The Distributive Implications of Patents on Indivisible Goods[1]

April 8, 2004

Abstract

Patents raise the price and reduce consumption of the patented good, but the resulting deadweight loss is thought to be worth bearing when patent protection is required as an incentive to invention. The newly-invented good generates a residual surplus, making people better off than they would be if the good had not been invented. This well-known argument is usually framed in a context where people are identical, everybody's demand curve for the newly-invented good is the same and everybody shares to some extent in the residual surplus. However, when the newly-invented good is indivisible - like a heart transplant or the treatment of AIDS, where, in effect, a person consumes either one full unit of the good or none - the effect of a patent is to concentrate the entire benefit of the patented good upon the rich, leaving the poor no better off than if the good had not been invented.

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By providing the inventor with monopoly power over his invention, patents generate an incentive to invent, but, at the same time, reduce the consumer surplus from the availability of the newly-invented goods, leaving consumers worse off than they would be if the newly-invented good were available un-monopolized but better off than they would be without the invention. Usually told with reference to the representative consumer, this story of the effects of patents on the economy tends to convey the impression that the benefit from the newly-invented good is widely, perhaps universally, shared, though the quantity demanded of the new good, and the corresponding consumer's surplus, would typically be larger for the rich than for the poor.

These distributional implications of this story are often, but not invariably correct. This note is about a case where the story is distinctly wrong, where every penny of benefit from the appearance on the market of a newly-invented good accrues to people whose incomes exceed

[1]I appreciate helpful suggestions by my colleagues Klaus Stegemann, Marvin McInnis and Sumon Majumdar

some critical amount, where everybody else is excluded altogether, and where patents focus all benefit onto an even smaller and wealthier segment of the population.

Patents direct all benefit to the wealthy when the newly-invented good is what might be called *indivisible*. A good is indivisible in this sense of the term when each person wants to buy either one unit of the good or none at all because less than one full unit is useless and any amount over and above one unit is of no additional benefit. Indivisible goods are exemplified by medical procedures which you adopt entirely or not at all. You cannot have half an appendectomy and wouldn't want a second. You cannot have half a heart transplant. Nor can you have half an AIDS treatment or half a course of antibiotics; you must take the entire treatment if it is to do you any good.

What distinguishes an indivisible good in this sense of the term from an ordinary *divisible* good is not physical indivisibility, but a technically-prescribed quantity required to attain a desired result. A course of antibiotics may contain many, many pills and each pill may be broken into many parts, but the course itself is indivisible because the number and mix of pills is for all practical purposes predetermined by the disease and by the requirements for a cure. By contrast, oranges are divisible not just because you can enjoy half an orange, but because you can enjoy many oranges in the course of the year and because your demand for oranges would expected to vary almost continuously with the price. Housing is also divisible because, though you can only live in one house at a time, that house may be larger or smaller, more or less comfortable, and more or less expensive.

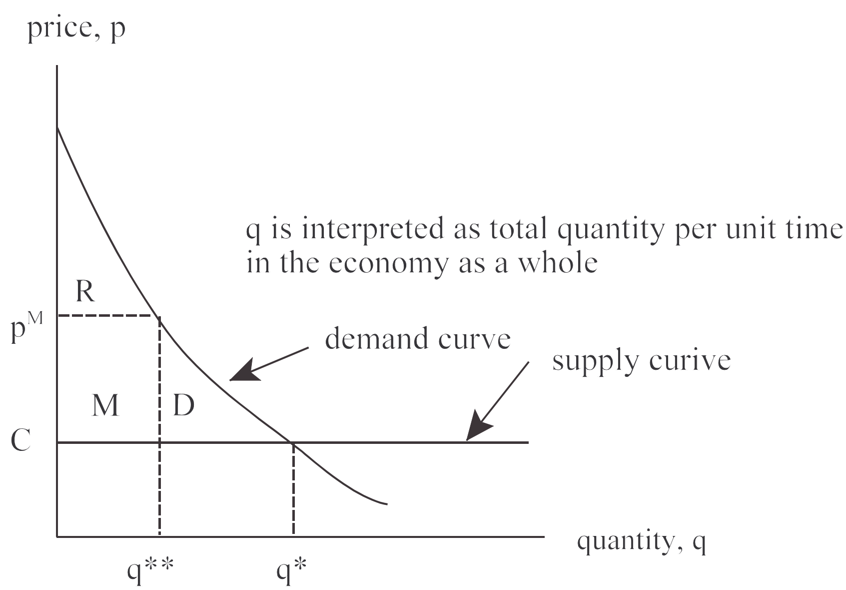
Indivisibility lies at an extreme of a continuum; it is a property that goods possess to a greater or lesser extent. You either have a kidney transplant or you don't, but your surgeon may or may not be the best and most expensive available. In general, goods differ in quality as well as in quantity. A person's consumption of a purely and completely indivisible good would differ in neither dimension. While it is hard to imagine goods with no variation in quality whatsoever, some goods are close enough to complete indivisibility and the implications of indivisibility for the effect of patents on society are striking enough to make the analysis of indivisible goods interesting and useful.

When a good is indivisible, every consumer has his own reservation price. He buys one unit of the good if the market price is below his reservation price, and he buys none if the market price is above his reservation price. When everybody's utility function is the same and as long as the indivisible good is normal, a person's reservation price must be an increasing function of his income. The indivisible good is purchased by everybody whose income is above some critical level and by nobody else. Each purchaser's surplus from the indivisible good is an increasing function of his income.

A patent is a monopoly of a newly-invented good conferred by the state on the inventor as an inducement to invent. Demand and supply curves for a newly-invented good are illustrated in figure 1 with quantity, q, on the horizontal axis and price, p, on the vertical axis. For simplicity, the supply curve is assumed to be flat at a height, C, equal to the cost of production. Without patent protection, the market-equilibrium price and quantity would be C and q* as indicated by the crossing of the demand and supply curves. The surplus from the availability of the newly invented good - the amount people would be prepared to pay for the right to buy any amount of the good at a price of C - is the area between the demand and supply curves, the sum of the areas marked as R, M and D, mnemonic for residual surplus, monopoly revenue and deadweight loss.

The patent-holder chooses the profit-maximizing price and quantity along the demand curve. The monopoly price, p^M , and the corresponding quantity, q**, are chosen to maximize monopoly revenue, M, equal to (p^M - C)q**. The rise in the price from C to p^M , and the corresponding reduction in quantity from q* to q**, leave the consumer worse off than if the newly-invented good were available at its cost of production, C, but better off than if the good were not available at all. Society's benefit from the availability is reduced from the original surplus of R + M + D to the remaining surplus R + M, of which M accrues to the patent-holder and R accrues to consumers. The deadweight loss, D, is wasted altogether. In this demonstration, the quantity, q, is always graduated per unit of time but could equally-well be thought of as "per person" or "in total", as long as R, M and D are graduated accordingly. It is convenient here to think of q as total output in the economy as a whole. That the patent expires after a certain time is not relevant to the argument in this note. Our analysis is entirely static.

Figure 1: Surplus, Residual Surplus and Deadweight Loss



Strictly speaking, the argument is valid regardless of whether people's demands for the newly-invented good are the same or different and regardless of how many units of the newly-invented good each person is prepared to buy at any given price. There is, however, a fundamental difference between two extreme possibilities: a) where all consumers are alike and the demand curve is a reflection of each and every person's response to price, and b) where a person buys at most one unit of the newly-invented good depending on whether or not the market price exceeds his reservation price. The former case, where the patented good is what we are calling "divisible", is what most people have in mind when the welfare implications of patents are assessed.[2] The latter case arises when the newly-invented good is indivisible.

When the newly-invented good is indivisible, the demand and supply curves of figure 1 must be thought of as pertaining to the community rather than to the individual. Quantity is transformed from a typical person's consumption (in circumstances where he can consume more or less of the good in question) to the number of people who choose to buy the indivisible good, and price corresponding to any given quantity must be the reservation price of the least enthusiastic buyer. Though not strictly necessary, it is certainly plausible and will be assumed from here on that a person's reservation price is an increasing function of his income. On this assumption, the variable q on the horizontal axis of figure 1 becomes a measure of the number of people ordered by income, the richest first, the next richest second, and so on, and the height of the demand curve above any point q is the reservation price for the newly-invented good of the q^{th} richest person in society. If people's reservation prices were exactly proportional to their incomes - for instance, if each person's reservation price were 2% of his income - the demand curve for the newly-invented good would become a mirror image of the distribution of income in society as a whole, a variant of the Gini curve plotted from richest to poorest rather than the other way round.

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[2]I analyzed this case in "The Welfare Economics of Invention", *Economica*, (believe it or not) 1964, 279-287. The analysis there was conducted with reference to indifference curves and a production possibility curve. Invention created a new production possibility curve rather than a new supply curve. Taste was represented by indifference curves rather than by a demand curve. The formulation is that paper is translatable into figure 1.

If the newly-invented good were available unpatented at its cost of production, C, it would be purchased by the q* richest people exclusively. If the newly-invented good were patented and available at the monopoly price p^M, it would be purchased by the q** richest people exclusively, where q** < q*. With divisible goods and identical consumers, the patent reduces everybody's purchase of the patented good. With indivisible goods and consumers with different incomes, the patent withdraws the good altogether from people with incomes between q* and q**.

Quantifying the Distributive Implications of the Monopolization of Indivisible Goods

For an economy where people's incomes differ but their utility functions are the same, the consequences of the monopolization of an indivisible good can be summarized in seven propositions:

1) For any given price, there is a critical income such that people whose income is above the critical income purchase the indivisible good, and people whose income is below do not.

2) Monopolization raises the critical income, confining consumption of the indivisible good to the richer portion of those who would consume the good if it were not monopolized.

3) The aggregate demand curve for an indivisible good can be inferred from the form of the common the utility function and the distribution of income.

4) For some, not implausible, forms of the common utility function, the demand curve for the indivisible good is nothing more than a rotation of the plot of the distribution of income.

5) The proportion of the population consuming the indivisible good - when the good is monopolized and when it is not - can be read off the demand curve.

6) The welfare loss from monopoly can be can be computed from the parameters of the distribution of income.

7) The measure of welfare loss can be modified to account for diminishing marginal utility of income.

From her on, these propositions will be established with varying degrees of generality. The proportion of users of the monopolized good, the welfare loss from monopoly, and a utilitarian extension of the measure of welfare loss will be computed from postulated parameters of the economy.

Imagine an economy with two goods, a divisible good that people may consume in smaller or larger amounts, and an indivisible good such that a person consumes either one unit of the good or none at all. The divisible good is representative of all goods except the one indivisible good. People have different incomes but identical utility functions ranking all possible bundles of the two goods. The distribution of income can be approximated by a Pareto distribution. Specifically,

a) The common utility function is

$$u = u(\delta, x) = \delta\ln(\alpha) + \ln(x) \qquad (1)$$

where x is consumption of other goods, the parameter α must be larger than 1, and the parameter δ is either 0 or 1. It is equal to 1 if a person consumes the newly-invented good and it is equal to 0 if otherwise. δ is equal to 0 if the person does not consume the newly-invented good.

b) People are endowed with different incomes, y.

c) Prices are expressed in units of the divisible good so that the price of the divisible good is automatically equal to 1. Consider a person whose income is y when the price of the indivisible good is P. That person's consumption of the divisible good is equal to y - P if he chooses to buy the indivisible good and is equal to y otherwise.

d) The distribution of income is approximated by a Pareto distribution from which it follows that the proportion, Q(y), of the population with incomes greater than y is[3]

$$Q(y) = (y^{\#}/y)^{\mu} \tag{2}$$

where μ is an indicator of the extent of equality in the distribution of income. The larger μ, the smaller the proportion of the population with incomes in excess of any given multiple, y/y#, of the minimal income, y#.

e) The cost of production of the indivisible good is C; the technology of the economy is such that each unit of the divisible good produced is at the expense of C units of the divisible good.

From equation 1, a person's reservation price for the indivisible good can be derived as a function of income. One's reservation price is the highest price one is prepared to pay when the alternative is to do without the good altogether. At that price, a person must be just as well off if

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[3]The Pareto distribution is characterized by the density function

$$f(y) = \mu(y^{\#})^{\mu}\,(y)^{-\mu-1}$$

where $y^{\#}$ is lowest income, there is no upper limit on y and μ is a parameter representing the degree of equality in the income distribution. The larger μ, the more equal the distribution of income must be. The parameter μ has been observed to be about 3 in many countries, and it is assumed here to be 3 exactly. It follows at once that the proportion of the population, $\pi(y)$, with incomes between $y^{\#}$ and y is

$$\pi(y) = \int_{y^{\#}}^{y} \mu(y^{\#})^{\mu} y^{-\mu-1} dy = 1 - (y^{\#}/y)^{\mu}$$

so that $\pi(y) = 1$ when y rises to infinity. When the proportion of the population with incomes between $y^{\#}$ and y is $1 - (y^{\#}/y)^{\mu}$, the proportion, Q(y), of the population with incomes equal to or greater than y must be $(y^{\#}/y)^{\mu}$ as indicated in equation 2.

he buys the good as if he does not. In other words, the reservation price, $P(y)$, for a person with an income of y is identified by the equation

$$u(1, y - P(y)) = u(0, y) \tag{3}$$

where 1 means that the person consumes a unit of the indivisible good, and 0 means that he does not. Equivalently

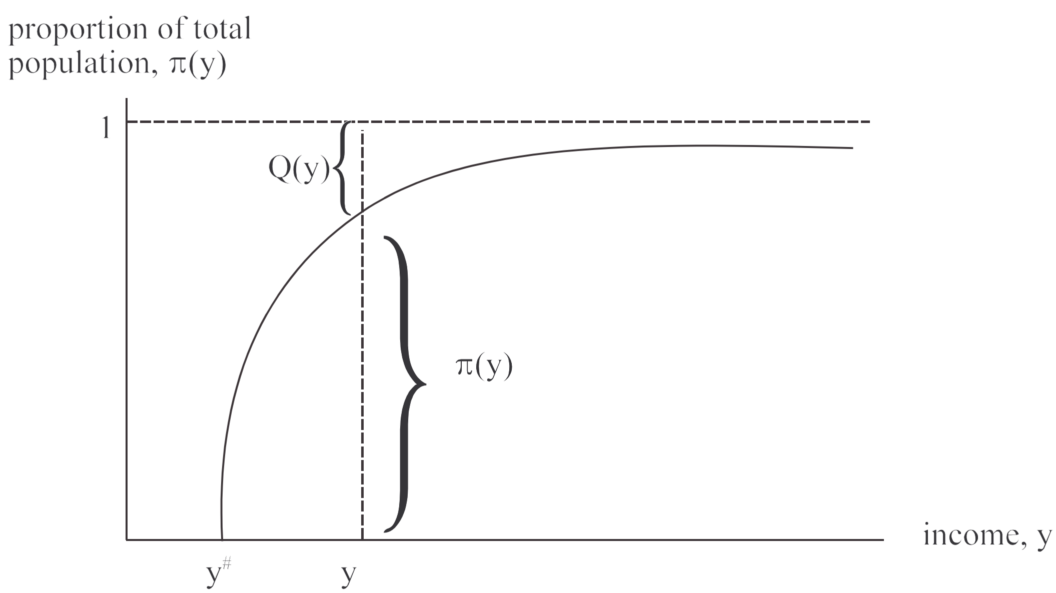
$$\ln(\alpha) + \ln(y - P(y)) = \ln(y) \tag{4}$$

from which it follows at once that

$$P(y) = [(\alpha-1)/\alpha]y \tag{5}$$

This proportionality between income and reservation price transforms the relation between numbers of people and their incomes on the graph of the distribution of income into a relation between numbers of people and their demand prices on the demand curve for the indivisible good. The demand curve for the indivisible good in figure 1 is nothing more than a rotation of the appropriately dimensioned distribution of income in the community as a whole. Of course, a different utility function would have yielded a less convenient demand curve, but one would normally expect people's reservation prices to be increasing in income as long as their utility functions are the same.

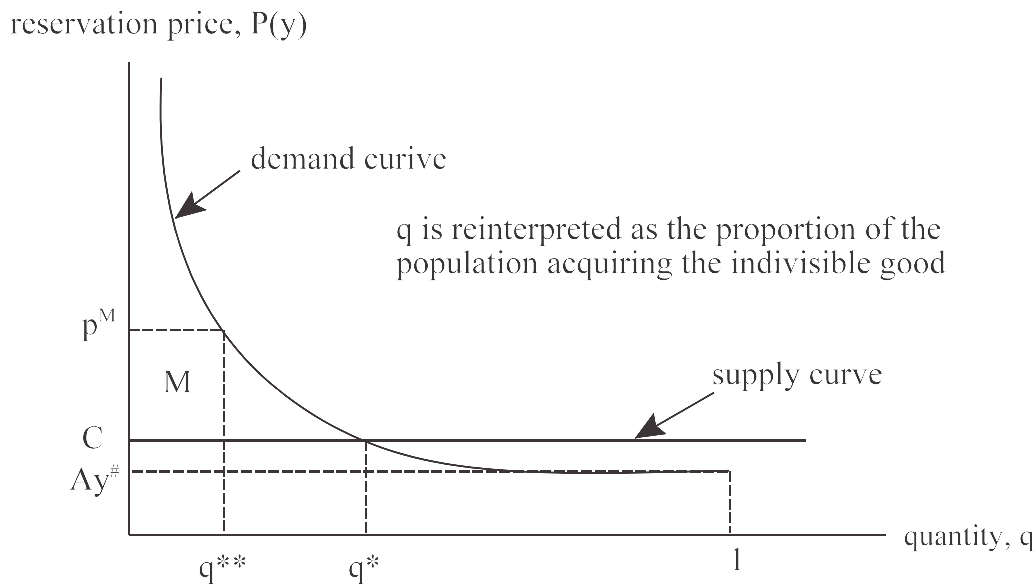
Figure 2: The Cumulative Pareto Distribution of Income



The distribution of income is illustrated in the usual way in figure 2 with income, y, on the horizontal axis and the proportion of the population, $\pi(y)$, with incomes less than or equal to y on the vertical axis. The curve depicting the distribution of income begins at 0 for, $y^{\#}$, the lowest income in the distribution, and rises gradually toward 1 as y increases indefinitely. For any given y, Q(y) is the distance between the curve and 1, as shown in the figure.

A 90 degree rotation transforms the distribution of income in figure 2 into the demand curve for the indivisible good in figure 3. The horizontal line at a height of 1 above the horizontal axis of figure 2 becomes the vertical axis of figure 3, with a change in units to represent the reservation price of the indivisible good rather than income itself. The height of the vertical axis becomes the reservation price, $P(y)$, which is equal, as indicated in equation 5, to $[(\alpha - 1)/\alpha]y$ and is directly proportional to y. Points on the horizontal axis of figure 3 represent not total population, but the proportion of total population, $Q(y)$, with a reservation price greater than Ay, where A is a constant set equal to $(\alpha - 1)/\alpha$. The proportion, $Q(y)$, reaches a maximum of 1 when income falls to $y^{\#}$ (and the corresponding reservation price falls to $Ay^{\#}$) because 100% of the population have incomes greater or equal to $y^{\#}$.

Figure 3: Demand and Supply Curves Reflecting the Distribution of Income



Also shown on figure 3 are the supply curve of the indivisible good, the monopoly price and the revenue per person of the patent holder as monopolist. The supply curve is assumed to be flat at a height C above the horizontal axis. The monopoly price is p^M. The area of the box, labeled M is the monopoly revenue, per person rather than in total because points on the horizontal axis represent proportions of total population rather than numbers of people. If total population were one million, actual monopoly revenue would be 1,000,000,000M. A monopolist maximizing M would automatically be maximizing total monopoly revenue as well. In accordance with this reinterpretation of the horizontal axis, the terms q* and q** are not the numbers of people who would buy the newly-invented good at prices C and p^M, but are instead the proportions of total population who would buy the newly-invented good at those prices.

The monopolist can equally-well be thought of as choosing a monopoly price, p^M, when the corresponding quantity is number of people for whom the reservation price exceeds p^M, or as supplying the good to the wealthiest proportion Q(y) of the population when the price must rise to the reservation price of the poorest person within that group. The latter is analytically simpler. For any given y, the revenue of the monopolist is

$$Q(y)[P(y) - C] = (y^{\#}/y)^{\mu} [Ay - C] \quad (6)$$

where Q(y) equals $(y^{\#}/y)^{\mu}$ as shown in equation (2), and P(y), the reservation price, equals Ay when A is defined as $(\alpha - 1)/\alpha\}$. The value of y at which revenue of the monopolist is maximized becomes[4]

$$y^M = C\mu/A(\mu - 1) \quad (7)$$

The corresponding monopoly price is

$$p^M = Ay^M = [C\mu/(\mu - 1)] \quad (8)$$

The proportion of the population for which the reservation price exceeds the monopoly price

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[4]Maximizing revenue of the monopolist with respect to y, we see that

$$\delta\{Q(y)[P(y) - C]\}/\delta y = (y^{\#}/y)^{\mu}A + y^{\#\mu} (-\mu)y^{-\mu-1}(Ay - C) = 0$$

from which it follows that

$$\mu A - \mu c/y = A$$

which, rearranged, becomes equation 7.

becomes

$$Q(y) = (y^{\#}/y^{M})^{\mu} = [y^{\#} A(\mu\text{-}1)/C\mu]^{\mu} \qquad (9)$$

From equations (6) to (9), it follows immediately that, for any given A and C, an *increase* in the equality of the income distribution, μ, leads at once to a *decrease* in the monopoly price, p^M, and to an increase in the proportion, $Q(y^M)$, in the proportion of the population choosing to consume the monopolized indivisible good.[5]

A simple numerical example shows what is at stake. Suppose a) the equality parameter in the income distribution is 3, which is not too far off what has been estimated in fitting the Pareto distribution to actual data for many countries, b) everybody's intensity of preference, A, for the indivisible good is .05, indicating that each person's reservation price for the indivisible good is 5% of his income, and c) the cost of production, C, of the indivisible good turns out to be just equal to the reservation price of the poorest person in society, so that everybody would consume the indivisible good if it were made available at its cost of production. This last supposition is that

$$C = Ay\# \qquad (10)$$

On these assumptions, the monopoly price becomes

$$p^M = Ay\#\mu/(\mu\ \text{-}1) \qquad (11)$$

the critical income above which people buy the monopolized indivisible good and below which they do not becomes

[5]The ratio μ/μ - 1) automatically decreases and its inverse $(\mu$ - $1)/\mu$ increases together with an increase in μ.

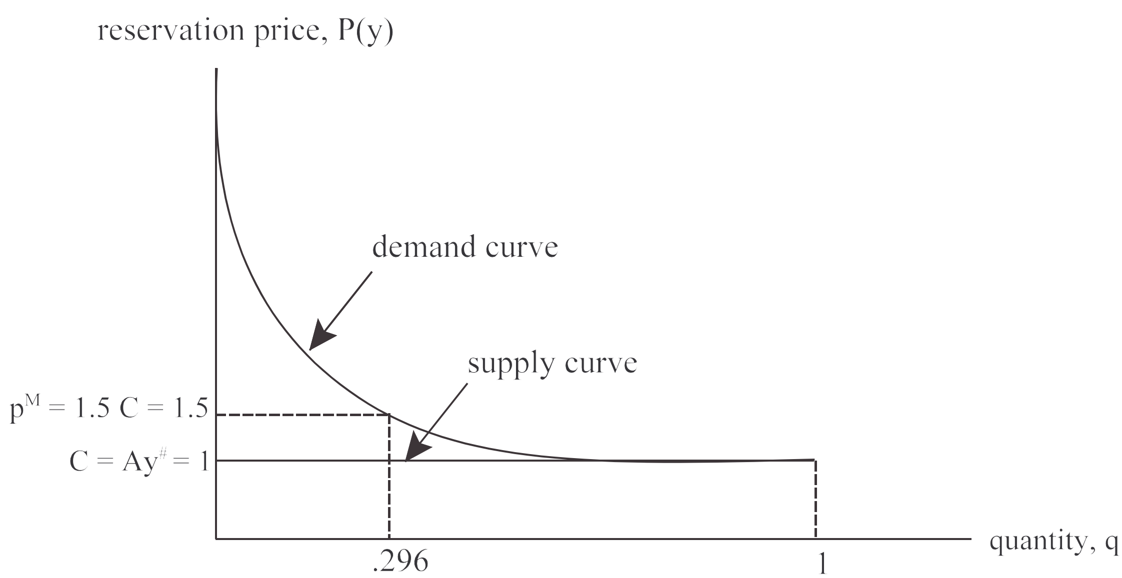
$$y^M = y\#\mu/(\mu -1) \tag{12}$$

and the proportion of the population buying the indivisible good becomes

$$Q(y) = (y^{\#}/y^M)^{\mu} = [(\mu\text{-}1)/\mu]^{\mu} = (2/3)^3 = .2962 \tag{13}$$

or about 30%.

Figure 4: How a Patent Deprives 70% of the Population of Access to the Newly-invented Good



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gure 4 is a repetition of figure 3 with the additional assumption that the reservation price of the poorest person just happens to equal the cost of production of the newly-invented good, so that everybody would buy a unit of the good if it were made available at its cost of production. Reflecting the Pareto distribution of income, the demand curve in figure 4 is sufficiently bowed in that a 50% rise in the price over and above the cost of production is sufficient to reduce consumption by about 70%. The patent concentrates all benefit from the newly-invented good upon the richest 30% of the population whose reservation price exceeds the monopoly price. The rest of the population is out of luck.

Efficiency and Utility in the Measurement of Deadweight Loss as a Proportion of Total Potential Surplus

For ordinary divisible goods, it would seem natural to measure the econo0mic significance of deadweight loss from patents or any other source of monopoly as a proportion of the total potential surplus from the monopolized good. With reference to figure 1 above, the measure of relative deadweight loss would be

$$\frac{D}{S} = \frac{D}{R+M+D} = \frac{\int_{q^{**}}^{q^{*}}(p^{D}(q)-p^{S}(q))dq}{\int_{0}^{q^{*}}(p^{D}(q)-p^{S}(q))dq} \tag{14}$$

where D, S, M and R are respectively deadweight loss, total potential surplus from the availability of the good, monopoly revenue and residual surplus, where $p^{D}(q)$ and $p^{D}(q)$ are respectively the demand and supply prices for any given quantity, q, and where q** and q* are respectively quantities consumed when the good is monopolized and when it is not. It makes no difference to the measure of D/S whether q is interpreted as average quantity consumed or as quantity per head. By assumption, the supply curve is flat so that $p^S(q)$ is invariant at C, the cost of production. The analogous measure for an indivisible good is

$$\frac{D}{S} = \frac{D}{D+M+R} = \frac{\int_{y^\#}^{y^M}[P(y)-C]f(y)dy}{\int_{y^\#}^{\infty}[P(y)-C]f(y)dy} \tag{15}$$

where P(y) is the reservation price of a person with an income of y and the integral is over income rather than quantity. Income substitutes for quantity in equation (15) as long as each person consumes no more than one unit of the monopolized good and as long as income is weighted by the density function, f(y), of the distribution of income. Employing the expressions for the terms in equation (15) as set out in the preceding equations, and adhering to the assumption that C is just high enough that everybody would buy the good if it were not monopolized, we can solve for S/D explicitly.

It turns out that relative deadweight loss in equation (15) is a function of μ, the equality parameter in the Pareto distribution *and nothing else*. Specifically, as demonstrated in the appendix,

$$S = \int_{y^\#}^{\infty}(P(y)-C)f(y)dy \tag{16}$$

$$D = \int_{y^\#}^{y^\#\frac{\mu}{\mu-1}}(P(y)-C)f(y)dy = S\{1-[(\mu/(\mu-1)^{-\mu}][(2\mu-1)/(\mu-1)]\} \tag{17}$$

so that $D/S = \{1 - [(\mu/(\mu - 1)^{-\mu}] [(2\mu - 1)/(\mu - 1)]\}$ (18)

which is equal to about 26% when $\mu = 3$ as we have been assuming. If every one of our long list of assumptions were strictly true, monopolization would waste about a quarter of the potential surplus of the monopolized good.[6]

However, there is some question as to whether relative surplus as defined above is the appropriate measure of the importance of surplus for an indivisible good. For ordinary divisible goods, it is not unreasonable to suppose that monopolization strikes rich and poor alike. For most such goods, the demand curve of a rich person than the demand curve of a poor person, but deadweight loss as a proportion of total surplus might be expected to be more of less the same. Not so for indivisible goods, for which monopolization may take away a part of the surplus of the rich and the entire surplus of the poor.

This consideration suggests a different measure of relative deadweight loss. Total surplus and deadweight loss could be redefined from a utilitarian point of view. People's benefits from the availability of the indivisible good could be compared not dollar for dollar regardless of whether one is rich or poor, but in accordance with some notion of utility where a person's utility of an extra dollar's worth is deemed to be a decreasing function of his income. Each person's monetary surplus, P(y) - C, in equations (10), (11) and (12) would be replaced by a utilitarian surplus, (du/dy)(P(y) - C), in which additional term, du/dy, is the marginal utility of income for a person whose income is y, based on some postulated utility of income function. A simple and convenient utility function for this purpose is

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[6]Actually relative deadweight loss is not very sensitive to the postulated value of μ. For a constant y#, both total surplus and the monopoly price fall when the distribution of income becomes more equal, leaving their ratio more or less unchanged.

$u(y) = y^{\delta}$ (19)

where δ is the postulated elasticity of utility with respect to income, so that $du/dy = \delta y^{\delta - 1}$. With this substitution, the utilitarian variant of the total surplus , S^U, becomes

$$S^U = \int_{y^\#}^{\infty} \frac{du}{dy}(P(y) - C)f(y)dy \tag{20}$$

and the utilitarian variant of deadweight loss, D^U, becomes

$$D^U = \int_{y^\#}^{y^\# \frac{\mu}{\mu-1}} \frac{du}{dy}(P(y) - C)f(y)dy \tag{21}$$

Solving these integrals with the appropriate substitutions from preceding equations, it turns out, as shown in the appendix, that

$$\frac{D^U}{S^U} = 1 - \left[\left(\frac{\mu}{\mu - 1} \right)^{-\mu + \delta} \left(2 - \frac{\delta}{\mu} \right) \right] \tag{22}$$

which reduces to the formula in equation (18) above when $\delta = 1$ signifying that income and utility are essentially the same.

With the equality parameter, μ, maintained at 3 and with the elasticity of utility with respect to income, δ, set at ½, the relative deadweight loss rises from 26% to 69%. Behind the veil of ignorance, where one does not know whether one will emerge into society as a rich person or a poor person, one has more to fear from monopolization a good when that good is indivisible than when that good is divisible.

Qualifications and Conclusions

Our seven propositions about the monopolization of indivisible goods are, like all such propositions, sensitive to changes in the assumptions on which they are based. Among these, is the assumption that everybody's utility function is the same. An interesting modification of this assumption restricts the demand for the indivisible good to a subgroup of the population, as would normally be the case in the medical examples we have been employing as illustrations of indivisible goods. Not everybody wants a heart transplant, or an appendectomy, or a course of antibiotics! The analysis carries over unchanged from goods used by everybody to goods only used by a portion of the population as long as the income distribution of the users is useful is an accurate reflection of the income distribution in the population as a whole. Imagine a drug that cures appendicitis. The story in figures 2, 3 and 4 remains valid as long as appendicitis shows no respect for the income of its victim and is as likely to inflict the rich as the poor. A good only used by a small portion of the population good may even be converted to a good used by everybody through product-specific insurance. The patent-holder on a drug to cure appendicitis may choose to sell not the drug itself, but an insurance contract entitling the buyer to a sufficient supply of the drug if and when the buyer needs it. Everybody can be expected to have a positive reservation price for the insurance contract even though most people will not need the drug at any time during their lives.

The analysis must be modified to some extent when the distribution of income among the users of an indivisible good differs significantly from the distribution of income in the population as a whole. Suppose, for example, that appendicitis were a disease of the poor exclusively so that a patent on a drug to cure appendicitis restricts the drug not to the wealthier members of the community as a whole, but to the wealthier members of the group at risk of appendicitis who may, nevertheless, be among the poorer half of the population as a whole. With the postulated utility function in equation (1), the demand curve for the drug to cure appendicitis would become a rotation of the distribution of income among people at risk of appendicitis.

On the other hand, the first proposition - that a patent restricts usage of the patented good to the wealthy - may break down completely in the market for product-specific insurance contracts. One's reservation price for a drug-specific insurance contract would be an increasing function of both one's income and one's probability of needing the drug. That presents no problem as long as one's probability of needing the drug is either independent of income or an increasing function of income. The proposition breaks down when one's probability of needing the drug is higher for the poor than for the rich, sufficiently higher to outweigh the effect of income on the on one's reservation price. In that case, there may be no monotonic relation, positive or negative, between income and reservation price.

A final observation: The argument in this note seems to be widely recognized in public discussion of the provision of medical services, especially the treatment of AIDS in poor countries. This paper can be looked upon as a rationalization for the commonly-heard recommendation that patents on key drugs be expropriated by the state, making them available at their cost of production while at the same time compensating drug companies for the cost of research, on the general principle that all public taking should be compensated.

Appendix: **Derivations of Equations (18) and (22)**

Equation (18)

Deadweight loss and surplus - being different portions of the area between the demand and supply curves - can be represented by one and the same integral, but with different upper limits. Designate the common integral as

$$\int_{y^{\#}}^{U} (P(y) - C) f(y) dy \tag{A1}$$

where U is the upper limit, equal to ∞ in the measure of surplus, S, and equal to y#μ/(1 - μ) in the measure of deadweight loss, D. Substituting for the reservation price, P(y) from equation (5), for the distribution function of the Pareto distribution, f(y), from footnote (3) and for the cost of production, C, from equation (10), the integral becomes

$$\int_{y^{\#}}^{U} A(y - y^{\#}) \mu (y^{\#})^{\mu} y^{-\mu-1} dy \tag{A2}$$

which may be expressed as

$$g(U) - g(y\#) \tag{A3}$$

for the function

$$g(y) = A\mu (y^{\#})^{\mu} \left(\frac{y^{-\mu}}{-\mu} - \frac{y^{\#} y^{-\mu+1}}{-\mu+1} \right) \tag{A4}$$

By construction,

$$S = g(\infty) - g(y^{\#}) \tag{A5}$$

and

$$D = g(y^{\#}\mu/(\mu-1)) - g(y^{\#}) \tag{A6}$$

where $\quad g(\infty) = 0 \quad$ (A7)

$$g(y\#) = -Ay^{\#}/(\mu-1) \tag{A8}$$

and

$$g(y\#\mu/(\mu-1) = -[Ay^{\#}/(\mu-1)]\left[\left(\frac{\mu}{\mu-1}\right)^{-\mu}\left(\frac{2\mu-1}{\mu-1}\right)\right] \tag{A9}$$

so that

$$D/S = \frac{g(y^{\#}\mu/(\mu-1)) - g(y^{\#})}{g(\infty) - g(y^{\#})} = 1 - \left[\left(\frac{\mu}{\mu-1}\right)^{-\mu}\left(\frac{2\mu-1}{\mu-1}\right)\right] \tag{A10}$$

which is equation (18).

Equation (22)

The derivation of this equation is essentially the same as the derivation of equation (18), except that the expression P(y) - C in equation (18) is replaced by the expression $\delta y^{\delta-1}$(P(y) - C), where $\delta y^{\delta-1}$ is the marginal utility of income for the postulated utility of income function. The common integral is

$$\int_{y^{\#}}^{U} \delta y^{\delta-1}(P(y)-C)f(y)dy \tag{A11}$$

with the same pair of upper limits as before: U = ∞ for the measure of S and U = y#μ/(μ - 1) for the measure of D. The integral can be expressed as

h(U) - h(y#) (A12)

where, for any y,

$$h(y) = \delta\mu y^{\#}\left[\frac{y^{-\mu+\delta}}{-\mu+\delta} - \frac{y^{\#}y^{-\mu+\delta-1}}{-\mu+\delta-1}\right] \tag{A13}$$

By construction, the revised measures of surplus and deadweight loss, S^U and D^U, become

$$S^U = h(\infty) - h(y\#) \tag{A14}$$

and $$D^U = h(y\#\mu/(\mu - 1) - h(y^{\#}) \tag{A15}$$

where $$h(\infty) = 0 \tag{A16}$$

$$h(y^{\#}) = \frac{-\delta\mu A(y^{\#})^{\delta}}{(\mu-\delta)(\mu-\delta+1)} \tag{A17}$$

and

$$h(y^{\#}\mu)/(\mu-1) = \frac{-\delta\mu A(y^{\#})^{\delta}}{(\mu-\delta)(\mu-\delta+1)}\left[\left(\frac{\mu}{\mu-1}\right)^{-\mu+\delta}\left(2-\frac{\delta}{\mu}\right)\right] \tag{A18}$$

so that

$$D^U/S^U = \frac{h(y^{\#}\mu/(\mu-1))-h(y^{\#})}{h(\infty)-h(y^{\#})} = 1-\left[\left(\frac{\mu}{\mu-1}\right)^{-\mu+\delta}\left(2 - \frac{\delta}{\mu}\right)\right] \tag{A19}$$

which is equation (22).