

**Comment on “The Optimal supply of Public Goods and the Distortionary Cost of Taxation” by Louis Kaplow (National Tax Journal, 1996, 513-32)<sup>1</sup>**

Dan Usher, May 2004

The government is choosing whether to undertake a project yielding a benefit of \$B and with a cost of \$C, where B is the sum of the benefits to each and every person in the land, where C is the government’s required expenditure on the project and where B and C may be present values. Should the project be undertaken a) if and only if  $B > C$ , or b) if and only if  $B > \alpha C$  for some  $\alpha$  greater than 1 reflecting taxpayers’ response to taxation? The former (a) is commonly referred to as the original Samuelson rule. The latter (b) is commonly referred to as the modified Samuelson rule. Kaplow argues in favour of the former procedure, opposing what he sees as an erroneous consensus among economists - a “conventional view” - in favour of the modified Samuelson rule.

Justification for this belated critique of Kaplow is three fold: that there are interesting situations in which Kaplow is right, that, as a guide to cost-benefit analysis, he is broadly-speaking wrong, and that the unresolved dispute creates uncertainty in the practice of cost-benefit analysis.

The conventional view rests upon the distinction between cost to the government and cost to the tax payer. Recall that B is defined as benefit to the taxpayer while C is defined as cost to the government. To draw an appropriate comparison, both values have to be attributed to the taxpayer. The cost of to the government must be scaled up account for the taxpayers’ burden of taxation of taxation over and above the actual tax he pays, cost of distortions in the economy arising from tax evasion, extra public expenditure to deter tax evasion, and the tax payer’s maneuvers to minimize his tax bill by switching from taxed labour to untaxed leisure and do-it-yourself activities as well as from more-taxed investment to less-taxed consumption. The full cost of taxation to finance a project in the public sector is not just the additional revenue required to finance the additional expenditure. It is the additional revenue plus additional deadweight loss - per dollar of additional revenue acquired. To say that the full cost to the tax payer exceeds the amount of tax he pays is to place a restriction on benefit-cost analysis, to undertake public projects - roads, bridges, hospitals, schools and so on - if and only if benefits exceed cost to the government by a margin sufficient to compensate for tax-induced distortions. It is difficult to say in practice just how large the margin should be, but there is a significant consensus that tax-induced distortions cannot be ignored altogether; public projects should not be undertaken unless B is significantly larger than C.<sup>2</sup>

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<sup>1</sup>Thanks to my colleague Robin Boadway and to Likun Liu for discussion and suggestions about the issues raised in this paper.

<sup>2</sup>See, for example, Charles Ballard and Don Fullerton, “Distortionary Taxes and the Provision of Public Goods”, *Journal of Economic Perspectives*, 1992, 117-31.

Kaplow disagrees. Against the conventional view that the critical benefit-cost ratio is significantly greater than 1, he presents his “unconventional view” that it is equal to 1 after all, that the original Samuelson supplies the appropriate test for benefit-cost analysis and that opinion to the contrary is a large misunderstanding of the principles of public finance.

Kaplow states that “the conventional view of economists is that the optimal supply of public goods is not determined by the simple cost-benefit test - which asks whether the sum of individuals’ benefits exceeds a project’s direct cost - because financing of public goods involves the use of distortionary taxation, notably, the income tax.....the literature uses estimates of the elasticity of labor supply to compute an approximate “marginal cost of public funds” for purposes of adjusting the simple cost benefit formula.” He “reexamines the relationship between the optimal supply of public goods and the distortionary cost of income taxation.” He “first demonstrates that, under standard simplifying assumptions (including the separability of leisure), there exists a way to modify the income tax to finance a public good such that there is no additional distortion...”

“The second part of the analysis addresses the situation in which a public good is not financed by the foregoing tax adjustment that offsets the benefits of the public good....In such cases there will be an appropriate change in redistributive benefits....In general it will be explained that the act of supplying a public good financed with a non-offsetting tax adjustment can be decomposed into 1) financing the public good with a tax adjustment that does offset the benefits of the public good (as described above) and 2) a further *purely redistributive* adjustment to the tax system.” (page 514) Kaplow devises a proof of the validity of the original Samuelson rule and this is followed by a general discussion of how the rule is distributionally neutral and of how deviation from the rule might be warranted in the interest of redistribution.

It would have been helpful for Kaplow to have described the conventional view more completely. At one level, the contrast between Kaplow’s view and the conventional alternative is clear enough. For governments choosing the amount of public goods or for deciding which projects, programs and activities to undertake, Kaplow advocates strict adherence to the original Samuelson rule and he speaks against those who would modify the rule by the imposition of a more stringent criterion. Kaplow’s paper lacks a detailed critique of the logic of the conventional view, specifying where exactly the conventional derivation of the modified Samuelson rule is mistaken.

The reader is not told whether Kaplow sees a) the modified Samuelson rule as incorrectly deduced from the assumptions of the model from which it is allegedly derived, b) the assumptions of the conventional model as so far from the facts of the world that no inference from the model can be other than worthless, or c) the modified Samuelson rule as picked from the air with no support from a coherent model of the taxpayer’s response to tax collection. In this context, it is not enough to prove yourself right on the strength of some model of the economy. You must also prove the other fellow wrong, at least in the sense that your assumptions fit the world better than his.

What exactly is the conventional view to which Kaplow objects, and who does he see as its adherents? These questions are difficult for a critic to answer because no official academy prescribes a conventional view for subservient economists to follow. Every author has his own way of looking at the world, and nobody likes to think of himself as conventional. Nonetheless, if Kaplow's views are to be taken seriously, it must be against a background of some presumably conventional alternative. For the purposes of this article, I take it upon myself to supply the missing doctrine. At some risk of imposing my own views upon others who Kaplow would classify as conventional, I shall present what I presume to be the conventional model, a model based upon a much-cited article by Atkinson and Stern<sup>3</sup>. I shall refer to this as the conventional view because it is how I see the convention and because I believe many other economists would accept the model as representative of their views too.

Broadly speaking, the original Samuelson rule is to produce public goods up to, but not beyond, the point where

$$[\text{the sum of the marginal benefits}] > [\text{the marginal cost}] \quad (1)$$

In this formulation, "the sum of the marginal benefits" is the sum of every person's benefit from the addition of one unit of the public good, and "the marginal cost" is the required expenditure by the government to procure an extra unit of the public good. By contrast, the modified Samuelson rule, to be developed precisely below, is to produce public goods up to, but not beyond, the point where

$$[\text{the sum of the marginal benefits}] > [(\text{the marginal cost}) \times (\text{the marginal cost of public funds}) \times (\text{the shadow price per dollar of benefit of the project})] \quad (2)$$

In this formulation, the "sum of the marginal benefits" and "the marginal cost" are exactly as defined in equation (1). The "marginal cost of public funds" is the cost to the taxpayer per dollar of public expenditure taking account of the extra deadweight loss per dollar of public

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<sup>3</sup>A. B. Atkinson and N. Stern, "Pigou, Taxation and Public Goods", *Review of Economic Studies*, 1974, 119-128. Interestingly enough, both papers - the Atkinson-Stern paper and Kaplow's critique - refer to the other's view as conventional. The switch in usage may be attributed in part to the universal predilection for designating the other people's views as conventional, but it may be due to the passage of time. Atkinson and Stern's paper appeared in 1974 against a background where the original Samuelson rule was looked upon as the proper foundation for cost-benefit analysis. Kaplow's paper appeared over twenty years later, by which time the original Samuelson rule had been displaced in many usages by the modified Samuelson rule, with its correction for the marginal cost of public funds. Perhaps the original Samuelson rule really was conventional (in the sense of being widely applied) in 1974, and the modified rule had become conventional by 1996.

expenditure from the distortions in the economy brought about when tax payers rearrange their affairs to reduce the amount of tax they pay. The “the shadow price per dollar of benefit of the project” is the effect on deadweight loss of people’s actions brought about by the provision of an extra unit of the public good.<sup>4</sup>

The contrast between these formulas suggests three introductory questions.

- 1) Where exactly does Kaplow’s model of taxation and public expenditure differ from the model or models from which the modified Samuelson rule is conventionally derived? When one model is claimed to yield the original Samuelson rule and another is claimed to yield the modified Samuelson rule, then either at least one of the two models is defective or differences in outcomes can be attributed to differences in assumptions. Assuming the latter, what precisely are the differences in assumptions yielding these differences in outcomes?
- 2) How sweeping a change is Kaplow recommending? Is he arguing that the original Samuelson rule should be applied immediately without necessarily changing the tax structure as well, or is his recommendation conditional on accompanying changes in the tax structure.
- 3) Might an economy ever be on the wrong side of the Laffer curve, and, if so, what becomes of the original Samuelson rule as a criterion for expenditure on public projects, programs and activities?<sup>5</sup>

From here on, our critique consists of three parts: a review of the derivation and assumptions of the modified Samuelson rule, a proof that the modified rule boils down to the original Samuelson rule in one especially interesting case, and a comparison of the

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<sup>4</sup>Deadweight loss arises in a classic prisoners’ dilemma. When I devote resources worth \$1.00 to reducing my tax bill by \$1.10, I become better off by 10¢ but the rest of society is worse off by \$1.10. This is true no matter what the nature of my manoeuvre to reduce my tax bill: taking more leisure, engaging in more do-it-yourself activities, saving less or avoiding tax by legal or illegal means. When each person in a community of 100 people devotes \$1.00 to reducing his tax bill by \$1.10, there is a loss of \$100.00 to society as a whole. The usage of resources to reduce one’s tax bill is a loss both to the community and to oneself, while each person’s gain in tax saved is at the expense of the rest. The key assumption in the analysis of deadweight loss is that people cannot enforce an agreement among themselves to avoid such tax-saving manoeuvres. That the marginal cost of public funds exceeds 1 is a consequence of deadweight loss, regardless of its source.

<sup>5</sup>For the pros and cons of the proposition that the American economy is already on the wrong side of the Laffer curve, see Martin Feldstein, “The Effects of Marginal Tax Rates on Taxable Income: A Panel Study of the 1986 Tax Reform”, *Journal of Political Economy*, 1995, 551-72 and Austin Goolsbee, “What happens when You Tax the Rich? Evidence from Executive Compensation”, *Journal of Political Economy*, 2000, 352 -78.

redistributional neutrality of the income tax and the lump sum tax. Our conclusion will be that, though Kaplow is actually right on certain assumptions, those assumptions are much more restrictive than the reader of his paper might suppose. The upshot of Kaplow's critique is not to overturn the conventional view but to strengthen it by clarifying the assumptions on which it is based and by identifying considerations that might otherwise be overlooked.

### Derivations of the Original Samuelson Rule and the Modified Samuelson Rule

There are many routes by which taxation may distort the economy: the choice between labour and leisure, the choice between paid work and do-it-yourself activities, the choice between consumption and investment, the choice between paying tax and evading tax. For analytical convenience rather than because of its intrinsic importance, this literature focuses exclusively on the labour-leisure choice. The diversity of publicly-supplied projects, programs and activities is suppressed by the assumption the government produces one and only public good. Government's only choice is the amount of the public good and the corresponding tax to pay for it. Public revenue is assumed to be acquired by a proportional income tax at a rate  $t$ .

Consider an economy with  $N$  identical people with utility functions

$$u = u(x, L, G) \quad (3)$$

where  $x$  is a person's consumption per unit of time,  $L$  is his supply of labour and  $G$  is the total supply of a public good. The technology of this economy is represented by a production possibility frontier

$$Nx + P_G G = NwL \quad (4)$$

where  $w$  is the real wage (consumption goods produced per hour of labour) and  $P_G$  is the cost of the public good (graduated in terms of the consumption good).

An omnipotent, omniscient and benevolent planner would choose  $x$  and  $L$  as well as  $G$  to maximize utility in equation (3) when constrained by the production possibility frontier in equation (4). The planner's choice is in accordance with the original Samuelson rule in equation (1) because no such planner need tolerate tax-induced distortions. The planner chooses  $x$ ,  $L$  and  $G$  to maximize  $u(x, L, G)$  subject to the constraint that  $Nx + P_G G = NwL$ . The first order conditions of this constraint are

$$-u_L/u_x = w \quad (5)$$

and 
$$N(u_G/u_x) = P_G \quad (6)$$

where equation (6) is the precise specification in this context of the original Samuelson rule of equation (1).

Tax-induced distortions arise when  $x$  and  $L$  are chosen by the taxpayer rather than by the government. Think of the taxpayer as choosing  $x$  and  $L$  when presented with a supply of the public good,  $G$ , and a tax rate,  $t$ , both seen as invariant not because the taxpayer is ignorant of the role of the government in the economy, but because each taxpayer knows himself to be too small a part of the economy for his actions to have any significant effect on outcomes in the economy as a whole. By contrast, the government chooses  $t$  and  $G$  in conformity with the production possibility frontier and in full recognition of how the taxpayers' self-interested choice of  $x$  and  $L$  responds to the government's choice of  $t$  and  $G$ .

For any given  $t$  and  $G$ , the taxpayer chooses  $x$  and  $L$  to maximize utility subject to *his* budget constraint

$$x = wL(1 - t) \quad (7)$$

where  $t$  is the rate of the income tax. Specifically, the taxpayer chooses  $x$  and  $L$  to maximize the Lagrangian

$$\mathcal{L} = u(x, L, G) - \alpha[x - wL(1-t)] \quad (8)$$

The first order conditions become

$$u_x = \alpha \quad \text{and} \quad -u_L = \alpha w(1-t) \quad (9)$$

from which it follows that

$$-u_L/u_x = w(1 - t) \quad (10)$$

which differs from the planner's first order condition in equation (5) because the rate of trade-off in production between  $x$  and  $L$  differs from the taxpayer's rate of trade-off in use.

From equations (7) and (9) it follows that

$$x = x(G, t) \quad \text{and} \quad L = L(G, t) \quad (11)$$

because  $u_x$  and  $u_L$  are both functions of  $x$ ,  $L$  and  $G$ .

Since the taxpayer's budget constraint (7) is true for any and every  $t$  and  $G$ ,

$$x_t = w(1 - t)L_t - wL \quad \text{and} \quad x_G = w(1 - t)L_G \quad (12)$$

Then, recognizing how taxpayers respond to its choice of  $t$  and  $G$ , the government chooses  $t$  and  $G$  to maximize its Lagrangian

$$\mathcal{L} = u(x, L, G) - \beta[P_G G - twLN] \quad (13)$$

subject to its budget constraint<sup>6</sup>

$$P_G G = t w L N \quad (14)$$

The first order condition with respect to  $t$  becomes

$$u_x x_t + u_L L_t + \beta [t w N L_t + w L N] = 0 \quad (15)$$

From equations (9) and (12), it follows that

$$u_x x_t + u_L L_t = \alpha x_t - \alpha w (1-t) L_t = -\alpha w L, \quad (16)$$

allowing equation (15) to be rewritten as

$$(\beta N / \alpha) [1 + (t/L) L_t] = 1 \quad (17)$$

where  $(t/L) L_t$  can be interpreted as the elasticity,  $\epsilon_{Lt}$ , of leisure with respect to the tax rate.

Similarly, the first order condition of the government's Lagrangian with respect to  $G$  is

$$u_x x_G + u_L L_G + u_G - \beta [P_G - t w N L_G] = 0 \quad (18)$$

which, recognizing that  $u_x x_G + u_L L_G = 0$  and dividing both sides of the equation by  $N/u_x$ , simplifies to

$$N(u_G / u_x) = (N\beta / \alpha) [P_G - t w N L_G] \quad (19)$$

Replacing the expression  $N\beta / \alpha$  in equation (19) by its value in equation (16) yields the modified Samuelson rule.

$$\begin{aligned} N(u_G / u_x) &= P_G \{1 / [1 + (t/L) L_t]\} \{1 - (G/L) L_G\} \\ &= P_G \{1 / [1 + \epsilon_{Lt}]\} \{1 - \epsilon_{LG}\} \end{aligned} \quad (20)$$

where  $\epsilon_{Lt}$  and  $\epsilon_{LG}$  are the elasticities of labour supply with respect to the tax rate and the supply of public goods. Equation (20) is precisely the modified Atkinson and Stern modification of the Samuelson rule in equation (2) above. The original Samuelson rule is adjusted by the two

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<sup>6</sup>Note that , together, the equations for the production possibility frontier [ $Nx + P_G G = NwL$ ] and the budget constraint of the taxpayer [ $x = wL(1-t)$ ] imply equation (14).



expressions in curly brackets.<sup>7</sup> The first, called “the marginal cost of public funds”, is the correction for the effect of taxation on the supply of labour; this expression must be greater than 1 as long as  $L_t$  is negative, as it would be whenever leisure is a normal good. This expression reflects the taxpayer’s response to taxation regardless of the amount of public goods supplied. Generalizing somewhat, we would expect the expression to remain as it is regardless of the nature and composition of the projects, programs or activities of the government. The second expression, called “the shadow price of the public goods”, is the correction for the effect of the provision of the public good upon the taxpayers’ supply of labour; it may be greater or less than 1 depending on whether an increase in the supply of the public good augments or diminishes the taxpayer’s willingness to work.

Five aspects of Atkinson-Stern’s formula should be noted before proceeding to Kaplow’s refutation.

1) The Identical Twins Assumption: All citizens are identical and are treated identically by the government. This assumption supplies a clear and unambiguous criterion for public policy with no need for interpersonal comparison and no recognition of the possibility that the best policy for one person is not the best policy for others.

2) The Proportional Income Tax Assumption: All public revenue is raised by a proportional income tax.

3) The Double Correction: The original Samuelson Rule is modified in two respects, for the marginal cost of public funds - represented by the term  $[1/(1 + (t/x)\delta x/\delta t)]$  - and for the shadow price of public expenditure - represented by the term  $[1 - N(t/P_G)\delta x/\delta G]$ . The modified Samuelson rule is reduced to the original rule if the marginal cost of public funds and the shadow price of public expenditure can be ignored ( that is, if they are both equal to 1) or if they cancel out.

4) The Single Public Good: The model abstracts from the nearly infinite diversity of public expenditure. The assumption is important in this context because different items of public expenditure have different degrees of substitutability with labour. Some public expenditures induce a switch in private usage of time from labour to leisure, accentuating the tax-induced distortion associated with the marginal cost of public funds. Other public expenditures do just the opposite

5) The Single Tax-induced Distortion: In this model, the modified Samuelson rule differs from the original rule because, and only because, the increase in the supply of the public good requires an increase in the tax rate generating an increase in the tax-induced diversion of taxpayer’s time from taxed labour to untaxed leisure. Abstracted away in this formulation are the tax-induced diversions of time from labour to do-it-yourself activities, of time from productive

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<sup>7</sup>Equation (20) is a rearrangement of equation (3) in Atkinson and Stern, *op cit*.



labour to be used for tax evasion or tax avoidance and of expenditure from saving to current consumption.

The assumptions of the Atkinson-Stern model are quite restrictive in one sense but significantly less so in another. The assumptions, as stated above, wipe away much that is relevant for the conduct of cost-benefit analysis, but the formula in equation (19) generalizes well beyond the narrow context in which it is derived. The formula shows the original Samuelson rule modified in accordance with two elasticities, the elasticity,  $\epsilon_{Lt}$ , of tax base to tax rate in the marginal cost of public funds, and the elasticity  $\epsilon_{LG}$ , of tax base to the supply of the public good. Strictly speaking, the tax base is income,  $wL$ , rather than labour supply,  $L$ , but the two are the same for all practical purposes because the wage,  $w$ , is assumed to be invariant. The modified Samuelson rule is in practice considerably broader than its derivation might suggest:

The modified Samuelson rule applies with modest changes to all sources of distortion in the tax system: the choice between work-for-pay and do-it-yourself activity, the choice between consumption and investment in human and physical capital, the choice of how much tax to evade, legally or illegally, when avoidance is for one reason or another costly. Any tax-induced contraction of the tax rate gives rise to a modified Samuelson rule similar to that in equation (19).

The modified Samuelson rule boils down to the original Samuelson rule when either a) the tax base (whatever it may be) is invariant to the tax rate and the supply of public goods, or b) the elasticities  $\epsilon_{Lt}$  and  $\epsilon_{LG}$  are such that the corrections to the original Samuelson rule cancel out. If  $\epsilon_{Lt} = 0$ , then the marginal cost of public funds is equal to 1. If  $\epsilon_{LG} = 0$ , then the shadow price of the public good is equal to 1. If  $(1 + \epsilon_{Lt}) = (1 - \epsilon_{LG})$ , then the original Samuelson rule is restored.

The formula remains approximately valid when there are many different kinds of taxes and many different kinds of public goods, but the two extra expressions have to be interpreted differently. The marginal cost of public funds would be the same for all sorts of taxes because the taxpayer is made as well off as he can be when marginal distortions per additional dollar of revenue acquired are the same. The shadow price of public projects and public goods would differ from one project to the next because projects have different impacts upon the tax base. Some projects induce taxpayers to devote time to the acquisition of highly taxed goods. Other projects induce taxpayers to devote time to the acquisition of less taxed or untaxed goods. The greater the effect of the project on the taxpayer's propensity to acquire highly taxed goods, the larger  $\epsilon_{LG}$  and the smaller the shadow price of the project must be.

It is in the light of the potential generalization of the modified Samuelson rule in equation (19) that one should assess the mismatch, bordering on blatant contradiction, between the identical twins assumption (1) and the proportional income tax assumption (2). If people really were identical, they would not be so stupid as to raise public revenue through a proportional income tax. Rather they would levy a lump sum tax for which there really is no deadweight loss,

and for which the original Samuelson rule would be strictly valid. That however is no real criticism of the model where the identical twins assumption is introduced for analytical convenience, and the proportional income tax assumption is employed because we know full-well that the identical twins assumption is never true and that tax systems must be restricted to what might be appropriately imposed in a world where people are far from identical.<sup>8</sup> It is hardly a valid criticism of this model that the identical twin assumption and the proportional income tax assumption are unrealistic, for *all* economic analysis - indeed all clear thinking - is based on strategic assumptions that are imperfectly descriptive of the world.

### Kaplow's Attack on the Conventional View

Kaplow's major premise is that the combination of additional public goods and additional tax to pay for them is non-distortionary if each person's additional tax payment is covered, or more than covered, by his benefit from the additional public good. "When a public good is financed using an offsetting adjustment to the income tax scheme, each individual's monetary benefit from the public good just equals the additional tax he pays. If individuals who earn \$20,000 have a benefit of \$126, they will be paying an additional \$126 in taxes; if those earning \$21,000 have a benefit of \$135, their additional tax will be \$135. Thus, the direct effect of the public good combined with such a tax is to leave everyone as well off as before, as long as each individual chooses to earn the same level of income." (517) Kaplow then argues in the following paragraph that "it can readily be demonstrated that there can be no effect on behaviour when the tax adjustment is offsetting, *if one assumes that individual's utility functions are weakly separable in the disutility of labor.* (Italics added).

As a general proposition, and ignoring the qualification in italics, Kaplow's assertion that "there can be no effect on behaviour when the tax adjustment is offsetting" would be completely wrong. It would be wrong by ignoring the non-exclusive property of public goods. Even if every person's benefit from a project exceeds his portion of the additional tax imposed to pay for it, there remains there remains an incentive to alter one's behaviour so as to reduce one's tax bill, for each person's benefit from a project is what it is regardless of his behaviour, while any benefit from a person's manoeuvres to reduce his tax bill accrue to that person alone. It is precisely this dichotomy that accounts for the displacement of the original Samuelson rule by the

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<sup>8</sup>Another consideration becomes important when a social-welfare-maximizing government chooses a tax structure for a two class society to maximize the welfare of the poor for any given level of welfare of the rich and any given provision of the public good. In that case the original Samuelson rule must be modified for differences between rich and poor in their marginal valuations of the public good in terms of the private good. Broadly speaking, the sum of the marginal valuations of the public good should exceed the marginal cost if and only if the marginal valuation of the poor exceeds the marginal valuation of the rich. For a thorough analysis of this proposition and its qualifications, see Boadway, R. And Keen, M., "Public Goods, Self-Selection and Optimal Income Taxation", *International Economic Review*, 1993, 463-78.

modified Samuelson rule in what Kaplow calls the conventional view.

What then is the power of “weak separability” to nullify this line of argument? Note particularly that the identical twins assumption of the Atkinson-Stern model guarantees that “the direct effect of the public good combined with such a tax is to leave everyone as well off as before, as long as each individual chooses to earn the same level of income.” Our problem becomes to ferret out the economic significance of weak separability and then to determine whether it really does bear the weight that Kaplow places upon its shoulders.

Though the mathematical interpretation of weak separability is clear enough, its economic significance is less evident. Is weak separability a trivial assumption introduced for analytical convenience, or does it entail a major restriction upon the role of public goods and public projects in the economy? Does the introduction of the weak separability modify the conclusions of the Atkinson-Stern model so profoundly as to restore the original Samuelson rule as the proper criterion for cost-benefit analysis?

I shall argue in the following two sections that 1) weak separability is almost equivalent to the assumption that the public good is an intermediate product, 2) that this restriction on the role of the public good in the economy does cause the Atkinson-Stern model to yield the original rather than the modified Samuelson rule and 3) regardless of the role of public goods in the economy, the original Samuelson rule emerges if the proportional income tax assumption of the Atkinson-Stern model is replaced by personalized taxation. The first two assertions will be discussed in the following section. The third assertion will be discussed afterwards.

### Does the Double Correction Cancel Out?

There is a sense in which proponents and critics of the conventional view, as described above, are talking at cross purposes. Proponents of the conventional view focus primarily upon the marginal cost of public funds,  $1/[1 + \epsilon_{Lt}]$ , which is the first correction to  $P_G$  on the right hand side of equation (19). They claim the marginal cost of public funds to be greater than 1 because the elasticity of tax base to tax rate - the term  $\epsilon_{Lt}$  - is negative. The tax base shrinks as the tax rate increases as a reflection of people’s incentives to rearrange their affairs to minimize their tax bill. Such rearrangements are always expensive, but, the higher the tax bill, the more one is prepared to spend to reduce the tax base. That is proved in the preceding section for a proportional income tax influencing the tax payer’s labour-leisure choice. It is believed to true much more generally: for other tax structures and for other opportunities to influence one’s tax base.

By contrast, Kaplow is concerned with the entire gap between the sum of the marginal benefits and the marginal cost: between  $N(u_G/u_x)$  and  $P_G$  in equation (19). He argues, in effect that there is no such gap, an argument that can only be valid on one of three conditions: that the logic of the derivation of equation (19) is faulty, that one or more of the assumptions on which the equations is derived is unreasonable, or that the two corrections in equation (19) cancel out, leaving the original Samuelson rule in tact.

Why, it may be asked at this point, do the proponents of the conventional view put so much emphasis on the first correction and so little on the second? The reason, as mentioned briefly above, is that the marginal cost of public funds is seen as being what it is for all public investment, while the shadow price is seen as differing from one project to the next. If there are two sources of taxation, tax rates for each source should be such that their marginal cost of public funds are the same. There is no analogous rule for public projects. The shadow price of one may be high; the shadow price of another may be low. My favourite example of this is the contrast between a marina and a park. To make use of a marina, one must buy a yacht. To afford a yacht, one must work more, earn more, take less leisure and, in the process, pay extra tax on the extra hours of paid labour. When the public project is a marina, the term  $\epsilon_{LG}$  in the expression for the shadow price in equation (19) must be positive and the shadow price itself - the expression  $\{1 - \epsilon_{LG}\}$  must be less than 1. To make use of a park, one must merely take the time to smell the flowers or participate in the community baseball game. One works less and takes more leisure so that the term  $\epsilon_{LG}$  in the expression  $\{1 - \epsilon_{LG}\}$  is negative and the expression itself is greater than 1. negative. Nothing in the usual presentation of the modified Samuelson rule precludes the possibility that the shadow price of a public expenditure may sometimes cancel out with the marginal cost of public funds, leaving the original Samuelson rule intact, exactly as Kaplow claims. But to admit that the shadow price and the marginal cost of public funds *might* cancel out for some projects is not to claim that they *must* cancel out for all projects. The conventional view is that shadow prices of some projects neutralize the marginal cost of public funds, while shadow prices of other projects magnify the marginal cost of public funds.

It has always been recognized that the marginal cost of public funds and the shadow price of public expenditure may, for some expenditures, cancel out. In their derivation of the modified Samuelson rule, Atkinson and Stern stated that their analysis threw “no light on whether output levels in the full optimum are greater or less than the optimum levels when expenditure is financed by distortionary taxation”(126). On the other hand, authors writing about taxation have tended to ignore the shadow price of public expenditure , not denying its existence but reasoning instead that the two corrections are best kept in separate boxes and analyzed separately. One box contains the marginal cost of public funds, defined as the gap between the value to the consumer of a dollar in his own hands and the full cost to the consumer of a dollar in the hands of the government, where the full cost to the consumer of a dollar in the hands of the government is the sum of marginal tax revenue and marginal deadweight loss associated not just with the choice between labour and leisure choice but with the choices between saving and investment, between work for pay and do-it-yourself activity and between tax payment and legal or illegal tax avoidance. The other box contains a wide range of public expenditures, each with its own impact on the behaviour of the tax payer and each with its own shadow price of public expenditure.

In short, if Kaplow’s “unconventional” view is right and if the modified Samuelson rule in equation (20) is not wrong, then the two expressions in curly brackets in equation (20) must cancel out. Kaplow’s defense of his “unconventional” view - his proof of the validity of the original Samuelson rule in the appendix to his article - can be interpreted as showing that the

expressions do cancel out in one important case. To get this result, Kaplow postulates a utility function of the form

$$u = v(wL(1 + g)(1 - t)) - z(L) \quad (21)$$

where  $u$ ,  $v$  and  $z$  refer to utility functions,  $L$  is the supply of labour,  $w$  is the going wage rate,  $t$  is the tax rate and  $g$  is a measure of the productivity of the public good. Equation (21) exemplifies what Kaplow referred to as weak separability in the quotation presented above. However equation (21) is equivalent to a pair of equations

$$u = u(x, L) \quad (22)$$

and 
$$x = wL(1 + g)(1 - t) \quad (23)$$

the main point being that utility is in the first instance a function of two arguments,  $x$  and  $L$ , rather than three,  $x$ ,  $L$  and  $G$ , as in the Atkinson-Stern model. The term  $g$  must be a measure of the productivity of the public good rather than a measure of the public good itself.

In developing Kaplow's argument formally, it is helpful to generalize equation (23), making  $x$  a function of the total supply of the public good,  $G$ , rather than of the productivity of public goods,  $g$ . To do so, we define pre-tax income per hour of labour as  $W(G)$ , and, on that assumption, we retrace the steps in the derivation of equation (20). Once again, an omnipotent, omniscient and benevolent planner would choose  $x$ ,  $L$  and  $G$  to maximize the typical person's utility subject to the constraint of a production possibility

$$Nx + P_G G - NW(G)L = 0 \quad (24)$$

The Lagrangian and its first derivatives are

$$\mathcal{L} = u(x, L) - \alpha[Nx + P_G G - NW(G)L] \quad (25)$$

$$\mathcal{L}_x = u_x - \alpha N = 0 \quad (26)$$

$$\mathcal{L}_L = u_L - \alpha w(G)N = 0 \quad (27)$$

and 
$$\mathcal{L}_G = \alpha[P_G + NW'(G)L] = 0 \quad (28)$$

from which it follows immediately that

$$-u_L / u_x = w(G) \quad (29)$$

and 
$$N[w'(G)L] = N(u_G / u_x) = P_G \quad (30)$$

which reproduces the Samuelson rule in equation (6) as long as  $u_G$  is understood as  $du/dG$

derived by differentiating  $\mathcal{L}$  with respect to  $G$ .<sup>9</sup>

Now reconsider the double maximization procedure where, for any  $t$  and  $G$ , the taxpayer chooses  $x$  and  $L$  to maximize utility, and then the welfare-maximizing planner, taking account of the taxpayer's behaviour, maximizes the taxpayer's utility in the choice of  $t$  and  $G$ . The taxpayer choosing  $x$  and  $L$  in an environment where  $t$  and  $G$  are looked upon as invariant. He chooses  $x$  and  $L$  to maximize  $u(x, L)$  subject to a budget constraint

$$x = rL \quad (31)$$

where  $r$  is his net after-tax wage

$$r = w(G)(1 - t) \quad (32)$$

which he looks upon as invariant regardless of his choice of  $x$  and  $L$ . Utility becomes

$$u(x, L) = u(rL, L) \quad (33)$$

so that 
$$u_x r + u_L = 0 \quad (34)$$

or equivalently 
$$-u_L/u_x = r \quad (35)$$

From these equations it follows that

$$x = x(r), \quad L = L(r), \quad (36)$$

and 
$$\delta r/\delta G = w'(G)(1 - t), \quad \delta r/\delta t = -w(G),$$

$$\delta x/\delta G = (\delta x/\delta r)(\delta r/\delta G), \quad \delta L/\delta G = (\delta L/\delta r)(\delta r/\delta G),$$

$$\delta x/\delta t = (\delta x/\delta r)(\delta r/\delta t) \quad \text{and} \quad \delta L/\delta t = (\delta L/\delta r)(\delta r/\delta t).$$

Recognizing the effects of  $t$  and  $G$  upon the taxpayer's choice of  $x$  and  $L$ , the government now chooses  $t$  and  $G$  to maximize

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<sup>9</sup>Differentiating the Lagrangian in equation (25) with respect to  $G$  yields the equation

$$du/dG = \alpha NW'(G)L = (u_x / N)NW'(G)L$$

from which it follows that  $(du/dG)/u_x = w'(G)L$

$$\mathcal{L} = u(x, L) - \beta[Nx + P_G G - NW(G)L] \quad (37)$$

which is the same as the objective function of the omniscient planner except for the substitution of  $\beta$  for  $\alpha$  as the Lagrange multiplier. Differentiating with respect to  $t$ ,

$$\mathcal{L}_t = u_x \delta x / \delta t + u_L \delta L / \delta t - \beta N[\delta x / \delta t - w(G) \delta L / \delta t] = 0 \quad (38)$$

Differentiating with respect to  $G$ ,

$$\mathcal{L}_G = u_x \delta x / \delta G + u_L \delta L / \delta G - \beta N[\delta x / \delta G - w(G) \delta L / \delta G] + \beta \{N[w'(G)L] - P_G\} = 0 \quad (39)$$

which, using properties of the taxpayer's optimization problem, implies that

$$\mathcal{L}_G = \mathcal{L}_t \{(\delta r / \delta G) / (\delta r / \delta t)\} \{-w'(G)(1-t) / w(G)\} + \beta \{N[w'(G)L] - P_G\} = 0 \quad (40)$$

Since  $\mathcal{L}_t = 0$ , equation (40) reduces to

$$N[w'(G)L] = P_G \quad (41)$$

which is precisely the original Samuelson rule as derived above for the omniscient and omnipotent planner.

Figure 1 is a simple representation of this result. The axes of the figure are hours of leisure,  $H$ , and total consumption,  $x$ . The utility function in equation (21) can be reformulated as

$$u = u(x, L) = u(x, 24 - H) = u(x, H) \quad (42)$$

and is illustrated by the dashed indifference curves on the figure. Also shown on the figure is a production possibility frontier for  $x$  and  $H$  constructed on the assumption that, for any given  $H$ , the value of  $G$  is chosen to maximize  $x$ .<sup>10</sup> The production possibility curve must cut the vertical

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<sup>10</sup> From the production possibility frontier in equation (21), it follows that

$$Nx + P_G G - NW(G)L = Nx + P_G G - NW(G)(24 - H) = 0$$

Fix  $H$  (and therefore  $L$ ) and take the total derivative of the left hand side of this expression with respect to  $x$  and  $G$ .

$$Ndx + [P_G - NW'(G)L]dG = 0$$

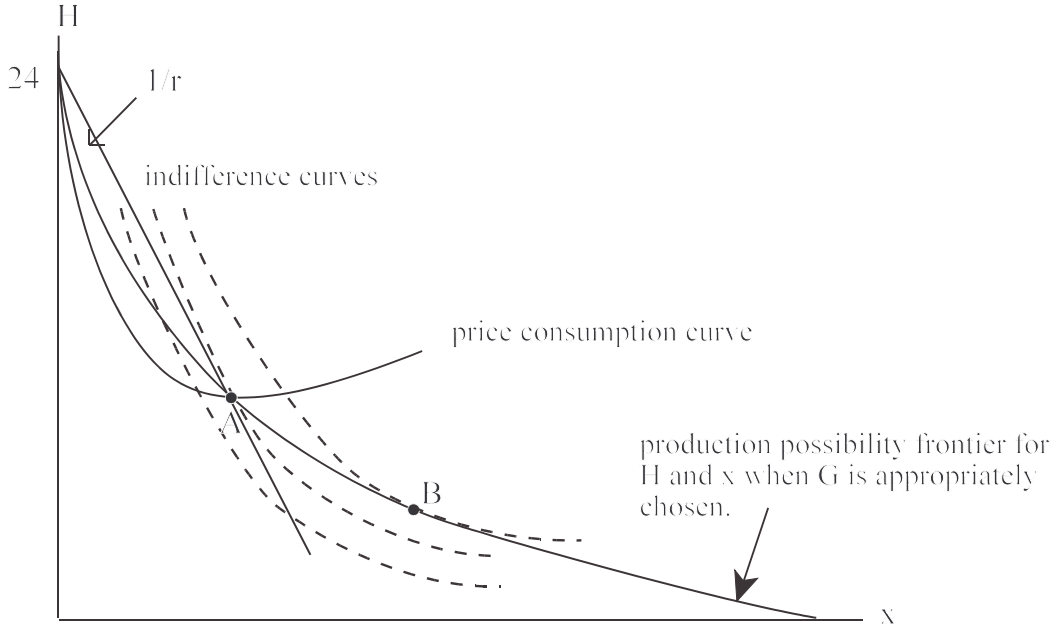
so that

$$dx/dG = [NW'(G)L - P_G]/N$$

which is a hump-shaped function of  $G$  as long as  $w''(G) < 0$ . Thus, for any  $H$ ,  $x$  is maximized with respect to  $G$  when  $NW'(G)L = P_G$  which is precisely the Samuelson rule.



**Figure 1: The original Samuelson Rule as a Criterion for the Unconstrained Planner and for a Government Constrained by the Taxpayers Response to the Tax Rate and the Supply of a Public Good when the Public Good is an Input to the Private Good rather than a Variable in the Utility Function**



axis at 24 hours because, when one takes 24 hours of leisure, there is no time left for work and no income can be earned. For any real wage  $r$ , the budget constraint of the consumer is

$$rL = r(24 - H) = x \quad (43)$$

and is representable by a downward-sloping straight line. For every value of  $r$ , the consumer chooses his best point on his budget constraint, the point for which the budget constraint is tangent to an indifference curve. The locus of all such points of tangency is the price-consumption curve. Clearly, the taxpayer is better off the higher  $r$  and the farther along the price-consumption curve he happens to be.

A government constrained by the behaviour of the taxpayer as represented in the price-consumption curve chooses  $t$  and  $G$  to place the taxpayer at the point where his price-consumption curve cuts the production possibility curve. This is marked on the figure as A. No better point is attainable. Every other attainable point leaves the taxpayer worse off than he would be at A. But points unattainable to a government constrained by the behaviour of the taxpayer may be attainable to the planner. As illustrated in the figure, an omnipotent planner

would not choose the point A. He would choose the point B at which an indifference curve is tangent to the production possibility curve.<sup>11</sup>

The figure highlights features of the Samuelson rule that are not quite so evident from the mathematical formulation. The Samuelson rule is neither an indicator of the socially-optimal supply of the public good nor a guarantee that full efficiency - as represented by the planner's choice of  $x$ ,  $L$  and  $G$  - is attained. It is one of two efficiency conditions, the other being the equality between the rates of substitution in production and in use between goods and leisure. Both conditions are necessary, but neither alone is sufficient. When  $G$  is an intermediate product, the Samuelson rule that carries over, from the planner choosing  $x$ ,  $L$  and  $G$  to the government choosing  $t$  and  $G$  in recognition of the taxpayers response in their choice of  $x$  and  $L$ . The other first order condition does not carry over from one context to the other. The planner would choose  $x$  and  $L$  to ensure that  $-u_L/u_x = w(G)$ , while the taxpayer would choose  $x$  and  $L$  to ensure that  $-u_L/u_x = w(G)(1 - t)$ .<sup>12</sup>

Nor is the Samuelson rule a unique determinant of the supply of the public good. This is immediately evident from the fact that the rule holds at every point on the locus of attainable combinations of  $H$  and  $x$ . As  $x$  increases along that locus, together with the decrease in  $H$ , the corresponding conditionally optimal value of  $G$  must increase as well. The optimal  $G$  must be 0 at the extreme left-hand side of the locus where  $H = 24$ , and it increases steadily as more and more  $H$  is traded off for  $x$ . To say that the Samuelson rule holds, is not to specify a unique value of  $G$ .

Bear in mind what has been proved and what has not been proved. The original Samuelson rule carries over from the unconstrained planner to the constrained government if and only if the public good is intermediate, providing an indirect contribution to utility by increasing the marginal product of labour, but providing no direct contribution in utility in and of itself. The Atkinson and Stern modification to the Samuelson rule remains valid, but reduces to the original rule if, but only if, that the two extra expressions in the formula - representing the marginal cost of public funds and the shadow price of public projects - cancel out. Nothing in the preceding argument would suggest a canceling out of these two expressions unless the public good is

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<sup>11</sup>The knowledgeable reader will notice that figure 1 above is a variant of figure 5 in Peter Diamond and James Mirlees, "Optimal taxation and Public Production I: Production and Efficiency", *American Economic Review*, 1971, 8-25. Thanks to my colleague Robin Boadway who spotted this connection.

<sup>12</sup>Edgar Browning and Liqun Liu emphasized this point in their critique of Kaplow's paper. ["The Optimal Supply of Public Goods and the Distortionary Cost of Taxation: Comment", *National Tax Journal*, 1998.] Employing a diagram not unlike Figure 1, they showed that, despite the validity of the Samuelson rule, the omnipotent planner would produce more public goods and would leave people better off than a government controlling  $t$  and  $G$  but not  $x$  or  $L$  directly.

intermediate. Otherwise, the sign of  $L_G$  may be positive or negative depending on the nature of the public good itself.

Note finally that the argument is restricted to deadweight loss originating from distortions in the labour-leisure choice and has no bearing upon other tax-induced contractions of the tax base. Intermediacy of public goods does not forestall the contraction of the tax base from the increased incentive for tax evasion and tax avoidance brought about by an increase in tax rates to finance extra public goods. The argument is almost tautological: With the labour-leisure trade-off as the only source of deadweight loss, and with no imbalance between  $x$  and  $G$  originating from that source, there can be no imbalance at all. True, but hardly enlightening.

### Lump Sum Taxation: Comprehensive, Marginal and Personalized

A different objection to the Atkinson and Stern formula stems from the mismatch between the identical twins assumption and the proportional income tax assumption. The proportional income tax assumption is open to the objection that actual tax systems are never proportional, that they are almost always progressive to some degree and that they are idiosyncratic favouring some groups at the expense of others. While true, this objection has little bearing on the subject of this paper because deviations from proportionality are more likely than not to magnify rather than reduce tax-induced distortions and the marginal cost of public funds. A more relevant objection is that, if people really were identical as is assumed in the Atkinson and Stern model, they would not be so foolish as to impose upon themselves a proportional income tax. They would surely opt for a lump sum tax instead.

The question becomes whether a distortionless tax system can still be devised for an economy where people's incomes are far from identical. Our present tax system gives rise to a marginal cost of public funds that is both greater than 1 and an increasing function of total public expenditure. Could our present tax system be replaced with a different tax system in which the marginal cost of public funds is always equal to 1 regardless of the magnitude of total public expenditure? Bear in mind the reason why the marginal cost of public funds is thought to be greater than 1. It is because an increase in any tax rate causes the tax base to shrink. Rearranging one's affairs so as to reduce one's tax base is costly, but, the higher the tax rate, the larger is the cost worth bearing to for that purpose. This mechanism can only be circumvented by taxation with an unshrinkable base, taxation for which the term  $\epsilon_{L_t}$  in equation (19) is 0. In fact, any of three tax systems might do the job, but all are, for one reason or another, infeasible or undesirable..

The first is a comprehensive and uniform head tax. Ideal in a world where everybody's income is the same, a uniform lump sum tax might still be imposed in a world where people's incomes differ. If each and every person, regardless of income or wealth, were confronted with the very same tax bill, then none of the tax-avoiding maneuvers that cause the marginal cost of public funds to exceed 1 would be of any use to the tax payer. With a uniform lump sum tax, the tax payer could no longer reduce his tax bill by diverting time from taxable work to untaxable

leisure or do-it-yourself activities, by switching expenditure from investment to consumption, by redirecting effort from producing goods to schemes for tax avoidance, or by rearranging his affairs to reduce his tax bill. We do not have such a tax now because of our concern for the distribution of income. Were we willing to give up that objective completely, all deadweight loss in taxation could be eliminated and the marginal cost of public funds would automatically be equal to 1 regardless of the share of public expenditure in the economy.<sup>13</sup>

A second variant of lump sum taxation might be called marginal. Each person's tax bill would be the sum of two parts: ordinary taxation and a lump sum supplement. Ordinary taxation is based upon a tax schedule,  $t(y)$ , that may be progressive but that is *invariant* with respect to changes in total public expenditure. In this arrangement, the total tax bill,  $T(y)$ , of a person with income  $y$  would be

$$T(y) = t(y) + (E - R)/N \quad (44)$$

where  $E$  is total expenditure,  $R$  is total revenue from ordinary taxation,  $N$  is total population and the expression  $(E - R)/N$  is the universal lump sum tax or lump subsidy as the case may be. Clearly, since the ordinary income tax schedule is independent of  $E$  and  $R$ , additional revenue can only be acquired by additional lump sum taxation.

Such a tax regime is surely distortionary, for people would have the same incentives as now to alter their behaviour in order to reduce their tax assessments. But a regime of marginal lump sum taxation is not distortionary at the margin. The *marginal* cost of public funds falls to 1 because additional public expenditure would be financed by additional lump sum taxation bearing no additional deadweight loss. With such a tax regime in force, a \$1 increase in public expenditure per head leads to an increase of \$1 in everybody's tax bill or to a decrease in everybody's lump sum subsidy as the case may be regardless of whether one is rich or poor.<sup>14</sup>

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<sup>13</sup>Actually, that is not quite true. Tax avoidance could not be completely eliminated under a lump sum tax, but it would take the form of disappearance. Poor people would have an enormous incentive to conceal their existence from the tax collector. There is some question about the appropriate base of the lump sum tax. Should it be per person (so that a newly-born child bears the same dollar value of tax as a prosperous middle-aged bachelor) or per family, or age-dependent. The reader can easily imagine how the incentives for marriage and child-bearing might be affected under each tax system. A uniform lump sum tax would be especially inconvenient to a person who, for example, earns less than \$20,000 in a society where the average income is over \$80,000 and the share of public expenditure in the economy is 25%. Any attempt to alleviate that inconvenience would revive precisely those incentives to alter one's behavior so as to reduce one's tax bill that cause the marginal cost of public funds to exceed 1.

<sup>14</sup>Note that none of these three tax systems are optimal in the usual sense of the term. An optimal tax system is designed to maximize some notion of social welfare incorporating average income per head and the distribution of income. Not even "optimal taxation", as the term is

The financing of additional public projects, programs or activities would never be distortionary, the original Samuelson rule would be appropriate and the unconventional view would be triumphant, exactly as Kaplow claims.

A third system of lump sum tax is the personalized ability tax. People would be taxed not according to their actual incomes, but according to their abilities to earn income. The all-wise, all-seeing planner identifies each person's innate capacity to earn income, regardless of whether his actual income is large or small. Each person's tax would then be levied in accordance with innate ability. The tax system would be free of deadweight loss and the marginal cost of public funds would never exceed 1 because nothing anybody might do would alter his tax bill. By contrast with the uniform lump sum tax, this system could be administered so as not to harm the poor excessively. Low-skilled people would pay low taxes and high skilled people would pay high taxes, no matter what they actually earn. Regardless of whether it is what Kaplow has in mind in his defense of the ordinary Samuelson rule, this third variant of lump sum taxation would be sufficient to procure Kaplow's result. The ability tax is ideal "in theory" and impossible "in practice", for the tax collector does not possess the required insight into our innate abilities, and most of us would prefer things that way.<sup>15</sup>

Of the three variants of lump sum taxation, the marginal lump sum tax seems to be the only option with even a remote possibility of working in practice, but it has major defects. The establishment of marginal lump sum taxation would require a constitutional amendment. In no other way could a country commit itself to financing all new public expenditures by extra lump sum taxation or by diminishing lump sum subsidies. Voters would be most unlikely to tolerate such rules. One way or another, tax rates would be raised when total public expenditure is increased and lower tax rates when public expenditure is reduced, creating a mechanism that is, broadly-speaking, like that postulated in Atkinson-Stern. Voters would want the cost of a war,

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commonly employed, do the trick. Optimal taxation minimizes the total distortion for any given distribution of post-tax income, but the distortion is not eliminated altogether, and the minimal attainable distortion would normally be an increasing function of total public expenditure. See R. Boadway and M. Keen, "Public Goods, Self-Selection and Optimal Income Taxation", *International Economic Review*, Vol. 34, No 3, 1993, 463-478,

<sup>15</sup>The process is downright theological. God in his wisdom looks into the souls of men and determines how much tax each person should pay for any given menu of public expenditures. The process is non-distortionary because God bases his decision on people's characteristics that nobody can adjust in order to induce God to reduce his tax bill. God might levy an ability tax as long as each man's ability is assessed early enough in life - perhaps in the womb - for the tax system to have no influence his or his parents' decisions about how much education to procure. God would not resort to an income tax because he knows that men act to adjust taxable income in the light of the tax schedule imposed.

for example, to be financed not by an equal dollar value of tax increase per person as would be required under marginal lump sum taxation, but by tax increases proportional to (or more than proportional to) people's incomes.

There is good reason for not passing such an amendment. The financing of marginal public expenditure by increases in lump sum taxation is typically (though not invariably) regressive. Such a procedure can be expected to place a very nasty burden upon the poor. A new road can be constructed at a cost of \$10 per person. If it just so happens that the dollar value of each person's benefit from the road (where a person's benefit is to be understood as his reservation price, the most he would be prepared to pay as the alternative to not having the road at all) is the same, regardless of whether one is old or young, rich or poor, then, as Kaplow claims, the financing of the road by a marginal lump sum tax can have no distributive consequences and the original Samuelson rule is appropriate for deciding whether to build the road or not. But that is a most atypical situation. Normally public goods, though made available on the same terms to all, are valued differently by different people, depending to a large extent on whether one is rich or poor. If the new road saves time, people's monetary value of its benefit would depend on their values of time, on the amounts of money they would be prepared to pay per hour saved. If the new road averts fatal accidents, people's monetary values of its benefits would depend on their values of life, on the amounts of money they would be prepared to pay to avoid a small risk of death. Values of time and life are both higher for the rich than for the poor, and the values of the benefits of a new road would vary accordingly. For most (though not all) public goods, a person's valuation of an increase  $\Delta G$  in the supply of the public good is  $w'(G)L\Delta G$  where  $w'(G)$  can be expected to vary among people in accordance with  $w(G)$ . The greater one's skill or income, the higher one's valuation of a given supply of the public good.

As long as people's valuations of the public good are more or less proportional to their incomes, the financing of additional public goods by marginal lump sum taxation is regressive, for, when an increase in the lump sum tax is just sufficient to cover the cost of a road, for example, the presence of the road is worth more than the additional tax to the rich and less than the additional tax to the poor. The median voter (whose income is invariably less than the average income) would never go for such a rule. Nor would a utilitarian planner who weighs benefits and costs of a projects according to the incomes of people affected by it. Typically, marginal lump sum taxation is a trade-off of efficiency for equality. Distortions in taxes are avoided, but only at the cost of accentuating the gap between rich and poor. Any attempt to alleviate that cost by extra redistribution must necessarily reintroduce the distortionary taxation which it was the purpose of marginal lump sum taxation to avoid.<sup>16</sup>

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<sup>16</sup>Since tax-induced distortions depend on marginal rather than average tax rates, it is usually possible to increase public expenditure without at the same time creating additional distortions by increasing intra-marginal tax rates. Such a procedure is equivalent to financing additional public expenditure by lump-sum taxation. However, the equivalent lump sum taxation is universal rather than individualized, and is not distributionally-neutral except in the unlikely case where everybody's marginal valuation of the additional public expenditure is the same. For



Kaplow recognizes this point, but the reader is somehow left with the impression that lump sum taxation is distributionally neutral, while income taxation is distortionary but redistributive. That would only be so if public goods yielded equal benefits to everybody and if equality of provision signified equality of monetary valuation as well. It seems to me that, as it is more reasonable to suppose monetary equivalents of benefits of public goods are proportional to incomes than to suppose they are the same per person, it would be best to deal with the whole question through adjustments to the shadow price of public good. The Atkinson and Stern formula allows the shadow price of public goods to be either greater or less than 1 depending on the public good's impact upon the tax base. Dropping the identical twins assumption allows for a second determinant of the shadow price, namely the relative benefits of the public good to the rich and to the poor. Thought there in only one type of public good in the model as set out above, the model and the corresponding modified Samuelson rule can be constructed to allow for a diversity of public expenditures, each with its own shadow price as a reflection of its particular impact upon the economy. It is difficult to say how much of the specifics of the different public goods should be recognized in practice, for it is by no means obvious who the beneficiaries of any given road, for example, will turn out to be or how benefits to particular people are to be identified. Rules of thumb may be best in the circumstances.

The question of whether taxation is distortionary is quite distinct from the question of whether each person's additional tax to finance a new project or an increase in the supply of the public good is fully covered by that person's benefit from the new project or increase in the supply of the public good. The marginal cost of public funds is conventionally thought to exceed 1 because all feasible tax systems are thought to be distortionary, regardless of the distributional impact of new projects or increases in public goods. In choosing what to supply, a government may well take the distribution of taxes and expenditures into account, but the marginal cost of public funds is what it is regardless. If personalized lump taxes were feasible (and perhaps under less stringent conditions as well), it would make sense to choose projects on no other criterion than to maximize the difference between benefits and costs "to whomsoever the benefits accrue" and to conduct all redistribution through taxation. That may remain as the desirable procedure in practice when the distribution of benefits is difficult to identify and less than perfectly correlated with income. None of this has any bearing upon the marginal cost of public funds or upon the validity of the modified Samuelson rule.

There are two conditions on which Kaplow is right. He is right if the labour-leisure choice is the only source of deadweight loss in the tax structure and if the public good is intermediate. He is also right if a God-like planner can levy individualized, ability-based lump sum taxes and can adjust each person's tax bill to reflect his benefit of each and every project. Otherwise he is wrong, and what has become the conventional view - to adjust the original Samuelson rule for a uniform marginal cost of public funds, and to recognize shadow prices that vary from project to project - emerges unscathed from Kaplow's critique. In practice, it may be

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a thorough analysis of these considerations, see Bev Dahlby, "Progressive taxation and the Social Marginal Cost of Public Funds", *Journal of Public Economics*, 1998, 105-22.



best to look upon the entire tax system as distortionary, upon the marginal cost of public funds as greater than 1 and upon the allocation among people of the benefits of public expenditure as unavoidably project-specific. Shadow prices of public projects, programs and activities may be greater or less than 1 depending on the distribution of their benefits and their impact on the tax base. Determination of the exact magnitude of the marginal cost of public funds at any time and place is a difficult empirical question about which we have nothing to add here to the literature cited at the outset of this paper. Despite its simplifications, the modified Samuelson rule points benefit-cost analysis in the right direction, and the conventional view, in Kaplow's sense of conventional, seems more or less correct.