Chapter 4: Putting Demand and Supply Curves to Work

You can teach a parrot to be an economist. Just get it to repeat over and over again: supply and demand, supply and demand.

The simple model of demand and supply is at once a reminder of how resources are guided by prices and a device for the analysis of public policy. The price mechanism was the subject of chapter 3. This chapter is about the analysis of public policy. The chapter begins with the technology of demand and supply curves, introducing the concepts of *deadweight loss*, *surplus* and *the full cost to the taxpayer per additional dollar of tax revenue*, all representable as areas on the demand and supply diagram. Deadweight loss is the harm to society from the taxpayer's diversion of consumption from more taxed to less taxed goods. Surplus is the benefit of having access to a commodity over and above the cost of producing it. The full cost to the taxpayer per additional dollar of tax revenue is central in determining whether public expenditure - on roads, public buildings, education or anything else - is warranted. Economic arguments are clarified when shapes of demand and supply curves are signified by their *elasticities*. The second part of the chapter employs these concepts in expounding some of the lessons of economics: the superiority of income taxation over excise taxation, the virtues of free trade, the harm from monopoly, the logic of patents, identifying circumstances where these lessons would seem to be valid together with important exceptions and limitations. The chapter concludes with a close examination of some properties of demand curves.

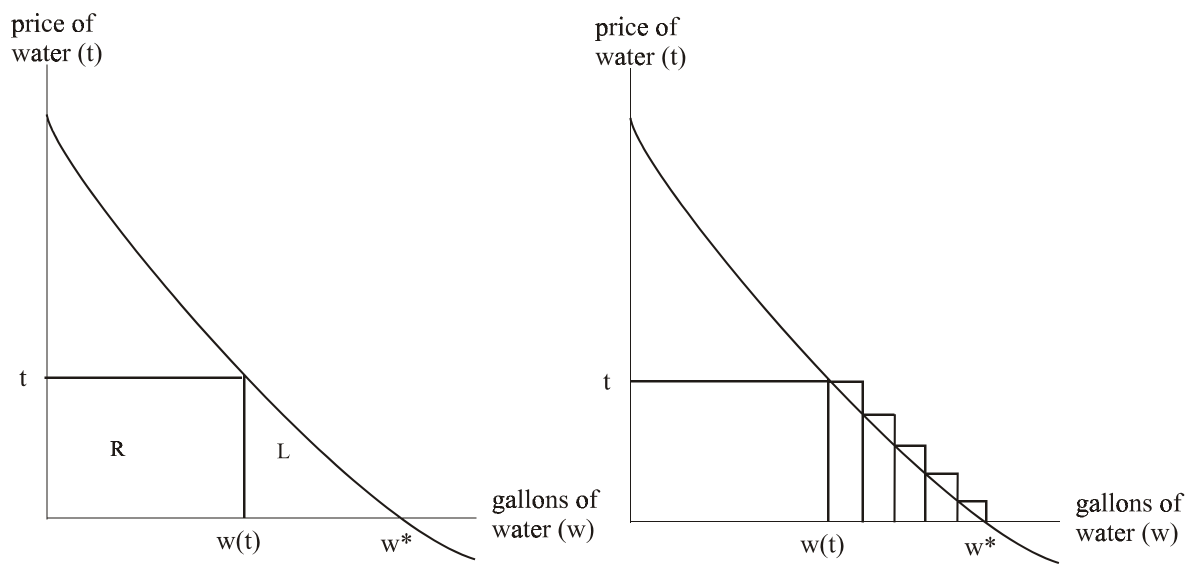
A: The Technology of Demand and Supply Curves

The Excess Burden of Taxation when the Taxed Good is Costless to Produce

Throughout most of this chapter, it will be assumed, as was assumed in chapter 3, that there are only two good and that those goods are bread and cheese. We begin, however, with an even simpler assumption. Suppose people consume, not bread and cheese, but bread and water where the significant difference between cheese and water is that water is a free good with no alternative cost of production in terms of bread. Imagine an economy where people are either farmers or policemen. Each farmer produces b^{max} loaves of bread and nothing else. As in chapter 2, a police force is required to protect people from one another. A fixed number of police - no more, no less - is required to maintain order, and the police are paid enough that they are just as well off as farmers. Water is a free good in the sense that it is available in unlimited amounts - as much as anyone would ever want to drink - from a well in the town square. The key assumption is that water can be taxed but bread cannot. Think of farmers as widely dispersed throughout the land in places where the tax collector cannot find them. By contrast, as there is only one well, the tax collector has no difficulty in determining how much water each person takes or in collecting tax. Everybody, including the police, pays the tax on water. The tax is assessed in loaves per gallon. A tax revenue of R loaves per person is required to finance the police force.

The assumption that bread cannot be taxed is representative of the fact that some goods are taxable and others are not. The question at hand is whether this restriction matters. Given that a tax on water can be collected fairly and expeditiously, does it matter to the representative consumer that public revenue could not be acquired by a tax on bread instead? One might be inclined to suppose that the restriction is of no importance because one tax is as good as another as long as everybody is affected identically and the required revenue is obtained. That would be mistaken. To see why, consider the demand for water as illustrated in figure 1.

Figure 1: The Deadweight Loss from a Tax on Water



The demand curve for water is illustrated in figure 1 with the "price" of water, in loaves per gallon, on the vertical axis and gallons per person on the horizontal axis. For any given quantity of water, the corresponding demand price is the number of loaves of bread per gallon one would be prepared to pay for one extra glass of water. The demand curve is illustrated twice, side by side, to emphasize different aspects of the taxation of water. It cuts the horizontal axis at w*, signifying that w* gallons of water per head would be consumed if water were available free of charge. Strictly speaking, all demand curves must cut the vertical axis at a point of satiation for the person or group to which the demand curve refers, but that is normally of no practical importance unless goods are free.

A tax on water is more burdensome to the taxpayer than an equivalent (in the sense of generating equal revenue) tax on bread. One way or another, R loaves of bread per person must be procured. When the R loaves of bread are procured by a tax on bread, everybody's consumption of bread is reduced by R loaves, but nobody's consumption of water is affected. Everybody consumes as much water as before. By contrast, when the R loaves of bread are procured by a tax on water, every person's consumption of bread is once again reduced by R loaves, but everybody's consumption of water is reduced as well. Taxation of water makes water expensive, reducing the amount of water each person chooses to drink. The amount of bread one would be prepared to give up to avoid this tax-induced reduction in the amount of water consumed is the excess burden, or deadweight loss, from the tax on water. It is a cost to the taxpayer over and above the cost of the tax he actually pays. Thus the full burden of taxation to the taxpayer includes not just the bread he actually pays as tax, but the water he is induced not to drink, despite the fact that his cutback in consumption of water is of no use to the policeman or anybody else. In short,

> The full cost of taxation (measured in loaves of bread)
> = the reduction in the consumption bread (the tax revenue)
> \+ the value in terms of bread of the reduction in the
> consumption of water (deadweight loss) (1)

For any given tax on water, the revenue from the tax and the deadweight loss from taxation can be represented as areas on figure 1. Since water would be free in the absence of the tax, the price of water and the tax on water are one and the same, and the demand curve for water can be represented by the equation $w = w(t)$ where t is the height of the demand curve when w gallons of water are consumed. Everybody, including the policeman, is taxed at a rate of t loaves of bread per gallon of water taken from the well. From the point of view of the user of water, the tax on water is a price. With a tax of t loaves per gallon, the revenue from the tax becomes $tw(t)$, represented on figure 1 by the rectangle R with base $w(t)$ and height t.

When the required revenue is extracted by a tax on bread, each person consumes $b^{max} - R$ loaves of bread and w^* gallons of water which is all anybody wants to drink when water is free. When the required revenue is extracted indirectly by a tax on water, each person consumes $b^{max} - R$ loaves of bread as before, but only $w(t)$ gallons of water. The source of deadweight loss is that what is in reality a transfer of bread from each tax payer to the rest of society - a transfer triggered by consumption of water - is seen by the tax payer as equivalent to a genuine cost of production. If water had to be produced and if the production of each gallon of water required the use of resources that might have been used to produce t loaves of bread instead, then people would be better off acquiring $w(t)$ rather than w^* gallons, for only when consumption of water is reduced to $w(t)$ would an extra gallon be worth the bread forgone to acquire it. Taxation induces people to look upon water as though it had been produced despite the fact that acquisition of water entails no loss of bread at all. The magnitude of the deadweight loss is the value in terms of bread of the tax-induced wastage of $w^* - w(t)$ gallons of water per tax payer when public revenue is acquired by the taxation of water rather than bread. The deadweight loss is an amount of bread just sufficient to compensate the representative consumer for the tax-induced wastage of water.

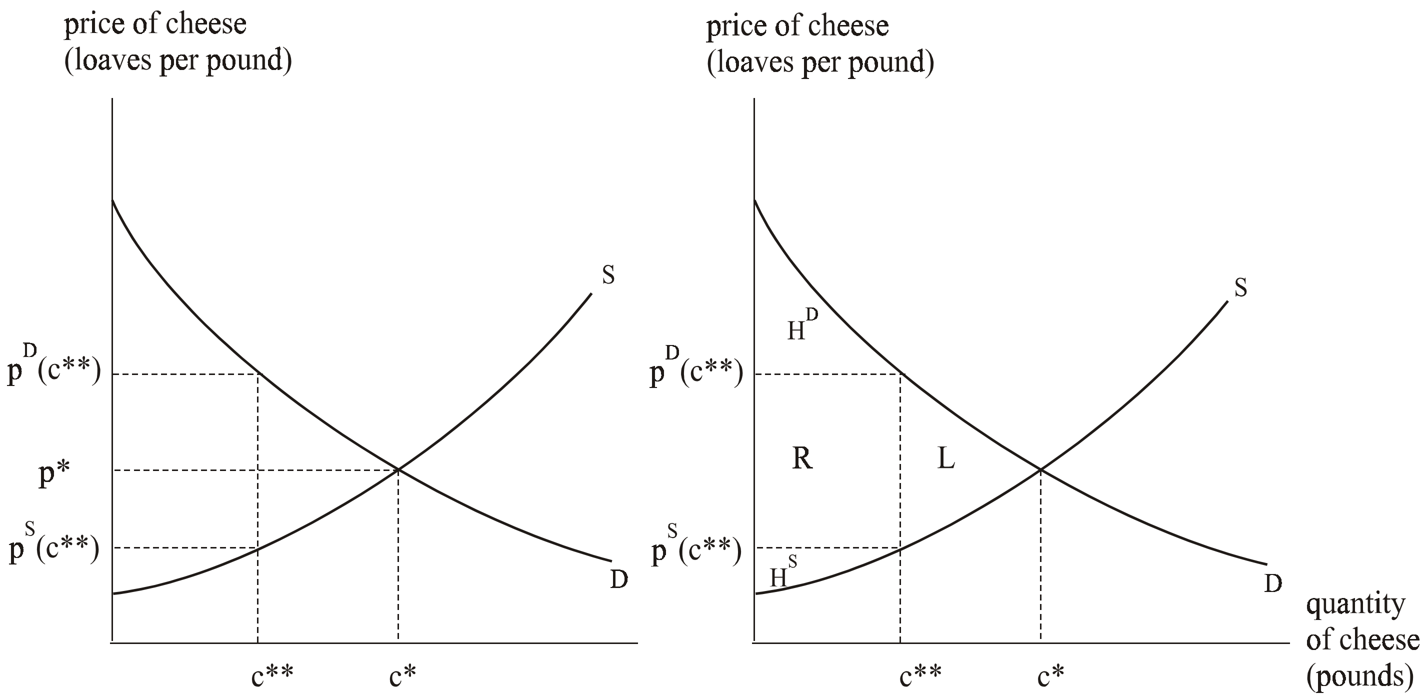
Deadweight loss is represented on the left-hand side of figure 1 as the triangular area L. To see why this is so, turn to the right-hand side of the figure. The distance from $w(t)$ to w^* is divided into equal segments. In the figure, there are five such segments, but the choice of the number of segments is arbitrary. When there are n segments, the width,) w, of each segment must be $[w^* - w(t)]/n$. Over each segment, a thin rectangle is constructed, equal in height to the demand curve at the beginning of the segment. By definition, the height of the demand curve over any point on the horizontal axis is the value of water - expressed as loaves per gallon - at that point. Thus the area of the thin rectangle constructed over the range from w to w +) w is the value of an extra) w gallons of water, the amount of bread one would be prepared to give up in exchange for the extra water, when one has w gallons already. The height of the rectangle over the first segment to the right of w(t) is the value of an extra gallon of water when one has w(t) gallons already, the height of the rectangle over the next segment is the value in terms of bread of an extra gallon of water when one has w(t) +) w gallons already, and so on. The tiny triangles above the demand curve may be ignored because the sum of the areas of all these triangles approaches 0 when n becomes large. The sum of all the areas of all the rectangles from w(t) to w* is the triangular area L on the left hand side of the figure, the full value in terms of bread of an extra w* - w(t) gallons of water when one has w(t) gallons already.

Taxation yielding a revenue of R imposes a cost on the taxpayer of R + L. Since R and L are defined as amounts of bread, the ratio of L/R is dimensionless and may equally be thought of as loaves of deadweight loss per loaf of tax revenue, or as dollars of deadweight loss per dollar of tax revenue. On the latter interpretation, the full cost of taxation per dollar of tax revenue is (R + L)/R. If R is 1000 loaves of bread and L is 200 loaves of bread, then the full cost per dollar of tax revenue becomes $1.20. The police force should be hired if and only if the benefit of the police force exceeds $1.20 for every dollar of taxation required to finance it.

Illustrating Tax Revenue, Deadweight loss and Surplus as Areas on the Demand and Supply Diagram

Return now to the bread and cheese economy, and suppose that cheese can be taxed but bread cannot. In principle, the tax on cheese could be assessed in pounds of cheese or in loaves of bread. Of every pound of cheese produced, one might be required to pay, for instance, an ounce of cheese or, alternatively, a half a loaf of bread to the tax collector. Assume for convenience that the numeraire in this economy is bread; the price of cheese is reckoned in loaves per pound and the tax on cheese is assessed in loaves per pound as well.

Figure 2: Tax Revenue, Deadweight Loss and Surplus



The impact of taxation is shown on figure 2, a standard demand and supply curve for cheese with price, p, graduated as loaves per pound on the vertical axis and quantity, c, graduated as pounds per person on the horizontal axis. Once again, the demand and supply curves are shown twice, side by side, each version conveying slightly different information. In the absence of taxation, the quantity of cheese produced and consumed would be c* where the demand and supply curves intersect and the price of cheese would be p*. With the imposition of a tax of t loaves per pound, the quantity of cheese falls from c* to c** at which the gap between the demand and supply prices is just equal to the tax. The demand price - the amount of bread people would be willing to give up to acquire an extra pound of cheese - rises from p* to $p^D(c^{**})$ and the supply price - the amount of bread that must be sacrificed to acquire an extra pound of cheese - falls from p* to $p^S(c^{**})$ where

$$p^D(c^{**}) - p^S(c^{**}) = t \tag{2}$$

The effect of the tax on cheese is to divert resources from the production of cheese to the production of bread, lowering the cost of cheese in terms of bread and raising its valuation as shown in the figure.

All prices and quantities are shown on the left hand side of the figure. The right hand side divides the area between the demand and supply curve into smaller areas - R, L, H^D and H^D - with important economic implications.

1) The area R is the revenue from the tax on cheese.

2) The area L is the deadweight loss, or excess burden, of taxation. It is the harm, assessed in loaves of bread, from the tax-induced diversion of production and consumption from taxed cheese to untaxed bread.

3) The area H^D is the remaining benefit to consumers from the availability of cheese, even though cheese is made more expensive by the imposition of the tax.

4) The area H^S is the remaining benefit of being able to produce cheese, even though the producer's price of cheese is reduced by the imposition of the tax.

5) The total area between the demand and supply curves - $R + L + H^D + H^S$ - is the benefit to people of being able to produce both bread and cheese rather than bread alone when production and consumption of cheese is not restricted by taxation. Called the *surplus* from the availability of cheese, it is the amount of extra bread one would require, over and above what people could produce for themselves, to compensate for the loss of the option to produce cheese as well. Together H^D and H^S, are the residual surplus when cheese is taxed. By definition,

$$\text{Total Surplus} = \text{Revenue} + \text{Deadweight Loss} + \text{Residual Surplus} \quad (3)$$

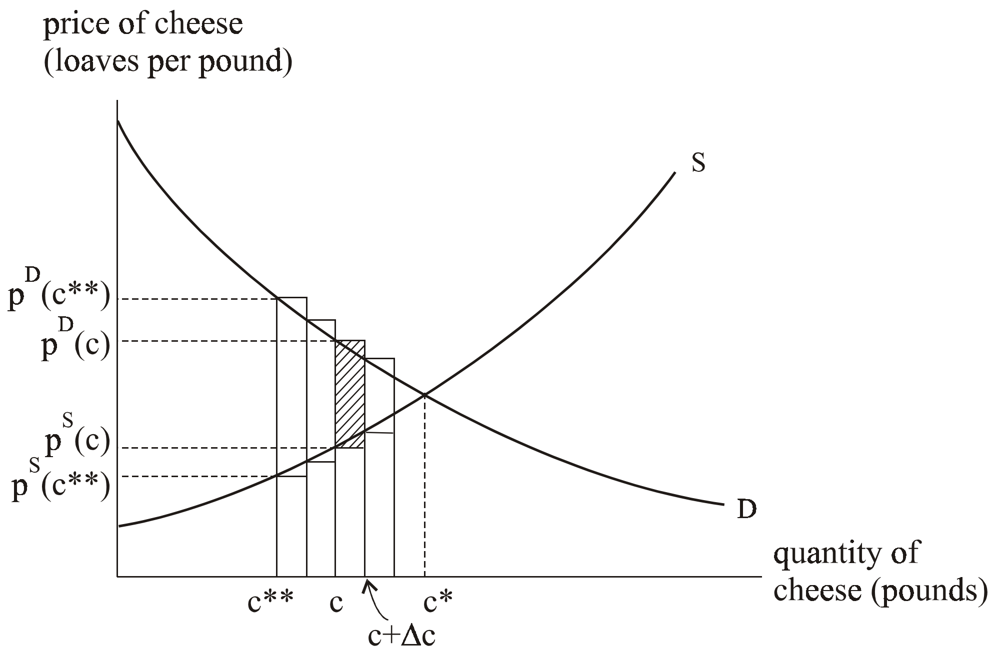
These interpretations of the areas on the demand and supply diagram are not immediately obvious, but will be explained in the course of this chapter. Note, however, that the interpretation of areas on the demand and supply diagram as amounts of bread is an immediate consequence of the definition of price. Since the dimension of area is quantity x price, the dimension of quantity is pounds of cheese, and the dimension of price is loaves of bread per pound of cheese, the dimension of area must be loaves of bread - pounds x (loaves/pounds).

The interpretation of R as tax revenue is straightforward. The revenue from the tax is tc** which, using equation (2), is equal to $[p^D(c^{**}) - p^S(c^{**})]c^{**}$ which is precisely R. The interpretation of the area L as waste requires some explanation.

Deadweight Loss

A minor extension of the bread and water example establishes the area L in figure 2 as the deadweight loss from the tax on cheese. This is shown with the aid of figure 3 pertaining to the bread and cheese economy but combining features of both figure 1 and figure 2. Once again,

Figure 3: The Deadweight Loss from the Taxation of Cheese



a tax on cheese reduces consumption from c* to c**, increasing the output of bread accordingly, raising the demand price of cheese from p* to $p^D(c^{**})$ and lowering the supply price of cheese from p* to $p^S(c^{**})$. As in figure 1, the distance from c** to c* is divided into n equal segments. Then, above each segment construct a pillar extending to the demand curve but originating not from the horizontal axis as in figure 1, but from the supply curve. One such pillar is shaded. It is the pillar over the segment from c to c +) c. The height of that pillar is $p^D(c) - p^S(c)$, its width is) c, and its area must therefore be [$p^D(c)$) c - $p^S(c)$) c]. From the definitions of demand and supply curves, it follows at once that the expression $p^D(c)$) c is the amount of extra bread that leaves a person as well off as before and the expression $p^S(c)$) c is the amount of extra bread produced when the output of cheese is reduced from c +) c pounds to c pounds per person and the output of bread is increased accordingly. The difference [$p^D(c) - p^S(c)$]) c must therefore be the net loss, measured in loaves of bread, from the reduction in the output of cheese when resources freed up from the production of cheese are devoted to the production of extra bread.

Since the area of each pillar in figure 3 is the bread-equivalent of the consumer's loss as the consumption of cheese is reduced along the segment at its base, the entire area between the demand curve and the supply curve over the range from c** to c*, being the sum of the areas of all the pillars, must equal the total loss defined as the amount of extra bread one would require to make the representative consumer as well off as he would be without the tax-induced reduction in the production and consumption of cheese. Ignore the tiny triangles above the demand curve and below the supply curve in figure 3 because these shrink to insignificance as the number of segments placed between c* and c** becomes very large. Taxation provokes people to produce and consume less cheese and correspondingly more bread even though everybody would be better off if nobody responded that way.

Surplus: The Discovery of Cheese

The full surplus from cheese is defined as the amount of extra bread required, over and above the amount people could produce for themselves, to compensate for the loss of the option of producing and consuming cheese as well. It is measured by the entire area between the demand and the supply curves in figure 2 over the range of c from the origin to c*. This can be demonstrated in two ways, an easy but slightly imprecise way and a more complex way that identifies the exact conditions under which the equivalence is valid. We shall consider both. The simple way involves nothing more than the observation that the deadweight loss L would occupy the entire area between the demand and supply curves if the tax on cheese were high enough to drive out consumption of cheese altogether. Also, having interpreted the total area between the demand and supply curves as the total surplus, the interpretations of H^D and H^S in figure 2 become obvious. Together, they must be the residual surplus after the loss of the tax revenue, R, and the deadweight loss, L. The consumers' surplus H^D is the full surplus as it would be if the supply curve were flat at a distance $p^D(c)$ above the horizontal axis. The producers' surplus, H^S, is the full surplus as it would be if the demand curve were flat at a distance $p^S(c)$ above the horizontal axis.

The other demonstration is a development of figure 5 of chapter 3 explaining how demand and supply curves on the bottom part of the figure are derived from indifference curves and the production possibility curve on the top. For the purpose of exposition, it is convenient to begin with an economy where only bread can be produced and then to measure the gain from the discovery of how to make cheese.

The gain cannot be expressed as utility directly because utility is "ordinal." Recall the conceptual experiment in which Robinson Crusoe's indifference curves were discovered. He was asked a long series of questions of the general form, "Do you prefer this to that?" where this and that were bundles of bread and cheese. From the answers to such questions, there could be drawn boundary lines separating all bundles that are preferred to some given bundle from all bundles that are dispreferred. The boundary lines themselves were the indifference curves. What is important to emphasize now is that the process of questioning and answering that yielded the shapes of the indifference curves did not at the same time yield any natural numbering of indifference curves. Numbers had to be imposed arbitrarily if they were to be obtained at all, the only constraint being that higher numbers be attached to higher curves. Thus three indifference curves numbered as 1, 2, and 3 could equally well be numbered as 7, 8, and 100, or as any increasing sequence at all. Similarly, the postulated utility function in the last chapter, $u = bc$, should be interpreted as nothing more than an assumption about the shapes of indifference curves and no significance should be attached to the absolute value of utility. The functions $u = (bc)^2$ or $u = A(bc)$, where A is any positive parameter, or $u = bc + K$, where K is any parameter at all, would contain exactly the same information. Any transformation of the function u = bc would do as long as the transformed value of u is an increasing function of bc. A function is said to be ordinal when meaning can be attached to the direction but not the magnitude of change. Utility is ordinal in that sense. An increase in utility is an ambiguous measure of the gain from public policy. But to claim that utility is ordinal is not to claim that improvement cannot be measured at all or that utility has no bearing on such measurement. People's benefit from the discovery of cheese can be measured as a utility-equivalent amount of bread, the amount of extra bread required to make one as well off as one would become from the discovery of how to make cheese.

Construction of such a measure requires a change in our working assumption about the shapes of indifference curves. The postulated utility function u = bc served us well in the last chapter because it yielded important results simply and because its special properties did not lead us seriously astray. Now we run into trouble. The difficulty with this utility function is the implication that both goods are indispensable. No amount of bread could can ever compensate people for the loss of the opportunity to consume cheese as well, for u = 0 whenever b = 0 or c = 0. This implication is sometimes quite realistic as, for instance, if c is interpreted as all food and b is interpreted as all clothing. Invented goods cannot be indispensable in that sense because people must have survived prior to the invention. To represent the gain from invention, a new representation of utility is required.

The new postulated utility function of the representative consumer is

$$u = 2 / c + (1-2) / b \qquad (4)$$

where b and c are his consumption of bread and cheese and where 2 is a parameter assumed to lie between 0 and 1. Like the utility function u = bc, the new utility function in equation (4) is bowed inward implying that, if a person is indifferent between two slices of bread and two slices of cheese, he must prefer a combination of one slice of bread and one slice of cheese. It is easily shown that the associated demand price of cheese in terms of bread becomes[1]

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[1] Rewrite equation (4) as $u = u^b + u^c$ where $u^b =(1-2) / b$ and $u^c =2 / c$. Then let $)u^b$ and $)u^c$ be the changes in u^b and u^c resulting from an increase $)c$ and a decrease $)b$ that leave u invariant so that the points {b, c} and {b -)b, c +)c} lie on the same indifference curve. Necessarily,

$$u^b +)u^b = (1-2) / (b -)b), \qquad u^c +)u^c = 2 / (c +)c) \qquad \text{and} \qquad)u^b = -)u^c$$

Squaring both sides of the first of these three equations, we see that

$$(1-2)^2(b -)b) = [u^b +)u^b]^2 = [u^b]^2 + 2u^b)u^b + [)u^b]^2$$

As in the derivation of the demand price in the last chapter, we can ignore terms that are the product of two first differences because such terms become very, very small relative to other terms, and, in the limit, vanishing altogether. Also, since $u^b =(1-2) / b$ by definition, the first terms on both sides of the equation cancel out, so that preceding equation reduces to $- (1-2)^2)b = 2u^b)u^b$, or

$$)u^b = - (1-2))b/2 / b$$

$$p^D = [2/(1-2)] / (b/c) \qquad (5)$$

The new utility function in equation (4) is a little more complicated than the utility function in the last chapter, but it has two properties that will prove useful here: (1) utility does not fall to 0 when one of the two goods is unavailable, and (2) the parameter 2 is an indicator of the relative importance of cheese as compared with bread in the sense that, for any given production possibility curve, more cheese is consumed and less bread when 2 is large than when 2 is small.

Suppose, for example, that people originally consumed 9 loaves of bread and 9 pounds of cheese (that is, b = 9 and c = 9) and consider how much extra bread would be required to compensate for the loss of all cheese. We are seeking to discover an amount of bread, b, such that a combination of b loaves of bread and no cheese yields the same utility as a combination of 9 loaves of bread and 9 pounds of cheese. By equation (4), the utility of 9 loaves of bread and 9 pounds of cheese is 3, regardless of the value of the parameter 2. To acquire a utility of 3 with no cheese at all, the quantity of bread, b, must be such that (1-2) / b = 3, or, equivalently, b = $[3/(1-2)]^2$. It follows immediately that the larger 2, the greater is the amount of bread required to compensate for the total loss of cheese. If 2 = 1/4, then b = 16 loaves. If 2 = ½, then b = 36 loaves. If 2 = 3/4, then b = 144 loaves. As 2 increases, cheese becomes ever more important in one's preferences in the sense that ever more bread would be required as compensation for its absence.

Initially, people do not know how to make cheese and must subsist on bread. Eventually, it is discovered how to make cheese and, from then on, people may consume a combination of bread and cheese, rather than just bread. Our problem is to determine how much better off people becomes as a consequence of the discovery. Before the discovery, each person produced b^{max} loaves of bread per day, and consumed all that he produced. The discovery itself can be interpreted as the acquisition of a production possibility curve for bread and cheese. On learning how to make cheese, people do not forget how to make bread. If b^{max} loaves of bread could be produced before the discovery, they could be produced afterwards too if people chose not to produce some cheese instead. The new production possibility curve for bread and cheese together must be consistent with his original productivity at bread-making before the discovery.

Suppose the new production possibility curve is

$$b^2 + c^2 = D \qquad (6)$$

A similar line of reasoning establishes that

$$) u^c = 2) c/2 / c$$

Finally, since $) u^b = -) u^c$, it follows that

$$p^D(b, c) =) b/) c = [2/(1-2)] / (b/c)$$ which is equation (5).

where the value of D in equation (6) must be $(b^{max})^2$ if capacity for bread-making remains undiminished. As shown in the last chapter, the corresponding supply price becomes

$$p^S = c/b \tag{7}$$

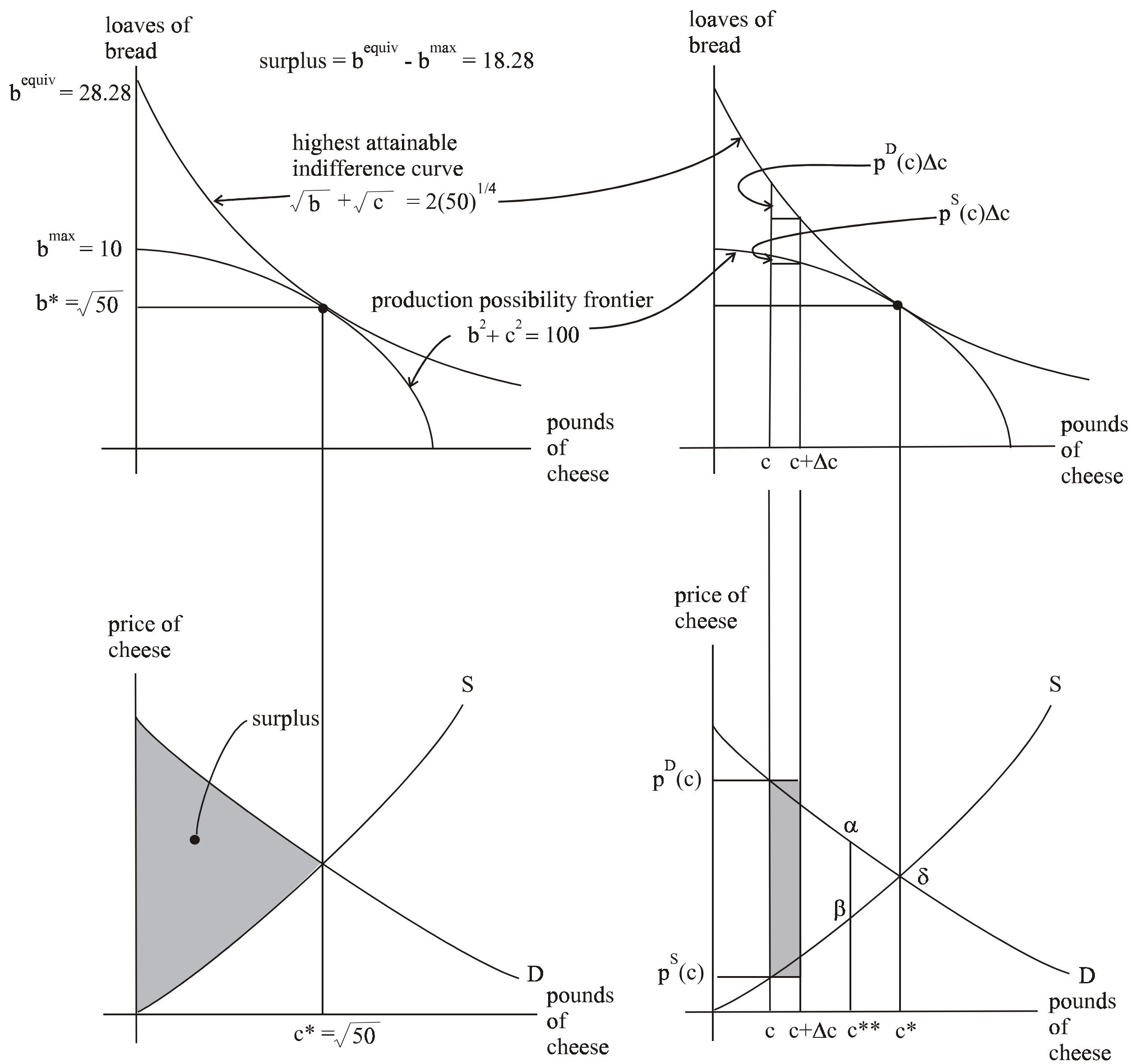
The gain from the discovery of how to produce cheese is illustrated in figure 4, which is a development of figure 5 in the last chapter. In effect, a variant of figure 5 is reproduced twice, side by side, with different information in each replication. On both sides of the figure, the production possibility curve in equation (6) is illustrated together with the highest attainable indifference curve in accordance with equation (4). As always, the chosen outputs of bread and cheese are represented by the point at which the two curves are tangent. This is the best people can do for themselves with the technology at their command. The chosen values of b and c depend on the shapes of the indifference curves which, in turn, depend on the chosen value of the parameter 2. For convenience of exposition, it is assumed in the construction of figure 5 that 2 = ½ so that 2 cancels out in the expression for the demand price in equation (5). It is also assumed that b^{max} equals 10 loaves of bread so that the value of D in equation (6) becomes 100.

Focus for the moment on the top left hand portion of the figure showing the production possibility frontier *after* the discovery of how to make cheese, together with the best attainable indifference curves for bread and cheese together. People make themselves as well off as possible by choosing a combination of bread and cheese {b*, c*} where the production possibility frontier is tangent to an indifference curve. Together, equations (5) and (7) imply that the common value of p^S and p^D must be 1 loaf per pound. A different value of 2 in the utility function or a differently shaped production possibility curve would have yielded a different equilibrium price. The symmetry in the assumptions about the shapes of the indifference curves and the production possibility frontier ensure that outputs of bread and cheese are equal. The chosen quantities of bread and cheese are b* = c* = /50. Employing the equation for the production possibility curve (6) to eliminate b in the equation for the supply price in equation (7), it is easily shown that the supply curve for cheese - the relation between quantity supplied and the relative supply price - becomes

$$c = 10\, p^S / / [1 + (p^S)^2] \tag{8}$$

Along the supply curve, the quantity of cheese, c, increases steadily with p^S, from 0 when $p^S = 0$, to /50 when $p^S = 1$, to 10 when p^S rises to infinity. With c* = b* = /50 and with 2 set equal to ½, the value of u in equation (4) become $(50)^{1/4}$ which is the highest attainable utility consistent with the production possibility curve in equation (6). Employing the utility function at those values of 2 and u to eliminate b from the demand curve in equation (5), it is easily shown that

Figure 4: The Measurement of Surplus



the demand curve for cheese, the relation between the quantity demanded and the demand price of cheese, becomes

$$c = 4\,/\,(50)/(1 + p^D)^2 \qquad (9)$$

Along the demand curve, the quantity of cheese, c, decreases steadily with p^D, from 4 / 50 when $p^D = 0$, to / 50 when $p^D = 1$, to 0 when p^D rises to infinity. These demand and supply curves are illustrated on the bottom left hand portion of figure 4. They cross at c = c* = / 50 where people are as well off as possible with the technology at their command.

Surplus is the measure of how much better off one becomes by learning how to make cheese. It is the amount of bread that would compensate for *not* learning how to make cheese. It is the extra bread required, over and above the original ten loaves, to become as well off as one would be on learning how to make cheese. It is the difference between b^{equiv} , the intersection with the vertical axis of the indifference curve through the point {b*, c*}, and b^{max} , the original production of bread which was 10 loaves per day. By definition,

$$/ b^{equiv} = / b^* + / c^* = (50)^{1/4} + (50)^{1/4} = 2\ (50)^{1/4}$$

so that $b^{equiv} = 4(50)^{½} = 28.28$. Thus

$$\text{Surplus} = b^{equiv} - b^{max} = 18.28$$

By learning how to make cheese, people becomes as well off as they would be if, instead, their production of bread increased from 10 loaves to 28.28 loaves per day.

In this example, the surplus from the invention is almost twice the original productive capacity (28.28 as compared with 10) and four times as large as the value (/ 50) in terms of bread of the cheese that is actually produced. Surplus is difficult to measure in practice, but there is reason to believe that it is often substantial. New products may convey benefits to the community well in excess of what people actually pay for them. Cars, air travel, television, and advances in medical technology have conveyed benefits to mankind far in excess of the cost of what we buy. Note that, like the equilibrium price, the surplus depends on taste as well as on technology. We have been assuming that the value of 2 in equation 1 is equal to ½ implying that bread and cheese are equally important as components of taste. Had it been assumed instead that 2 is smaller than 1/2, the surplus would have turned out smaller. Had it been assumed instead that 2 is larger than 1/2, the surplus would have been larger. If people did not like cheese there would be no surplus at all. The change in surplus becomes the indicator of gain or loss from the opening of trade, tariffs, taxation, monopolization and patents as will be shown below.

Though we may not have access to direct measures of distance between indifference curves, we can measure surplus indirectly, making use of an equivalence between *distances* in the upper part of the left hand side of figure 4 and *areas* in the lower part. Defined as the distance $b^{equiv} - b^{max}$ in the upper part of the figure, the surplus can be measured as the shaded area between the demand and supply curve in the lower part.

For an explanation of the equivalence, turn to the right hand side of figure 4 which is identical to the left hand side except for the removal of shading and the addition of a pipe is added over the range from c to c +) c where c is an arbitrarily chosen quantity of cheese within the span from the origin to c* on the horizontal axis. The pipe extends upward through both parts of the figure, cutting the production possibility frontier and the indifference curves on the upper part of the figure, and cutting the supply and demand curves on the lower part.

As shown on the upper part of the figure, the increase Δc in the output of cheese causes a *narrowing* of the gap between the indifference curve and the production possibility curve. From the definition of the demand price, it follows the height of the indifference curve is reduced by an amount $p^D(c)\Delta c$. From the definition of the supply price, it follows that the height of the production possibility curve is reduced by an amount $p^S(c)\Delta c$. Thus, the gap between the heights of the indifference curve and the production possibility curve narrows by an amount $[p^D(c) - p^S(c)]\Delta c$. The expression $[p^D(c) - p^S(c)]\Delta c$ must be positive when c is less than c* because $p^D(c)$ is necessarily equal to $p^S(c)$ when c equals c* and because the opposite curvatures of the indifference curve and the production possibility curve force $p^D(c)$ to increase more rapidly than $p^S(c)$ as c is reduced.

But the expression $[p^D(c) - p^S(c)]$ has already been identified in figure 3 as the difference between the heights of the demand and supply curves at the point c, and the expression $[p^D(c) - p^S(c)]\Delta c$ must be the area of the shaded pipe in the bottom part of the figure. To establish the equivalence between *distance* $b^{equiv} - b^{max}$ on the upper part of figure 4 and the shaded *area* between the demand and supply curves over the distance from 0 to c* on the lower part, divide the distance along the horizontal axis from the origin to c* into n equal segments, each of width Δc so that $n\Delta c = c^*$. Over each segment, the narrowing of the distance between the indifference curve and the production possibility curve on the top part of the figure is equal to the area of the between the demand and supply curves on the bottom part. The sum of the narrowings is the distance $b^{equiv} - b^{max}$. The sum of the areas is the total shaded area between the demand and supply curves. The surplus can be interpreted either way.

A similar line of argument leads to a measure of the loss of surplus when cheese production is reduced but not eliminated altogether and when resources withdrawn from the production of cheese are reallocated to the production of bread in accordance with the production possibility curve. As the bottom right-hand side of figure 4 is a virtual replication of figure 3, it is immediately evident that the area $\alpha\beta^*$ is the deadweight loss from the reduction in the output of cheese from c* to c**. Figure 4 identifies this loss as a vertical distance at the point c** between the production possibility curve and the highest attainable indifference curve.

There is a division of labour between these two essentially equivalent measures of surplus. The interpretation of surplus as a distance is best for establishing the meaning of the concept. The interpretation of surplus as an area is best for employing the concept in economic arguments and as a basis for measurement. Estimation based upon demand and supply curves requires information about price, quantity and elasticities of demand and supply for the commodity in question. Estimation based upon indifference curves and the production possibility curve would require information about the entire apparatus of production and the shapes of indifference curves, a considerably more formidable requirement in a world with a virtually infinite variety of goods than in the simple bread and cheese world of this chapter.

The Full Cost to the Taxpayer per Dollar of Additional Tax Revenue

The full cost per dollar of taxation to the taxpayer might be assessed as (R + L)/R, the sum of the tax actually paid and the deadweight loss from taxation expressed as a multiple of the tax paid, but for most purposes that ratio would be the right answer to the wrong question. When tax revenue is employed to pay for the police force, there is little advantage in knowing the full cost of the taxation required to pay for the police force because policemen must be hired regardless. The cost of anarchy in the absence of a police force would normally be far greater than the full cost of taxation to pay for the police force. The relevant question is not whether public services are costly, but when additional public projects, programs, or activities are warranted and how large the public sector ought to be.

Imagine a society that has already hired a certain number of policemen and is deciding whether to hire one more. The extra policeman would be helpful in reducing the incidence of crime but is not absolutely necessary for the preservation of society itself. Once, again, all tax revenue is acquired by a tax on cheese. The extra revenue to hire the additional policeman would have to be acquired by a slight increase in the tax rate on cheese, and there would be a corresponding increase in the deadweight loss from taxation. Suppose the benefit of the extra policeman is assessed at $x and the additional cost is assessed at $y, where y is the cost as seen by the accountant, the dollar value of the extra expenditure excluding the extra deadweight loss associated with the required increase in the tax rate to obtain the extra revenue.

If the public decision were whether or not to have a police force at all, and if the financing of the police force were the only object of public expenditure, then the right criterion would be whether the total benefit of the entire police force exceeds the total cost inclusive of the total deadweight loss. Had x and y been defined as average benefit and average cost per policeman, then a police force should be established if and only if x/y > (R + L)/R. But when the decision is about the enlargement of the police force, the right criterion becomes whether the additional benefit of an *extra* policeman exceeds the additional cost by the value of the extra deadweight loss. In other words, the extra policeman should be hired if and only if

x/y > ( ) R +) L)/) R (10)

where x and y are interpreted as extra benefit and cost rather than as average benefit and cost, where) R is the extra revenue required, and where) L is the extra deadweight loss generated by the required increase in the tax rate to finance the extra expenditure.

Precisely the same criterion is appropriate for any additional public expenditure. An enlargement of the police force, a new tank for the army, a new school or a new hospital is worth acquiring if and only if its ratio of benefit to cost exceeds the critical ratio, ( ) R +) L)/) R, of full cost to the tax payer, inclusive of deadweight loss, per additional dollar of tax revenue (or, equivalently, per additional dollar of public expenditure). As will be discussed in chapter 10, the economy-wide equilibrium value of this ratio - commonly referred to as the *marginal cost of public funds* - depends on the size of the public sector.

The meaning of this criterion is illustrated in figure 5 showing how revenue, R, and deadweight loss, L, change in response to the tax rate. To keep the story as simple as possible, the supply curve of cheese is assumed to be flat, indicating that the rate of substitution in production of bread for cheese is invariant no matter how much or how little cheese is produced. The analysis could be extended to allow for an upward-sloping supply curve.

As shown in figure 5, the supply price is invariant at p, that is, $p^S(c) = p$ for all c. The demand curve shows how the demand price, $p^D(c)$, varies with c. Since $p^D(c)$ is always equal to p + t in equilibrium, there is no harm in representing c itself as a function of t. Thus c(t) is the amount of cheese produced and consumed at a tax of t loaves per pound of cheese, and $c(t + \Delta t)$ is the amount of cheese produced and consumed when the tax is raised to $t + \Delta t$. As functions of the tax rate, total revenue, total deadweight loss, and the increments in revenue and deadweight loss can be represented as areas on the figure.

At the two tax rates, t and $t + \Delta t$, revenue and deadweight loss are:

$$R(t) = tc(t) = F + B \tag{11}$$

$$R(t + \Delta t) = (t + \Delta t)c(t + \Delta t) = F + A \tag{12}$$

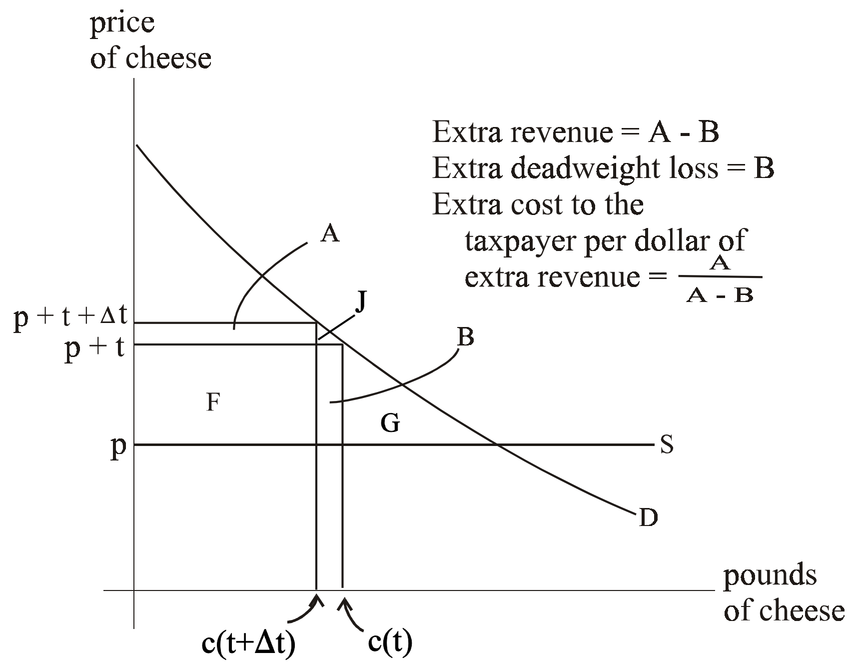
$$L(t) = G \tag{13}$$

and

$$L(t + \Delta t) = G + B + J \tag{14}$$

where F, B, A, G and J are areas in figure 5. The area J is the triangular area at the meeting of the areas A and B, It may be ignored because it is very small compared to A or B, small enough that the ratios J/A and J/B approach 0 when Δt approaches 0. The area J will be ignored from now on.

Figure 5: The Full Cost to the Taxpayer per Additional Dollar of Tax Revenue



It follows at once that

$$\Delta R(t) = R(t + \Delta t) - R(t) = A - B \tag{15}$$

$$\Delta L(t) = L(t + \Delta t) - L(t) = B \tag{16}$$

and

$$(\Delta R + \Delta L)/\Delta R = 1/(1 - B/A) \tag{17}$$

It is evident from inspection of figure 5 that, when the demand curve is approximately linear and as long as the ratio B/A remains less than 1, an increase in the tax rate leads to an increase in the ratio B/A which in turn leads to an increase in the full cost per additional dollar of public revenue.

The relation among tax rate, tax revenue and deadweight loss can now be illustrated in a simple example. In this example, it is convenient to express the price of cheese in money rather than in loaves of bread, so that revenue and deadweight loss can be expressed in money too. Fix the price of bread at $1 per loaf and let the supply price of cheese - the number of loaves of bread forgone per pound of cheese produced when resources are diverted from the production of bread to the production of cheese - be 2 loaves per pound or, equivalently, $2 per pound. The supply price is independent of the quantity of cheese produced because the supply curve of cheese is assumed to be flat.

Suppose the demand curve for cheese is

$$c = 5 - \tfrac{1}{2}p^{D} \tag{18}$$

In the absence of taxation, the demand and supply prices of cheese must be the same, so that c(0) becomes equal to 5 - ½p. A tax on cheese of t dollars per pound raises the demand price of cheese from p to p + t. Thus, as a function of the tax rate, the demand for cheese becomes

$$c(t) = 5 - \tfrac{1}{2}(p + t) = 4 - \tfrac{1}{2}(t) \tag{19}$$

Table 1 is a comparison of quantity of cheese demanded, tax revenue, deadweight loss, additional tax revenue and additional deadweight loss at several tax rates from $1 per pound to $8 per pound. In constructing the table, it is assumed that the demand curve for cheese is in accordance with equation (19) above. As a function of t, the revenue from the tax on cheese becomes R(t) = tc(t) and the deadweight loss becomes L(t) = ½[c(0) - c(t)] because the area L is a perfect triangle when, as assumed, the demand curve is a downward-sloping straight line. For any t,)R and)L can then be computed from equations (15) and (16).

The table is largely self-explanatory. Alternative tax rates are listed in the first column. The next four columns show demand price, quantity demanded, tax revenue, and deadweight loss, all as functions of the tax rate. The last three columns show the increase in tax revenue resulting from a dollar increase in the tax rate, the increase in the deadweight loss resulting from a dollar increase in the tax rate, and the full cost to the taxpayer per dollar of tax paid. The perverse result in last four rows of the final column will be explained below.

Table 1: How the Full Cost to the Taxpayer per Dollar of Tax Paid Increases with the Tax Rate when Cheese Can be Taxed but Bread Cannot

tax ($ per pound)	demand price ($ per pound)	quantity demanded (pounds per person	tax revenue ($ per person)	deadweight loss ($ per person)	additional tax revenue ($ per person)*	additional deadweight loss ($ per person)**	full cost to the taxpayer per additional dollar of tax revenue ($)***
t	p^D	c(t) = 4 - ½t	R(t) = tc(t)	L(t) = ½[c(0) -c(t)]t	) R(t) = R(t) - R(t-1)	) L(t) = L(t) - L(t-1)	[ ) R(t) +) L(t)] /) R(t)
0	2	4	0	0	-	-	-
1	3	3½	3½	37624	3½	1/4	1.07
2	4	3	6	1	2½	3/4	1.3
3	5	2½	7½	2 1/4	1½	1 1/4	1.83
4	6	2	8	4	½	1 3/4	4.5
5	7	1½	7½	6 1/4	- ½	2 1/4	-3.5
6	8	1	6	9	- 1½	2 3/4	-0.83
7	9	½	3½	12 1/4	- 2½	3 1/4	-0.3
8	10	0	0	16	- 3½	3 3/4	-0.07

*[area A less area B] in figure 5 **area B in figure 5 ***[area A]/[area A less area B] in figure 5

Several features of this table are interesting in themselves and can be generalized well beyond the confines of our bread-and-cheese economy. First, the higher the tax rate, the larger the deadweight loss. Deadweight loss increases steadily from 25¢ per person when the tax is $1 per pound to $16 per person when the tax is $8 per pound. The larger t, the larger the gap between the demand price and the supply price, and the larger the tax-induced distortion of the pattern of consumption as people are induced to consume less and less cheese even though the value of cheese to the consumer exceeds the cost of production. Second, the relation between the tax rate and the tax revenue is humped. One might suppose that every increase in the tax rate would yield an increase in tax revenue, but that turns out not to be so. Eventually, as the tax rate gets higher and higher, the shrinkage of the tax base outweighs the rise in the tax rate. Revenue peaks at $8 per head when the tax rate is $4 per pound, and revenue declines steadily thereafter. Third, as an immediate consequence of the foregoing, the full cost per additional dollar of tax revenue increases steadily with the tax rate. With a tax on cheese of $1 per pound, the full cost is $1.07. With a tax of $2 per pound, the full cost rises to $1.30. With a tax of $4 per pound, the full cost becomes as high as $4.50. Beyond that, there is no extra revenue from an increases in the tax and the full cost per additional dollar of tax revenue is negative, indicating that the government can increase revenue by *lowering* the tax rate.

The gradual rise, together with the tax rate, in the full cost per additional dollar of tax revenue has immediate implications for public expenditure on new projects or programs. Suppose the government is contemplating a new project - the establishment of a new hospitals, school or road - that would cost $130 million and would yield benefits deemed to be worth $150 millions dollars per year. Think of the benefits as spread out over the entire population so that distributional considerations may be ignored. Since the numbers in the table are per person and the costs and benefits of the project are for the nation as a whole, a conversion is required to bring the information in the table to bear on the problem at hand. Suppose that people consume only bread and cheese, that all public revenue is acquired by the taxation of cheese, and that the numbers in the table refer to purchases and revenues per person per *week* in a large economy with a millions people whose demand for cheese is represented by equation (19). Suppose also that the initial tax on cheese is $1 per pound, so that as shown in the second row, tax revenue is $3.50 per person per week, for a total of $182 million per year (3.5 x 52 x 1,000,000). Extra revenue to finance the new project can only be obtained by increasing the tax rate.

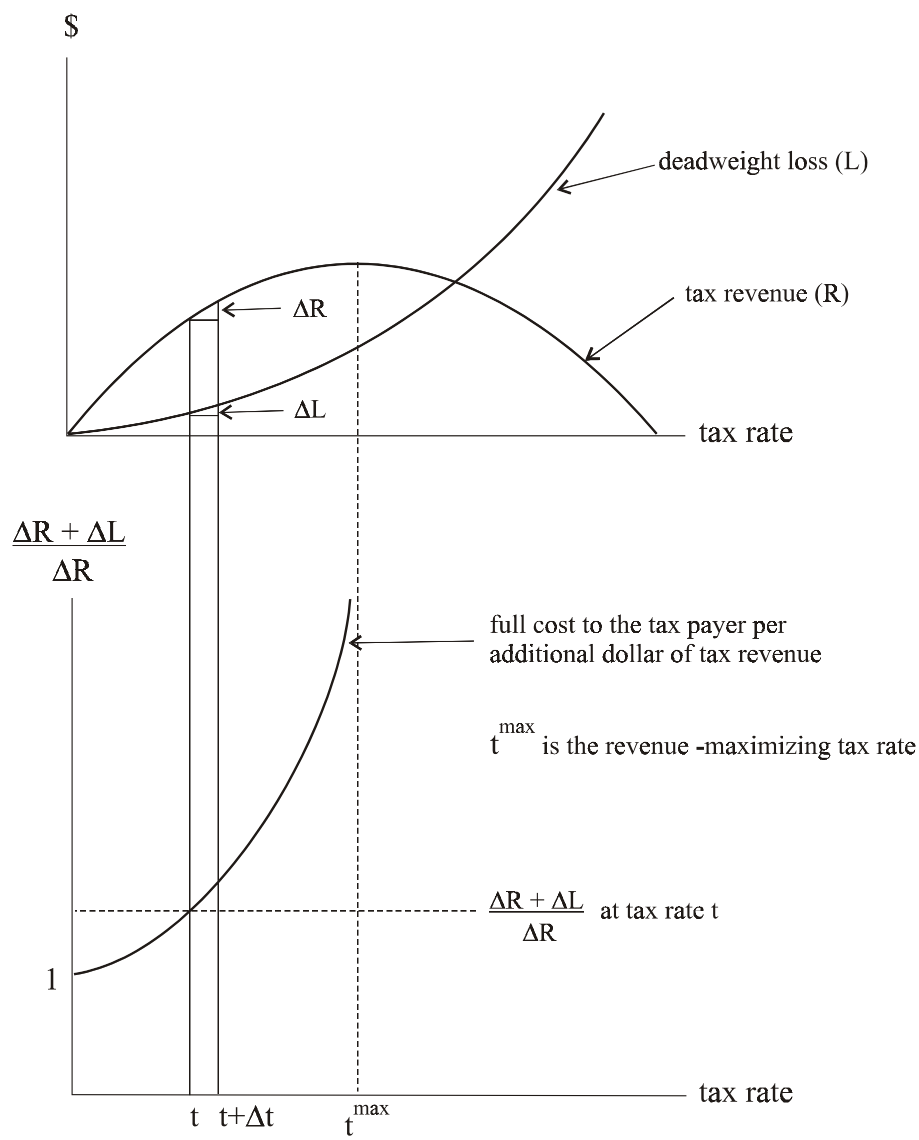
One's first thoughts on the matter would be that (1) a program costing $130 million and yielding $150 million worth of benefits is clearly advantageous and should be undertaken, and (2) if a tax of $1 per pound of cheese yields a total revenue of $182 million, then an extra $130 million of expenditure could be raised by increasing the tax on cheese from $1.00 per pound to $1.71 per pound [because (130/182) = 0.71].

Both inferences would turn out to be wrong, and for the same reason. As the tax rate on cheese rises, people shift purchases from taxed cheese to untaxed bread, buying less cheese and more bread. The acquisition of an extra $130 million of revenue requires an increase in tax revenue per person from $3.50 to $6.00 [(182 + 130) / 52 = 6] per week, requiring an increase in the tax rate on cheese not from $1.00 to $1.71, but from $1.00 to $2.00, as shown in the third row of the table. In other words, an increase in the tax on cheese from $1 per pound to $2 per pound raises revenue per head from $3.50 to $6.00 rather than to $7.00 as one who ignored the tax-induced diversion of purchasing power would automatically expect. The extra $2.50 per person per week is just sufficient to supply the extra $130 million (2.5 x 52) required to finance the new project or program.

The extra tax-induced diversion of purchasing power imposes costs over and above the cost of the tax-financed project itself. With a tax of $1 per pound, people consume cheese only to the point where the value of the last pound is equal to $3, a cost of production of $2 plus an additional $1 of tax. A rise in the tax from $1 to $2 per pound raises the price of cheese from $3 to $4, leading to a reduction of consumption of cheese to the point where its value to the consumer is, once again, equal to its price. Extra taxation induces people to reduce consumption of a good that has become worth $4 per pound even though it costs only $2 to produce. As shown in the second to last column in the table, the additional deadweight loss from this diversion of purchasing power is 75¢ per pound. As shown in the last column, the full cost per additional dollar of tax revenue is 1.30 [(2.5 + 0.75)/2.5]. Together, the last three columns show that an additional expenditure of $2.50 per person per week generates an extra deadweight loss of 75¢ per person per week, raising the total cost to $3.75 per person per week, for a total of $1.30 per dollar of public expenditure. Thus, to be worth undertaking, a program or project must cover its cost scaled up by a factor of 1.3. A project costing $130 million must yield benefits of at least $169 (130 x 1.3). A benefit of only $150 is not enough. The project ought to be rejected.

The relation between tax revenue and deadweight loss is illustrated on the top half of figure 6 with the tax rate on the horizontal axis and with dollars' worth on the vertical axis. The hive-shaped curve shows how tax revenue varies with the tax rate and tax revenue. The steadily

Figure 6: How Tax Revenue and Deadweight Loss Vary with the Tax Rate



rising curve shows how deadweight loss varies with the tax rate. The bottom half of the figure shows the full cost per additional dollar of tax revenue, ( ) R +) L)/) R, as a function of the tax rate. The components of the expression,) R and) L, are read off the top half of the figure. As long as $t < t^{max}$, the expression ( ) R +) L)/) R increases with t, rising to infinity at $t = t^{max}$ where, by definition,) R is equal to 0. These curves capture the essential features of table 1, but they are relevant to any and every tax system where the tax base shrinks because taxpayers can divert effort from more taxed to less taxed activities.

The hive-shaped relation between tax revenue and tax rate, commonly referred to as the *Laffer* curve, is characteristic of all taxation: the taxation of cheese in this example, excise taxation, income taxation, tariffs, and so on. No tax-setter - not a monopoly, not a predatory government, and certainly not a government with the interests of the citizens in mind - would deliberately raise taxes beyond the point where the tax revenue is as large as possible. It has sometimes been alleged that countries have occasionally placed themselves on the wrong side of the Laffer curve by mistake. Particular taxes may be on the wrong side of the Laffer curve when some public purpose is served by reducing consumption of the taxed good. The tax on tobacco may be on the wrong side of the Laffer curve to deter smoking, as discussed in the next chapter.

Elasticity as a Measure of the Steepness of Demand and Supply Curves.

It is immediately evident from the inspection of figure 2 that the steeper the demand and supply curves, the smaller is the deadweight loss as a proportion of the tax revenue. Thus, for choosing among taxes or for estimating the full cost per additional dollar of public expenditure, it would seem helpful to have at hand a standard measure of the steepness of curves. One’s first though on the matter is that steepness could be measured by the slope, by the change in price per unit change in quantity, or, equivalently, by the change in quantity per unit change in price. Suppose the price of bread is fixed at one dollar per loaf, so that the price of cheese, defined in the first instance as “loaves per pound” can be reformulated as $ per pound. If a reduction of 100 pounds in the quantity of cheese leads to an increase of $2 in the demand price together with a decrease of $1 in the supply price, we would say that the demand curve is twice as steep, on average, as the supply curve, and we would infer that the greater part of the burden of taxation is borne by consumers.

There are, however, several difficulties with the slope as a measure of the steepness of demand or supply curves. First, the indicator of steepness is affected by the units of measurement. A steepness of 2 when the quantity of cheese is graduated in pounds translates into a steepness of 32 when the quantity of cheese is graduated in ounces and into a steepness of 1/4,000 when the quantity of cheese is graduated in tons. Second, the steepness of the demand curves for different goods could not be compared. There would seem to be no intuitively meaningful basis for deciding whether the demand for cheese is more price-sensitive than the demand for oranges. Third, and most important, we would like the steepness of a market demand curve to inherit the steepness of the of the demand curves of the individuals in that market. If the demand side of the market for cheese consists of 1,000 identical buyers and if the steepness of each buyer’s demand curve for cheese is 2, then the steepness of the market demand curve for cheese would have to be 2/1,000. We would prefer an alternative measure of steepness for which these two numbers would have to be the same.

All three difficulties are circumvented when steepness is measured as an *elasticity* rather than as a slope. When two variables change together, the elasticity of one with respect to the other is defined as the percentage change in the former as a multiple of the percentage change in the latter. The elasticity of demand or supply - typically represented by the Greek letter ε - is defined as the *percentage* (or proportional) change in quantity in response to a *percentage* (or proportional) change in price along the demand curve or the supply curve as the case may be.

Formally, an elasticity, ε, of demand or supply is

$$\varepsilon = (\Delta c/c) \div (\Delta p/p) \tag{20}$$

where c now refers to quantity (of any good, not just cheese), p refers to price, Δp is a change in price and Δq is the corresponding change in quantity along the curve. As the supply curve is upward sloping, we expect Δp and Δq to have the same sign and the corresponding elasticity to be positive. As the demand curve is downward sloping, we would expect changes along the curve to have different signs, but, by convention, we speak of elasticities of demand as positive. It follows immediately from the definition of elasticity that a flat demand curve has an infinite elasticity of demand, that a vertical demand curve has a zero elasticity of demand, and that a clockwise rotation of a demand curve around some point translates into an decrease in the elasticity of the curve at that point. The same is true of elasticities of supply, except that a clockwise rotation leads to an increase in the elasticity of supply. A useful distinction may be drawn between *arc* elasticities and *point* elasticities. Arc elasticities are defined over finite portions of demand or supply curves. Point elasticities are defined, the name suggests, at particular points on demand or supply curves. The distinction is best clarified after demand and supply elasticities are formally defined.

Consider any two quantities of the good in question, c^1 and c^2. The elasticity of supply over the range from c^1 to c^2 is

$$\varepsilon^S = [\Delta c/c^1]/[\Delta p^S/p^S] \tag{21}$$

where Δc means $[c^1 - c^2]$, p^S means $p^S(c^1)$, and Δp^S means $[p^S(c^1) - p^S(c^2)]$. Similarly the elasticity of demand over the range from c^1 to c^2 is

$$\varepsilon^D = - [\Delta c/c^1]/[\Delta p^D/p^D] \tag{22}$$

where Δc means $[c^1 - c^2]$, p^D means $p^D(c^1)$, and Δp^D means $[p^D(c^2) - p^D(c^1)]$, ensuring that the elasticity of demand is positive even though the slope of the demand curve is negative. These are automatically arc elasticities when there is a finite distance between c^1 and c^2. They become point elasticities when c^1 and c^2 are placed very close together. When c^2 is close to c^1, the numerator,

[ ) c/c^1], and the denominators, [ ) p^S/p^S] and [ ) p^D/p^D], of the expressions for elasticity become very small, but the point elasticities themselves remain finite. Elasticities can be estimated from time series of prices, quantities and other variables governing the shifts of the curves over time. The measurement of elasticities is part of econometrics, a subject outside the range of this book but almost always included in undergraduate programs in economics. Though the concept of elasticity is most commonly employed in connection with demand and supply curves, the concept itself is more general. Whenever a variable x is a function of another variable y, an elasticity of x to y may be defined.

Suppose an increase in price of cheese from $2 per pound to $3 per pound leads to a decrease in a person's demand for cheese from 4 pounds to 3.5 pounds. The elasticity of demand over that range is 0.25 [(½ /4) ÷ (½)]. The numerator of the expression, (½ /4), is the proportional change in quantity demanded, and the denominator, (½), is the proportional change in price. Quantity units cancel out in the numerator, and price units cancel out in the denominator, leaving the elasticity itself with no dimension at all. Extending the example from one person to a market with 1,000 such people, the elasticity of demand remains equal to 0.25 because a decrease in each person's demand for cheese from 4 pounds to 3.5 pounds is a decrease in the market demand from 2 tons to 1 3/4 tons, which is exactly the same percentage change. Being dimensionless, the elasticity is comparable between goods. It is meaningful to say, if that be the case, that the elasticity of demand for cheese is greater than the elasticity of demand for potatoes.

A number of questions are best framed with reference to elasticities.

1) How is the burden of taxation divided between producers and consumers? As shown in figure 2 above, a tax on cheese of t loaves per pound reduces production and consumption of cheese from c* to c** pounds, raises the demand price of cheese from p* to $p^D(c^{**})$ and lowers the supply price of cheese from p* to $p^S(c^{**})$. Denote the fall in the output of cheese by) c, the rise in the price of cheese by) p^D and the fall in the price of cheese by) p^S where, by construction,

$$) c = c^* - c^{**}, \quad) p^D =) p^D(c^{**}) - p^* \quad \text{and} \quad) p^S = p^* - p^S(c^{**}) \tag{23}$$

and

$$) p^D +) p^D = t \tag{24}$$

Thus the consumers' share of the burden of the tax becomes) p^D/t and the producers' share becomes) p^S/t. It is easily shown that consumers' and producers' shares depend on elasticities of demand and supply. Specifically,

$$) p^D/t = ,^S/(,^S + ,^D) \quad \text{and} \quad) p^S/t = ,^D/(,^S + ,^D) \tag{25}$$

where ε^D and ε^S are the arc elasticities of demand and supply over the range from c** to c*.[2]

2) How large is the deadweight loss as a proportion of the tax revenue? Referring again to figure 2 and using linear approximations to demand and supply curves over the range from c** to c*, the tax revenue, R, is tc** and the deadweight loss, L, is ½ t Δc. The deadweight loss as a proportion of tax revenue becomes

$$L/R = \tfrac{1}{2}\,\Delta c/c^{**} = \tfrac{1}{2}\,\tau\,\varepsilon^D\,\varepsilon^S(\varepsilon^D + \varepsilon^S) \qquad (26)$$

where τ is the tax rate expressed as a proportion of the price of cheese as it would be in the absence of taxation.[3] Whatever the elasticities of demand and supply, an increase in the tax rate increases the deadweight loss as a proportion of the tax revenue. With a tax on cheese of 50%, the deadweight loss becomes one eighth of the tax revenue when the elasticities of demand and supply are both equal to 1. The ratio of deadweight loss to tax revenue rises from an eighth to a quarter when the elasticities of demand and supply rise from 1 to 2. Note also that if the elasticity of supply were infinite as assumed in figure 4, the ratio of deadweight loss to revenue, L/R, reduces to $\tfrac{1}{2}\tau\,\varepsilon^D$.

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[2]By definition, $\varepsilon^D = (\Delta c/c^*)/(\Delta p^D/p^*)$ and $\varepsilon^S = (\Delta c/c^*)/(\Delta p^S/p^*)$ so that

$\varepsilon^S/\varepsilon^D = \Delta p^D/\Delta p^S$ and $\varepsilon^S/(\varepsilon^D + \varepsilon^S) = \Delta p^D/(\Delta p^D + \Delta p^S) = \Delta p^D/t$.

[3]The change in production and consumption of cheese can be expressed as a function of the elasticities and the tax rate. From equation (24) and the definitions of the elasticities, it follows that

$$t = \Delta p^D + \Delta p^S = (\Delta c/c^{**})(p^*/\varepsilon^D) + (\Delta c/c^{**})(p^*/\varepsilon^S) = (\Delta c/c^{**})(p^*)[1/\varepsilon^D + 1/\varepsilon^S]$$

so that $\Delta c/c^{**} = (t/p^*)\,\varepsilon^D\varepsilon^S/(\varepsilon^D + \varepsilon^S)$ and $L/R = \tfrac{1}{2}\Delta c/c^{**} = \tfrac{1}{2}\tau\,\varepsilon^D\varepsilon^S/(\varepsilon^D + \varepsilon^S)$.

3) What is the full cost to the taxpayer per additional dollar of tax revenue? Like the ratio of total deadweight loss to total tax revenue, this turns out to depend straightforwardly on the tax rate and the elasticities. Continue to assume, as in figure 4, that the supply curve of cheese is flat. It then turns out that

$$()R +)L)/)R = 1/(1 - ,_{ct}) = 1/(1 - J,^{D}) \qquad (27)$$

where $,_{ct}$ is the elasticity of tax base to tax rate and is just equal to $J,^{D}$.[4]

Equation (27) also identifies the appropriate mix of tax rates on some commodities when taxation of other commodities is blocked. If there were three goods, bread, cheese and oranges, if only cheese and oranges could be taxed, and if each tax shifted consumption away from the taxed good toward untaxed bread rather than toward the other taxed good, then a government seeking to minimize the total cost of taxation to the tax payer would choose rates of tax on cheese and oranges to so that their full costs per additional dollar of tax revenue - indicated as $1/(1 - ,^{D}J)$ in equation (27) - would be the same. That in turn requires the tax rate on each good to be inversely proportional to its elasticity of demand; the higher the elasticity of demand , the lower the rate of tax. This rule is consistent with the general principle from the preceding section that, if all goods can be taxed, then their rates of tax should be the same. The absence of the elasticity of supply in equation (27) is because, for convenience of exposition, the supply curve in figure 5 has been assumed to be flat. Let the supply curve be upward-sloping, and equation (28) must be modified to account for both the elasticity of demand and the elasticity of supply.

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[4] Recall from equation (17) that $()R +)L)/)R = 1/(1 - B/A)$ where, as illustrated in figure 4, B is defined as $t)c$ and A is defined as $c)t$. Thus $B/A =()c/c)/()t/t)$ which is the elasticity of tax base to tax rate, called $,_{ct}$ in equation (28). But

$$,_{ct} = ()c/c)/()t/t) = (t/p)()c/c)/()t/p)$$

where t/p is the tax rate as a proportion of the initial price p, and $()c/c)/()t/p)$ is the ordinary elasticity of demand.

B: The Income Tax, Monopoly, Patents, Tariffs and the Gain from Trade

The Superiority of the Income Tax over the Excise Tax

The incautious reader might suppose that, if the taxation of cheese causes a deadweight loss, the imposition of a tax on bread as well would magnify the loss, increasing the gap between the cost of taxation to the tax payer and the revenue from the tax. That is not so. Deadweight loss arises not because cheese is taxed, but because bread is exempt from tax. Deadweight vanishes when all goods are taxed at the same rate. Furthermore, in so far as the income tax is equivalent to a set of excise taxes at equal rates on all goods consumed, the income tax must be free of deadweight loss.

To illustrate these propositions, consider a society where 1) a proportion T of the population is required to serve on the police force, 2) everybody else is a farmer who can produce either b^{max} loaves of bread per day, or c^{max} pounds of cheese per day, or any combination of the two, and 3) the salary of the police is financed by taxation of farmers at a level just high enough that policemen and farmers are equally well off. Assumption 2 ensures that the production possibility curve identifying all feasible outputs of bread and cheese is a downward-sloping straight line and that the supply curve of cheese is horizontal as shown in figure 5.

Diversion of a proportion T of the population from farming to policing reduces total outputs of bread and cheese accordingly. The maximal output of bread *per person* falls from b^{max} to $b^{max}(1-T)$. The maximal output of cheese *per person* falls from c^{max} to $c^{max}(1-T)$. The production possibility curve identifying all feasible outputs of bread and cheese *per person* (not per farmer) becomes

$$P^{\$B} b + P^{\$C} c = Y(1 - T) \qquad (28)$$

where $P^{\$B}$ and $P^{\$C}$ are the money prices of bread and cheese, graduated as dollars per loaf and dollars per pound, Y is the value of a farmer's production, the common value of $b^{max}P^{\$B}$ which is the amount of money a farmer could earn by producing all bread and $c^{max}P^{\$C}$ which is the amount of money a farmer could earn by producing all cheese. As explained in the last chapter, money prices may be high or low depending on the money supply. But whatever money prices turn out to be, the proportion between them must be invariant as shown in equation (29).

$$p = b^{max}/c^{max} = P^{\$C}/P^{\$B} \qquad (29)$$

where p is the supply price of cheese (the rate of trade-off in production between bread and cheese regardless of how much of each good is produced) and where $P^{\$B}$ and $P^{\$C}$ are the money prices of bread and cheese.

When public revenue is raised by an income tax, the required tax rate must be equal to the proportion, T, of the economy's resources transferred from the tax payer to the government. though equation (28) was constructed as a production possibility curve, it can equally well be seen as a person's post-tax budget constraint displaying the set of options for the consumption of bread and cheese. From among all combinations of bread and cheese consistent with equation (28), a person chooses a combination to place himself on the highest attainable indifference curve, a combination of bread and cheese at which an indifference curve is tangent to the budget constraint, the demand price is just equal to the supply price, and there is no tax-induced distortion in the consumer's allocation of production and consumption between bread and cheese. Income taxation generates no deadweight loss because the productive capacity remaining after the diversion of some workers from farming to policing is employed to make the representative consumer as well off as possible with the resources at hand.

Suppose instead that public revenue is raised by a pair of excise taxes at rates J_B and J_C on bread and cheese. A distinction must now be drawn between producer prices and consumer prices. Redefine $P^{\$B}$ and $P^{\$B}$ as pre-tax money prices of bread and cheese received by producers, prices for which equation (29) must remain valid. The excise tax on bread raises the money price of bread to the consumer from $P^{\$B}$ to $P^{\$B}(1 + J_B)$. The excise tax on cheese raises the price of money cheese to the consumer from $P^{\$C}$ to $P^{\$C}(1 + J_C)$. Together, these taxes generate a demand price of cheese - the consumers' rate of trade off between bread and cheese - of $[P^{\$C}(1 + J_C)]/ [P^{\$B}(1 + J_B)$. The demand price is either higher or lower than the supply price depending on whether or not $J_B < J_C$.

In the discussion surrounding the demand curve for cheese in figure 5, a tax on cheese was graduated in loaves per pound. By contrast, the excise taxes J_B and J_C are expressed as proportions of demand prices. Look once again at figure 5 where a deadweight loss emerges because of the imposition of a tax on cheese of t loaves per pound. The deadweight loss may be thought of as arising because of a divergence between the *true* supply curve of cheese represented by the flat line at a height p above the horizontal axis and the consumers' *perceived* supply curve represented by a flat line at a height p + t above the horizontal axis. With an income tax there would be no such divergence because the demand and supply prices of cheese are the same. With a pair of excise taxes, the implicit tax on cheese, assessed in loaves per pound, in accordance with equation (2), becomes

$$t = p^D - p = [P^{\$C}(1 + J_C)]/[P^{\$B}(1 + J_B)] - P^{\$C}/P^{\$B} = [P^{\$C}/P^{\$B}][(1 + J_C)/(1 + J_B) - 1] \qquad (30)$$

which is positive if $J_C > J_B$, which is negative if $J_C < J_B$ and which is zero if $J_C = J_B$. A pair of equal excise taxes is equivalent to an income tax and gives rise to no deadweight loss because no gap is interposed between the demand and supply prices of cheese in terms of bread.

Another way of thinking about excise taxation is that it may create a divergence between the *true* production possibility curve in equation (28) and the consumers' *perceived* options as represented by their budget constraint,

$$P^{\$B}(1 + J_B)b + P^{\$C}(1 + J_C) = Y \qquad (31)$$

As long as the excise taxes are imposed on all consumption of bread and cheese, including what the farmer reserves for his own use, the farmer may be thought of as producing Y dollars worth of bread or cheese and then using his income to buy bread or cheese at market prices $P^{\$B}(1 + J_B)$ and $P^{\$C}(1 + J_C)$. To provide the required public revenue for the police force, excise taxes must be set high enough to force consumers to choose a combination of bread and cheese on the net production possibility curve in equation (28). If in addition the excise tax rates are the same, equations (28) and (31) become identical, the excise taxes become equivalent to an income tax, and there can be no deadweight loss. On the other hand, if the excise tax rates differ, the budget constraint in equation (31) cuts the production possibility curve in equation (28), the perceived supply price of cheese differs from the true supply price, and the consumer is induced by taxation to choose the wrong point on the production possibility curve. If only one of the two goods is taxed or if the goods are taxed at different rates, the consumer is driven to an inferior point on the production possibility curve where an indifference curve is tangent not to the production possibility curve itself, but to the perceived budget constraint in equation (31).

The core of the "proof" of the superiority of the income tax over the excise tax is the avoidance under the income tax of a tax-induced diversion of consumption from taxed to untaxed goods. The proof is subject to qualifications. As discussed above, the proof becomes irrelevant when excise taxes are imposed at equal rates on all goods, for, in that case, income taxation and excise taxation are indistinguishable. More importantly, the income tax becomes like an excise tax when some goods are excluded from the tax base. Just as the tax on cheese diverts consumption from cheese to bread, so too does the income tax divert consumption from taxed commodities to untaxed leisure, from paid work to do-it-yourself activities, and from investment to consumption. The tax-induced shift from labour to do-it-yourself activity will be discussed in detail in chapter 9 where it becomes a major impediment to the redistribution of income. The tax-induced shift from investment to consumption can be explained by example. Suppose I earn a dollar in circumstances where the tax rate on earned income is 50%, leaving me 50¢ to spend today or save as I please. If I spend the dollar immediately, my initial 50¢ of tax is all that I have to pay. If I save the remaining 50¢ at, say, a rate of interest of 10%, I trade my 50¢ today for an income of 5¢ a year forever on which I must pay an additional tax of 2.5¢ per year. Spending today entails at tax of 50¢ immediately. Saving to spend tomorrow entails the same 50¢ of tax today plus an extra tax of 2.5¢ per year forever. That is called the *double taxation of saving*. Spending today is like consuming bread, though with low tax rather than with no tax. Saving to spend tomorrow is like consuming cheese. A rise in the rate of the income tax diverts purchasing power to one from the other. In fact, deadweight loss reemerges under the income tax whenever the tax base shrinks in response to an increase in the tax rate not just because expenditure is diverted from more taxed to less taxed goods, but because resources are diverted from the production of goods to the concealment of taxable income and to the investigation and punishment of tax evasion.

Virtually all taxation induces the tax payer to alter his behaviour in ways that are advantageous to the tax payer himself but disadvantageous to society as a whole. Taxation is an involuntary contribution from the tax payer to the rest of the community. As citizen and as voter, a person recognizes that his contribution is reciprocated. As self-interested consumer, a person seeks to minimize his own contribution if and to the extent that others' contributions are what they are regardless of how he personally behaves. One way or another, the acquisition public revenue requires that goods be taxed. Taxes induce diversion of purchasing power from taxed to untaxed goods, lowering one's tax bill at any given rate of tax, but mandating higher-than-otherwise tax rates for the acquisition of any given amount of public revenue, and making everybody worse off than if everybody could agree not to alter their consumption patterns.

The Gain from Trade

International trade is a substitute for invention. The surplus from the invention of cheese, illustrated in figure 4 above, is equally attainable when cheese can be purchased abroad. Consider a "small" country - small in the sense that its trade is too small a part of the world market to have a significant effect on world prices - where people's taste for bread and cheese is in accordance with the utility function in equation (4) above, where the parameter 2 in the utility function is set equal to ½, where people can produce nothing but bread and where the output of bread is b^{max} per person. People never learn to make cheese, but can buy cheese instead. The country is situated in a world-wide market where bread can be sold and cheese can be bought at fixed money prices, $P^{\$B}$ dollars per loaf of bread and $P^{\$C}$ dollars per pound of cheese. To say that prices are fixed is to say that people can buy as much cheese as they pleases as long as the values at world prices of their imports and their exports are the same.

$$b^{ex}P^{\$B} = c^{im}P^{\$C} \qquad (32)$$

where b^{ex} and c^{im} are exports of bread and imports of cheese per head. Consumption of bread and cheese become

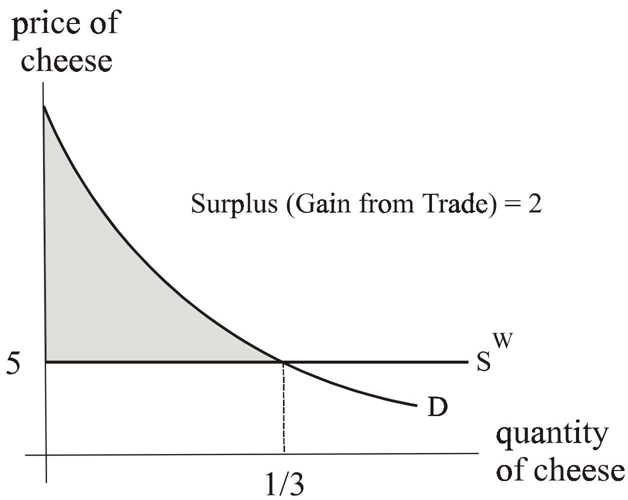
$$b = b^{max} - b^{ex} \quad \text{and} \quad c = c^{im} \qquad (33)$$

From equations (33) and (34) together, it follows that

$$b^{max} = b + pc \qquad (34)$$

where $p = P^{\$C}/P^{\$B}$, the relative price of cheese on the world market. Equation (34) is, in effect, the trade-created equivalent of a production possibility curve for bread and cheese together. The only difference between the trade-created equivalent and the actual production possibility curve in figure 6 is in the shape of the curve. The trade-created equivalent curve is a downward-sloping straight line rather than a quarter circle, implying that the corresponding supply curve of cheese is flat. With that qualification, the shaded area in figure 7 can be equally well interpreted as the surplus from invention or the gain from trade.

Figure 7: The Gain From Trade



Suppose that production of bread is 10 loaves, and that world prices of bread and cheese are \$2 per loaf and \$10 per pound, so that the relative price, p, of cheese is 5 loaves per pound. Options for the consumption of bread and cheese become

$$10 = b + 5c \tag{35}$$

which is a special case of equation (34) above. With the parameter 2 in the utility function of equation (4) is set equal to ½ , the demand price of cheese becomes

$$p^D = / (b/c) \tag{36}$$

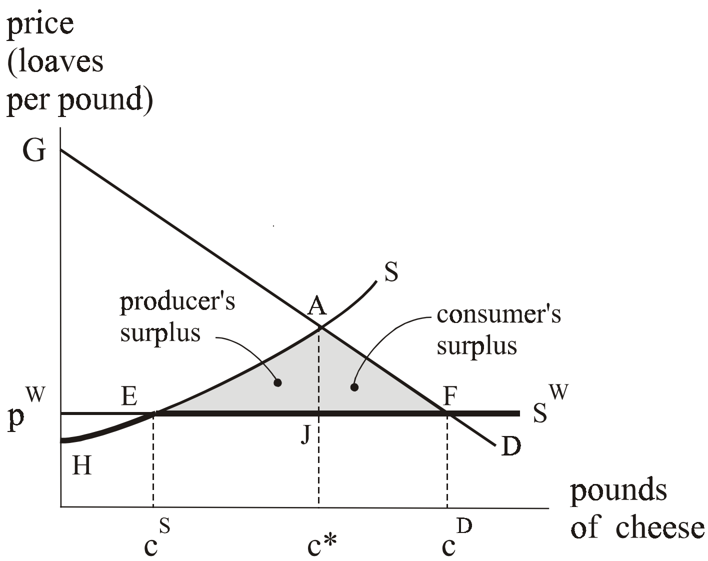
The chosen quantities of bread and cheese, b* and c*, are identified by equating the domestic demand price to the world price, and then substituting for b or c in equation (35). Since the world relative price of cheese is 5 and the demand price is / (b/c), it must be the case that b = 25c. Substituting for b in equation (35), we see that the chosen consumption of cheese, c*, is 1/3 pounds, and that the chosen consumption of bread, b*, is 25/3 loaves. Utility in accordance with equation (4) becomes 6/ / 3, [/ (1/3) + / (25/3)], and the amount of bread, b^{equiv}, that would compensate for the loss of the opportunity to trade bread for cheese is $(6/ / 3)^2$ = 36/3 = 12 loaves. The gain from trade, b^{equiv} - b^{max}, is 2 loaves of bread per person.

The gain from trade is represented as an area in figure 7, showing demand and supply curves for cheese when only bread can be produced but cheese may be acquired by trade. The demand curve, D, may be thought of as carried over from figure 4 because taste for cheese, as represented by the postulated utility function, is the same. The supply curve is S^W where the superscript is mnemonic for "world trade." It differs from the supply curve in figure 4 in two respects: It is a reflection of opportunities for transforming bread into cheese through trade rather than through production. It is flat rather than upward sloping because opportunities for acquiring cheese are as represented by the linear production possibility curve in equation (34) rather than by the bowed out production possibility curve in equation (6) Nevertheless, as in figure 4, the surplus from the availability of cheese is the shaded area between the demand curve and the supply curve. Note that the surplus from the acquisition of cheese exceeds the value of the cheese acquired. As computed above the surplus is 2 loaves. As is immediately evident from figure 7, the value of the cheese (the amount of bread forgone in trade to acquire the cheese) is only 5/3 which is slightly less than 2.

The magnitude of the surplus from trade depends upon the world price. If the world relative price of cheese were higher, the supply curve of cheese would be higher too and the surplus would be less than 2. If the world relative price of cheese were lower, the supply curve of cheese would be lower too and the surplus would be greater than 2.

The gain has been described so far for the import of goods that a country cannot make for itself at all. Though the magnitude of the gain diminishes, a gain from trade remains when imports become a supplement rather than a substitute for domestic production. It assumed above that cheese cannot be produced domestically. Now assume instead that both bread and cheese can be produced domestically in accordance with the production possibility curve in figure 4. In the absence of the opportunity to trade the surplus would once again be the shaded area between the demand curve and the supply curve in the bottom left-hand portion of the figure. These demand and supply curves are reproduced as the curves D and S in figure 8. Once again, the supply curve shows opportunities for acquiring cheese through production rather than trade. Without the opportunity to trade, people would produce c* pounds of cheese per head, and the surplus per head from the availability of cheese would be the triangular area GAH.

Figure 8: The Gain from Trade when Both Goods Can Be Produced at Home



Trade provides a second supply curve, allowing people to convert bread to cheese through either or both of two routes represented by their supply curve in production, S, and by their supply curve in world trade, S^W. The best strategy is to go some distance along both routes. Consume c^D pounds of cheese, for which the demand price (the height of the demand curve) is just equal to the world price of cheese, p^W. Produce c^S, pounds of cheese, for which the supply price (the height of the supply curve) is just equal to the world price of cheese, p^W. Buy c^D - c^S pounds of cheese on the world market.

This strategy may be looked upon as combining the two supply curves into a single new supply curve, the kinked curve HEJF, tracing S until c^S and tracing S^W thereafter. This strategy increases consumption from c*, as it would be in the absence of trade, to c^D, enlarging his surplus from the availability of cheese from GAH, as it would be in the absence of the opportunity to trade, to GFEH. The additional surplus acquired by trade is the shaded area EAD. That is the new gain from trade.

When trade supplements production, the gain from trade may be thought of as composed of two parts. One part, the area AEJ, is the saving of bread by the substitution purchase for production, buying c* - c^S pounds of cheese on the world market rather than producing it at home. The other part, the area AJD, is the gain from the acquisition of an extra c^D - c* pounds of cheese, worth more to than the cost of acquisition if and only if it can be acquired through trade rather than through production. This demonstration of the gain from trade is constructed on the assumption that the world relative price of cheese is lower than the domestic price (the height of the point A) as it would be without the opportunity to trade. There is a similar gain from trade in the opposite case where the world relative price of cheese exceeds the domestic price as it would be in the absence of trade. Cheese would be exported rather than bread. Either way, trade is advantageous.

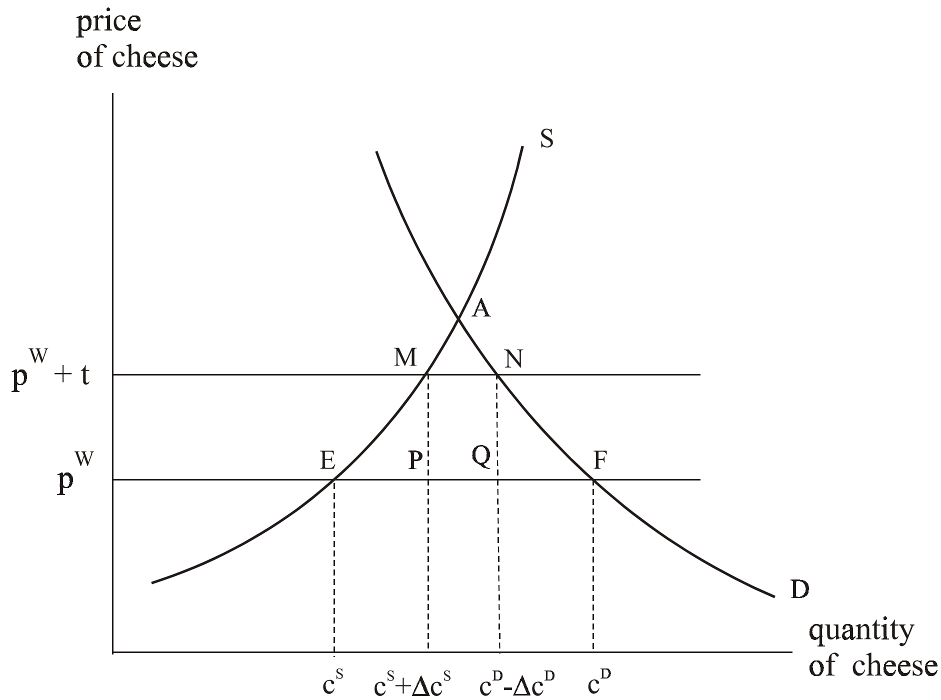
That trade is beneficial is as true for countries in a world market as for people in a local market. People produce a few things and they want many. People specialize in production to maximize income, and then spend their income on a great variety of goods they require. One person grows peas, another sells cosmetics at the local drug store, another delivers babies, another lectures on economics. Everybody consumes an almost infinite variety of goods and services, drawing on the entire technology and all of the available resources in every country in the world. Trade makes this possible. Trade remains advantageous even when autarchy is feasible - when all of the goods one wants to consume could be produced at home - as long as world relative prices differ from relative prices as they would be if all goods consumed had to be produced at home.

Tariffs

Figure 8 showed the gain from trade when the world price of cheese, p^W, is less than the domestic price - indicated by the crossing of the demand curve and the supply curve at A - as it would be if all foreign trade were blocked. In the absence of trade, c* pounds of cheese would be produced and consumed. Unrestricted trade led to a reduction in domestic production to c^S, an increase in consumption to c^D, and a surplus to the representative consumer represented by the shaded area, AEF. The imposition of a tariff on the import of cheese has three effects upon the economy: It diminishes the gain from trade by raising the domestic price above the world price, it yields revenue, and it may (but need not) lead to a reduction in the world price itself. Concentrate for the moment on the first two effects.

When the world price of cheese is invariant, the impact of the tariff upon the domestic economy is illustrated in figure 9, which is an extension of figure 8 above. The tariff is set at t loaves of bread per pound of cheese (or $t per pound when the price of bread is held invariant at $1 per loaf), and the domestic price of cheese rises from p^W to $p^W + t$. Responding to the rise in the domestic price, domestic suppliers increase production of cheese from c^S to c^S +) c^S and domestic consumers reduce consumption of cheese from c^D to c^D -) c^D. The import of cheese is reduced from $c^D - c^S$ to (c^D -) c^D) - (c^S +) c^S), which is equal to the distance PQ (or MN) in figure 9. The revenue from the tariff is t loaves per pound imported, equal to the area of the box PQMN in figure 9. The impact of the tariff on domestic producers and consumers of cheese is as though the world price of cheese had risen from p^W to $p^W + t$.

Figure 9: A Tariff on the Import of Cheese



By raising the domestic price of cheese, the tariff reduces the gain from trade from AEF to AMN. The reduction in the gain from trade is represented by the area MNFE. Balancing the reduction in the gain from trade (area MNFE) against the revenue from the tariff (area PQNM), we see that there is a net loss equal to the sum of the two triangles, MPE and NFQ. The tariff would seem to be unambiguously harmful because the loss of surplus exceeds the gain in tax revenue.

In short, tariffs chuck away a significant portion of the gain from trade. When the import of cheese is taxed at a rate t, the benefit, or surplus, to the taxpayer from the availability of cheese is doubly reduced, once by the revenue from the tariff and again by the deadweight loss from the tariff-induced reductions in the production of bread and in the consumption of cheese. The first of these costs of the tariff is balanced by the acquisition of public revenue to pay for police, roads, schools, and other public facilities. The second is pure waste.

The moral of the story would seem to be that trade taxes should be avoided altogether because citizens are better off when public revenue is acquired by income taxation or by taxing consumption of all goods at the same rate, regardless of whether goods are produced at home or abroad. That is part of the reason why most federal countries forbid states or provinces from levying taxes on inter-state or inter-provincial trade. That is part of the rationale for the European Union and the North American Free Trade Agreement where taxes on trade flows between signatories are limited or banned altogether.

However, one hesitates to accept this sweeping condemnation of all tariffs because tariffs were once the principal source of revenue in many countries and are still imposed by almost every country today. Why, it may be asked, are tariffs and other impediments to trade ever imposed if they are harmful to the countries that impose them? There would seem to be three main reasons.

The first is that tariffs may be relatively inexpensive to collect. Recall the explanation for the tax on water in the discussion surrounding figure 1. Throughout most of history, it was difficult and expensive for the tax collector to measure each person's production, and it was virtually impossible to determine each person's income as a basis for the imposition for an income tax. Better to rely for public revenue on the taxation of imports and exports which could at least be observed with some degree of accuracy. All things considered, trade taxes may have been less burdensome than any other form of public revenue.

The second reason is that the excess burden of a moderate tariff may not exceed the excess burden of other taxes the government might collect. Recall the discussion surrounding figure 5 and table 1 above of the full cost to the taxpayer per additional dollar of tax revenue from a tax on cheese, as illustrated in figure 4 and expressed by the ratio $(\Delta R + \Delta L)/\Delta R$ where R is revenue and L is deadweight loss. A comparable ratio may be identified for the tariff with revenue measured by the area PQNM in figure 9 and deadweight loss measured by the sum of the areas MPE and NFQ. It should be evident from the diagram that as the tariff increases from 0 to something high enough to choke off imports altogether, the ratio of deadweight loss to revenue varies between 0 and infinity. There must then be some rate low enough that the full cost per additional dollar of tax revenue acquired by the tariff is no greater than the full cost of revenue acquired through ordinary taxation.

The third reason is to shift the burden of taxation abroad. Tariffs may be beneficial to the countries that levy them if and to the extent that the burden of the tax is borne by one's trading partners. This consideration was deliberately abstracted away by the assumption that the home country is too small a player in the world market to affect the price of cheese. The assumption was that the world price of cheese was fixed at p^W loaves per pound regardless of how much or how little cheese people in the home country choose to buy. But world prices need not be invariant. A tariff on cheese reduces the import of cheese. If, contrary to what we have been assuming, the home country were a large player in the world market, the tariff-induced reduction in the demand for cheese abroad might lead to a fall in the world price of cheese as suppliers of cheese abroad compete vigorously with one another over shares of the diminished volume of business. Were that so, the imposition of a tariff at a rate t could cause a fall in the pre-tariff price of cheese and a correspondingly smaller rise in the domestic price, $p^W + t$, than if the world price were invariant. If a tariff of t causes the world price to fall by x from p^W to $p^W - x$, then the tariff-inclusive domestic price of cheese becomes $p^W - x + t$. Think of figure 9 as it would be if the price of cheese rose from p^W to only $p^W - x + t$. Imports would be somewhat larger than MN, the revenue from the tariff would be somewhat larger too, and the loss of surplus, MNFE, would be somewhat less. Depending on the magnitude of x, the gain in revenue may or may not be sufficient to overbalance the remaining loss of surplus. A tariff may be advantageous to the country that imposes it as long as other countries do not retaliate.

But that is exactly what other countries would be inclined to do. If there is to be a tax on international trade, I want it to be levied by my country rather than by yours. Effects of trade taxes on the on the volume of trade, on prices in both our countries and on both countries' residual surplus are the same no matter where trade taxes are levied, but the revenue from a tax on trade accrues to the country that levies it. I want to turn prices in my favour. You want to turn prices in yours. When we both act on such considerations, we may both end up worse off than if we agreed between ourselves to keep our tariffs low. Furthermore, from an initial position where we have both levied optimal trade taxes in response to the other trade taxes, there may be a wide range of bargains we might strike - some bargains relatively advantageous to you and others relatively advantageous to me - that make us both better off. Countries bargaining over tariff reduction are like Mary and Norman in the last chapter. The determinacy of the price mechanism gives way to the indeterminacy of negotiation.

Monopoly and Patents

In competition, firms respond to prices looked upon by each and every firm as invariant regardless of how little or how much the firm chooses to produce. By contrast, a monopolist - defined as the one and only producer of some good - knows that it can affect prices through its choice of how much to produce, and it acts accordingly to maximize its profit. Monopoly profit is like a private excise tax set levied for the benefit of the monopolist. Figure 2 above can be reinterpreted as describing a monopoly of the market for cheese. The height of the supply curve is the cost in terms of bread of an extra pound of cheese. The height of the demand curve becomes the monopolist's attainable price of cheese as dependent on the quantity supplied. The area R is the monopoly profit. The monopolist need not tax cheese directly. Instead, monopoly revenue is maximized by the choice of either c (the quantity of cheese produced) or p^D (the price at which cheese is offered for sale) to make the monopoly profit, R, as large as possible. Monopoly is inefficient in exactly the same way that an excise tax is inefficient, by creating a deadweight loss, L, which is harmful to consumers without at the same time yielding any corresponding benefit to the monopolist. The benefit of the monopoly to the monopolist is the monopoly profit, R. The cost to users of the monopolized product is the sum of R and L. In addition, monopoly is deemed harmful to society because the transfer of income, R, to the monopolist from the rest of society is unbalanced by any social gain and because competition among would-be monopolist to acquire monopoly power may turn out to be wasteful. Patents are an exception to the rule for reasons to be discussed below.

Monopoly power may arise naturally, may be conferred by the state, or may be acquired privately. A road or railroad sufficient to accommodate all traffic is a natural monopoly and would for that reason be normally owned or heavily regulated by the state. A monopolist road-owner would charge a revenue-maximizing fee for the use of the road, reducing the flow of traffic in circumstances where the deterred traffic would have impose no burden on anybody. (Tolls imposed to reduce congestion are another matter, but there is no presumption that the revenue-maximizing toll and the appropriate congestion-reducing toll are the same.) Monopoly may be conferred by the state for a variety of reasons. Monarchs have sold monopoly rights to raise money for wars, or have granted monopolies to their relatives, courtiers, and friends. Patents and trade unions are monopolies conferred by the state for various reasons. Monopoly may be acquired without the connivance of the state through voluntary coordination among firms within an industry or by one firm replacing all the rest. A firm may acquire title to all known sources of a raw material by buying up rival firms or driving them out of business. Sellers may collude to drive up prices by restricting supply without any one firm acquiring all the rest. Standard Oil's acquisition of a monopoly of petroleum in the United States during the late nineteenth century is an example of monopoly by acquisition. OPEC is an example of collusion to drive up prices. In most countries, deliberate monopolization by acquisition or by collusion is illegal. Governments maintain anti-trust departments that break up monopoly and punish monopolists for certain kinds of behaviour.

A patent is a special kind of monopoly. It is a monopoly granted by the state to an inventor on the use of his invention. Typically, a patent is valid for a fixed term of years and subject to a battery of subsidiary conditions such as, for example, that the patent-holder of a very beneficial drug may not, willingly and capriciously, keep the drug off the market altogether until the patent runs out. The justification for the granting of this special kind of monopoly is that it may be the only feasible, or least expensive, way of providing an incentive for invention, and that the consumer is better off having the newly invented product monopolized than if he did not have the newly invented product at all. Once the invention appears, the patent is inefficient for the same reason than any monopoly may be inefficient; the full cost of the patent to the user of the invention exceeds the revenue to the patent-holder. Despite this cost, patents are awarded as inducements to invention, for, even when the revenue from the patent is maximized, there always remains a residual surplus for the users and the producers of the new product. The residual surplus is represented by the sum of the areas H^D and H^S in figure 2 when R is reinterpreted as the largest attainable revenue of the patent-holder whose income from the patent is just like an ordinary tax on the parented product and who is entitled by the patent to choose the revenue-maximizing tax.

A patent is a monopoly conferred by the government on an inventor to make invention profitable. Consider once again the discovery of cheese. Whoever makes the discovery must first devote resources to invention, resources which would otherwise be devoted to the production of bread. To induce the invention of cheese, the rest of the community must compensate the inventor. The market does not compensate him automatically because, once cheese has been invented and the knowledge of how to make cheese becomes readily available, the inventor has no edge over anybody else in its manufacture and he can expect no reward for his effort in creating it. He requires some special privilege as a reward for the invention. The inventor might be compensated with a salary financed by taxation. That is how basic scientific research is financed in universities and in government labs. Alternatively, the inventor might be compensated by a prize set in accordance with an estimate of the value to society of the invention. Such compensation would be difficult to administer, unfair and capricious because nobody can be sure what a discovery is worth. Where the invention consists of the discovery of a well-specified product or process, the granting of a patent rewards the inventor without the government having to decide on the value to society to the invention. If the invention turns out to be highly beneficial in the sense of leading to a large surplus, then the revenue from the patent is likely to be large too. Otherwise the revenue will be small. In practice, patents are issued for a term of years after which the monopoly to the inventor is rescinded and anybody is free to use the invention without compensating the inventor.

There is a great deal at stake here. People are better off today than they were two hundred years ago in part because we have learned to make goods with a smaller input of labour, but primarily because we have learned to make new types of goods that our ancestors did not have at all, including electricity, aeroplanes, cars, telephones, radios, television, and the medicines that make our lives longer and more comfortable. Innovation may not have been forthcoming if inventors were not rewarded for their inventions. We return to the subject in the chapter on technology.

C: Alternative Interpretations of the Demand Curve

The Constant Money Income Demand Curve

The demand curve was defined in the last chapter for an isolated person constrained by a production possibility curve. A similar, though not quite identical curve may be constructed for a person with a given income and confronted with given market prices. For such a person, the demand curve for cheese shows how his purchase of cheese changes in response to changes in the price of cheese when his money income and all other prices remain constant. Consider a person Consider a person in an economy with many people but only two goods, bread and cheese. He has a money income of Y dollars, and he is confronted with market prices of $P^{\$C}$ dollars per pound of cheese and $P^{\$B}$ dollars per loaf of bread to be spent on the purchase of bread and cheese. He chooses quantities, c pounds of cheese and b loaves of bread, to make himself as well off as possible with the money at his disposal. In other words, he maximizes his utility, u(c, b), subject to his budget constraint

$$bP^{\$B} + c\,P^{\$C} = Y \tag{28}$$

which, dividing through by $P^{\$B}$, can be rewritten as

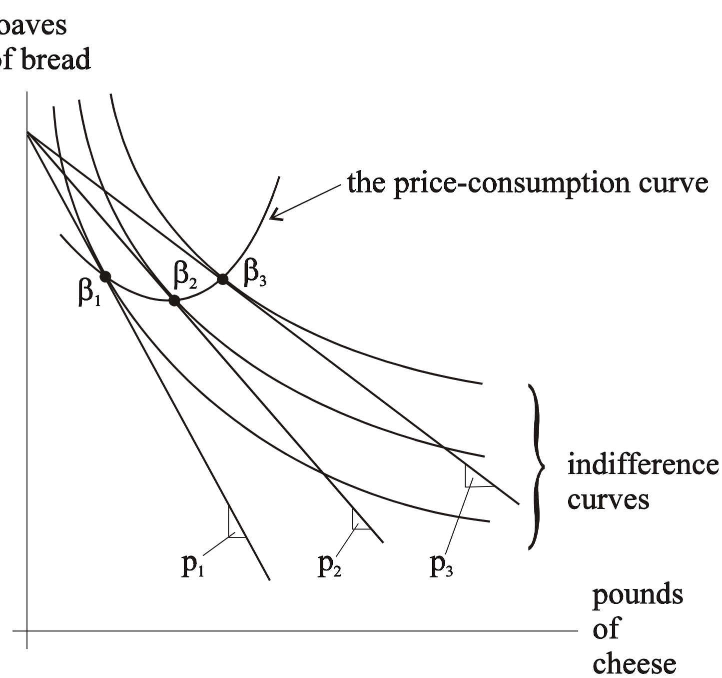
$$b + pc = y \tag{37}$$

where p, defined as $P^{\$C}/P^{\$B}$ is the market-determined relative price of cheese, and y, defined as $Y/P^{\$B}$, is the amount of bread the person can buy when he spends all his money on bread.

Consider a person with a money income, Y, of \$20,000 per year when the price of bread, $P^{\$B}$ is \$4 per loaf and the price of cheese, $P^{\$C}$, is \$8 per pound. That person may equally well be said to have an income, y, of 5,000 loaves from which he may purchase cheese at a price of 2 loaves per pound. The advantage of this second version of the budget constraint is that it can be represented together with indifference curves on the bread-and-cheese diagram we have been using throughout this chapter, and a person choosing b and c can be represented as placing himself on the highest indifference curve attainable within his budget constraint.

Figure 10: The Price-consumption Curve for the Constant Income Demand Curve

[The downward-sloping straight lines originating at y are alternative budget constraints]



Now the demand curve shows how a person's purchase of cheese varies when the price of cheese varies but his income and the price of bread remain the same. His behaviour is illustrated on the bread and cheese diagram in figure 10. Income (in units of bread) is shown as a distance, y, on the vertical axis. For any given relative price of cheese, all attainable bundles of bread and cheese are represented by a "budget constraint", a downward-sloping straight line beginning at y and with slope equal to p, the relative price of cheese. As long as the money price of bread is assumed constant, any change in the money price of cheese, $P^{\$C}$, is a change in its relative price, p, as well, but only the latter can be illustrated on figure 10. Three budget constraints are shown for three distinct relative prices of cheese, p_1, $p_{2,}$ and p_3. Three indifference curves are also shown, each tangent one of the three indifference curves. Points of tangency, are labelled $\$_1$, $\$_{2,}$ and $\$_3$. Each point of tangency represents the person's best (utility maximizing) combination of bread and cheese when his income and the price of bread are held constant. A curve, called the "price-consumption" curve, is drawn through all points such as $\$_1$, $\$_2$, and $\$_3$.

As will be explained in the next section, this demand curve is slightly different from the demand curves employed earlier in the book. It is relatively easy to estimate from data on prices, quantities and income. It is useful as a component of a large model of the economy where changes in income can be accounted for within the rest of the model. It is easily generalized from the bread-and-cheese economy to a many-good economy where a person's demand curve for any particular good shows how his quantity demanded of that good varies in response to its price when his income and all other prices remain invariant. It may be interpreted as showing how the quantity demanded of some good responds to a change in the technology of the economy in the special case where the production possibility curve is a downward-sloping straight line and where the change in technology can be represented by a rotation of that line from its intersection with the vertical axis. A straight-line production possibility curve may be characterized completely by its slope (represented by p) and the height its intersection with the vertical axis (represented equivalently by b^{max} or y). Any technical change can be represented as a change in one or both of these parameters.

As quantity is assumed to depend on price and income, the consumer's behaviour may summarized in two key elasticities, the price elasticity of demand and the income elasticity of demand. For our bread-and-cheese economy, the *price elasticity of demand* for cheese is

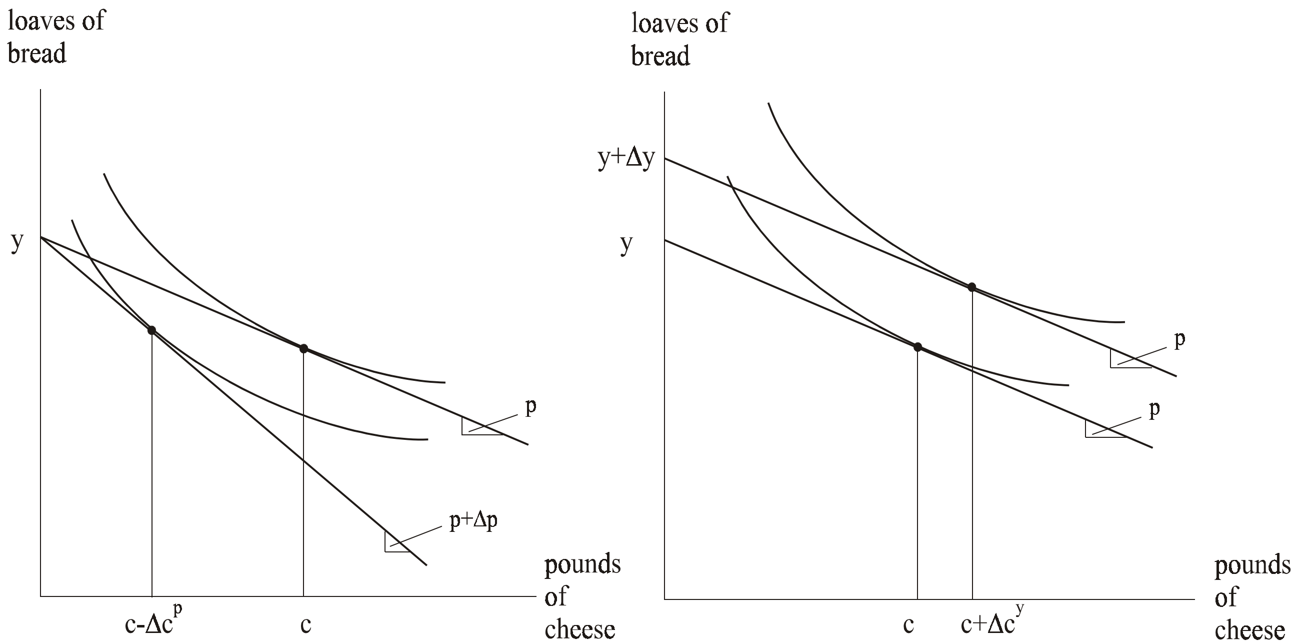
,^{cp} = (percentage change in quantity of cheese consumed)/(percentage change in price)
= ( ) c^p/c)/( ) p/p)

where) c^p is the number of extra pounds of cheese consumed in response to an increase in the price of cheese from p to p +) p and where income remains constant, and the *income elasticity of demand* for cheese is

,^{cy} = (percentage change in quantity of cheese consumed)/(percentage change in income)
= ( ) c^y/c)/( ) y/y)

where) c^y is the number of extra pounds of cheese consumed in response to an increase in income from y to y +) y and where the price of cheese remains constant. The first of these elasticities is defined for a given income and the second is defined for a given price. In an economy with many goods, a price elasticity of demand for any particular good is the percentage change in quantity demanded in response to a percentage change in its price when income and all other prices remain constant.

Figure 11: Price and Income Elasticities of Demand for Cheese



The meanings of these changes in quantity, price and income are illustrated in the two sides of figure 11. The left-hand side of the figure shows price changing when income remains the same, and the right-hand side shows income changing when price remains the same. The changes in the relevant variables -)p,)y,)c and)c^y - can all be read off the figure. In principle, price and income elasticities of demand can be measured from observations of changes in prices, quantities, and incomes. Consider a person whose income is invariant at $50,000 per year. This week when the price of cheese is $2 per pound, he buys 3 pounds. Next week when the price has fallen to $1.75 a pound, he buys 3.5 pounds. His implied price elasticity of demand for cheese is 1 1/3, the absolute value of [(3½ - 3)/3]/[(2 - 1.75)/2]. His income elasticity of demand for cheese could be determined accordingly from evidence about how his quantity consumed changed when his income changed but the price of cheese remained invariant. Something more sophisticated than this would be required if quantities consumed changed randomly from time to time for reasons that had nothing to do with changes in prices or incomes. Estimation allowing for a degree of random behaviour would be discussed in a course on econometrics. Note that elasticities may vary along the demand curve. The price elasticity of demand may be high at low prices and low at high prices, or vice versa.

A Family of Demand Curves

In this chapter and in the preceding chapter, we have employed four distinct types of demand curves with marked similarities, but with some differences too. Every demand curve connects the quantity consumed of some good to its demand price, defined as the rate of trade-off in use between that good and a chosen numeraire good, and represented as the slope of the appropriately chosen indifference curve. For any particular good, types of demand curves may differ in their assumptions about variations along the curve in the quantities of other goods. In the last chapter, the demand curve was read off the production possibility frontier. That demand curve, which might be called the *constant technology demand curve*, showed how the demand price and quantity of cheese changed together as Robinson Crusoe altered amounts of bread and cheese produced in accordance with his technically given production possibility frontier. In this chapter, the demand curve was read off an indifference curve. To measure surplus and deadweight loss, it was appropriate to construct a demand curve, which might be called the *constant utility demand curve,* connecting demand price and quantity consumed where quantities of other goods were assumed to vary so as to keep the representative consumer on the highest attainable indifference curve. A third demand curve, which might be called the *constant income demand curve*, was discussed in the preceding section. Its focus is not upon the representative consumer, but upon the individual within a competitive market, who earns a certain income, who looks upon prices as externally given and whose preferences, as represented by the shapes of his indifference curves, may differ from those of other people in his community. The fourth type of demand curve, which might be called the *trade demand curve* shows a country's response to changes in the world price. Exemplified by the demand curve for cheese in figure 9 of the last chapter, a country's trade demand curve would be a scaled up version of a person's constant income demand curve, if the country produced only bread and imported all its cheese regardless of the world price, and if everybody's taste were in conformity with the utility function in equation (1) of the last chapter.

Each of the four demand curves - the constant technology demand curve, the constant utility demand curve, the constant money income demand curve, and the trade demand curve - has its own sphere of application. The constant technology demand curve may be the right demand curves from the vantage point of the policy-maker evaluating the impact on the economy as a whole and on the welfare citizens of restrictions on output of certain goods, taxes, tariffs, or other aspects of economic policy. The constant utility demand curve may be the right demand curve from the vantage point of a historian trying to assess the benefit of technical change. The constant income demand curve may be the right demand curve from the vantage point of the consumer in a large economy looking up at prices seen as fixed independently of their individual behaviour. The constant income demand curve would also be appropriate in estimating the change in quantity demanded of some good in response to a change in price brought about by technical change, where, for all practical purposes, the economy-wide production possibility curve may be thought of as a multidimensional plane that tilts appropriately when a good becomes relatively inexpensive to produce. As the name implies, the trade demand curve shows the response to world prices of a country engaged in international trade.

The four demand curves, and other types of demand curves as well, are differentiated from one another by what might be called their *auxiliary function*s showing how quantities of goods change together along the demand curve. For any set of indifference curves in our bread-and-cheese economy, there is a well-defined demand *function*, p(c, b), showing the demand price of cheese in terms of bread, p, as a function of the quantities of bread and cheese consumed. That there must be some such function is immediately evident from the representations of indifference curves in figures 3, 4, and the top half of figure 5 of the last chapter. At any point (c, b) in the top half of figure 5 of the last chapter, the demand price of cheese is the slope of the indifference curve at that point, and one might reasonably expect that slope to be a function of both b and c rather than of c alone. But the demand *curve*, as illustrated in the bottom half of the figure shows the demand price of cheese as dependent on the quantity of cheese alone, with no reference to the quantity of bread at all. The demand curve represents a relation of the form p(c), not p(c, b) as in the demand function. Somehow, the demand function p(c, b) must be transformed into a demand curve p(c).

Any such transformation can be represented as an auxiliary function, b(c), which, when plugged into the demand function for cheese, p(c, b), yields the demand curve, p(c), showing demand price dependent on c alone. For the constant technology demand curve, the auxiliary function is the production possibility curve. For the constant utility demand curve, the auxiliary function is an indifference curve, typically the highest indifference curve attainable with the technology at hand. For the constant income demand curve, the auxiliary function is the price consumption curve in figure 9 above. In principle, the choice of auxiliary functions is entirely unlimited, and every auxiliary function gives rise to its own unique demand curve. But the choice among demand curves is not entirely arbitrary. The appropriate demand curve and its auxiliary function are inherent in the problem at hand.

Recognition of the existence of conceptually distinct demand curves leads naturally to the question of magnitudes. Are the different types of demand curves close enough to one another that estimates of elasticities for one type of demand curve can be assumed for all practical purposes to be valid for other types of demand curves as well? Suppose a price elasticity of demand for cheese for the constant income demand curve has been estimated from data of prices, incomes, and quantities. The question is whether that elasticity can be appropriately employed in estimating the effect of a tariff on the quantity of cheese demanded in the nation as a whole (a problem for which the constant technology demand curve would seem most appropriate) or for estimating the corresponding deadweight loss (for which a constant utility demand curve would seem most appropriate)? The answer to this question is broadly speaking “yes” if and to the extent that cheese occupies a small share of the budget of the consumer, but “no” otherwise.

There are circumstances where the all types of demand curves fuse into one unique demand curve or where the different types of demand curves become close enough that remaining variations among them can be ignored. Consider the different types of demand curves for cheese in a bread-and-cheese economy. All such curves fuse and the auxiliary functions become irrelevant in the special case where indifference curves are stacked up, one on top of the other, so that their slopes depend on c but not on b. Since the price of cheese, p(c, b), is the slope of the indifference at the point (c, b), a vertical stacking of indifference curves means that $p(c, b^1) = p(c, b^2)$ for every b^1 and b^2. In effect, p(c, b) becomes independent of b, the auxiliary function becomes irrelevant and all types of demand curve become the same. However, vertical stacking of indifference curves, though not impossible, is quite unlikely and unusual. Neither of the two utility functions discussed in this chapter - u = bc and u = %b + %c - conform to that pattern. As shown in equation (2) of chapter 3 and equation (5) of this chapter, both utility functions require the price of cheese to be an increasing function of the ratio of b to c. The price of cheese is b/c in one case and [2/(1 - 2)]%(b/c) in the other.

But the different demand curves do fuse together - not completely but sufficiently for most contexts where demand and supply curves are employed - for goods occupying a small part of the budget of the consumer. This will be demonstrated for two of the four demand curves we have examined, the constant income demand curve and the constant utility demand curve. That these curves fuse for goods occupying a small part of the budget can be shown analytically or by example. An example will be presented first, to be followed by a general demonstration yielding a formula for converting one price elasticity to another. The example is a comparison between food and pears, one constituting a large share of the budget of the representative consumer and the other constituting a small share of the budget of the representative consumer. Suppose food and pears are alike in their income elasticities of demand and in their price elasticities along the constant utility demand curve, but very different in their shares of the budget of the consumer. It will be shown that the price elasticity of demand for food along the constant money income demand curve differs significantly from the price elasticity of demand for food along the constant utility demand curve, but that the correspondingly elasticities for pears are approximately the same.

Their common income elasticity is 2. Their common price elasticity along the constant utility demand curve is ½. The share of food in the budget of the consumer is 30%. The share of pears in the budget of the consumer is only 1/100 %. The representative consumer has an annual income of $50,000 of which $15,000 (30%) is spent on food and $5 (1/100 %) is spent on pears. To keep matters simple, suppose that the supply prices of food and pears are both invariant at $1 per pound, so that the preceding values are immediately translatable into quantities. Finally, imagine that the price of a commodity (food or pears as the case may be) is increased by 10% by an excise tax of 10¢ per pound, raising the price of the commodity in question from $1.00 to $1.10. Demand curves can be said to fuse if the resulting changes in quantity along the both demand curves are the same. That is nearly so for pears, but not for food.

Consider food first. Since the price elasticity of demand along the constant utility demand curve is ½, a 10% increase in the price of food must induce a 5%, or 750 pounds, reduction in the quantity of food consumed, from 15,000 pounds to 14,250 pounds. To keep the representative consumer on the same indifference curve, he must be compensated by *at least* the value of the tax he pays. The revenue from the tax is 10% of the value of the new quantity consumed - (.1)(15,000 - 750) - equal to $1,425. If the revenue from the tax is not returned to the consumer, as is assumed to be the case in the construction of the constant money income demand curve, then, in effect, his income is reduced by 2.85%, [(1425/50,000)(100)]. With an assumed income elasticity of 2, his consumption of food must be reduced an additional 5.7%, or 855 pounds (0.057)(15,000) over and above the direct reduction from the 10% increase in price. Thus the total reduction in the quantity of food consumed from an uncompensated tax-induced 10% increase in price must be 1,605 pounds (750 + 855). Along the constant utility demand curve, the price-induced reduction in the quantity of food consumed is 750 pounds, or 5% of its original amount. Along the constant money income demand curve, the price-induced reduction in quantity of food consumed is 1,605 pounds or 10.7% of its original amount. A 10 % increase in the price of food reduces the amount of food consumed by 5% along the constant utility demand curve and by 10.7% along the constant money income demand curve. The implied price elasticity of the constant money income demand curve is 1.07.

Now consider pears. Since the price elasticity of demand along the constant utility demand curve is equal to ½, a 10% increase in the price must lead to a 5% reduction, or 0.25 pound reduction in quantity of salt consumed, from 5 pounds to 4.75 pounds. Once again the revenue from the tax is 10% of the value of the new quantity consumed - (.1) (5 - 4.75) - equal to 12.5¢. If the revenue from the tax is not returned to the consumer, as is assumed to be the case in the construction of the constant money income demand curve, then, in effect, his income is reduced by only 0.00025%, [(0.125/50,000)(100)]. With an assumed income elasticity of 2, his consumption of pears must be reduced by an additional 0.00025%, or 0.0000125 pounds [(0.0000125)(5)] over and above the direct reduction from the 10% increase in price. Thus the total reduction in the quantity of pears consumed from an uncompensated tax-induced 10% increase in price must be 0.2500125 pounds (0.25 + 0.0000125). Along the constant utility demand curve, the price-induced reduction in the quantity of pears is 0.25 pounds, or 5% of its original amount. Along the constant money income demand curve, the price-induced reduction in quantity of food consumed is 0.2500125 pounds or 5.00025% of its original amount. A 10 % increase in the price of pears reduces the amount consumed by 5% along the constant utility demand curve and by 5.00025% along the constant money income demand curve. Unlike food, the reductions along the two demand curves are virtually the same. The implied elasticity of the constant money income demand curve is 0.500025.

For any commodity, the formula connecting the three elasticities of demand - the price elasticity along the constant utility demand curve, the price elasticity along the constant money income demand curve, and the income elasticity of demand - is

$$,^{M} - ,^{U} = s\ ,^{Y} \qquad (38)$$

where

$,^{M}$ is the price elasticity of demand along the constant income demand curve,

$,^{U}$ is the price elasticity of demand along the constant utility demand curve,

s is the share of the commodity in the budget of the consumer,

and $,^{Y}$ is the income elasticity of demand,

Equation (89) is established with the aid of figure 12, which is an extension of figures 10 and 11. Once again, the representative consumer has an income of y loaves of bread and he can buy cheese at a price of p loaves per pound. He consumes quantities of bread and cheese represented by the point α_1. His consumption of cheese is c(p, y). When the price rises to $p + \Delta p$, his budget constraint swings clockwise around the point y on the vertical axis, and he consumes quantities of bread and cheese represented by the point α_2. So far the diagram is exactly like the left-hand side of figure 11.

A distinction must be drawn among three price-induced changes in the quantity of cheese. When the price of cheese rises from p to $p + \Delta p$,

.Δc^p is the reduction in the quantity of cheese consumed when income, y, remains unchanged,

Δc^u is the reduction in the quantity of cheese consumed when income increases by an amount Δy which is just sufficient to keep the consumer on the same indifference curve,

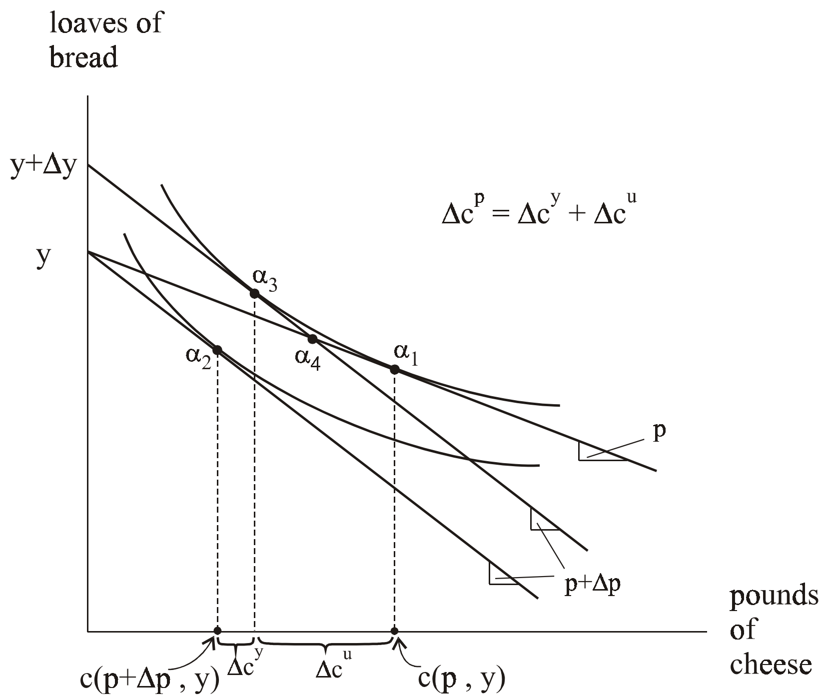
and

Δc^y is the reduction in the quantity of cheese consumed when income is decreased from $y + \Delta y$ to y while the price of cheese remains constant at $p + \Delta p$.

By construction,

$$\Delta c^p = c(p, y) - c(p + \Delta p, y) = \Delta c^y + \Delta c^u \tag{39}$$

Figure 12: Comparison of the Price Elasticities on the Constant Income Demand Curve and the Constant Utility Demand Curve



Consumption of cheese falls by an amount Δc^p which is represented in the diagram by the *sum* of Δc^y and Δc^u. The key step in the demonstration of equation (38) is the representation of Δy, the amount of extra bread one would need to be as well off after the rise in the price of cheese as before. The increase, Δy, is just sufficient to ensure that one's utility on choosing the best available combination of bread and cheese, represented by the point α_3, at the new higher price, $p + \Delta p$, and the new augmented income, $y + \Delta y$, is just equal to one's original utility at the original price, p, and the original income, y. In short, Δy is such that α_3 and α_1 lie on the same indifference curve.

With income held constant, the price increase, Δp, induces a shift in consumption from the point α_1 to the point α_3, reducing consumption of cheese from c(p, y) to $c(p + \Delta p, y)$. By definition, Δc^p is the fall in consumption of cheese along the constant money income demand curve in response to an increase of Δp in the price of cheese. As is evident from the figure, Δc^p is the sum of two parts, Δc^u the reduction in consumption of cheese between α_1 and α_3, and Δc^y the reduction in consumption of cheese between α_3 and α_2. Δc^u may be interpreted as the fall in consumption of cheese along the constant utility demand curve in response to an increase of Δp in the price of cheese because the points α_1 and α_3 lie on the very same indifference curve. Δc^y may be interpreted as the fall in consumption of cheese in response to a fall in income from $y + \Delta y$ to y because the slope of the indifference curves at the points α_1 and α_3 are exactly the same. With these interpretations of the terms) c^u and) c^y, equation (38) can now be derived from equation (39)[5].

From here on all distinctions among types of demand curves will be ignored on the supposition that the good in question occupies a small share of the resources of the economy or that the errors in inferences from demand and supply curves are not large enough to overshadow the main story in the analysis. One cannot think about society without the intermediation of gross simplifications, but neither can one desist from analysis. All one can do is to be aware of one's simplifications so as to recognize circumstances where one might be led seriously astray. Analysis of public policy with demand and supply curves is no exception.

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[5]Multiply both sides by p/(c) p). The equation becomes

(p/c)/( ) c) p) = (p/c)/( ) c^u/) p) + (p/c)/( ) c^y/) p)

where the expressions on either side of the equality sign are equal to , ^{M} and , ^{U}, the price elasticities for the constant money income demand curve and the constant utility demand curve. The equation can be rewritten as

, ^{M} - , ^{U} = (p/c)/( ) c^y/) p) = [(y/c)/( ) c^y/) y)][(p/y)( ) y/) p)] = , ^{Y} [(p/y)( ) y/) p)] = , ^{Y}s

as long as c) p =) y , so that the expression [(p/y)( ) y/) p)] can be rewritten as [(pc/y)] which is s by definition. As long as the change in price is small, the extra income required to keep one as well as before is the extra expenditure required to purchase the original quantity of good in question at the new higher price, the product of the change in price and the quantity of the good consumed. The equality c) p =) y is strictly true for values of c, p, y,) p and) y associated with the point "$_4$ in figure 12. It becomes true for the point "$_1$ as well when) p and the corresponding value of) y are very small because, in that case "$_3$, "$_4$ and "$_1$ are all bunched together. A more direct proof of equation (38) using calculus would be contained in most textbooks of micro economics. The equation itself is a variation of the Slutsky equation. The constant utility demand curve and the constant income demand curve are often referred to as *compensated* and *uncompensated*. That terminology was avoided here because it tends to obscure the fact that there are many kinds of demand curves, not just two. With numbers in the food and salt example, the estimated values of , ^{M} for food and for salt in accordance with equation (38) are 1.1 [0.5 - (0.3)(2)] and 0.50002 [0.5 - (0.0001)2] as compared with 1.07 and 0.500025 as estimated directly above. The discrepancies are minor and can be accounted for by the fact that the example is about finite changes in prices.