cdot \frac{-1 + 5z}{-6 + 35z} \]

Now we can graph the range of the solutions using the plot commands:

first define your functions:

\[
> z_1 := 2.500000000 \frac{1}{-6 + 35z};
\]
and
\[
> z_2 := 10 \cdot \frac{-1 + 5z}{-6 + 35z};
\]

\[
> \text{plot}(z_1, z = 0..0.3);
> \text{plot}(z_2, z = 0..0.3);
\]

As you can see Maple is quite powerful and if you want you need not solve your linear equations with any numbers rather just use symbols:

\[
> Q := \text{matrix}([[a, b], [c, d]]);
> r := \text{vector}([e, f]);
\]

Using \text{linsolve}(Q, r); yields:

\[
\text{vector}([-d\cdot e + f\cdot b]/(-c\cdot b + a\cdot d), (-c\cdot e + a\cdot f)/(-c\cdot b + a\cdot d))
\]

Since there are 6 unknowns we can't really graph it - but say if there were only 2 unknowns we could use \text{plot3d} and get a neat 3-Dimensional graphs. I leave that as an exercise to interested readers.

\text{QED}