Contracting in Vague Environments*

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Abstract

This paper investigates the implications for contracting problems with adverse selection of assuming a more general information structure than usual. The paper applies a choice theoretic framework introduced in Olszewski (2007) and Ahn (2008) - a special case of the vague environments in Vierø (2009a) - to a canonical principal-agent model with hidden information. The vague environment reflects that in many real-world contracting situations information is imprecise, and it gives rise to interesting effects. The intuition and mechanism behind the optimal contract fundamentally changes and so does the optimal contract. The results can, for example, explain patterns observed in franchising.

Keywords: contracts, vagueness, optimism, incentives, franchising

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1 Introduction

This paper applies a choice theoretic framework introduced in Olszewski (2007) and Ahn (2008) - a special case of the vague environments in Vierø (2009a) - to a canonical principal-agent model with hidden information. Many real-world contracting situations are characterized by imprecise information. Therefore, a natural thing to do is to directly allow for the contracting environment to reflect this. In the choice theoretic framework of vague or imprecise information, the contracting parties do not know the exact probabilities with which the different outcomes will occur. This vagueness is introduced into the canonical principal-agent model without changing any other assumptions. As I demonstrate below, the introduction of vagueness changes the incentive structure of the contracting problem and gives rise to non-standard results.

The environment in this paper is vague because the decision makers are assumed to only know a set of possible probability distributions, or lotteries, over outcomes, rather than a precise probability distribution as is usually assumed. I use Vierø's (2009a) Optimism-Weighted Subjective Expected Utility (OWSEU) representation of preferences in vague environments, which in the one-state version applied here also corresponds to one of the representations in Olszewski (2007). In the present context this representation characterizes a decision maker by his Bernoulli-utility and his optimism, and models him as if he evaluates an act (a contract in the present context) by computing the usual von Neumann-Morgenstern utility of the best lottery and of the worst lottery in the set of lotteries and weighting them together, where the weight on the best lottery can be interpreted as the decision maker's level of optimism.[1]

In the standard problem with risk neutral parties and no vagueness, the optimal contract is to 'sell the firm to the agent'. That is, the optimal contract gives the principal the same utility regardless of the agent's type, interpreted as the agent buying the firm for a fixed fee. Such a contract is optimal because there is no insurance need in the contracting relationship when the agent is risk neutral, and selling the firm to the agent completely solves the incentive provision problem. The agent will then on his own choose the effort levels that maximize total surplus.

Interestingly, in a world with vagueness a 'sell the firm to the agent' contract is not necessarily the optimal contract. An optimistic principal will never choose to sell the firm

[1] In the following I use the terminology from Vierø (2009a) and refer to the theory therein.

to the agent because the principal can do better by offering an alternative contract. For a pessimistic principal, a 'sell the firm to the agent' contract is optimal for some values of the agent's optimism but not for other values.[2]

These results can, for example, explain patterns observed in franchising. The 'sell the firm to the agent' contract is then interpreted as choosing to operate through a franchise. Not only do my results explain the coexistence of chains that operate exclusively through franchising and chains that operate exclusively through centrally owned manager operated outlets. The result that there exist principals who will choose a 'sell the firm to the agent' contract for some agents but not for others explains the existence of chains that operate through a mix of franchising and centrally owned manager operated outlets. The latter mix of operational form is, for example, used by some chains of gas stations.

The results are driven by a fundamental change in the intuition and mechanism behind the optimal contract, since the contracting parties' overall weights on the different final outcomes become endogenous as a result of the vague environment. This gives rise to the principal (she) often being able to exploit the presence of optimism or pessimism to offer contracts that are better from her point of view. This is possible because vagueness gives room for the principal to affect which final scenario the agent (he) puts most weight on through the design of the contract. Generally, vagueness gives rise to endogenous differences between the principal's and the agent's overall weights, which creates a motive for betting or trading in addition to the usual desire to provide incentives for and ensuring participation of the agent. The fact that this motive arises endogenously is a driving force behind the change in the mechanism that delivers the optimal contract, and hence also behind the new results.

Vagueness also provides insights into recruitment processes, since it illustrates the agent's ideal level of optimism from the point of view of different principals. The agent's optimism matters for the compensation needed to attract him to the job. When deciding whether to accept a job offer, the agent evaluates the tradeoff between the disutility he will suffer from doing the job and the compensation he will receive. If there is vague information about how difficult it will be to get the job done, the agent's optimism influences his perception of the

[2]The relevance of pessimism among decision makers is well accepted based on Ellsberg (1961). Andersen, Fountain, Harrison, and Rutström (2009) provide experimental evidence of the relevance of optimism.

disutility he will suffer. The more optimistic he is, the easier he will think it is to do the job, and the smaller is the compensation needed for him to accept the job.

The present paper is related to a group of papers that consider contracting when the parties have heterogeneous beliefs. These include Adrian and Westerfield (forthcoming) who consider a continuous-time dynamic model with moral hazard where the parties have heterogeneous beliefs, and Carlier and Renou (2005, 2006) who consider specific static problems with heterogeneous beliefs. When beliefs are heterogeneous, the parties also want to place side-bets on the resolution of uncertainty, but there is no possibility for the principal to influence the agent's weight on the different final scenarios. With precise information and heterogeneous beliefs, all differences between the contracting parties are exogenous. Vagueness, on the other hand, creates an endogenous difference in the weights the principal and agent assign to the different final scenarios.

Grant and Karni (2005) demonstrate the importance of the principal using the agent's actual beliefs when designing contracts. Mukerji (1998), Rigotti (2006), and Vierø (2009b) analyze other contracting problems in non-standard choice theoretic settings. Mukerji (1998) considers a moral hazard problem with firms in a vertical relationship and a discrete choice set and shows that ambiguity aversion among the parties can rationalize incomplete contracts. Rigotti (2006) considers a principal-agent model with moral hazard in which the agent has incomplete preferences. Vierø (2009b) considers contracts that are conditional on vague signals when the contracting environment is itself precise.

A different line of related research looks at corporate culture, motivation, and low-powered incentives, see for example MacLeod (2003), Levin (2003), and Van den Steen (2005, 2007). These papers are concerned with the optimal match of employee characteristics to firms and/or the use of incentive pay as a function of the match of characteristics. Since my paper illustrates the agent's ideal level of optimism as a function of the principal's optimism it adds to this line of research as well.

Note that the notion of optimism in the present paper is different from the notion of overconfidence in people's own abilities that we see in e.g. Benabou and Tirole (2002). In the present paper, optimism is a consistent feature of the individual's personality, which affects his or her perspective on life in general.

The paper is organized as follows: Section 2 presents the model with vagueness. Section

3 analyzes the problem with asymmetric information and contains the results. Section 4 concludes. The proof of Proposition 2 is given in the appendix.

2 Model

Consider the canonical principal-agent problem with hidden information. A risk neutral principal wants to hire a risk neutral agent to complete a task. It is assumed that the agent's utility depends on a variable, measuring how well suited to the required task he will find himself, the value of which is realized after the contract is signed. For convenience, I will refer to this variable as the agent's efficiency level, but it could be interpreted in a variety of ways. Suppose the agent's effort can be measured by a one-dimensional variable $e \in [0, \infty)$. The principal's gross profit is a function of the agent's effort, $\pi(e)$, with $\pi(0) = 0$, $\pi'(e) > 0$, and $\pi''(e) < 0$ $\forall e$. Her Bernoulli utility function is given by her net profits,

$$u_P(w, e) = \pi(e) - w,$$

where w denotes the wage she pays to the agent.

The agent's Bernoulli utility function depends on his wage w, how much effort he chooses to exert, and his efficiency x, which affects how much disutility, denoted $g(e, x)$, he experiences from effort. It is assumed that there are only two possible values of x: the agent is either of high-efficiency type x_H or of low-efficiency type x_L. The efficiency level is unobservable for the principal, while effort is assumed to be observable and contractible.

Assume further that the agent is risk neutral with a Bernoulli utility function of the form

$$u_A(w, e, x) = v(w - g(e, x)) = w - g(e, x),$$

The disutility $g(e, x)$ is assumed to satisfy the following standard conditions: $g(0, x_H) = g(0, x_L) = g_e(0, x_H) = g_e(0, x_L) = 0$, such that the agent suffers no disutility if he does not exert any effort, $g_e(e, x) > 0$ $\forall e > 0$ and $g_{ee}(e, x) > 0$ $\forall e$, such that his disutility from effort is increasing at an increasing rate, and $g(e, x_L) > g(e, x_H)$ $\forall e > 0$ and $g_e(e, x_L) > g_e(e, x_H)$ $\forall e > 0$, such that his disutility and marginal disutility from effort are higher if he is of low-efficiency type. Finally, let $\overline{u}$ denote the agent's reservation utility.

The innovation in this paper is to change one assumption in the standard canonical principal-agent model with hidden information: instead of assuming that the contracting parties know the precise probability with which the agent will be of high-efficiency type x_H, I assume that the parties only know a possible interval $Q = [a, b] \subseteq [0, 1]$, with $a < b$, of this probability.[3] Hence the parties have common but vague knowledge of the probability of x_H.

This information structure is an example of a vague environment as defined in Vierø (2009a). Vierø's choice theory maintains the standard assumptions about preferences, while it relaxes the assumption that the environment is precise. Rather than having acts (contracts in the present context) be mappings from states into singleton lotteries over outcomes, acts are mappings from states into sets of such lotteries (Q in the present context). The one-state version of a vague environment used here can also be found in Olszewski (2007) and Ahn (2008).

Vierø (2009a) shows that if preferences satisfy the standard axioms for subjective expected utility, properly expanded to the generalized domain, plus two additional axioms that are natural extensions of the standard axioms when considering vague environments, then decision makers can be modeled as Optimism-Weighted Subjective Expected Utility (OWSEU) maximizers, who maximize

$$OWSEU(h) = \sum_{s=1}^{n} \mu(s) \Big[\alpha \sum_{i=1}^{m} \overline{h}^{s}(z_i) u(z_i) + (1-\alpha) \sum_{i=1}^{m} \underline{h}^{s}(z_i) u(z_i) \Big].$$

Here $u(\cdot)$ is the decision maker's Bernoulli utility function over outcomes z_i, μ denotes his beliefs over states, and α is a parameter that captures the decision maker's level of optimism. $\underline{h}^s$ and $\overline{h}^s$ are, respectively, the worst and best lotteries in the set of lotteries the act h returns in state s.

The Optimism-Weighted Subjective Expected Utility representation shows that we can model the decision maker as if he evaluates an act by computing for each state the usual von Neumann-Morgenstern utility of the best lottery and of the worst lottery in that state's set and weighing them together, where the weight α on the best lottery can be interpreted as the decision maker's level of optimism. The decision maker assigns unique subjective probabilities to the states and computes his overall utility using these and the weighted utility for each state.

I assume that the contracting parties have such OWSEU-preferences. In the present one-state context this implies that both the principal and the agent maximize utility of the

[3]Note that if Q were a singleton, this reduces to the standard principal-agent model.

following form:

$$OWSEU_j(h) = \alpha_j \sum_{i \in \{L,H\}} \overline{p}_j(h)(z_i)u_j(z_i) + (1-\alpha_j) \sum_{i \in \{L,H\}} \underline{p}_j(h)(z_i)u_j(z_i),$$

where each sum is over the support of the relevant lottery, $j \in \{P, A\}$, u_j is j's Bernoulli utility function defined over outcomes $z_i = (w_i, e_i, x_i)$, $\alpha_j \in (0,1)$ is a parameter that captures j's degree of optimism, while $\overline{p}_j(h)$ and $\underline{p}_j(h)$ are, respectively, the best and worst lotteries from j's point of view – given the contract h – in the set Q of possible probabilities. The decision maker computes the von Neumann-Morgenstern utility of the best lottery and of the worst lottery in the set of lotteries, and weighs them together with weight α_j on the best lottery.

Contracting is assumed to take place ex-ante, i.e. before the agent knows his type. Ex-ante contracting has two stages: the agent first agrees to a menu of contracts, and then, once he learns his type, selects one of the contracts in the menu. I assume that the principal is unable to observe the agent's efficiency level at any point in time, hence there is asymmetric information ex-post.

It is important to note that which lotteries are best and worst depend on the contract offered. This implies that the weights the parties assign to the different final scenarios, i.e. to high-efficiency type and low-efficiency type, are influenced by the contract as well, and are therefore endogenous. This is the key consequence of vagueness, which substantially alters the intuition and mechanism of the contracting problem. For instance, this often allows the principal to offer a contract that makes her better off than she would be in the absence of vagueness.

3 Contracting in vague environments

As is usually the case under asymmetric information, the principal must rely on the agent to reveal his efficiency level. Therefore, the first important step to analyze the problem with asymmetric information is to show that the revelation principle also holds in a world with vagueness.[4]

[4]The proof of Proposition 1 does not require that the agent is risk neutral. Proposition 1 therefore generalizes to situations where the agent is risk averse, i.e. where $v(\cdot)$ is strictly concave.

Proposition 1 (Revelation principle for vague environments)**.** *In a vague environment, any general incentive compatible contract can be implemented with a truthful revelation mechanism.*

Proof: To see that the revelation principle also holds in vague environments, note that the only thing that is really different from when there is no vagueness, and therefore might cause problems, is that the best and worst lotteries in each state might not be the same for a general contract and a truthful revelation mechanism. However, it is shown next that the best and worst lotteries will be the same for the two mechanisms, and hence that the revelation principle will hold.

Suppose the optimal incentive compatible contract specifies the set T of strategies for the agent and an outcome $f(t)$, where $f : T \to (W, E)$ and (W, E) is the space of possible wages and effort levels. Denote the agent's optimal strategy with this contract by t^*.

Alternatively, the principal could use a revelation mechanism and let the agent announce his type: $\tilde{x}_H$ denotes an announcement of high-efficiency type, while $\tilde{x}_L$ denotes an announcement of low-efficiency type. Let $\tilde{w}(x)$ and $\tilde{e}(x)$ be defined by

$$\tilde{w}(\tilde{x}_H) = w(t^*(x_H)),\ \tilde{e}(\tilde{x}_H) = e(t^*(x_H)),\ \tilde{w}(\tilde{x}_L) = w(t^*(x_L)),\ \text{and}\ \tilde{e}(\tilde{x}_L) = e(t^*(x_L)).$$

Then, since the original contract $\{(t, f(t)) : t \in T\}$ was incentive compatible it follows that $v(\tilde{w}(\tilde{x}_H), \tilde{e}(\tilde{x}_H), x_H) \geq v(\tilde{w}(\tilde{x}_L), \tilde{e}(\tilde{x}_L), x_H)$ and $v(\tilde{w}(\tilde{x}_L), \tilde{e}(\tilde{x}_L), x_L) \geq v(\tilde{w}(\tilde{x}_H), \tilde{e}(\tilde{x}_H), x_L)$, i.e., that the agent will tell the truth. Also,

$$v(f(t^*(x_H)), x_H) \geq v(f(t^*(x_L)), x_L) \Leftrightarrow v(\tilde{w}(\tilde{x}_H), \tilde{e}(\tilde{x}_H), x_H) \geq v(\tilde{w}(\tilde{x}_L), \tilde{e}(\tilde{x}_L), x_L),$$

which means that the best and worst lotteries for the revelation mechanism are the same as for the original contract.$\blacksquare$

It follows from Proposition 1 that we can restrict our search for optimal incentive compatible contracts to truthful revelation mechanisms.

3.1 The principal's problem

The optimal contract will be either a separating or a pooling contract. Having established that the revelation principle holds, the principal can find the best separating contract by maximizing her OWSEU subject to a participation constraint and two incentive compatibility constraints for the agent:

$$\begin{aligned} \max\nolimits_{w_L,e_L\geq 0,w_H,e_H\geq 0} \alpha_P\Big\{\overline{p}_P(h)\big(\pi(e_H)-w_H\big)+(1-\overline{p}_P(h))\big(\pi(e_L)-w_L\big)\Big\} \\ +(1-\alpha_P)\Big\{\underline{p}_P(h)\big(\pi(e_H)-w_H\big)+(1-\underline{p}_P(h))\big(\pi(e_L)-w_L\big)\Big\} \end{aligned}$$

subject to

$$\begin{aligned} \alpha_A\Big\{\overline{p}_A(h)\big(w_H-g(e_H,x_H)\big)+(1-\overline{p}_A(h))\big(w_L-g(e_L,x_L)\big)\Big\} \\ +(1-\alpha_A)\Big\{\underline{p}_A(h)\big(w_H-g(e_H,x_H)\big)+(1-\underline{p}_A(h))\big(w_L-g(e_L,x_L)\big)\Big\} \geq \overline{u}, \end{aligned} \tag{PC}$$

$$w_H-g(e_H,x_H)\geq w_L-g(e_L,x_H), \tag{IC_H}$$

$$w_L-g(e_L,x_L)\geq w_H-g(e_H,x_L). \tag{IC_L}$$

where $\overline{p}_P(h)$ and $\underline{p}_P(h)$ are the lotteries in Q that are best and worst, respectively, from the principal's point of view given the contract h, and $\overline{p}_A(h)$ and $\underline{p}_A(h)$ are the lotteries in Q that are best and worst, respectively, from the agent's point of view given the contract h.

If the optimal contract is a pooling contract, it solves

$$\max_{w,e\geq 0} \pi(e)-w$$

subject to the participation constraint

$$\begin{aligned} \big(\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)\big)\big(w-g(e,x_H)\big) \\ +\Big[1-\big(\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)\big)\Big]\big(w-g(e,x_L)\big)\geq \overline{u}. \end{aligned} \tag{PC_P}$$

The principal chooses between all the feasible contracts she could offer, while the agent chooses between accepting the offered contract or taking the outside option.

Importantly, the best and worst lotteries, $\overline{p}_j$ and $\underline{p}_j$, for both the principal and the agent depend on the contract offered. Hence the contracting problem with vagueness does not reduce to a standard problem with heterogeneous beliefs. Since the best and worst lotteries depend on the contract, so do the weights the parties assign to the different final scenarios. Thus, vagueness creates an endogenous difference in the parties' weights. With precise information and heterogeneous belies, any differences in the weights are, on the contrary, exogenous. This is a fundamental difference between vagueness and a situation with precise information and heterogeneous beliefs, and is the key observation to understand the effects of vagueness and the results that follow.

3.2 Optimal contracts

In the standard problem with a risk neutral agent and no vagueness, the optimal contract $(w_H^*, e_H^*, w_L^*, e_L^*)$ is to 'sell the firm to the agent'. That is, the optimal contract gives the principal the same utility regardless of the agent's type, interpreted as the agent buying the firm for a fixed fee. This fee is the one that exactly gives the agent his reservation utility. Such a contract is optimal because when the agent is risk neutral there is no insurance need in the contracting relationship and selling the firm to the agent completely solves the incentive provision problem. The agent will then choose the effort levels that maximize total surplus given his type, i.e. the levels that maximize $\pi(e) - g(e, x)$.

Interestingly, when there is vagueness a 'sell the firm to the agent' contract that implements the effort levels that maximize total surplus given the agent's type is not necessarily the optimal contract. Rather, depending on the parties' degrees of optimism it can be optimal for the principal to distort effort away from these levels. Often she can do better by offering an alternative contract, and 'sell the firm to the agent' contracts are only optimal for values of the optimism parameters for which none of these alternatives are implementable.

With vagueness there are three classes of optimal contracts: The first class are agreement contracts where the parties are both better off if the agent turns out to be of type x_H than if he turns out to be of type x_L and therefore agree on the best and worst lotteries: $\overline{p}_P(h) = \overline{p}_A(h) = b$ and $\underline{p}_P(h) = \underline{p}_A(h) = a$. The second class are disagreement contracts where the contracting parties disagree on which final scenario is the best, and thus the best lottery from the principal's point of view is the worst lottery from the agent's point of view and vice versa: $\overline{p}_P(h) = \underline{p}_A(h) = a$ and $\underline{p}_P(h) = \overline{p}_A(h) = b$. The final class are the 'sell the firm to the agent' contracts where the principal is equally well off regardless of the agent's type, and hence $\overline{p}_P(h)$ and $\underline{p}_P(h)$ can be any lotteries in Q. Proposition 2 characterizes the optimal contracts, which are also illustrated in Figure 1.

Proposition 2 (Optimal contracts)**.** *In vague environments, selling the firm to the risk-neutral agent is dominated whenever $\alpha_A \leq \alpha_P$ or $\alpha_A \geq 1 - \alpha_P$.*

An optimistic principal ($\alpha_P \geq \frac{1}{2}$) offers either a disagreement or an agreement contract and never sells the firm to the agent. There exists $\hat{\alpha} \in (1-\alpha_P, \alpha_P)$, such that the optimal contract is a disagreement contract for all $\alpha_A \geq \hat{\alpha}$, and the optimal contract is an agreement contract for all $\alpha_A < \hat{\alpha}$.

A pessimistic principal ($\alpha_P < \frac{1}{2}$) offers either a disagreement contract or an agreement contract or sells the firm to the agent. If $\alpha_A \geq 1-\alpha_P$, the optimal contract is a disagreement contract. If $\alpha_A \leq \alpha_P$, the optimal contract is an agreement contract. If $\alpha_P < \alpha_A < 1-\alpha_P$, the principal sells the firm to the agent.

Outline of proof: In solving the principal's problem, one needs to take into account that vagueness causes points of non-differentiability because of the dependency of the best and worst lotteries on the contract. Therefore, analysis of the problem's first-order conditions must be supplemented by comparison of internal candidates for solution and potential corner solutions. I first show that the constraint set for the pooling problem is a subset of the constraint set for the separating problem, so that attention can be restricted to the separating problem. If a pooling contract is optimal, this will emerge as a solution to the separating problem.

As in the standard model with no vagueness, the participation constraint binds such that the agent gets exactly his reservation utility. The optimal contract will be a separating contract with $e^*_H > e^*_L > 0$. It follows directly from the incentive compatibility constraints that the agent will always be best off when he is of type x_H. Rather than solving the principal's problem explicitly, I use the first-order conditions as well as properties of the potential corner solutions to establish in which regions of (α_A, α_P)-space the different classes of contracts are implementable. Implementability hinges on whether the principal will be best off when the agent is of type x_H or x_L under the contracts that satisfy the first-order conditions. This establishes which type of contract is optimal in the different regions of (α_A, α_P)-space. The proof of Proposition 2 is in the appendix. ■

The intuition behind the results in Proposition 2 is that which lotteries are best and worst for each of the contracting parties depend on the contract offered. Hence, the presence of vagueness gives room for the principal to affect which final scenario the parties put most emphasis on through the design of the contract. The principal can exploit the presence of optimism or pessimism to offer contracts that are better than a 'sell the firm to the agent' contract from her point of view.

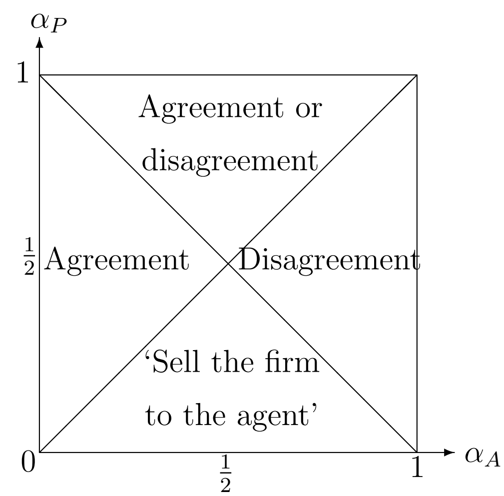
That is, because vagueness can create endogenous differences in emphasis, the contract fulfills two purposes. On one hand, it serves the usual purpose of ensuring participation of and providing incentives for the agent to undertake the desired level of effort. On the other hand, the parties are betting or trading on their differences in emphasis through the contract. Thus, in addition to ensuring participation of the agent, the wage paid to him contains side bets that exploit the parties' differences in emphasis. The two incentive compatibility constraints together ensure that the solution is bounded: the principal can only exploit the differences in emphasis as long as incentive compatibility is satisfied. If she tries to push beyond that, the type of agent receiving the lowest payment will lie about his type, and the principal will lose money.

To understand Proposition 2, first note that in order to be incentive compatible, a contract must eventually make the agent best off if he turns out to be of high-efficiency type. Depending on the optimism parameters, either (IC_H), (IC_L), or none of the incentive compatibility constraints binds. Note secondly that the principal's objective function is non-differentiable at the points (e_H, e_L, w_H, w_L) for which $\pi(e_H) - w_H = \pi(e_L) - w_L$, which is where $\overline{p}_p$ and $\underline{p}_p$ switch value.

Let e_L^o denote the total surplus maximizing effort level for the low-efficiency type, i.e. the level that maximizes $\pi(e) - g(e, x_L)$, and let e_H^o denote the total surplus maximizing effort level for the high-efficiency type, i.e. the level that maximizes $\pi(e) - g(e, x_H)$. Which class the optimal contract belongs to varies across (α_A, α_P)-space as follows.

Disagreement contracts: If $\alpha_A \geq \hat{\alpha} > 1-\alpha_P$ (for the optimistic principal) or $\alpha_A+\alpha_P > 1$ (for the pessimistic principal) such that joint optimism is high, the optimal contract is a disagreement contract where the principal is eventually best off if the agent turns out to be of the low-efficiency type. The intuition is that the joint optimism is large enough that the principal can exploit that they emphasize different things and therefore it is worthwhile for her to generate such a difference. Since the agent is eventually best off when he is of the high-efficiency type, the best lottery in Q from the agent's point of view is b, the one that assigns the highest possible probability to him being of high-efficiency type. Since the principal is eventually best off when the agent is of the low-efficiency type, the best lottery in Q from the principal's point of view is a, the one that assigns the lowest possible probability to the agent being of high-efficiency type. The parties' joint optimism ensures that their difference in emphasis will be large enough that the principal can take advantage of it and offer a contract that makes herself better off than she would be if she were to sell the firm

Figure 1: Optimal contracts



to the agent. With a disagreement contract the principal does not mind compensating the agent a lot if he is of type x_H because the principal assigns higher weight to not having to compensate the agent very much if he is of type x_L. The agent as usual gets exactly his reservation utility and therefore does not care. In this case (IC_L) binds and the contract has $e_L^* = e_L^o$ and $e_H^* > e_H^o$. Effort for the high-efficiency type is distorted upwards from the level that maximizes total surplus (given a high-efficiency type) to ensure incentive compatibility.

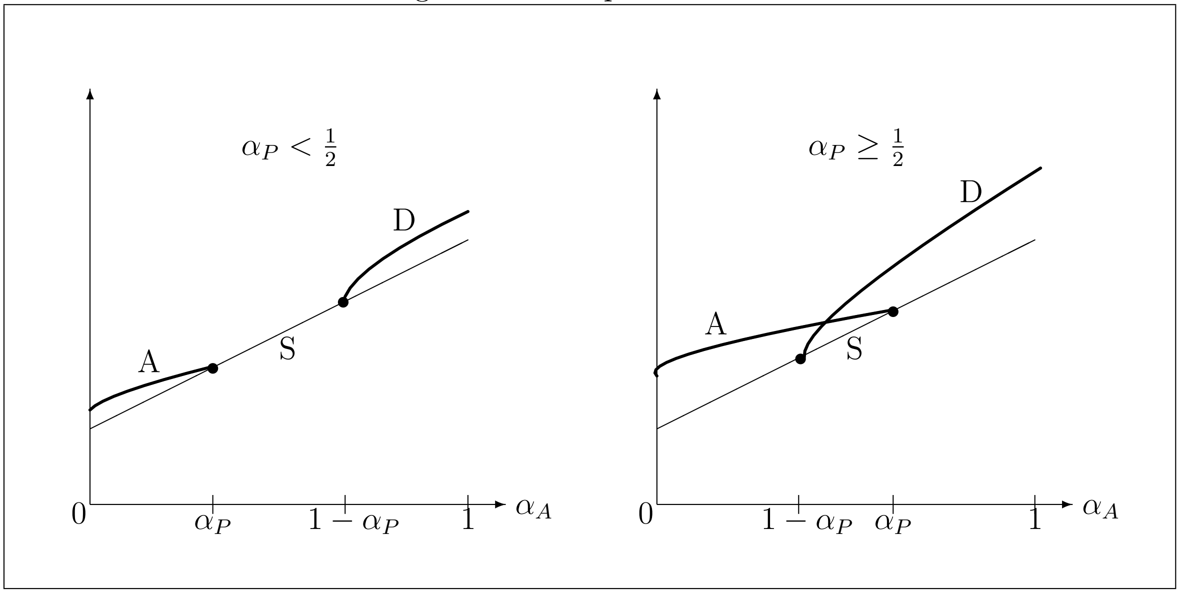
Agreement contracts: If $\alpha_A < \hat{\alpha} < \alpha_P$ (for the optimistic principal) or $\alpha_A < \alpha_P$ (for the pessimistic principal), an agreement contract where both parties are eventually best off if the agent turns out to be of the high-efficiency type can be implemented. When $\alpha_A < \alpha_P$ it is worthwhile to offer an agreement contract, because with such a contract the relatively pessimistic agent will put more emphasis than the principal on the worst final scenario, which is that he is of type x_L. The principal can exploit this to make herself better off than she would be if she were to sell the firm to the agent. The reason for the requirement that $\alpha_A < \alpha_P$ is the following: Since we have an agreement contract, the best lottery is the same from both the principal's and the agent's points of view, namely the one that assigns highest possible probability to x_H. Therefore, if $\alpha_A > \alpha_P$ the principal's overall weight on the agent being of low-efficiency type would be greater than the agent's under the agreement contract. Hence, there would be room for betting: The principal could provide higher utility for the agent when he is of high-efficiency type in return for getting higher utility for herself when the agent is of low-effieciency type, and do so at a favorable rate. Therefore, the principal would want to deviate from the agreement contract and move towards a disagreement contract to exploit this.

'Sell the firm to the agent' contracts: Finally, if $\alpha_A + \alpha_P < 1$ and $\alpha_A > \alpha_P$, neither a disagreement nor an agreement contract can be implemented. To see why, consider first a disagreement contract. Since the disagreement contract results in the agent eventually being best off if he is of type x_H, while the principal will be best off if the agent is of type x_L, $\alpha_A < 1 - \alpha_P$ means that the principal puts more emphasis on x_H than the agent does. Hence the principal would like to give the agent more if he is of type x_L in return for getting more for herself when the agent is of type x_H, which means that the principal would like to deviate from the disagreement contract and move in the direction of an agreement contract. An agreement contract, on the other hand, results in both parties eventually being best off if the agent is of type x_H, and thus $\alpha_A > \alpha_P$ means that the agent puts more emphasis on x_H than the principal does. Hence the principal would like to give the agent more if he is of type x_H in return for getting more for herself when the agent is of type x_L, which means that the principal would like to move away from the agreement contract in the direction of a disagreement contract. As a result, the only option for the principal is to offer a contract that makes herself equally well off regardless of the agent's type, that is, to sell the firm to the agent. Such a contract enables that the parties' emphasis is exactly equal, thereby giving no further desire for betting. None of the incentive compatibility constraints bind and the optimal contract has $e_L^* = e_L^o$ and $e_H^* = e_H^o$. Thus, when the parties have exactly the same overall weights on the final scenarios, the information asymmetry does not distort effort away from the levels that would maximize total surplus given the agent's type.

If $\alpha_A = 1 - \alpha_P \geq \frac{1}{2}$, either a 'sell the firm to the agent' contract or a disagreement contract with $e_L^* = e_L^o$ and $e_H^* = e_H^o$ can be implemented, since they give the principal the same utility. If $\alpha_A = \alpha_P \leq \frac{1}{2}$ either a 'sell the firm to the agent' contract or an agreement contract with $e_L^* = e_L^o$ and $e_H^* = e_H^o$ can be implemented, since they give the principal the same utility for these values of the preference parameters.

Interestingly, Proposition 2 implies that in a world with vagueness, the optimal contract

Figure 2: Principal's OWSEU



in a standard model with no vagueness, namely selling the firm to the risk-neutral agent, is only optimal when none of the other types of contracts can be implemented. For an optimistic principal, the standard optimal contract is, in fact, always dominated. The principal does better by creating an endogenous difference in emphasis. The effect of being able to exploit this difference in emphasis outweighs the effects of the incentive provision problem. Importantly, there also exist principals who will choose a 'sell the firm to the agent' contract for some agents, but not for other agents. If 'sell the firm to the agent' contracts are interpreted as franchising, the latter result explains the existence of chains that operate through a mix of franchising and centrally owned manager operated outlets.

Figure 2 outlines the principal's OWSEU as a function of the agent's level of optimism. The thicker curves labeled A and D show the principal's utility if she offers an agreement contract respectively a disagreement contract, while the thinner curve labeled S shows her utility if she sells the firm to the agent. For any level of α_A, the principal's utility of the optimal contract is given by the maximal of the curves.

The figure illustrates the answer to the question of what the agent's ideal level of optimism is from the principal's point of view. It would always be best for the principal to hire the most optimistic agent possible, i.e. an agent with optimism level α_A arbitrarily close to 1. The intuition for why optimistic agents are preferred is that optimism affects profits through the compensation of the agent. When deciding whether to accept an offered contract, the agent evaluates the tradeoff between the disutility he will suffer from doing the job and the compensation he will receive. If there is vague information about how difficult it will be to get the job done, the agent's optimism influences his perception of the disutility he will suffer. The more optimistic he is, the easier he will think it is to do the job, and the smaller is the compensation needed for him to accept the contract offered.

4 Concluding remarks

The above analysis has shown that in a world with vagueness and contracting between risk neutral parties, a 'sell the firm to the agent' contract is not necessarily the optimal contract. An optimistic principal will never choose to sell the firm to the agent because she can do better by offering an alternative contract. For a pessimistic principal, a 'sell the firm to the agent' contract is optimal for some values of the agent's optimism but not for other values.

When interpreting the 'sell the firm to the agent' contract as choosing to operate through a franchise, the results not only explain the coexistence of chains that operate exclusively through franchising and chains that operate exclusively through centrally owned manager operated outlets. The result that there exist principals who will choose a 'sell the firm to the agent' contract for some agents but not for others explains the existence of chains that operate through a mix of franchising and centrally owned manager operated outlets. Such a mix is, for example, used by some chains of gas stations.

The basic intuition and mechanism behind the effects is that vagueness gives room for the principal to affect which final scenario each of the parties puts most emphasis on through the design of the contract. This can sometimes be to the principal's advantage, because she can then exploit the presence of optimism or pessimism. The analysis has also illustrated the agent's ideal level of optimism from the point of view of the principal.

Appendix: Proof of Proposition 2

Suppose for simplicity (and without loss of generality) that $\overline{u} = 0$.

I first argue that pooling contracts are special cases of separating contracts. To see this, note first that a contract that satisfies the participation constraint (PC_P) for the pooling problem also satisfies the participation constraint (PC) for the separating problem. Furthermore, a contract with $w_H = w_L = w$ and $e_H = e_L = e$ satisfies both of the separating problem's incentive compatibility constraints (IC_H) and (IC_L) with equality. Hence, the pooling contract satisfies all the constraints for the separating problem, i.e. the constraint set for the pooling problem is a subset of the constraint set for the separating problem. Therefore, pooling contracts need not be considered separately. If the optimal contract is a pooling contract, this will emerge as the solution to the separating problem.

The principal's objective function is non-differentiable at the points (e_H, e_L, w_H, w_L) for which $\pi(e_H) - w_H = \pi(e_L) - w_L$, which is where $\overline{p}_P(h)$ and $\underline{p}_P(h)$ switch value. The value of the objective function at any interior candidate for solution therefore has to be compared to its value at these non-differentiability points. Below, I first find solutions to the first-order necessary conditions for interior solutions to the principal's problem, then I do comparison of values.

The Lagrangian for the principal's problem of finding the best separating contract when she faces a risk neutral agent is

$$\begin{aligned}\mathscr{L} = {}&\big(\alpha_P\overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big(\pi(e_H) - w_H\big) + \big[1 - \big(\alpha_P\overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big]\big(\pi(e_L) - w_L\big)\\ &+\gamma\Big[\big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big(w_H - g(e_H,x_H)\big) + \big[1 - \big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big]\big(w_L - g(e_L,x_L)\big)\Big]\\ &+\lambda_H\big[w_H - g(e_H,x_H) - w_L + g(e_L,x_H)\big] + \lambda_L\big[w_L - g(e_L,x_L) - w_H + g(e_H,x_L)\big].\end{aligned}$$

The first-order conditions for the problem are:

$$\big(\alpha_P\overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big) - \gamma^*\big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big) = \lambda_H^* - \lambda_L^*, \tag{1}$$

$$\big[1 - \big(\alpha_P\overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big] - \gamma^*\big[1 - \big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big] = \lambda_L^* - \lambda_H^*, \tag{2}$$

$$\begin{gathered}\big(\alpha_P\overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\pi'(e_H^*) - \gamma^*\big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)g_e(e_H^*,x_H)\\ -\lambda_H^* g_e(e_H^*,x_H) + \lambda_L^* g_e(e_H^*,x_L) \le 0,\end{gathered} \tag{3}$$

$$\begin{gathered}\big[1 - \big(\alpha_P\overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big]\pi'(e_L^*) - \gamma^*\big[1 - \big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big]g_e(e_L^*,x_L)\\ -\lambda_L^* g_e(e_L^*,x_L) + \lambda_H^* g_e(e_L^*,x_H) \le 0,\end{gathered} \tag{4}$$

$$\begin{gathered}\big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big(w_H^* - g(e_H^*,x_H)\big)\\ +\big[1 - \big(\alpha_A\overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big]\big(w_L^* - g(e_L^*,x_L)\big) \ge 0,\end{gathered} \tag{PC}$$

$$w_H^* - g(e_H^*, x_H) \geq w_L^* - g(e_L^*, x_H), \qquad (IC_H)$$

$$w_L^* - g(e_L^*, x_L) \geq w_H^* - g(e_H^*, x_L), \qquad (IC_L)$$

where (3), (4), (PC), (IC_H), and (IC_L) hold with equality if, respectively, e_H^*, e_L^*, γ^*, λ_H^*, and λ_L^* are strictly greater than zero.

It follows from (1) and (2) that $\gamma^* = 1$. Furthermore, since $\pi'(0) > 0$, $g_e(0, x_L) = g_e(0, x_H) = 0$, it follows from (3) and (4), respectively, that $e_H^* > 0$ and $e_L^* > 0$. Thus we have equality in (3), (4), and (PC). Equality in the latter means that the agent receives exactly his reservation utility. Note that since $g(e, x_H) < g(e, x_L)$ for all e, it follows from (IC_H) and (IC_L) that in any contract the agent is always best off when he turns out to be of the high-effieciency type. Note also that (IC_H) and (IC_L) together ensure that the principal cannot exploit differences in emphasis between the parties ad infinitum, thus guaranteeing that the wage payments w_H^* and w_L^* are bounded and that the problem has a solution. That is, (PC) with equality, (IC_H), and (IC_L) constitute a compact constraint set.

Party j's overall weight on the agent being of high-efficiency type is given by $\alpha_j \overline{p}_j(h) + (1 - \alpha_j)\underline{p}_j(h)$. The analysis of the first-order conditions can be broken down into 4 cases.

Case 1: $\lambda_\mathbf{L} > 0$ and $\lambda_\mathbf{H} = 0$. With $\lambda_H = 0$, (2) and (4) imply that $e_L^* = e_L^o$, and (IC_L) then implies that

$$w_L^* - g(e_L^o, x_L) = w_H^* - g(e_H^*, x_L). \qquad (5)$$

Equations (1) and (3) imply that e_H^* satisfies

$$\big(\alpha_P \overline{p}_P(h) + (1 - \alpha_P)\underline{p}_P(h)\big)[\pi'(e_H^*) - g_e(e_H^*, x_H)] = \lambda_L[g_e(e_H^*, x_H) - g_e(e_H^*, x_L)]$$

and therefore that $e_H^* > e_H^o$, since the right hand side of this expression is negative when $\lambda_L > 0$, and thus e_H^* is on the downward sloping part of the function $\pi(e) - g(e, x_H)$. Together (PC) and (5) now imply that

$$w_H = \big(\alpha_A \overline{p}_A(h) + (1 - \alpha_A)\underline{p}_A(h)\big) g(e_H^*, x_H) + \Big(1 - \big(\alpha_A \overline{p}_A(h) + (1 - \alpha_A)\underline{p}_A(h)\big)\Big) g(e_H^*, x_L). \quad (6)$$

By equations (1) and (2),

$$\lambda_L = \big(\alpha_A \overline{p}_A(h) + (1 - \alpha_A)\underline{p}_A(h)\big) - \big(\alpha_P \overline{p}_P(h) + (1 - \alpha_P)\underline{p}_P(h)\big),$$

so $\lambda_L > 0$ gives that

$$\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h) > \alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h). \tag{7}$$

That is, the principal's overall emphasis (or weight) on high-efficiency type is lower than the agent's.

Using (5) and (6), the principal's Bernoulli utility can now be calculated to be

$$\pi(e_H^*) - w_H^* = \pi(e_H^*) - g(e_H^*, x_L) + \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big(g(e_H^*, x_L) - g(e_H^*, x_H)\big) \tag{8}$$

when the agent is of type x_H and

$$\pi(e_L^o) - w_L^* = \pi(e_L^o) - g(e_L^o, x_L) + \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big(g(e_H^*, x_L) - g(e_H^*, x_H)\big) \tag{9}$$

when the agent is of type x_L. Since e_L^o is the argmax of the function $\pi(e) - g(e, x_L)$, it follows from (8) and (9) that the principal will eventually be best off if the agent turns out to be of the low-efficiency type x_L. Thus, we have a disagreement contract with $\overline{p}_P(h) = \underline{p}_A(h) = a < \underline{p}_P(h) = \overline{p}_A(h) = b$. Then (7) implies that $\alpha_A > 1 - \alpha_P$.

Using (8) and (9), the principal's utility in case 1 can now be written as

$$\begin{aligned} OWSEU_P \;=\;& \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big[\pi(e_H^*) - g(e_H^*, x_H)\big] \\ &+ \big[1 - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big]\big[\pi(e_L^o) - g(e_L^o, x_L)\big] \\ &+ \Big[\big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big) - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\Big]\Big[g(e_H^*, x_L) - g(e_H^*, x_H)\Big]. \end{aligned} \tag{10}$$

The right-hand side of equation (10) is non-decreasing in the agent's level of optimism for $\alpha_A \in (1-\alpha_P, 1]$. To see why, consider the constraint set. Denote the optimal contract when the agent's optimism is $\hat{\alpha}_A > 1 - \alpha_P$ by $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$. Since this is a solution, it satisfies (IC_H) and (IC_L), and (PC) holds with equality. Consider now an agent with optimism $\tilde{\alpha}_A > \hat{\alpha}_A$. The contract $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$ obviously still satisfies (IC_H) and (IC_L) for the $\tilde{\alpha}_A$-agent, since these constraints do not depend on α_A. To see that (PC) will also be satisfied for the $\tilde{\alpha}_A$-agent, note that (IC_H) and (IC_L) imply that $\hat{w}_H - g(\hat{e}_H, x_H) > \hat{w}_L - g(\hat{e}_L, x_L)$, such that $\overline{p}_A(h) > \underline{p}_A(h)$, and therefore $\tilde{\alpha}_A > \hat{\alpha}_A$ implies that

$$\begin{aligned} & \big(\tilde{\alpha}_A \overline{p}_A(h) + (1-\tilde{\alpha}_A)\underline{p}_A(h)\big)\big(\hat{w}_H - g(\hat{e}_H, x_H)\big) + \big[1 - \big(\tilde{\alpha}_A \overline{p}_A(h) + (1-\tilde{\alpha}_A)\underline{p}_A(h)\big)\big]\big(\hat{w}_L - g(\hat{e}_L, x_L)\big) \\ > \;& \big(\hat{\alpha}_A \overline{p}_A(h) + (1-\hat{\alpha}_A)\underline{p}_A(h)\big)\big(\hat{w}_H - g(\hat{e}_H, x_H)\big) + \big[1 - \big(\hat{\alpha}_A \overline{p}_A(h) + (1-\hat{\alpha}_A)\underline{p}_A(h)\big)\big]\big(\hat{w}_L - g(\hat{e}_L, x_L)\big) \\ = \;& 0. \end{aligned}$$

Hence $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$ also satisfies (PC) for the $\tilde{\alpha}_A$-agent. Since the $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$-contract satisfies all the constraints for the $\tilde{\alpha}_A$-agent, the optimal contract must make the principal at least as well off. It follows that $OWSEU_P$ is non-decreasing in the agent's level of optimism for $\alpha_A \in (1-\alpha_P, 1]$.

Case 2: $\lambda_{\mathbf{H}} > \mathbf{0}$ and $\lambda_{\mathbf{L}} = \mathbf{0}$. With $\lambda_L = 0$, (1) and (3) imply that $e_H^* = e_H^o$, and (IC_H) then implies that

$$w_H^* - g(e_H^o, x_H) = w_L^* - g(e_L^*, x_H). \tag{11}$$

Equations (2) and (4) imply that e_L^* satisfies

$$\big(1 - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big)[\pi'(e_L^*) - g_e(e_L^*, x_L)] = \lambda_H[g_e(e_L^*, x_L) - g_e(e_L^*, x_H)]$$

and therefore that $e_L^* < e_L^o$, since the right hand side of this expression is positive when $\lambda_H > 0$, and thus e_L^* is on the upward sloping part of the function $\pi(e) - g(e, x_L)$. Together (PC) and (11) now imply that

$$w_L = \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big) g(e_L^*, x_H) + \big(1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big) g(e_L^*, x_L). \tag{12}$$

By equations (1) and (2),

$$\lambda_H = \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big) - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big),$$

so $\lambda_H > 0$ gives that

$$\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h) < \alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h). \tag{13}$$

That is, the principal's overall emphasis (or weight) on high-efficiency type is higher than the agent's.

Using (11) and (12), the principal's Bernoulli utility can now, similarly to case 1, be calculated to be

$$\pi(e_H^o) - w_H^* = \pi(e_H^o) - g(e_H^o, x_H) + \big(1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big)\big(g(e_L^*, x_H) - g(e_L^*, x_L)\big) \tag{14}$$

when the agent is of type x_H and

$$\pi(e_L^*) - w_L^* = \pi(e_L^*) - g(e_L^*, x_H) + \big(1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\big)\big(g(e_L^*, x_H) - g(e_L^*, x_L)\big) \tag{15}$$

when the agent is of type x_L. Since e_H^o is the argmax of the function $\pi(e) - g(e, x_H)$, it follows from (14) and (15) that the principal will eventually be best off if the agent turns out to be of the high-efficiency type x_H (recall that it is always the case that in order for the contract to be incentive compatible, it must eventually make the agent best off if he turns out to be of high-efficiency type). Hence, we have an agreement contract with $\overline{p}_P(h) = \overline{p}_A(h) = b > \underline{p}_P(h) = \underline{p}_A(h) = a$. Then (13) implies that $\alpha_A < \alpha_P$.

Using (14) and (15), the principal's utility in case 2 can now be written as

$$\begin{aligned} OWSEU_P \;=\; & \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big(\pi(e_H^o) - g(e_H^o, x_H)\big) \\ & + \big[1 - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big]\big(\pi(e_L^*) - g(e_L^*, x_L)\big) \\ & + \Big[\big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big) - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\Big]\big(g(e_L^*, x_L) - g(e_L^*, x_H)\big). \end{aligned} \tag{16}$$

The same argument that was used to show that $OWSEU_P$ in (10) is non-decreasing in α_A for $\alpha_A \in (1-\alpha_P, 1]$ can be used to show that $OWSEU_P$ in (16) is also non-decreasing in the agent's level of optimism for $\alpha_A \in [0, \alpha_P)$.

Case 3: $\lambda_{\mathbf{H}} = \mathbf{0}$ and $\lambda_{\mathbf{L}} = \mathbf{0}$. With $\lambda_L = \lambda_H = 0$, (1) and (3) imply that $e_H^* = e_H^o$ and (2) and (4) imply that $e_L^* = e_L^o$, hence the contract specifies the first-best levels of effort, which maximize total surplus given the agent's type. Equations (1) and (2) imply that

$$\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h) = \alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h), \tag{17}$$

i.e. the parties must assign the same weight to the agent being of type x_H. Equation (PC) now gives that

$$w_L^* = g(e_L^o, x_L) + \frac{\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)}{1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)}\big(g(e_H^o, x_H) - w_H^*\big),$$

and (IC_H) and (IC_L), neither of which binds, then give that

$$\begin{aligned} & g(e_H^o, x_H) + \Big(1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\Big)\big(g(e_L^o, x_L) - g(e_L^o, x_H)\big) \\ \leq \;\; & w_H^* \leq g(e_H^o, x_H) + \Big(1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\Big)\big(g(e_H^o, x_L) - g(e_H^o, x_H)\big). \end{aligned}$$

If $\pi(e_H^o) - w_H^* > \pi(e_L^o) - w_L^*$, i.e. with an agreement contract, (17) requires that $\alpha_A = \alpha_P$, since $\overline{p}_P(h) = \overline{p}_A(h) = b$ and $\underline{p}_P(h) = \underline{p}_A(h) = a$. If $\pi(e_H^o) - w_H^* < \pi(e_L^o) - w_L^*$, i.e. with a disagreement contract, (17) requires that $\alpha_A = 1 - \alpha_P$, since $\overline{p}_P(h) = \underline{p}_A(h) = a$ and $\underline{p}_P(h) = \overline{p}_A(h) = b$. If $\pi(e^o_H) - w^*_H = \pi(e^o_L) - w^*_L$, we are at the corner where the objective function is non-differentiable. The potential corner solutions will be considered below.

The principal's utility in case 3 is

$$\begin{aligned} OWSEU_P &= \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\Big[\pi(e^o_H) - g(e^o_H, x_H)\Big] \\ &\quad + \Bigg(1 - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\Bigg)\Big[\pi(e^o_L) - g(e^o_L, x_L)\Big] \\ &= \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\Big[\pi(e^o_H) - g(e^o_H, x_H)\Big] \\ &\quad + \Bigg(1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)\Bigg)\Big[\pi(e^o_L) - g(e^o_L, x_L)\Big], \end{aligned} \tag{18}$$

where the last equality follows from (17).

Case 4: $\lambda_H > 0$ and $\lambda_L > 0$. With $\lambda_H > 0$ and $\lambda_L > 0$, (IC_H) and (IC_L) imply that $e^*_H = e^*_L$. At the same time, (1) and (3) imply that e^*_H satisfies

$$\big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)[\pi'(e^*_H) - g_e(e^*_H, x_H)] = \lambda_L[g_e(e^*_H, x_H) - g_e(e^*_H, x_L)]$$

and therefore that $e^*_H > e^o_H$, while (2) and (4) imply that e^*_L satisfies

$$\big(1 - \big(\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h)\big)\big)[\pi'(e^*_L) - g_e(e^*_L, x_L)] = \lambda_H[g_e(e^*_L, x_L) - g_e(e^*_L, x_H)]$$

and therefore that $e^*_L < e^o_L$, which together imply that $e^*_H > e^*_L$. Therefore, this case leads to a contradiction.

Corner contracts: $(\mathbf{e_H}, \mathbf{e_L}, \mathbf{w_H}, \mathbf{w_L})$ for which $\pi(\mathbf{e_H}) - \mathbf{w_H} = \pi(\mathbf{e_L}) - \mathbf{w_L}$. At these points, the principal's objective function is non-differentiable. Since these are 'sell the firm to the agent' contracts where the principal is equally well off regardless of the agent's type, $\overline{p}_P(h)$ and $\underline{p}_P(h)$ can take any value in the interval Q. Specifically, they can take the value $\overline{p}_P(h) = \underline{p}_P(h) = \alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h) = \alpha_A b + (1-\alpha_A)a$. Then $\alpha_P \overline{p}_P(h) + (1-\alpha_P)\underline{p}_P(h) = \alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)$, i.e. the parties have equal emphasis on the two final scenarios and the principal has no incentive to deviate.

The best the principal can do at the corner is to set $e^*_H = e^o_H$ and $e^*_L = e^o_L$. The contract still has to satisfy the constraints, so (PC) gives that

$$w^*_L = g(e^o_L, x_L) + \frac{\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)}{1 - \big(\alpha_A \overline{p}_A(h) + (1-\alpha_A)\underline{p}_A(h)\big)}\big(g(e^o_H, x_H) - w^*_H\big),$$

and (IC_H) and (IC_L) then give that

$$\begin{aligned} & g(e_H^o,x_H)+\Big(1-\big(\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)\big)\Big)\big(g(e_L^o,x_L)-g(e_L^o,x_H)\big) \\ \leq\ & w_H^* \leq g(e_H^o,x_H)+\Big(1-\big(\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)\big)\Big)\big(g(e_H^o,x_L)-g(e_H^o,x_H)\big). \end{aligned}$$

The principal's utility at the corner is

$$\begin{aligned} OWSEU_P &= \big(\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)\big)\Big[\pi(e_H^o)-g(e_H^o,x_H)\Big] \\ &\quad +\Big(1-\big(\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)\big)\Big)\Big[\pi(e_L^o)-g(e_L^o,x_L)\Big], \end{aligned} \tag{19}$$

since $\alpha_P\overline{p}_P(h)+(1-\alpha_P)\underline{p}_P=\alpha_A\overline{p}_A(h)+(1-\alpha_A)\underline{p}_A(h)$. Note that (19) is also increasing in the agent's optimism.

I now have to establish which type of contract is optimal in the different areas of the (α_A,α_P)-space. First note that if $\alpha_A=\alpha_P$, then (16)=(18) and is equal to (19) in value. On the other hand, if $\alpha_A=1-\alpha_P$, then (10)=(18) and is equal to (19) in value.

In general, subtracting (16) from (19) gives the difference between the principal's utility with the corner contract and an agreement contract. Denote this difference by Δ_P^{SA}. It is given by

$$\begin{aligned} \Delta_P^{SA} &= \big(\alpha_Ab+(1-\alpha_A)a-\big(\alpha_Pb+(1-\alpha_P)a\big)\Big[\pi(e_H^o)-g(e_H^o,x_H)+g(e_L^*,x_L)-g(e_L^*,x_H)\Big] \\ &\quad +\Big(1-\big(\alpha_Ab+(1-\alpha_A)a\big)\Big)\Big[\pi(e_L^o)-g(e_L^o,x_L)\Big] \\ &\quad +\Big(1-\big(\alpha_Pb+(1-\alpha_P)a\big)\Big)\Big[\pi(e_L^*)-g(e_L^*,x_L)\Big]. \end{aligned} \tag{20}$$

Taking the derivative of (20) with respect to α_A gives

$$\begin{aligned} \frac{\partial\Delta_P^{SA}}{\partial\alpha_A} &= (b-a)\Big[\pi(e_H^o)-g(e_H^o,x_H)+g(e_L^*,x_L)-g(e_L^*,x_H)-\big[\pi(e_L^o)-g(e_L^o,x_L)\big]\Big] \\ &\quad +\Big[\big(\alpha_Ab+(1-\alpha_A)a-(\alpha_Pb+(1-\alpha_P)a)\big)\big(g_e(e_L^*,x_L)-g_e(e_L^*,x_H)\big) \\ &\quad -\big(1-(\alpha_Pb+(1-\alpha_P)a)\big)\big(\pi'(e_L^*)-g_e(e_L^*,x_L)\big)\Big]\frac{\partial e_L^*}{\partial\alpha_A} \\ &= (b-a)\Big[\pi(e_H^o)-g(e_H^o,x_H)+g(e_L^*,x_L)-g(e_L^*,x_H)-\big[\pi(e_L^o)-g(e_L^o,x_L)\big]\Big], \end{aligned} \tag{21}$$

where the latter equality follows from the first order conditions. The derivative in (21) is positive. Therefore, since $\Delta_P^{SA}=0$ when $\alpha_A=\alpha_P$, (21) gives that (16) is greater than (19) for all $\alpha_A<\alpha_P$.

Subtracting (10) from (19) instead gives the difference between the principal's utility with the corner contract and a disagreement contract. Denote this difference by Δ_P^{SD}. It is given by

$$\begin{aligned}
\Delta_P^{SD} \;=\;& \big(\alpha_A b + (1-\alpha_A)a\big)\Big[\pi(e_H^o) - g(e_H^o, x_H)\Big] - \big(\alpha_P a + (1-\alpha_P)b\big)\Big[\pi(e_H^*) - g(e_H^*, x_H)\Big] \\
&-\Big(\alpha_A b + (1-\alpha_A)a - \big(\alpha_P a + (1-\alpha_P)b\big)\Big]\Big[\pi(e_L^o) - g(e_L^o, x_L) + g(e_H^*, x_L) - g(e_H^*, x_H)\Big].
\end{aligned} \tag{22}$$

Taking the derivative of (22) with respect to α_A and using an envelope theorem argument as for $\frac{\partial \Delta_P^{SA}}{\partial \alpha_A}$, it follows that

$$\frac{\partial \Delta_P^{SD}}{\partial \alpha_A} = (b-a)\Big[\pi(e_H^o) - g(e_H^o, x_H) - \big[\pi(e_L^o) - g(e_L^o, x_L) + g(e_H^*, x_L) - g(e_H^*, x_H)\big]\Big], \tag{23}$$

which is negative. Therefore, since $\Delta_P^{SD} = 0$ when $\alpha_A = 1-\alpha_P$, (23) gives that (10) is greater than (19) for all $\alpha_A > 1-\alpha_P$.

Finally, subtracting (16) from (10) gives the difference between the principal's utility with a disagreement contract and an agreement contract. Denote this difference by Δ_P^{DA}. It is given by

$$\begin{aligned}
\Delta_P^{DA} \;=\;& \big(\alpha_P a + (1-\alpha_P)b\big)\big[\pi(e_H^*) - g(e_H^*, x_H)\big] + \big[1 - \big(\alpha_P a + (1-\alpha_P)b\big)\big]\big[\pi(e_L^o) - g(e_L^o, x_L)\big] \\
&+\Big[\big(\alpha_A b + (1-\alpha_A)a\big) - \big(\alpha_P a + (1-\alpha_P)b\big)\Big]\Big[g(e_H^*, x_L) - g(e_H^*, x_H)\Big] \\
&-\big(\alpha_P b + (1-\alpha_P)a\big)\big(\pi(e_H^o) - g(e_H^o, x_H)\big) - \big[1 - \big(\alpha_P b + (1-\alpha_P)a\big)\big]\big(\pi(e_L^*) - g(e_L^*, x_L)\big) \\
&-\Big[\big(\alpha_A b + (1-\alpha_A)a\big) - \big(\alpha_P b + (1-\alpha_P)a\big)\Big]\big(g(e_L^*, x_L) - g(e_L^*, x_H)\big).
\end{aligned} \tag{24}$$

Taking the derivative of (24) with respect to α_A and using an envelope theorem argument as for $\frac{\partial \Delta_P^{SA}}{\partial \alpha_A}$ gives that

$$\frac{\partial \Delta_P^{DA}}{\partial \alpha_A} = (b-a)\Big[g(e_H^*, x_L) - g(e_H^*, x_H) - \big(g(e_L^*, x_L) - g(e_L^*, x_H)\big)\Big], \tag{25}$$

which is positive.

Now, consider first an optimistic principal with $\alpha_P > \frac{1}{2}$. Then $1-\alpha_P < \alpha_P$. Therefore, when $\alpha_A = \alpha_P$, $\Delta_P^{SA} = 0$ and $\Delta_P^{SD} < 0$, implying that the optimal contract is a disagreement contract. Also, when $\alpha_A = 1-\alpha_P$, $\Delta_P^{SA} < 0$ and $\Delta_P^{SD} = 0$, implying that the optimal contract is an agreement contract. Because (10) is a continuous upward sloping function of α_A on $[1-\alpha_P, 1]$ and (16) is a continuous upward sloping function of α_A on $[0, \alpha_P]$, this implies that there exists $\hat{\alpha} \in (1-\alpha_P, \alpha_P)$ such that the optimal contract is a disagreement contract for all $\alpha_A > \hat{\alpha}$ and is an agreement contract for all $\alpha_A < \hat{\alpha}$, see also the right panel of Figure 2. For $\alpha_A = \hat{\alpha}$, an agreement contract and a disagreement contract are equally good.

Note that, although it is always possible to implement the effort levels that will maximize total surplus given the agent's type, it never optimal for the optimistic principal to do so, since the corner contract is dominated by either an agreement or a disagreement contract. The reason is that by offering one of these two contracts, the principal can make the agent put most emphasis on the final scenario that the principal cares the least about and that way she can exploit their difference in emphasis. The right panel of Figure 2 illustrates the optimistic principal's utility given the different types of contracts.

Second, consider a pessimistic principal with $\alpha_P < \frac{1}{2}$. Then $1-\alpha_P > \alpha_P$ and for $\alpha_A \in (\alpha_P, 1-\alpha_P)$ neither a disagreement nor an agreement contract is implementable. Also, the utility at the corner contract is lower when $\alpha_A = \alpha_P$ than when $\alpha_A = 1-\alpha_P$, since (19) is upward sloping. Using (21), (23), and (25), one can now see that the optimal contract is a disagreement contract for all $\alpha_A \geq 1-\alpha_P$, an agreement contract for all $\alpha_A \leq \alpha_P$, and the corner contract with which the principal is equally well off regardless of the agent's type for all $\alpha_A \in (\alpha_P, 1-\alpha_P)$.

Note that it is only optimal to implement the effort levels that will maximize total surplus given the agent's type when neither an agreement or a disagreement contract can be implemented (except when $\alpha_A = \alpha_P$ or $\alpha_A = 1-\alpha_P$). The corner contract is dominated by the other types of contracts when they can be implemented. The left panel of Figure 2 illustrates the pessimistic principal's utility given the different types of contracts.

Finally, when $\alpha_P = \frac{1}{2}$, the optimal contract is a disagreement contract for all $\alpha_A > \alpha_P$ and is an agreement contract for all $\alpha_A < \alpha_P$, and of either class for $\alpha_A = \alpha_P$.

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