Lecture 3: Other Selling Mechanisms

by

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1. Introduction to Bargaining and Posted Prices

- Posted Prices & Take-It-or-Leave-It Offers
  -- Posted prices: usually the seller posts the prices
  -- Take-it-or-leave-it offers: can be made by the seller or the buyer

  e.g. A seller and a buyer discussing the price of transaction
  $V_b=$buyer’s valuation, $V_s=$seller’s valuation
  $V_b \sim F(v), \ V_s \sim G(v)$; both are private information

  Posted price: seller maximizes $(p-V_s)[1-F(p)]$
  Take-it-or-leave-it offer made by the buyer:
  buyer maximizes $(V_b-p)G(p)$
Simultaneous Bargaining
-- buyer and seller make offers simultaneously
\[ P_b = \text{buyer’s offer} \]
\[ P_s = \text{seller’s offer} \]
-- determination of transaction price
\[ \text{If } P_b < P_s, \text{ no trade;} \]
\[ \text{If } P_b > P_s, \text{ transaction at } p = (P_b + P_s)/2 \]
-- this is fully efficient if \( P_b = V_b \) and \( P_s = V_s \)
   (truthful reporting of valuation)
   not an equilibrium without subsidies or surpluses
-- Characterization of equilibrium strategy
buyer \( P_b = B(V_b) \), seller \( P_s = S(V_s) \)
buyer maximizes \[ \int_{V_s}^{S^{-1}(P_b)} \left( V_b - \frac{P_b + S(V_s)}{2} \right) g(V_s) dV_s \]
seller maximizes \[ \int_{B^{-1}(P_s)}^{{\bar{V}_b}} \left( \frac{B(V_b) + P_s}{2} - V_s \right) f(V_b) dV_b \]

=> 2 simultaneous differential equations
=> equilibrium not fully efficient
=> it is second best: the most efficient solution with budget balance

-- If there is no uncertainty (i.e., \( V_s \) and \( V_b \) are known), then there are multiple equilibria with full efficiency.
   (There are also equilibria with no trade => not efficient)
2. Alternating-Offer Bargaining Models (perfect info)

- Rubinstein bargaining model
  - 2 players sharing a pie
  - Player 1 makes an offer, player 2 says “Yes” or “No”
  - If “Yes”, negotiation ends
  - If “No”, player 2 makes a counter-offer
  - Player 1 can now say “Yes” or “No”

........................
- Order of players making offers: 1, 2, 1, 2, 1, 2,........
- Game ends when an agreement is reached, or when pre-specified number of rounds (T) is reached
- If T=∞, negotiation could last indefinitely
3. Delays in Bargaining Models (private info)

- Delays or costly actions can be used to reveal private information
e.g. Strikes, Protests

- A model of one-sided uncertainty
  - seller’s value = \( s \)  
  - buyer’s value = \( b \sim F(b) \)  
  - continuous time  
  - identical discount rate  
  - once buyer’s value is revealed, \( p = (s+b)/2 \)  
    (splitting the surplus of \( b-s \) )  
  - equilibrium strategy with delays  
    - seller makes an offer of \( p(t) \) at time \( t \)
a buyer of type \( b \) accepts the offer at time \( T(b) \)

\[ T^{-1}(t) \]

=> the type of buyer accepting an offer at \( t \) must be \( T^{-1}(t) \)

-- equilibrium conditions:

\[ p(t) = \frac{s + T^{-1}(t)}{2} \]

\[ t = T(b) \] maximizes

\[ (b - p(t)) e^{-rt} = \left( b - \frac{s + T^{-1}(t)}{2} \right) e^{-rt} \]

equivalently, if the buyer pretends to be type \( b^* \), then \( b^* = b \) maximizes

\[ \left( b - \frac{s + b^*}{2} \right) e^{-rT(b^*)} \]
First order condition:

\[ T'(b) = -\frac{r}{b - s} \]

Let \( B(t) \) be the inverse function of \( T(b) \). Then

\[ B'(t) = -\frac{B(t) - s}{r} \]

or

\[ (B(t) - s)' = -\frac{B(t) - s}{r} \]

This is a completely separating equilibrium. (There could be partially pooling equilibria in this game.)
A model of two-sided uncertainty

-- seller’s value = \( s \sim G(s) \)

-- buyer’s value = \( b \sim F(b) \)

-- completely separating equilibrium strategy:

\[
b = B(t) \quad \text{reveals his type at} \quad t
\]
\[
s = S(t) \quad \text{reveals his type at} \quad t
\]

-- First order condition:

\[
(B(t) - S(t))' = -\frac{B(t) - S(t)}{r}
\]
4. Auctions vs Posted Price Selling

● Model Setup
-- Continuous time, infinite horizon, no discounting
-- One seller, one object
-- Buyers arrive according to Poisson process with rate of arrival $\lambda$
-- A buyer’s valuation $v \sim F(v)$, i.i.d.
-- The seller can use either auctions or posted prices
-- If the object is not sold, the seller can sell it later
-- Selling by posted prices $\Rightarrow$ cost of display $\theta_d$
-- Selling by auctions $\Rightarrow$ cost of storage $\theta_s$ + auction fee $\Theta_a$
• Sell by Posted Prices

-- Suppose the price is \( p \)

-- An arriving buyer buys with probability \( 1-F(p) \)

-- The seller’s profit

\[
\Pi^S(p) = p - \frac{\theta_d}{\lambda(1-F(p))}
\]

-- First order condition:

\[
\frac{d}{dp} \Pi^S(p) = 1 - \frac{\theta_d f(p)}{\lambda(1-F(p))^2} = 0
\]

-- Define virtual function:

\[
J(v) = v - \frac{1-F(v)}{f(v)}
\]

-- At the optimal price, \( \Pi^S(p) = J(p) \)
Sell by Auctions

-- Auctioning off the object $T$ units of time apart

-- Buyers arriving during $[0,T]$ are notified of the auction to be held at $T$

$[T,2T] \Rightarrow$ auction held at $2T$, ……

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-- If $k$ buyers arrive during the period of time, with reserve price $p$, the seller’s profit becomes

$$
\Pi(k, p) = \int_{p}^{\tilde{v}} vk(k-1)[1 - F(v)]F^{k-2}(v)f(v)dv
$$

$$
+ pk[1 - F(p)]F^{k-1}(p) + F^k(p)\Pi^A
$$
-- where

\[ \Pi^A = \sum_{k=0}^{\infty} \Pi(k, p)P_k(T) - \theta_s T - \Theta_a \]

-- \( P_k(T) \) is the prob. that \( k \) buyers arrived within \( T \) units of time

● Conclusion:

-- Auctions more profitable when the valuation distribution is more dispersed
-- The dispersion concept is different from the variance of a distribution.
-- It is related to the slope of the virtual function \( J(v) \)
Figure 1. The Cumulative Distribution Functions and the Supports of Distributions
Plan for next lecture:
Mechanism Design

- Reading materials

