Calibration Results for a Generalized Model of Female Labour Force Participation

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Working Paper, November 2003

Abstract

The extent to which growth in human capital, accompanied by a change in the organizational structure of the firm and the adoption of new technologies in the sector that employed women, can account for an early transition of women into the workforce, is evaluated. The basic model from Adshade (2003) is generalized to include imperfect obsolescence of technology, sector-dependent returns to specialization parameters and productivity differentials between organizational modes. Using parameter estimates and changes in wages and employment over the period 1880 to 1970, the simulated results of the model are matched to changes in the relative wage of female clerical workers to male production workers, relative wage bill payments to workers in administration, the share of output produced by industries using the new organizational structure and the average annual growth rate of GDP. The basic model predicts the direction of change in employment and wages well but under predicts both the level of relative wages. The generalized version improves the models ability to match the rate of adoption of the new mode of organization but at the expense of matching wage levels.

1 Introduction

In Adshade (2003) early growth in the labour force participation of female workers is related to three changes in the economy; increased skill level of the workforce, increased division of labour in production and rapid adoption of new technologies. As high school attendance and graduation rates increased at the beginning of the twentieth century, firms that had previously found it unprofitable to hire administrative workers chose to reorganize in a way that incorporated skilled clerical workers into production. This adoption of a new organizational mode coincided with a redirection of investment toward technologies that complemented these workers. Compelled by increases in skill levels, both clerical employment and female labour force participation expanded over time. The model suggests that an increase in the supply of skilled workers that induces both a reorganization of the firm and increased adoption of complementary technologies should lead to higher levels of wages and employment for workers in administration and a falling wage relative to workers in administration in the short run.

In this paper estimated levels of male and female human capital for 1880 and 1970 are used to calibrate the basic model in Adshade (2003). The results of the calibration are compared to movements in the relative wage, the relative wage bill paid to administrative type workers and the proportion of output produced by industries using the new organizational structure. The model is then generalized to address three restrictions imposed in the basic model and comparative statics are undertaken to determine the impact of dropping each restriction on the outcome of the simulation. Finally a generalized version of the model is calibrated and the results are compared to movements in wages and employment over the period.

2 Calibration of the Basic Model

2.1 Description of the Algorithm

The algorithm of the simulation is constructed in the following manner. In every period, for a predetermined level of savings and human capital, the level of technology in Administration (M_A) and Goods (M_G) and the measure of suppliers in Administration using the new mode of organization ($\widetilde{M_A}$) is determined for every possible phase of the economy. The conditions required for an equilibrium are those given in the basic model. In the first phase no technology uses the new mode of production. For a phase one equilibrium, the level of labour demand in Administration must be such that the market wage, conditional on the phase one level of χ, falls below the reservation wage when no suppliers use the new mode of organization. In phase three some, but not all, technologies use the new mode of production, and pay a wage greater than the reservation wage. For a phase three equilibrium the market wage, conditional on the phase three level of χ, is greater than the reservation wage *and* the proportion of suppliers using the new mode of organization is less than one. If the proportion of suppliers using the new mode of organization for the phase three levels of technology and organization is greater than one *and* if the market wage conditional on the phase four level of χ is greater than the reservation wage, there is a phase four equilibrium. In phase four all technologies in Administration use the new mode of production and pay a wage greater than the reservation wage. If the market wage for the level of χ conditional on phase one is greater than the reservation wage and if the market wage for the level of χ conditional on phase three and phase four is less than the reservation wage, there is a phase two equilibrium. In a phase two equilibrium some, but not all, technologies use labour as an input and pay the reservation wage. See the appendix for a more detailed discussion for the algorithm used to simulate the model.

2.2 Determination of Parameter Values and Initial Levels of Human Capital

The simulation is run for 10 periods with each period lasting 10 years. The model is calibrated so that the initial period represents 1880 and the final period 1970. The reservation wage is chosen such that in 1880 firms are employing some clerical workers.[1] The final period represents 1970, by which time it is assumed that male and female human capital are growing at the same rate and the economy is on it's long run growth path. As the 1970's are the beginning of a rapid growth in university enrollment, changes in wages and employment are influenced by other factors other than those described in this model, 1970 seems like a suitable period in which to end the calibration.

The parameter values to be determined are $\alpha, \rho, \nu, \theta, \beta$ and the growth rates for male (g^m) and female (g^w) human capital. The initial levels of male and female human capital are chosen to match the estimated ratio of female to male human capital in 1880. The growth rate of male human capital is chosen the match the average 10 year growth rate of male human capital for the period 1880-1970 and the growth rate of female human capital is chosen to match the estimated ratio of female to male human capital in 1970. The values of ρ and ν are determined using the long run (1970) levels of relative wages and relative levels of female to male human capital.

The long run labour share of output is α. Although the historical data suggests that the labour share has varied over time (see Solow, 1979) the average level over the period 1929-1970 is close to the level consistent with the Real Business Cycle literature. With this in mind, I set $\alpha = 0.64$.

In order to establish the long run ratio of male to female human capital I first determine the level of male human capital in 1880. The stock of male human capital is:

$$H^m = a_u L_u^m + a_s L_s^m \tag{1}$$

where L_u^m is the stock of low-skilled male workers and L_s^m is the stock of skilled male workers. The stock of female human capital is:

$$H^w = b_s L_s^w, \tag{2}$$

where L_s^w is stock of skilled female workers. The total number of individuals in the population (age 25 and above)[2] who are considered skilled and unskilled is given in

[1] This assumption is consistent with Rotella's (1981) evidence that the initial increase in the demand for female clerical workers began at the beginning of the 1880-1890 period.

[2] This measure of skill level under-estimates the skill level of the workforce on two counts. Firstly it includes individuals who would have been of school age in a period in which high school graduation rates were very low. Most of these individuals would have dropped out of the workforce by the 1970 census, which reports that approximately 8.5% of males and 11.2% of females were over the age of 65 at the time of census. Secondly it does not include individuals in the age group 19 to 24 (approximately 8% for both men and women) who would have a higher probability of graduating from high school relative to the mean.

	Male	**Female**
Population	**51,869,770**	**58,029,589**
No School	852,851	914,902
1-4 Years Schooling	2,299,323	1,972,238
5-6 Years Schooling	3,082,912	3,134,180
7 Years Schooling	2,392,567	2,423,053
8 Years Schooling	6,708,041	7,307,323
High School (1-3)	9,633,537	11,652,385
Total Low Skilled (L_u)	**24,969,231**	**27,404,081**
High School (4)	14,365,218	19,792,833
College (1-3)	5,526,759	6,123,971
College Complete	7,008,562	4,708,704
Total Skilled (L_s)	**26,900,539**	**30,625,508**

Table 1: Total number of persons (age 25 and above) for every level of education

Table 3.1.[3]

Conventionally, the ratio of the wage paid to a skilled worker to the wage paid to an unskilled workers is known as the *wage premium* (the premium a skilled workers receives over the wage an unskilled workers receives). In the basic model the wage premium is equal to $\frac{a_S}{a_u}$. The 1970[4] estimated level of the wage premium to skilled labour (Acemoglu, 2002) is approximately 1.5.[5] In order to match this wage premium in the calibrated model I set $a_u = 1$, $a_s = 1.5$ and for consistency I set $b_S = 1.5$.[6]

Using these parameter values in (1) and (2) with L^m and L^w determined in Table 2.1, the 1970 ratio of male to female human capital is $\frac{H^w_{1970}}{H^m_{1970}} = 0.70$.

The initial ratio of female to male human capital is determined using the high school graduation rate from 1879 (U.S. Department of Education). In this year only 2.3% of boys and 2.5% of girls graduated from high school. Using this graduation rate as a proxy for the population as a whole, and using the parameter values for a_u, a_s and b_s from above, the ratio of female to male human capital in 1880 is $\frac{H^w_{1880}}{H^m_{1880}} = 0.03$. A summary of the estimated levels of human capital are given in Table 3.2.[78]

[3]Source: 1970 US Population Census.

[4]After 1970 the wage premium to skilled workers increased sigificantly.

[5]The wage premium in Acemoglu (2002) is defined as a *college premium*, the return to a college education. This number is consistent, however, with Goldin and Katz's (2002) estimate of the relative wage of female clerical workers to female production workers for the year 1923..

[6]This is consistent that Goldin and Katz's (1999) estimate of the relative wage of female clerical workers of 1.5.

[7]Source: Population numbers for adults over the age of 25, *Historical Statistics*, Series A119-134; High school graduation rates (U.S. Department of Education, 1993).

[8]For the 1880 period the high school graduation rate used is the percentage of 17 years olds who have graduated from high school. This measure ($< 3\%$) is used as a proxy for the level education of the population as a whole.

	Year	
	1880	1970
H^w	$304,485$	$45,938,262$
H^m	$10,905,704$	$65,320,039$
$\frac{H^w}{H^m}$	0.03	0.70

Table 2: Estimated levels of Male and Female Human Capital 1880 and 1970

The 10 year growth rate of male human capital, such that the level of male human capital grows from its estimated 1880 level to it's estimated 1970 level in 10 periods, is $g^m = .196$. The rate at which female human capital grows is chosen so that the 1970 level ratio of male to female human capital in the simulation matches the ratio determined by the data. Female human capital grows at a declining rate according to:

$$g^w = \max\left[1 - \frac{t}{16.22}, g^m\right]. \tag{3}$$

Where t is the period of the simulation ($t = \{1, 10\}$). Male and female human capital is in the simulation is given in Figure 1.

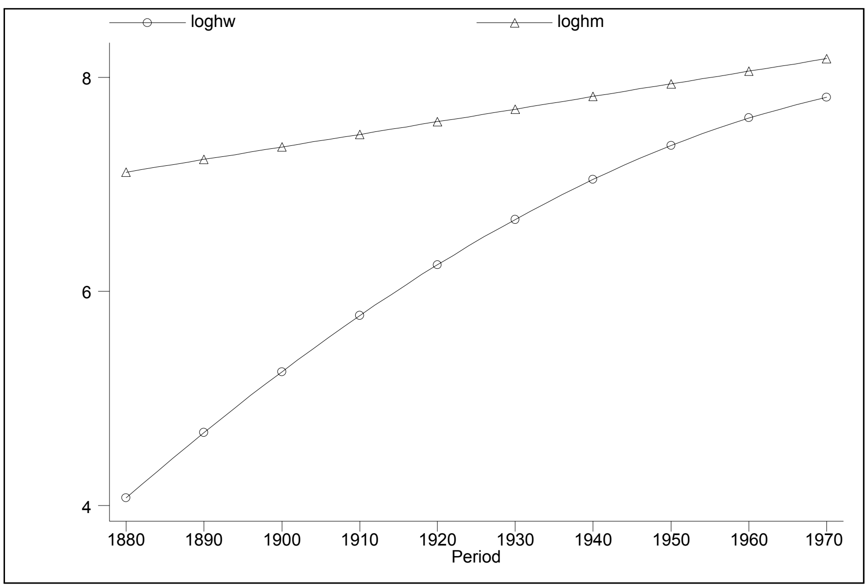

Figure 1: Growth rates of male and female human capital.

In order to determine the levels of ρ and ν, I assume that in the long run the economy is in phase four. In phase four the ratio of output A to G is determined by setting profits in Goods equal to profits and Administration and solving for:

$$\frac{A}{G} = \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}} \left(\frac{H^w}{H^m}\right)^{\frac{\alpha}{\alpha-\rho+\alpha\rho}}. \tag{4}$$

Prices of the inputs to final production are proportional to wages and human capital according to:

$$q_A = \frac{w_A H^w}{A} \text{ and } q_G = \frac{w_G H^m}{G}, \tag{5}$$

such that the ratio of input prices is:

$$\frac{q_A}{q_G} = \frac{w_A H^w}{w_G H^m} \frac{G}{A}. \tag{6}$$

From the derived demand for intermediates A and G:

$$q_A = (1-\nu)\left(\frac{Y}{A}\right)^{1-\rho} \text{ and } \; q_G = \nu\left(\frac{Y}{G}\right)^{1-\rho}, \tag{7}$$

and the ratio of input prices is:

$$\frac{q_A}{q_G} = \left(\frac{A}{G}\right)^{\rho-1} \frac{1-\nu}{\nu}. \tag{8}$$

Setting (6) equal to (8), substituting (4) for the ratio of output A and G, and rearranging, yields the long run relationship between wages and human capital:

$$\frac{w_A}{w_G} = \left(\frac{H^w}{H^m}\right)^{\frac{\rho-\alpha}{\alpha-\rho+\alpha\rho}} \left(\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}}. \tag{9}$$

In the basic model, m_A and m_G grow at the same rate in the long run provided that male and female human capital are growing at the same rate (see equation (2.62) in Chapter Two). Given a constant relative growth rate of m_A and m_G, the ratio of wages between the sectors is constant. Using the long run ratio of wages and the long ratio of male to female human capital determined above, (9) can be used to either determine a value for ν for a given level of ρ or to determine a value of ρ the level of ν.

There are several ways in which the long run wage ratio can be determined. The first would be to use the ratio of female to male employment earnings in manufacturing for 1970. Goldin determines that this ratio is 0.544 in manufacturing industries and .605 in all occupations (Goldin, 1990 page 60). Another way would is to use the ratio of wages of female clerical workers to male production workers. Using 1970 US census the average annual full time earnings of a female clerical work was \$4228. The annual average of a male production worker, taken as the weighted average of male craftsmen and operatives, was \$7614. Using these figures, the long run wage ratio is 0.555. Finally, the wage ratio can be computed using the relative wage of all clerical workers to all production workers. From the 1970 census data, this ratio is .746.

Table 3.3 gives the corresponding level of ν for four different values of ρ for each of the wage ratios above and for the 1970 ratio of male to female human capital. In the calibration I set $\rho = -0.5$ and $\nu = 0.873$.

	Implied Value of ν			
$\frac{w_A}{w_G}$	$\rho = -.25$	$\rho = -.5$	$\rho = -1$	$\rho = -3$
.544	.810	.877	.955	.999
.605	.786	.855	.941	.999
.555	.805	.873	.952	.999
.746	.733	.803	.904	.997

Table 3: Estimated Values of rho and upsilon

The discount factor, β, is chosen to match a 20% annual savings rate, $\beta = 0.25$. The productivity parameter, θ, and the cost of technology adoption, γ, are chosen so that for a given level of ρ and α the level of savings increases in the first period of the simulation: $\theta = .1$ and $\gamma = 1$. As these parameters are identical in both sectors, the level of and θ and γ have little impact on outcome of the simulation.

2.3 The Data

The data used to evaluate the predictions of the model are the relative wage bill paid to clerical labour, the growth rate of GNP, the ratio of female clerical wages to male production wages and the proportion of output in manufacturing produced by industries using the new mode of organization.

The relative wage bill of administrative workers and the relative average wage have been calculated using annual average earnings data for the period 1900-1930 from Goldin and Katz (1995),[9] annual average earning data for the period 1940-1970 (excluding 1950) are from *Historical Statistics* (series G 372-415)[10] and major occupational group data from *Historical Statistics* (series D182-232).[11] The relative wage bill is calculated in the form:

$$wage\ bill = \frac{H^w w_A}{H^m w_G}.$$

The relative wages are calculated in the form:

$$relative\ wages = \frac{w_A}{w_G}$$

Data used in the relative wage bill and relative average wage calculations are reported in Table 3.4.

[9] Wages for male production workers (Goldin & Katz, 1995 Table 3), female clerical workers (Goldin & Katz, 1995 Table 2).

[10] Male production workers annual average income is full-time salaries for Craftsmen, Foremen and Kindred Workers and for Operatives and kindred workers. Female Clerical workers annual average income is full-time salaries for Clerical and Kindred Workers.

[11] Major occupational groupings for male production workers are for Craftsmen, Foremen and Kindred Workers and Operatives and kindred workers. Major occupational groupings for female are for Clerical and Kindred workers.

Year	Male Operatives		Male Craftsmen		Female Clerical	
	Annual Wage	Number	Annual Wage	Number	Annual Wage	Number
1900	626	3720	626	3062	526	212
1910	641	5441	641	4315	668	688
1920	739	6587	739	5482	620	1614
1930	895	7691	895	6246	733	2246
1940	1268	9518	1562	6203	1072	4408
1960	4977	12846	5868	9241	3586	6497
1970	7623	13406	9254	10435	5551	9910

Table 4: Levels of Male Production and Female Clerical Workers and their Wages

The annual average growth rate of GNP is determined using real GNP, in constant 1958 prices, from *Historical Statistics* (Series F98-124 and F47-70) for the period 1880-1970 and is determined to be 3.53%.

In order to determine the proportion of output in manufacturing using the new mode of organization I calculate the percentage of manufacturing output (in terms of value added) that is produced by industries that have adopted a complex organizational structure over the period. These industries are identified by Chandler (1969) and are discussed in greater detail in Chapter Three. Data .for the period 1900 to 1940 (from *Historical Statistics (*Series P 1-12 and P 58-67)) is only available for six out of the nine industries. One caveat, while these industries are those that have in general adopted the new organizational structure, other industries, not specified here, will have adopted the new organizational mode to varying degrees. Therefore, it is probably that both the level and the growth rate of this measure are understated here.

2.4 Calibration Results of the Baseline Model

For the given growth rate of human capital, the average annual growth rate of GNP is matched reasonably well by the growth rate of output Y over the period. The average annual growth rate of output Y over the same period is 2.73% compared to 3.53% in the data.

Figure 2 plots the relative wage bill for both the simulation and for the data. The relative wage bill in the calibration matches the level and the growth rate of the relative well. Figure 3 shows that the relative wage of female clerical workers is falling in the calibration and in the data, but that the level of the relative wage is below that in the data and the rate at which the relative wage falls in the simulation is slower than that in the data..

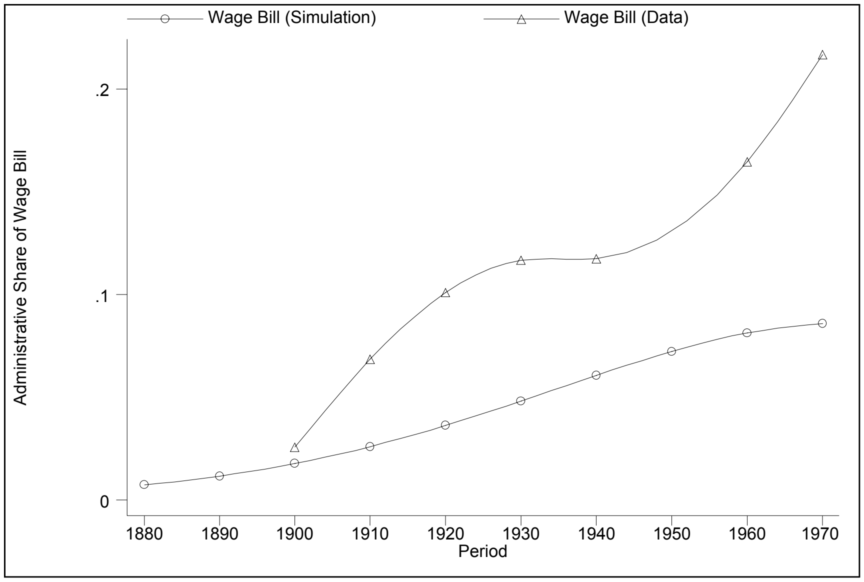

Figure 2: Calibration results for relative wage bill to administrative workers in the basic model.

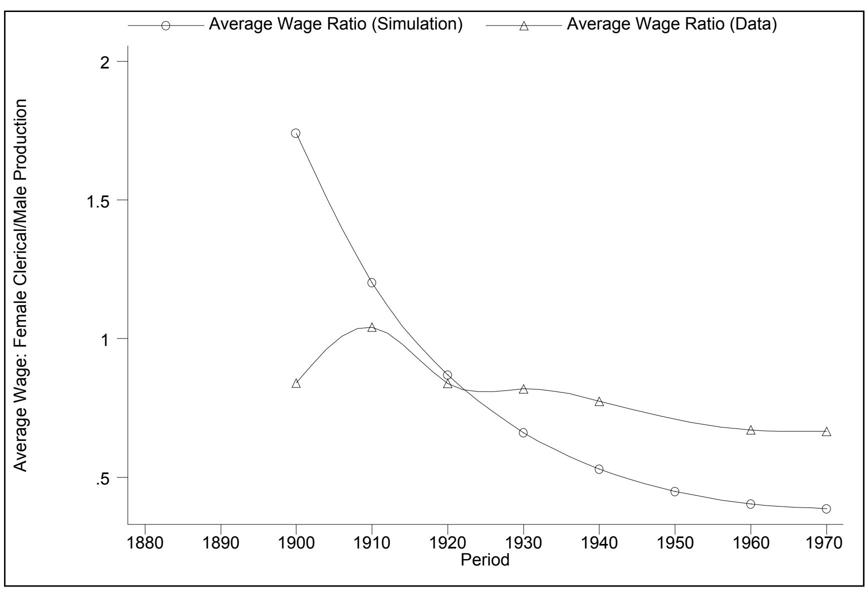

Figure 3: Calibration results for the relative wage of female clerical workers in the basic model.

Figure 4 plots the proportion of output of the final good attributable to suppliers using the new mode of organization. Where the share of output in Administration from suppliers using the new mode is:

$$o_{\widetilde{m}_A} = \frac{n\widetilde{m}_A}{m_{A+}(n-1)\,\widetilde{m}_A},$$

the share of output of the final good attributable to suppliers using the new mode is:

$$Y_{\widetilde{m}_A} = (1-v)\left(\frac{Y}{o_{\widetilde{m}_A}A}\right)^{\rho}.$$

$Y_{\widetilde{m}_A}$ is plotted with the proportion of output produced by industries using the new mode of production found in the data (all industries (1940-1970) and a subset of industries (1900-1970)). The simulated results match the growth rate but fall below the level of output produced by the six industries for which I have data for the whole period. Given the caveat above this growth rate understates the level of output produced by supplies using the new mode of production.

Figures 5 to 6 plots the full results of the calibration. Figure 5 plots the log differentials for m_A,m_G and $\widetilde{m}_A$. Technology in Administration and Goods grows at a fairly constant rate. The measure of producers using the new mode of production grows rapidly at the beginning of the period but slows over the long run to the growth rate of m_A.

The level of technology in sector A (m_A) and the ratio of technology in sector A to technology in sector G $\left(\frac{m_A}{m_G}\right)$ are plotted in Figure 6. The level of technology in A increases but falls relative to relative to technology in G. The share of final output attributable to output from sector A and from output from sector G is in Figure 7. Overtime sector A contributes an increasing share of output relative to sector G (by 1970 about 8% of output is produced by suppliers in sector A compared to less than 6.5% in 1880). The decline in the relative rate of technology adoption in A stems from the assumption of net complementarity between Goods and Administration. As the proportion of producers using the new mode of organization increases, output per producer in sector A increases, decreasing the relatively profitability of investing in that sector. The increase in average output per technology in A is such that, despite decreasing relative investment in that sector, the share of output from sector A is increasing over time. Admittedly, the increase in output from sector A is small in the calibration relative to what we would expect to see in the data.

3 The Generalized Model

The generalized model addresses three restrictions imposed in the basic model: identical productivity parameters across organizational modes, perfect obsolescence of

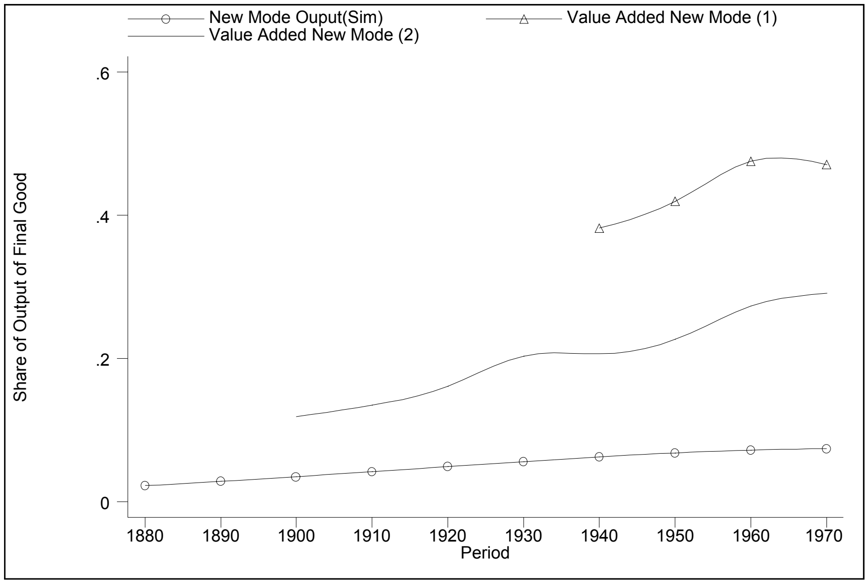

Figure 4: Calibration results for the share of output produced by suppliers using the new organizational mode in the basic model.

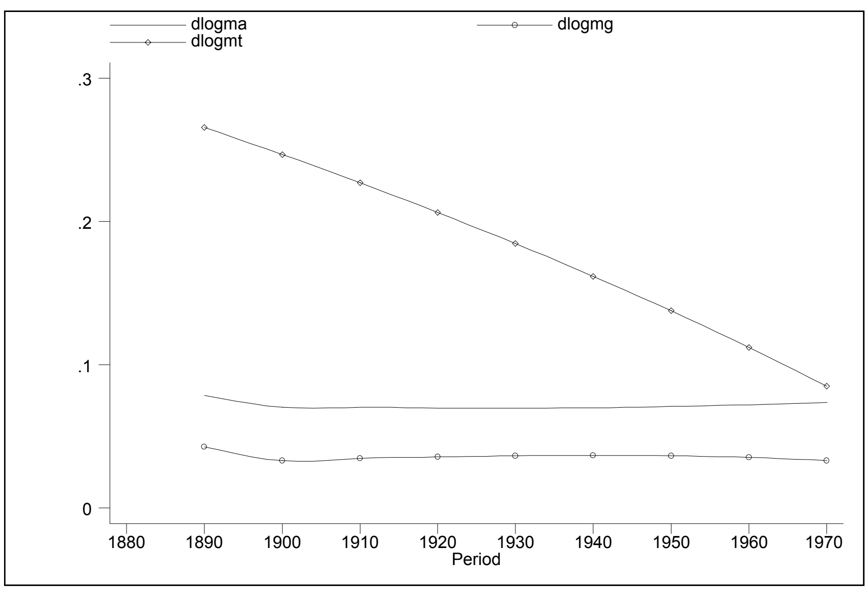

Figure 5: Growth rates from $m_A, \widetilde{m}_A$ and χ in the basic model.

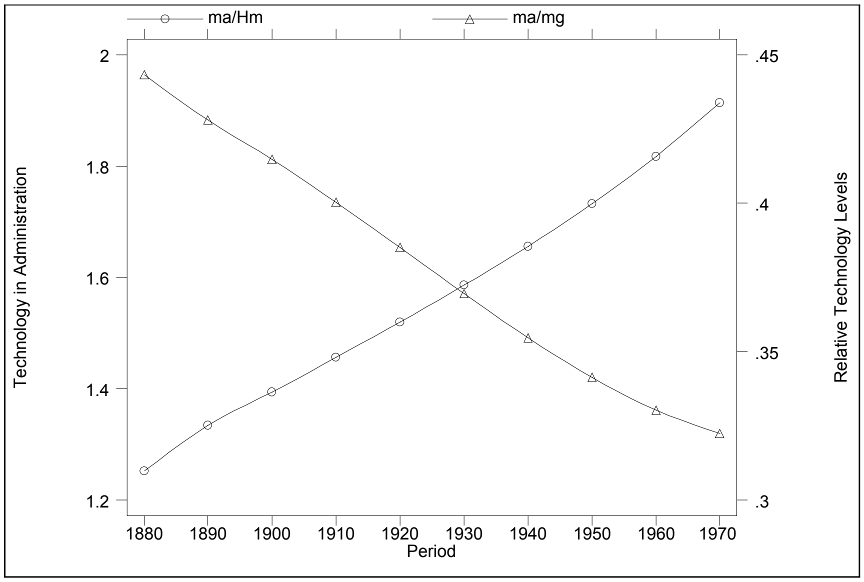

Figure 6: The level of technology in Administration and the ratio of technology in Administration to technology in Goods in the basic model.

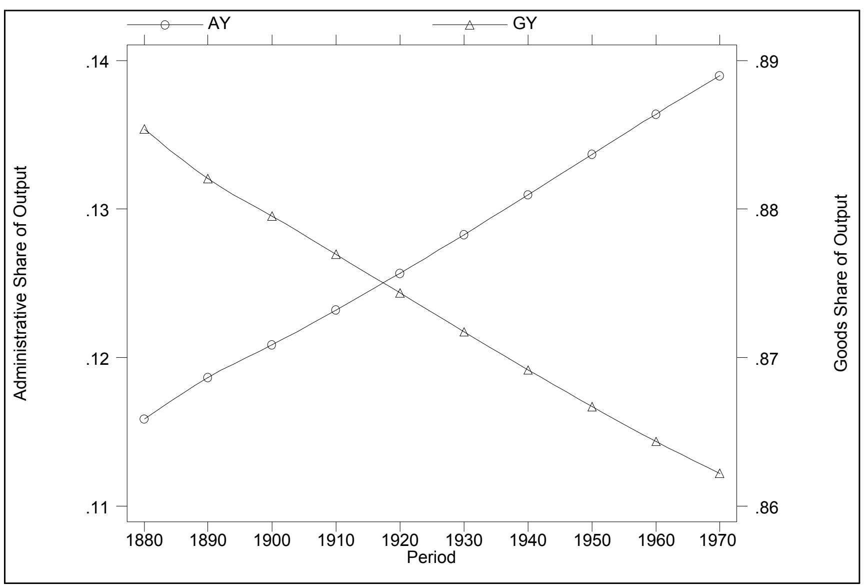

Figure 7: The share of output attributable to Administration and to Goods in the basic model.

technology and identical returns to specialization across sectors. The restriction imposed with identical productivity parameters across organization modes assumes a certain level of supervision costs to employing administrative workers. The main result of dropping this restriction is that for lower supervision costs both the rate of adoption of the new mode of organization increases and the level of labour input per technology decreases. While these effects offset each other in determining the share of output from suppliers using the new mode, the overall effect is to decrease the relative wage paid to workers in the administrative sector. The assumption of perfect obsolescence of technology imposes the restriction that suppliers only produce output in the period in which they arrive and at the monopolistic level. With a competitive labour market this leads to higher average levels of labour input and a slower rate of adoption of the new mode of production. In dropping the restriction that the level of substitutability between inputs is identical across sectors I determine that for relatively high returns to specialization in administration the wages of workers in that sector are lower and, as such, the rate of adoption of the new mode of organization is higher. In each of the following sections I discuss how each extension effects the outcome of the model and compare the simulation results to the baseline.

3.1 Incomplete Obsolescence

In the basic model we assume that new technologies adopted in this period become completely obsolete at the beginning of the next period. Where δ is the rate of obsolescence in technologies in both sectors, the generalized model considers a market in which $\delta < 1$: output in each sector is a function of both new technology adoptions and depreciated technologies from previous periods.

3.1.1 The Model with Incomplete Obsolescence

All suppliers earn monopoly profits in the period in which they enter. In subsequent periods competition drives the price and output to the competitive level. Where p^c is the competitive price, x^c is the competitive level of output and l^c is the level of labour input to competitive suppliers, the cost minimization problem yields the price and quantity of incumbent suppliers in the goods sector:

$$p_G^c = \frac{w_G}{\theta} \tag{10}$$

$$x_G^c = \left(\frac{\theta}{w_G} q_G\right)^{\frac{1}{1-\alpha}} G(=\theta l^c), \tag{11}$$

and of incumbent suppliers in the Administration using the new mode of organization:

$$p_A^c = \frac{w_A}{\theta} \tag{12}$$

$$x_A^c = \left(\frac{\theta}{w_A} q_A\right)^{\frac{1}{1-\alpha}} A(=\theta n^c). \tag{13}$$

Incumbent suppliers in Administration using the old mode of organization continue to produce θ but the price at which they sell their output is such that they earn zero profits, that is $p_A = 0$.

Total market output of G is a function of the number of entrants and incumbents. Where suppliers in Goods are price takers in the wage, the relative labour input to incumbents is :

$$l_t^c = \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} l_t, \tag{14}$$

the labour input to entrants is:

$$l_t = \frac{H_t^m}{M_{G,t} + (1-\delta) M_{G,t-1}\left(\left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} - 1\right)}, \tag{15}$$

and output in the Goods is:

$$G_t = \theta l_t \left[(1-\delta) M_{G,t-1}\left(\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} - 1\right) + M_{G,t}\right]^{\frac{1}{\alpha}}. \tag{16}$$

Output in the Administration is a function of both the number of entrants and incumbents using both the new and old mode of production. If suppliers in administration are price takers in the wage, the relative labour input to incumbents using the new mode of organization is:

$$n_t^c = \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} n_t, \tag{17}$$

the available labour input to entrants is:

$$n_t = \frac{H_t^w}{\widetilde{M}_{A,t} + (1-\delta) \widetilde{M}_{A,t-1}\left(\left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} - 1\right)}, \tag{18}$$

and output in Administration is:

$$A_t = \theta \left[(1-\delta) \widetilde{M}_{A,t-1}\left(\left(\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} - 1\right) {n_t}^{\alpha} - 1\right) + M_{A,t} + \widetilde{M}_{A,t}\left({n_t}^{\alpha} - 1\right)\right]^{\frac{1}{\alpha}}. \tag{19}$$

Allowing for a lower rate of obsolescence leads to a lower labour input per entrant in both Goods and Administration as competitive incumbents increase their labour input, decreasing available supply of labour. Additionally, as incumbents increase their output above the monopoly level the level of output in both G and A, for a given level of technology, is higher for lower rates of obsolescence.

The RTA Curve with $\delta < 1$ Let the level of input to G from incumbents divided by the labour input to entrants (l) and deflated by the level of male human capital be:

$$\mu_G = (1-\delta)\,\chi_{t-1}\left(\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} - 1\right), \tag{20}$$

and the level of input to A from incumbents deflated by the level of male human capital be:

$$\mu_A = (1-\delta)\,\widetilde{m}_{A,t-1}\left(\left(\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} - 1\right){n_t}^{\alpha} - 1\right).$$

The rate of technology adoption is determined by setting profits of entrants equal across sectors. The measure of technology in administrative services is a function of savings, the measure of suppliers using the new mode and the output of incumbents in both sectors G and A. When $\widetilde{m}_{A,t} < m_{A,t}$, this is $m_A\left(\widetilde{m}_{A,t}, s_{t-1}, m_{A,t-1}, \widetilde{m}_{A,t-1}\right)$ implicitly defined by:

$$m_A = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}}\left(\frac{1}{l_t}\right)^{\frac{\alpha\rho}{\alpha-\rho}}\left(\frac{1}{\gamma}s_{t-1} - m_A + \mu_G\right) - \widetilde{m}_{A,t}\left({n_t}^{\alpha} - 1\right) - \mu_A \tag{RTA-δ}$$

and when $\widetilde{m}_{A,t} = m_{A,t}$:

$$m_A = \left(\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}}\left(\frac{n_t}{l_t}\right)^{\frac{\alpha\rho}{\alpha-\rho}}\left(\frac{1}{\gamma}s_{t-1} - m_A + \mu_G\right) - \widetilde{m}_{A,t}\left({n_t}^{\alpha} - 1\right) - \mu_A \tag{RTA4-δ}$$

Where l_t is determined by (15) and is a function of savings, $m_{A,t}$ and $m_{G,t-1}$ and n_t is determined by (18) and is a function of $m_{A,t}$ and $m_{A,t-1}$.

The MRW Curve with $\delta < 1$ Where the wage in Administration, when $\delta < 1$, is:

$$w_{A,t} = \frac{\theta\alpha\,(1-\alpha)}{n_t^{1-\alpha}}vH_t^{m\frac{1-\alpha}{\alpha}}\left(v + (1-v)\left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\rho}{\alpha-\rho}}\left(\frac{1}{l_t}\right)^{\frac{\alpha\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}} l_t\left[m_{G,t} + \mu_G\right]^{\frac{1-\alpha}{\alpha}}, \tag{WAGE-δ}$$

the point at which the MRW-δ curve intersects the RTA curve is the level of m_A which solves (21) setting $w_{A,t} = \overline{w}$.

The MHC Curve with $\delta < 1$ The relative level of profits to entrants in A is unchanged when $\delta < 1$: the level of available labour at which an entrant chooses the new mode of production is:

$$\widetilde{n} = \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}}. \tag{21}$$

If $\widetilde{m}_A < m_A$, and if all human capital is employed by the $\widetilde{m}_A$ intermediates using the new organizational mode, then the level of labour input per supplier is as given in (18) and is equal to $\widetilde{n}$.

It follows that the maximum measure of intermediates using the new organizational mode is now:

$$\widetilde{m}_A = \begin{cases} (1-\alpha)^{\frac{1}{\alpha}} \frac{H_t^w}{H_t^m} - (1-\delta)\,\widetilde{m}_{A,t-1}\left(\left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} - 1\right) & \text{if } \widetilde{m}_A < m_A \\ n_t^{-1}\frac{H_t^w}{H_t^m} - (1-\delta)\,\widetilde{m}_{A,t-1}\left(\left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} - 1\right) & \text{if } \widetilde{m}_{A,t} = m_{A,t} \end{cases}. \tag{MHC-δ}$$

The MHC curve with incomplete depreciation is below the MHC curve with complete depreciation at every point below the 45° line.

The effect of the introduction of incomplete obsolescence on the position of the RTA curve is ambiguous. When $\delta < 1$, the RTA curve is decreasing in the level of $m_{A,t-1}$ and increasing in the level of $m_{G,t-1}$. If technology in Goods is high relative to Administration then the RTA curve with incomplete depreciation in technologies is above the RTA curve with complete depreciation. If, on the other hand, the level of technology in Administration is high relative to Goods, then the RTA curve with incomplete obsolescence in technologies is below the RTA curve with complete obsolescence.

3.1.2 Simulation Results for the Model with Incomplete Obsolescence

The results of the baseline model are compared to three rates of obsolescence, $\delta = \{0.75, 0.85, 0.95\}$. The results of these simulations are given in figures 8-12 and summarized in Figure 13.

The overall effect of a decrease in the rate of technology depreciation (a decrease in δ) is: the share of output produced by supplies using the new organizational mode decreases, the share of output of Y attributable to output from sector A increases, the relative wage bill to administrative workers falls, as does the relative wage of clerical to manufacturing workers, the overall level of technology in Administration increases but falls relative to technology in Goods.

At low rates of technology obsolescence the level of labour input available to suppliers who are entrants decreases as the labour input of suppliers who are competitors is above the monopolistic level. This decreases the profitability of adopting the new mode and decreases the share of output from producers using the new mode. As increases in $\widetilde{m}_A$ offset increases in technology sector (the RTA curve is negatively sloped) this reduction in $\widetilde{m}_A$ increases the profitability of investing in sector A and increases the share of output of the final good that is attributable to output from sector A.

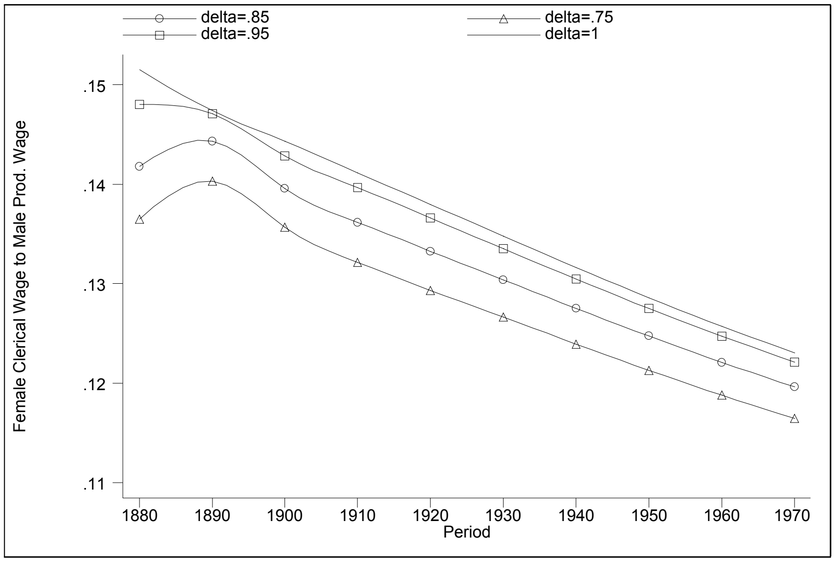

Figure 8: Comparative static results for the relative wage of female clerical workers in the generalized model for various levels of δ.

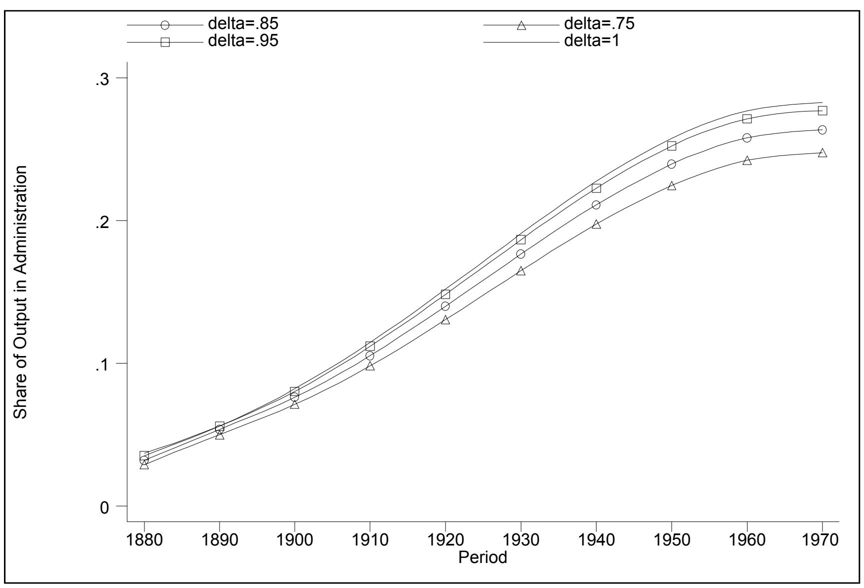

Figure 9: Comparative static results for the share of output in administration produced by suppliers using the new organizational mode in the generalized model for various levels of depreciation δ.

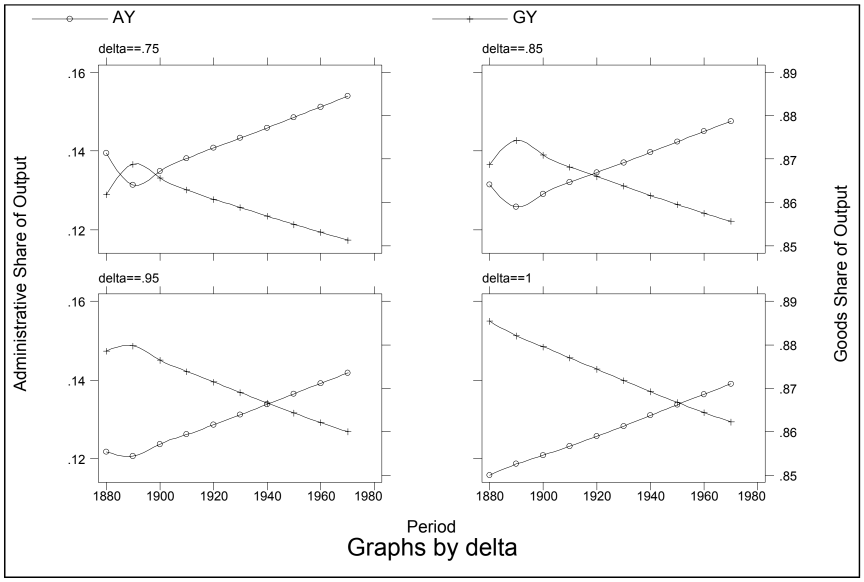

Figure 10: The share of output attributable to Administration and to Goods with various levels of depreciation, δ.

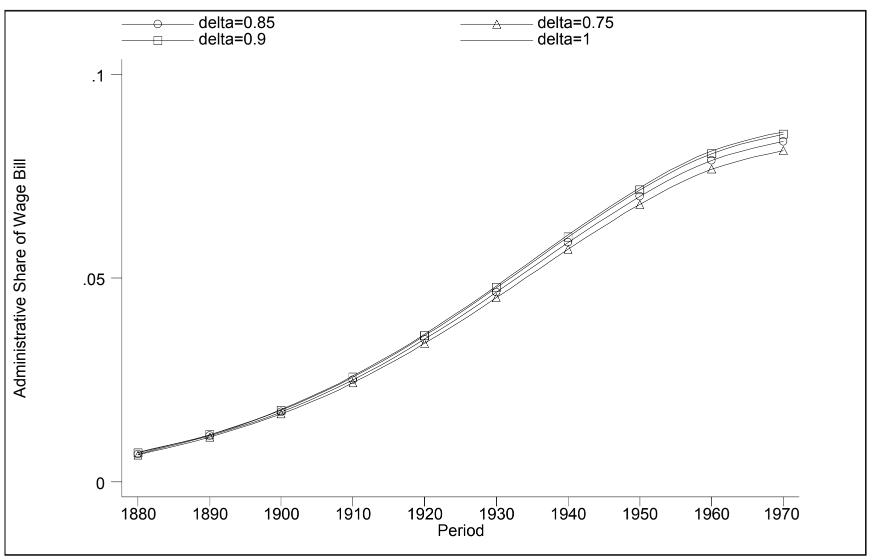

Figure 11: Comparative static results for relative wage bill to administrative workers in the generalized model with various rates of depreciation, δ.

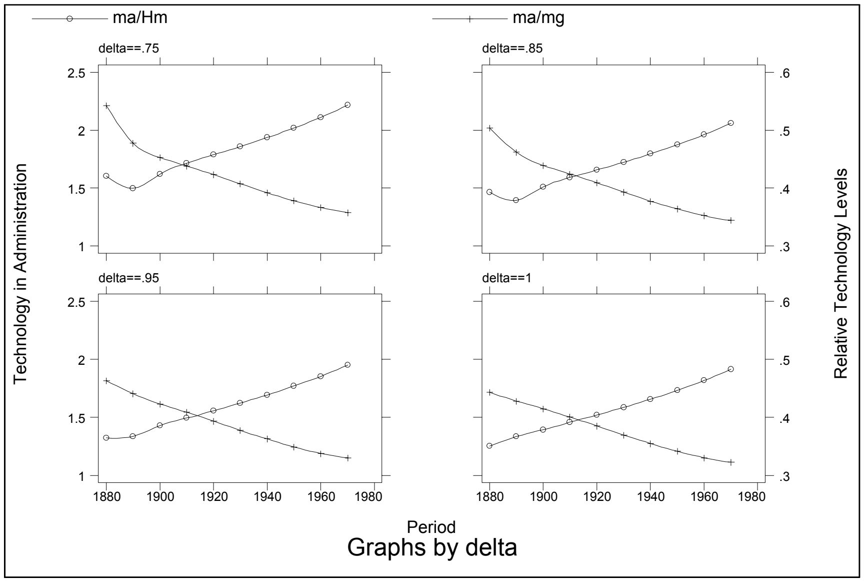

Figure 12: The level of technology in Administration and the relative level of technology in Administration for various rates of depreciation, δ.

	delta	kappa	lambda
Rate of Technology Adoption	↓	↑	↓
Adoption of the New Mode	↑	↓	↓
Proportionof the Wage Bill	↑	↑	↓
Share of Output of Final Good	↓	↑	↑

Figure 13: Simulation results when each of δ, λ, and κ are increased.

3.2 Changes in Productivity Differentials

The second restriction addressed in the generalized model is that the level of productivity is identical between modes of production. To see why it might be necessary to drop this restriction, consider the introduction of a new parameter, λ, such that output, when the old organizational mode is chosen is:

$$x_A^O = \theta\lambda, \tag{22}$$

Output, when the new organizational mode is chosen, is:

$$x_A^N = \theta n. \tag{23}$$

The relative level of productivity of the new to the old mode of production is simply:

$$\frac{x_A^N}{x_A^O} = \lambda n. \tag{24}$$

In the basic model, $\lambda = 1$ and the relative productivity of the new organizational mode is simply n. If $\lambda < 1$, however, the productivity of the new organizational mode increases relative to the old organizational mode. One way to think about this relationship is in terms of supervision costs. When a technology uses labour as an input the owner of the technology incurs a supervision cost in terms lost productivity. When $\lambda = 1$ the supervision cost of the first unit of effective labour employed by the technology is equal to the total productivity of technology. For example, consider a case in which the technology is owned by an entrepreneur. The entrepreneur chooses to product a good using either their own efforts alone or by combining their efforts with waged labour. If $\lambda = 1$, and the entrepreneur chooses to combine their own effort with waged labour, when $n = 1$ the net increase in output over choosing the old mode of production is zero: the cost of supervising the first worker is equal to the total productivity of the entrepreneur. For higher levels of n (> 1), average supervision cost declines.

3.2.1 The Model with Productivity Differentials

The RTA Curve with $\lambda \neq 1$ With $\lambda \neq 1$ output in Administration is now:

$$A_t = \theta \left[m_{A,t}\lambda^{\alpha} + \widetilde{m}_{A,t}\left(n^{\alpha} - \lambda^{\alpha}\right)\right]^{\frac{1}{\alpha}}. \tag{25}$$

The relative rate of technology adoption is determined by setting profits equal across sectors. When $\widetilde{m}_A < m_A$:

$$A_t = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}} \lambda^{\frac{\alpha}{\alpha-\rho}} \chi_t^{\frac{\alpha}{\alpha-\rho}} G_t, \tag{26}$$

and the relative rate of technology adoption is determined at $m_A(\widetilde{m}_{A,t}, s_{t-1})$ implicitly defined by:

$$m_A = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}} \lambda^{\frac{\alpha\rho}{\alpha-\rho}} \left(\frac{1}{\gamma}s_{t-1} - m_A\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \widetilde{m}_{A,t}\left(\frac{n_t^\alpha - \lambda^\alpha}{\lambda^\alpha}\right). \quad \text{(RTA-}\lambda\text{)}$$

When $\widetilde{m}_A = m_A$ the relative rate of technology adoption is as given in the basic model.

The MRW Curve with $\lambda \neq 1$ The wage in Administration, when $\lambda \neq 1$ is:

$$w_{A,t} = \frac{\theta\alpha(1-\alpha)}{n^{1-\alpha}}\nu H^{m\frac{1-\alpha}{\alpha}} \left(\nu + (1-\nu)\lambda^{\frac{\alpha\rho}{\alpha-\rho}}\left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\rho}{\alpha-\rho}} \chi_t^{\frac{\alpha\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}} \chi_t^{\frac{1-2\alpha}{\alpha}} \quad \text{(WAGE-}\lambda\text{)}$$

the point at which the MRW-λ curve intersects the RTA curve is just the level of m_A which solves (27) when $w_{A,t} = \overline{w}$.

The MHC Curve with $\lambda \neq 1$ The level of labour input per producer is determined by setting the profit to using the old mode equal to the profit to using the new mode. When $\lambda \neq 1$ the condition on the available labour supply is:

$$\widetilde{n}(\lambda) = \lambda\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}}, \tag{27}$$

so that when $\lambda < 1$ a higher level of labour per technology is necessary for a supplier to choose the new mode of organization.

The level of available female human capital per supplier using the new mode of organization is:

$$\widetilde{m}_A(n_t, H_t^w, H_t^m) = \begin{cases} \frac{1}{\lambda}(1-\alpha)^{\frac{1}{\alpha}}\frac{H^w}{H^m} & \text{if } \widetilde{m}_A < m_A \\ n_t^{-1}\frac{H_t^w}{H_t^m} & \text{with } n_t > \widetilde{n}, \text{ if } \widetilde{m}_{A,t} = m_{A,t} \end{cases}. \quad \text{(MHC-}\lambda\text{)}$$

The curve MHC-λ, at every point above the 45° line, is high relative to the basic model when $\lambda < 1$, and low relative to the basic model when $\lambda > 1$.

The effect of dropping the restriction that $\lambda \neq 1$ is unambiguous; $RTA - \lambda$ is decreasing in λ. If λ is low ($\lambda < 1$) then $RTA - \lambda$ will lie above RTA with $\lambda = 1$. If λ is high then $(RTA - \lambda)$ will lie below RTA with $\lambda = 1$. With both MHC and RTA higher when $\lambda < 1$, the model predicts that $\widetilde{m}_A$ will be greater for lower values of λ.

3.2.2 Simulation Results for the Model with Productivity Differentials

The results of the baseline model are compared to three rates of $\lambda = \{0.25, 0.5, 1.5\}$. The results of these simulations are given in figures 14-18 and summarized in Figure 13.

As predicted, the lower the level of λ, the faster the proportion of producers using the new mode of organization grows over time. The share of output in the administration sector from producers using the new mode of organization is higher when $\lambda < 1$. The share of output of the final good from Administration is lower, however, as the rate of technology adoption is slower, the level of output from suppliers using the old mode is lower and the labour input per technology is lower for suppliers using the new mode. The relative wage of workers in Administration is higher for low values of λ and falls at faster rate than when $\lambda = 1$.

3.3 Return to Division of Labour Differentials

The third restriction considered is the assumption that the level of benefit derived from increased specialization is equivalent between Goods and Administration sectors. This assumption is problematic if there is suspicion that the labour share of output to administrative goods is different than the labour share of output to manufactured goods. The effects of dropping this restriction, and allowing the parameter on the production function in the administrative sector to vary, are considered in this section.

3.3.1 The Model with Differential Returns to Specialization

In the basic model, the coefficient α determines the returns to specialization in production in both sectors. In the generalized version of the model the production functions for Administration and Goods are:

$$A = \left[\int_1^{M_A} x_A(j)^\kappa \, dj\right]^{1/\kappa} \text{ and } G = \left[\int_1^{M_G} x_G(j)^\alpha \, dj\right]^{1/\alpha} \tag{28}$$

where, as before, the measure of intermediates in A and G are M_A and M_G, and the output of intermediate j, in sectors A and G is $x_{A,j}$ and $x_{G,j}$. The coefficient that determines the benefit to specialization in Administration is κ and in Goods is α. A high κ relative to α indicates that intermediates in A are easily substitutable relative to those in G. A low κ relative to α corresponds to low substitutability and high returns to differentiation in A relative to G.

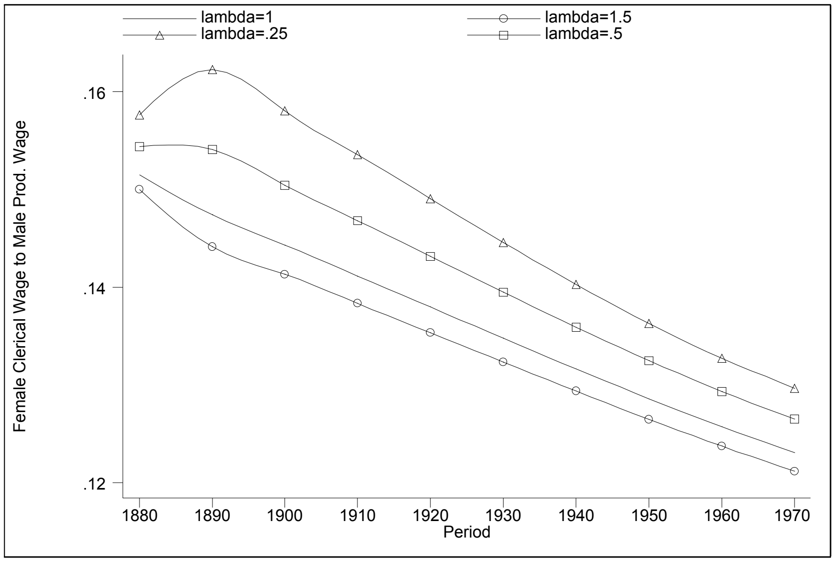

Figure 14: Comparative static results for the relative average wage of female clerical workers in the generalized model for various levels of λ.

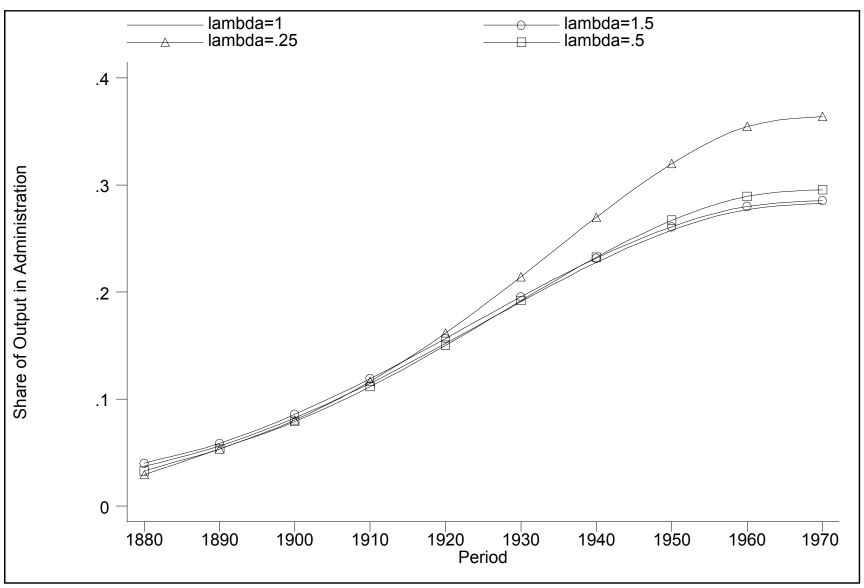

Figure 15: Comparative static results for the share of output in Administration produced by suppliers using the new organizational mode in the generalized model for various levels of productivity, λ.

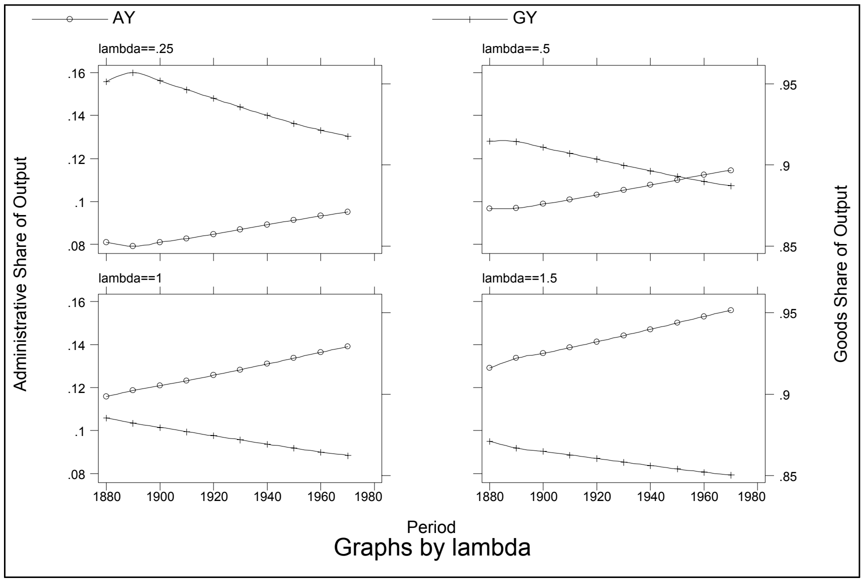

Figure 16: The share of output attributable to Administration and to Goods with various levels of productivity λ.

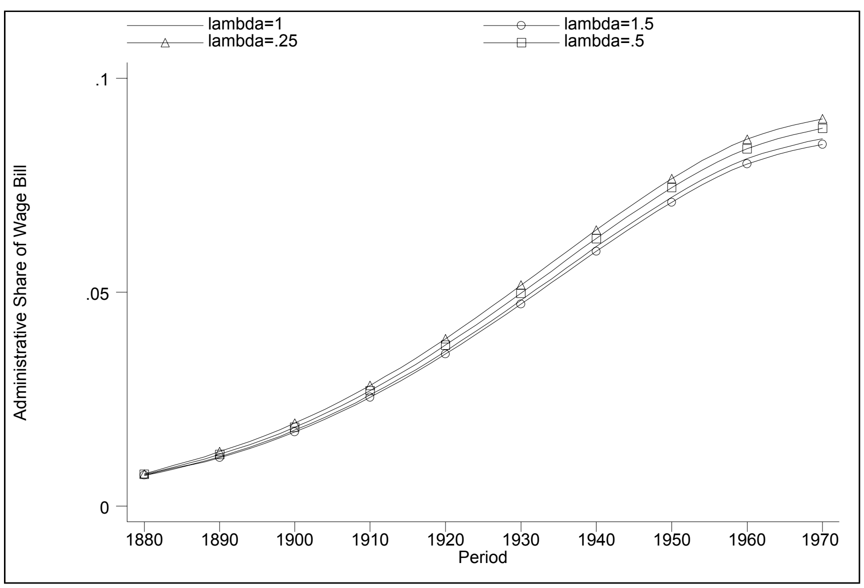

Figure 17: Comparative static results for relative wage bill to administrative workers in the generalized model with various levels of productivity, λ.

The RTA Curve with $\kappa \neq \alpha$ With $\kappa \neq \alpha$ output in Administration is now:

$$A_t = \theta \left[m_{A,t} + \widetilde{m}_{A,t} \left(n^{\kappa} - 1 \right) \right]^{\frac{1}{\kappa}}. \tag{29}$$

The relative rate of technology adoption is determined by setting profits equal across sectors. When $\widetilde{m}_A < m_A$:

$$A = \left(\frac{1}{1-\alpha} \frac{1-\nu}{\nu} \right)^{\frac{1}{\kappa-\rho}} \theta^{\frac{\kappa-\alpha}{\kappa-\rho}} G^{\frac{\alpha-\rho}{\kappa-\rho}} \chi^{\frac{\alpha}{\kappa-\rho}}, \tag{30}$$

and the relative rate of technology adoption is determined at $m_A \left(\widetilde{m}_{A,t}, s_{t-1} \right)$ implicitly defined by:

$$m_{A,t} = \left(\frac{1}{1-\alpha} \frac{1-\nu}{\nu} \right)^{\frac{\kappa}{\kappa-\rho}} H_t^{m\left(\frac{\kappa}{\alpha} \frac{\alpha-\rho}{\kappa-\rho} - 1 \right)} \left(\frac{1}{\gamma} s_{t-1} - m_{A,t} \right)^{\frac{\alpha-\rho+\alpha\rho}{\kappa-\rho} \frac{\kappa}{\alpha}} - \widetilde{m}_{A,t} \left(n^{\kappa} - 1 \right). \tag{RTA-κ}$$

When $\widetilde{m}_A = m_A$:

$$A_t = n_t^{\frac{\kappa}{\kappa-\rho}} \chi_t^{\frac{\alpha}{\kappa-\rho}} \left(\frac{1-\alpha}{1-\kappa} \frac{1-\nu}{\nu} \right)^{\frac{1}{\kappa-\rho}} \theta^{\frac{\kappa-\alpha}{\kappa-\rho}} G_t^{\frac{\alpha-\rho}{\kappa-\rho}},$$

and the relative rate of technology adoption is:

$$m_{A,t} = \left(\frac{H^w}{H^m} \right)^{\frac{\kappa\rho}{\kappa-\rho+\kappa\rho}} \left(\frac{1-\alpha}{1-\kappa} \frac{1-\nu}{\nu} \right)^{\frac{1}{\kappa-\rho}} \left(\frac{1}{\gamma} s_{t-1} - m_{A,t} \right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha} \frac{\kappa}{\kappa-\rho+kp\rho}}. \tag{RTA4-κ}$$

The MRW Curve with $\kappa \neq \alpha$ The wage in Administration, when $\kappa \neq \alpha$ is:

$$w_{A,t} = \theta \frac{\kappa (1-\kappa)}{n^{1-\kappa}} \nu H_t^{m \frac{1-\alpha}{\alpha}} \left(\nu + (1-\nu) \left(\frac{1-\nu}{\nu} \frac{1}{1-\alpha} \right)^{\frac{\rho}{\rho-\kappa}} H_t^{m \frac{\rho}{\alpha} \left(\frac{\rho-\alpha}{\rho-\kappa} - 1 \right)} \chi^{\frac{\rho}{\alpha} \left(\frac{\alpha-\kappa+\alpha\kappa}{\kappa-\rho} \right)} \right)^{\frac{1-\rho}{\rho}} \chi^{\frac{1-2\alpha}{\alpha}}. \tag{WAGE-κ}$$

The point at which MRW-κ intersects the RTA curve is just the level of m_A which solves (31) when $w_{A,t} = \overline{w}$.

The MHC Curve with $\kappa \neq \alpha$ The level of labour input per producer is determined by setting the profit to using the old mode equal to the profit to using the new mode. When $\kappa \neq \alpha$ the condition on the available labour supply is:

$$\widetilde{n} = \left(\frac{1}{1-\kappa} \right)^{\frac{1}{\kappa}}. \tag{31}$$

The level of available female human capital per supplier using the new mode of organization when $\kappa \neq \alpha$ is:

$$\widetilde{m}_A = \begin{cases} (1-\kappa)^{\frac{1}{\kappa}} \frac{H^w}{H^m} & \text{if } \widetilde{m}_A < m_A \\ n_t^{-1} \frac{H_t^w}{H_t^m} & \text{with } n_t > \widetilde{n}, \text{ if } \widetilde{m}_{A,t} = m_{A,t} \end{cases} . \tag{MHC-λ}$$

The curve MHC-κ, at every point above the 45° line, is high relative to the basic model when $\kappa < \alpha$, , and low when $\kappa > \alpha$.

3.3.2 Simulation Results for the Model with Differential Returns to Specialization

The results of the baseline model are compared to three rates of $\kappa = \{0.4, 0.5, 0.8\}$. The results of these simulations are given in figures 19-23 and summarized in Figure 13.

When $\kappa < \alpha$ the substitutability of inputs into the production of A is low and the return to specialization is high. This leads to a lower level of technology in A (m_A) and a higher proportion of producers using the new mode of organization. Despite the decrease in the labour input per technology, the share of output of the final good is greater for lower levels of κ. The relative wage of clerical workers is lower, as is the proportion of the wage bill paid to workers in administration. The growth rates of these factors are largely independent of the level of κ.

4 Calibration of the Generalized Model

The calibration of the generalized model will improve on the basic model's ability to match the data if; the relative wage of workers in administration falls more rapidly and is at a higher level and if either the proportion suppliers using the new mode of production or the level of output of the those suppliers is higher.

4.1 Parameter Values

Remembering that the relative average wage of workers is administration is:

$$\frac{w_{A,t}}{w_{G,t}} = (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{1}{\chi_t}\right),$$

any factor which increases the growth rate of χ will decrease the rate at which the relative wage falls over time. The results of the simulation suggest that either setting $\kappa < \alpha$ or $\lambda > 1$ will have this effect.

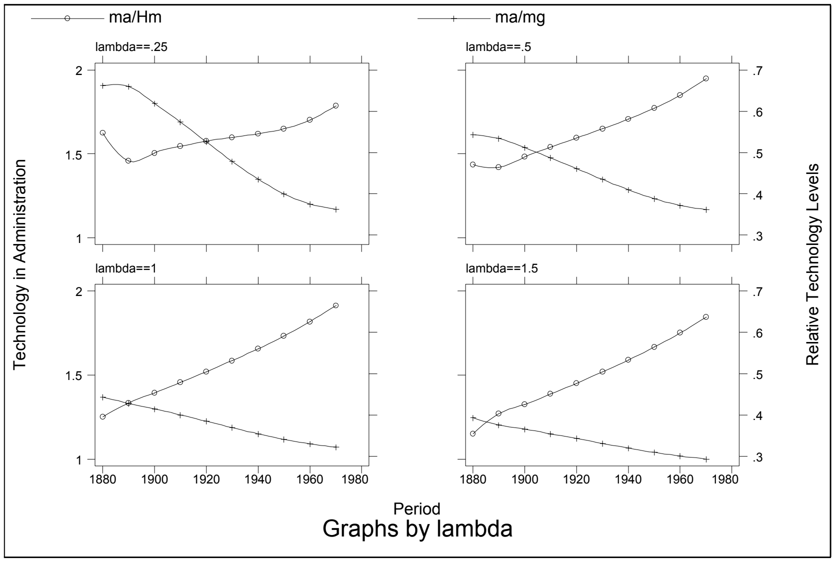

Figure 18: The level of technology in Administration and the relative level of technology in Administration for various levels of productivity, λ.

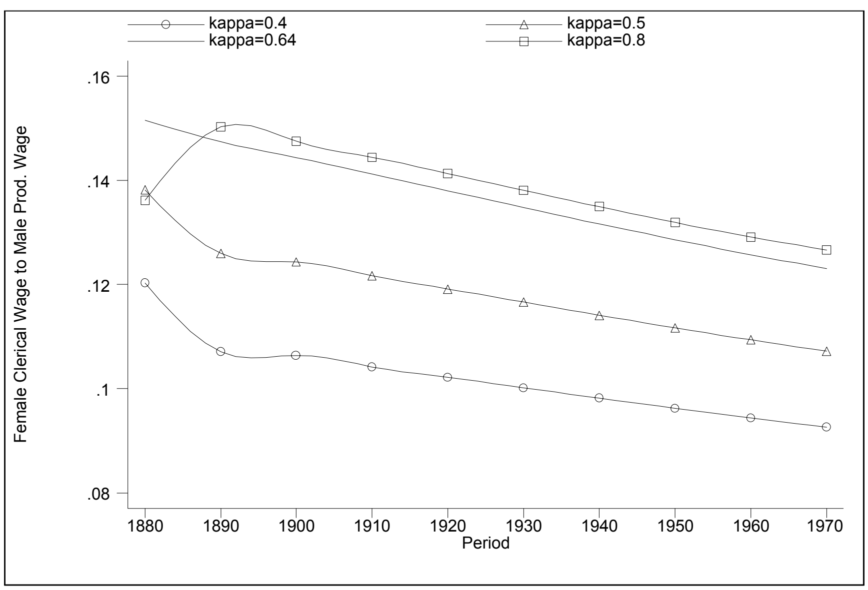

Figure 19: Comparative static results for the relative average wage of female clerical workers in the generalized model for various levels of κ.

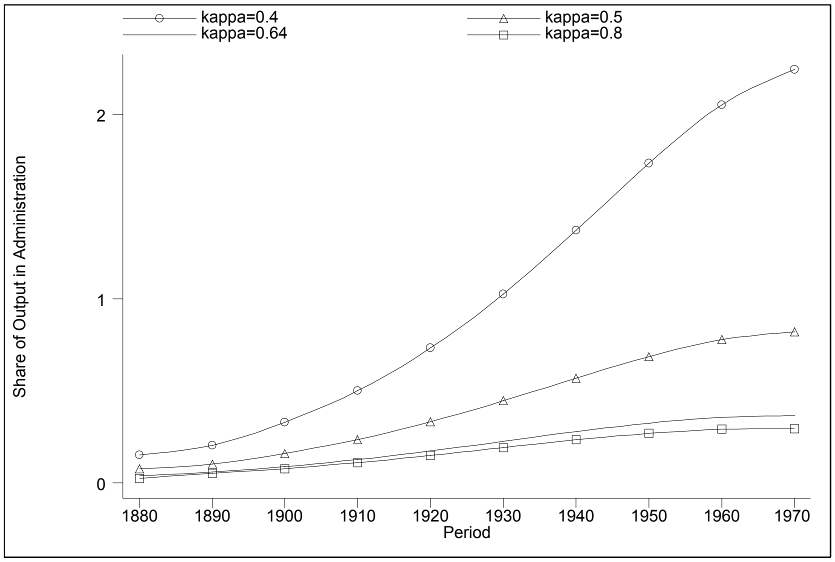

Figure 20: Comparative static results for the share of output in administration produced by suppliers using the new organizational mode in the generalized model for various levels κ.

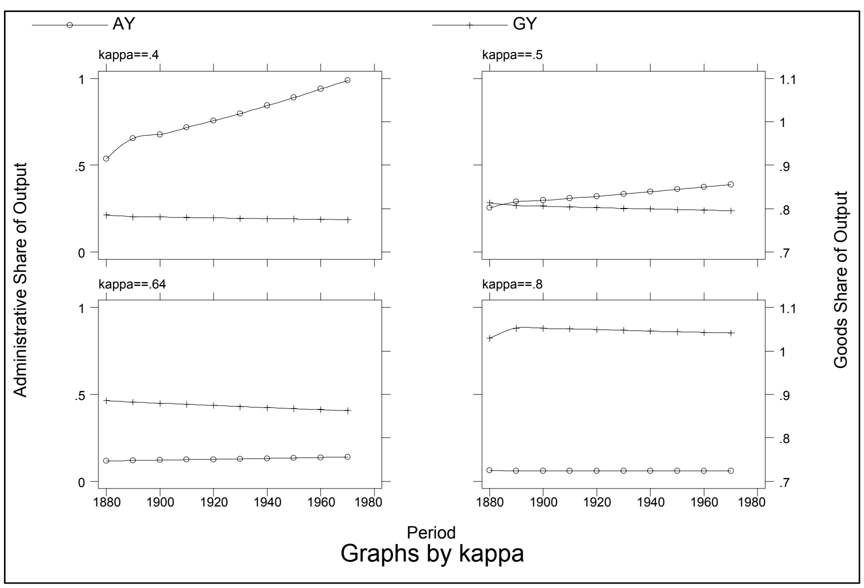

Figure 21: The share of output attributable to Administration and to Goods with various levels of specialization κ.

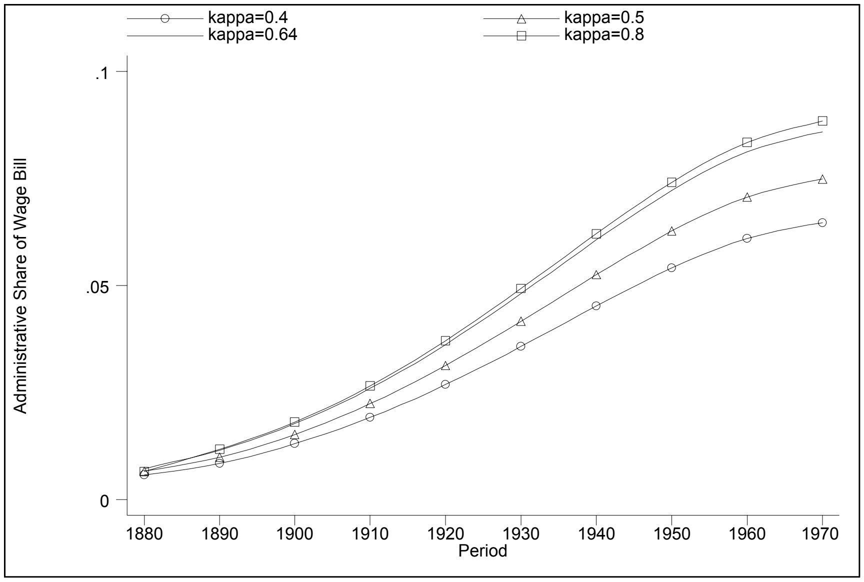

Figure 22: Comparative static results for relative wage bill to administrative workers in the generalized model with various levels of specialization, κ.

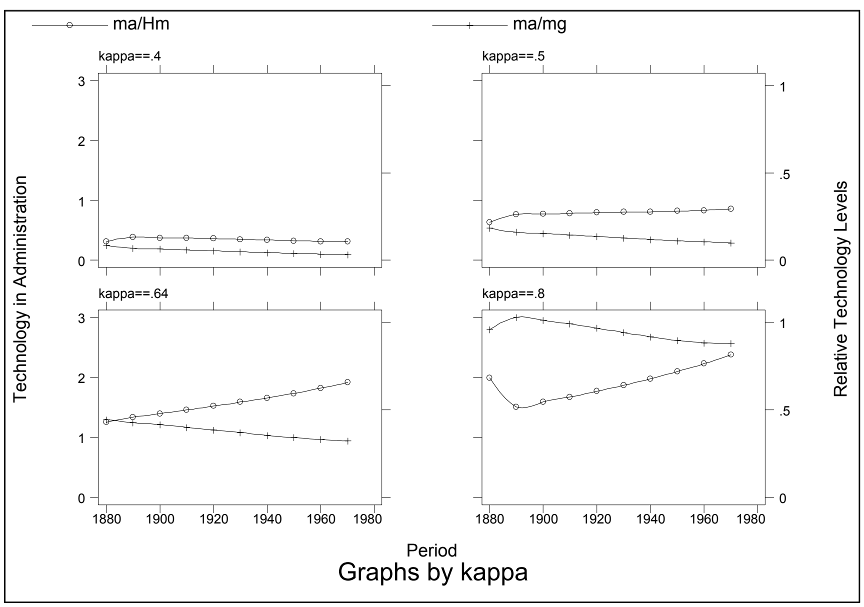

Figure 23: The level of technology in Administration and the relative level of technology in Administration for various levels of specialization, κ.

Output of individual producers using the new mode of production is a function of their labour input. Where each individual supplier uses $\widetilde{n}$ units of labour (in phase three), this is high when $\kappa > \alpha$ or when $\lambda > 1$.

Finally the proportion of suppliers using the new mode of organization increases, everything else held constant, for lower levels of λ, lower levels of κ and for higher levels of δ.

There is a trade off between levels of output from suppliers using the new mode and the relative wage. If κ is increased, such that the relative wage increases, the proportion of suppliers using the new mode of production decreases and the labour input per producer increases. If λ is decreased, such that the proportion of suppliers increases, both the labour input per producer falls proportionally and the relative wage decreases. The result is that in calibrating the general model it is possible to adjust parameter values to improve either the models fit with the relative wage or the share of output produced by suppliers using the new mode, but not both.

In choosing parameter values for the calibration I have attempted to improve the fit of the simulation results against the value of output produced by suppliers using the new mode of organization. This is achieved by choose values for κ and α that both lead to higher output from suppliers using the new mode, without greatly reducing the relative wage bill. The levels of λ and δ are chosen to be consistent with story. Increasing λ above the baseline level infers that the first worker employed reduces the overall output of the technology. If $\lambda < 1$, however, the productivity of the technology alone is lower then when it is combined with labour; the marginal product of the first worker is positive. In the calibration of the generalized model I set $\lambda < 1$. Given that a period of the model is 10 years it seems reasonable that some technology survive from one period to the next, such that $\delta < 1$.The parameter values used in the calibration of the generalized version of the model are in table 3.5.

4.2 Calibration Results of the Generalized Model

The results of the calibration are in figures 24 to 26. The improvement in the share of output from suppliers using the new mode of production from the basic model is significant. In the basic model, only 7% of output is produced by suppliers using the new mode in 1970. In the generalized version of the model, almost 20% of output in 1970 is produced by these suppliers (Figure 26). This increase in the share of output comes at a cost in terms of the relative wage bill. In the basic model the relative wage bill to administrative workers in 1970 is 15% (compared to 21% in the data). In the generalized version of the model this has dropped to just below 10% (Figure 24). This loss in the wage bill is mirrored in relative wages, from 22% in the basic model (compared to 66% in the data) to 14% in the generalized version of the model (Figure 25). The annual average growth rate of output of the final good is 3.14% compared with 3.53% in the data.

	Generalized
α	.70
κ	.56
ρ	-.5
θ	.5
λ	.75
δ	.9
β	.25
ν	.873
g_0^w	1
g^m	0.196
k	16.22
γ	1

Table 5: Parameter Values Used in the Calibration of the Generalized Model

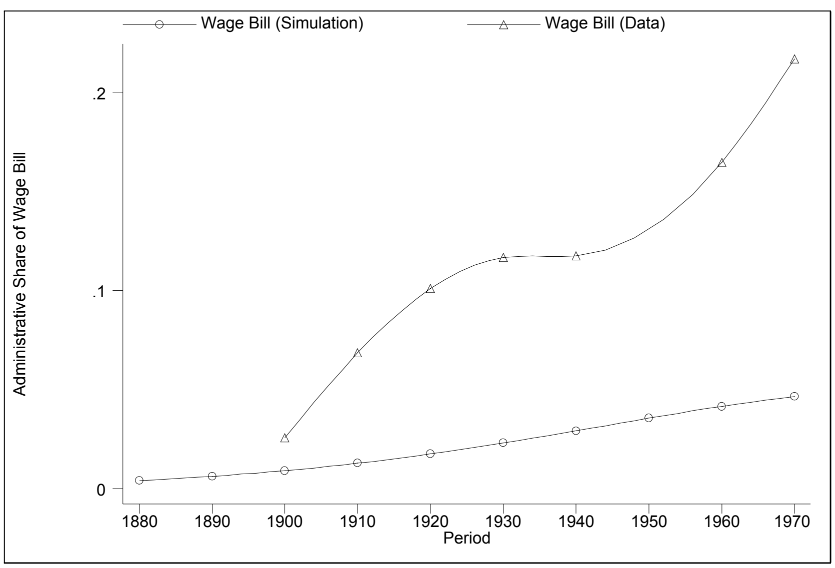

Figure 24: The ratio of the total wage bill paid to administrative workers to the total wage bill paid to production workers in both the simulation and the data in the generalized version of the model.

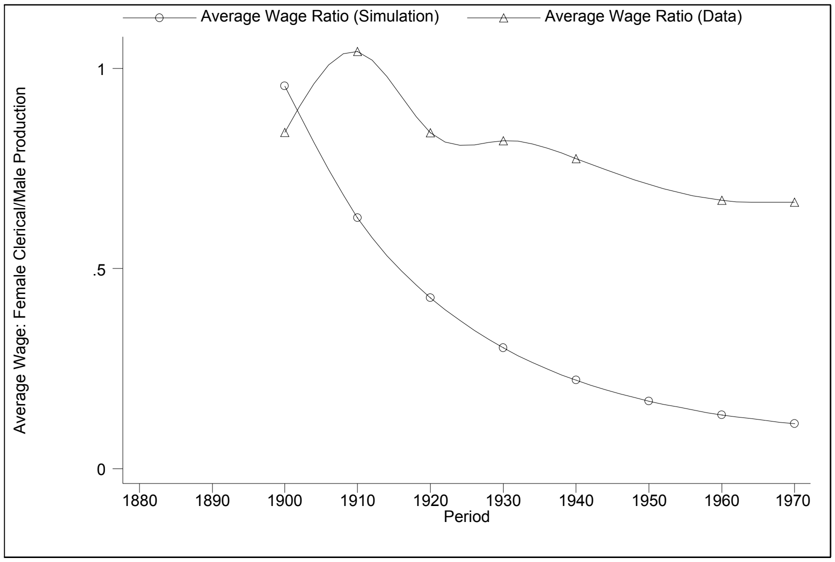

Figure 25: The ratio of average wages for female clerical to male production workers in the generalized version of the model.

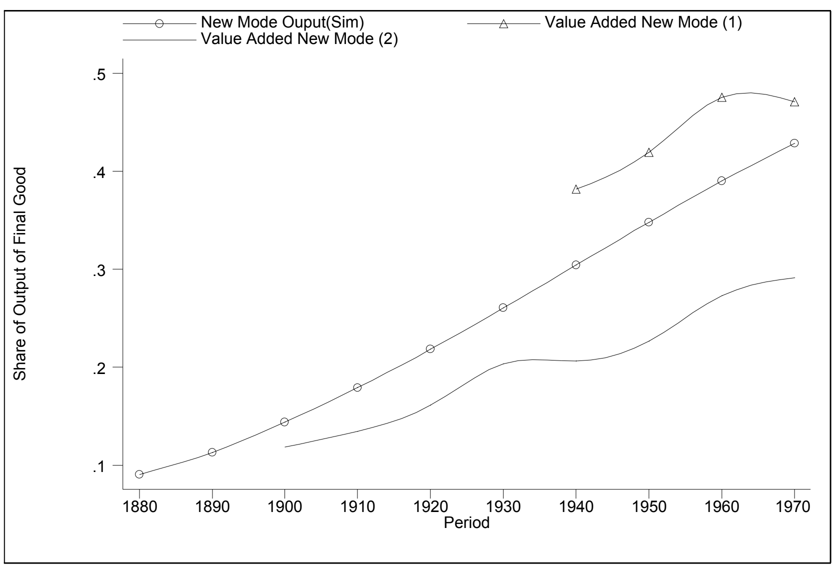

Figure 26: The share of of output of the final good produced by suppliers using the new organizational mode in the generalized version of the model.

5 Conclusion

The analytic results of the basic model are upheld by the calibration. As human capital increases both the level of employment and the wages of administrative workers increase. The wage bill paid to workers in administration increases relative to workers in manufacturing but the wage rate of workers in administration falls relative to workers in manufacturing. These results are consistent the movements in wages and employment in this period. The basic model falls short of predicting the level of the relative wages, however, and under predicts the rate at which industries adopt the new organizational structure. The generalized model improves on the ability to match increases in the share of output produced by suppliers in Administration using the new mode of production but in doing so reduces further the models ability to match relative wages. One possible extension might be to allow skilled men to enter the clerical work force. This would increase the rate at which the level of human capital available to the clerical sector grew relative to the manufacturing sector. The model predicts that doing so would increase the growth rate of the proportion of suppliers using the new mode of organization. The general equilibrium effects on the relative wages in the two sectors are difficult to predict however. In one respect, removing at least some of the skilled labour in the manufacturing sector could lead to lower average wages in that sector. If male clerical labour is perfectly substitutable for female clerical labour, however, and if labour demand increased by proportionally less than labour supply, the wages female clerical workers would also decrease. Taking the analysis one step further and allowing unskilled female labour to enter the manufacturing sector could potentially lead to a more complete picture of the movements of wages and employment in this period.

6 Appendix

6.1 Algorithm for solving the model

The level of χ, for each possible phase of the economy, is determined at the point at which the level of investment allocated to technology in Administration is equal to the level of savings allocated to technology in Administration. The of investment to technology in Administration in phases one and three, is the level of $m_A\left(\widetilde{m}_{A,t}, \chi_t\right)$ implicitly defined by:

$$m_{A,t} = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}} \left(\chi_t\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \widetilde{m}_{A,t}\left(\frac{\alpha}{1-\alpha}\right). \quad \text{(RTA)}$$

Where $n = \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}}$ when $\widetilde{m}_{A,t} < m_{A,t}$.

The level of savings allocated to technology in Administration is simply:

$$m_{A,t} = \frac{1}{\gamma} s_{t-1} - \chi_t. \tag{INV}$$

The level of χ is determined by setting the level of $m_{A,t}$ in (32) equal to the level of $m_{A,t}$ in (32). The solution is the level of χ which solves the equation:

$$(1-\alpha)^{\frac{\alpha}{\rho-\alpha}} (\chi)^{\frac{\alpha-\rho+\rho\alpha}{\alpha-\rho}} + \chi = \frac{1}{\gamma} s_{t-1} + \widetilde{m}_{A,t} \left(\frac{\alpha}{1-\alpha}\right), \tag{CHI}$$

Where the phase one level of χ is determined at the point at which $\widetilde{m}_{A,t} = 0$ and the phase level of χ is the point at which $\widetilde{m}_{A,t} = (1-\alpha)^{\frac{1}{\alpha}} \frac{H_t^w}{H_t^m}$.

The wage is:

$$w_A(\chi_t) = \frac{1-\alpha}{n_t^{1-\alpha}} \nu\theta H_t^{m\frac{1-\alpha}{\alpha}} \alpha \left(\nu + (1-\nu)\left((1-\alpha)^{-1}\left(\frac{1-\nu}{\nu}\right)\chi_t^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}} \chi_t^{\frac{1-2\alpha}{\alpha}}. \tag{WAGE}$$

The level of χ in phase two is the level of χ that solves (32) when $w_A = \overline{w}$.

The level of χ in phase four is determined by setting the $m_{A,t}$ in:

$$m_A = \frac{\left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}}{\left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}} + \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}} \frac{1}{\gamma} s_{t-1} \tag{RTA4}$$

equal to $m_{A,t}$ in (32) and solving for the level of χ_t^4 as:

$$\chi_t = \frac{\left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}}}{\left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}} + \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}} \frac{1}{\gamma} s_{t-1}. \tag{CHI-4}$$

The wage in phase four is:

$$w_A\left(\chi_t^4\right) = \alpha\nu\theta H_t^{m\frac{1}{\alpha}} \left(\frac{H_t^w}{H_t^m}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}} \left(\nu + (1-\nu)\left(\frac{1-\nu}{\nu}\right)^{\frac{\rho}{\alpha-\rho}} \left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}}\right)^{1-\rho} \left(\chi_t^4\right)^{\frac{1-\alpha}{\alpha}}. \tag{WAGE-4}$$

For a predetermined level of savings and male and female human capital, the algorithm used to simulate the model is as follows:

1. Determine if the economy is in phase one:

(a) Determine the level of χ_t^1 which solves (32) when $\widetilde{m}_{A,t} = 0$.

(b) Given χ_t^1 solve for $w_A\left(\chi_t^1\right)$ from (32).

(c) If $w_A\left(\chi_t^1\right) \leq \overline{w}$, the economy is in phase one: $\widetilde{m}_{A,t} = 0$ and $m_{A,t} = \frac{1}{\gamma}s_{t-1} - \chi_t^1$.

(d) If $w_A\left(\chi_t^1\right) \geq \overline{w}$, the economy is not in phase one.

2. Determine if the economy is in phase three:

   (a) Determine the level of χ_t^3, which solves (32) when $\widetilde{m}_{A,t} = (1-\alpha)^{\frac{1}{\alpha}} \frac{H^w}{H^m}$ and $n_t = (1-\alpha)^{-\frac{1}{\alpha}}$.

   (b) Given χ^3 solve for $w_A\left(\chi_t^3\right)$ from (32).

   (c) If both $w_A\left(\chi_t^3\right) > \overline{w}$ and $m_A\left(\chi_t^3\right) > (1-\alpha)^{\frac{1}{\alpha}} H^w$ the economy is in phase three: $\widetilde{m}_{A,t} = (1-\alpha)^{\frac{1}{\alpha}} \frac{H_t^w}{H_t^m}$, $n_t = (1-\alpha)^{-\frac{1}{\alpha}}$ and $m_{A,t} = \frac{1}{\gamma}s_{t-1} - \chi_t^3$.

   (d) If $w_A\left(\chi_t^3\right) < \overline{w}$ the economy is not in phase 3.

   (e) If both $w_A\left(\chi_t^3\right) > \overline{w}$ and $m_A\left(\chi_t^3\right) < (1-\alpha)^{\frac{1}{\alpha}} H^w$:

      i. Determine if the economy is in phase four:

         A. Determine the level of χ_t^4 from (32).

         B. Given χ_t^4 solve for the wage from (32).

         C. If both $w_A\left(\chi_t^4\right) > \overline{w}$ and $m_A\left(\chi_t^4\right) \leq (1-\alpha)^{\frac{1}{\alpha}} \frac{H^w}{H^m}$ the economy is in phase four: $\widetilde{m}_{A,t} = m_{A,t} = \frac{1}{\gamma}s_{t-1} - \chi_t^4$ and $n_t = \frac{H_t^w}{m_{A,t}}$.

         D. If $w_A\left(\chi_t^4\right) \leq \overline{w}$ the economy is not in phase four.

3. Determine if the economy is in phase two:

   (a) If $w_A\left(\chi_t^1\right) \geq \overline{w}$, $w_A\left(\chi_t^3\right) < \overline{w}$ and $w_A\left(\chi_t^4\right) < \overline{w}$, the economy is in phase 2.

   (b) The level of χ^2, is determined as the level of $\overline{\chi}_t$ which $w_A = \overline{w}$ in (32).

   (c) The level of $\widetilde{m}_{A,t}$ is the level of $\widetilde{m}_{A,t}$ that solves (32) when $\chi_t = \overline{\chi}_t$, $m_{A,t} = \frac{1}{\gamma}s_{t-1} - \overline{\chi}_t$ and $n = (1-\alpha)^{-\frac{1}{\alpha}}$.

4. Given the phase, and the levels of $m_{A,t}, \widetilde{m}_{A,t}, \chi_t, w_{A,t}, n_t$, determine $Y_t, G_t, A_t, w_{G,t}$, l_t and s_t.

5. Let $g_t^w = g_0^w - \frac{t}{k}$.

6. Let $H_{t+1}^w = (1+g_t^w) H_t^w$ and $H_t^m = (1+g^m) H_t^m$ if $g_t^w > g_t^m$ and $H_{t+1}^w = (1+g^m) H_t^w$ otherwise.

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