Policies for the Informal Sector with Undocumented Workers[1]

Katherine Cuff
McMaster University

Nicolas Marceau
Université du Québec à Montreal

Steeve Mongrain
Simon Fraser University

Joanne Roberts
University of Calgary

May 2, 2009

[1]Paper prepared for the Public Economics Conference in Honour of Robin Boadway, Queen's University, Kingston, Canada, 14-15 May 2009. All authors are grateful to the SSHRC for financial support. The usual disclaimer applies.

1 Introduction

The operation of the informal economy is important, not only in real terms, affecting the size and scale of productive output, but also in terms of optimal government policy. Illegal immigration and the presence of undocumented workers have a large impact on the nature of optimal tax and enforcement policy, and interact with standard tax evasion incentives, playing an important role not only in the determination of equilibrium wages, but also in the organization of production across the formal and informal sectors. It possibly also introduces a new role for minimum wages in limiting wage arbitrage between the formal and informal sectors. By now, it is well accepted in our profession that in developed countries, illegal immigration, tax evasion, and the informal economy are of sufficient importance to impact on the actual performance of the economy. This is because, and even if one should be extremely prudent with the interpretation of measures of illegal behaviour, most of the evidence leads to the conclusion that these phenomena are large and significant.

The issue of illegal immigration has received a lot of attention in recent years in the United States, and the perceived significant size of the phenomenon may explain why. According to a report of the Pew Hispanic Center,[1] the number of illegal immigrants living in the United States was 11.9 million in March 2008, of which 8.3 million participated in the U.S. labor force. Such numbers imply that unauthorized immigrants are 4% of the U.S. population and no less than 5.4% of its workforce. In Canada, estimates of the number of illegal immigrants by police and immigration personnel range between 50 000 and 200 000 according to the Canadian Encylopedia.[2] As for tax evasion, it also appears to be an important phenomenon according to the available evidence on tax evasion by individuals.[3] For example, Slemrod and Yitzhaki (2002) report that in the U.S., according to the Internal Revenue Service, 17% of personal income tax liabilities were simply not paid in 1992. Finally, while measuring the size of the informal sector is notoriously difficult, Schneider and Enste (2000) provide estimates for a large number of countries. According to them, in 1990-1993, the smallest informal sectors (8-10% of the economy) could be found in Austria, Switzerland, and the U.S.. At the other extreme were some developing countries where the informal sector represented 68-76% of the economy (e.g. Egypt, Nigeria, Thailand, Tunisia).

[1] http://pewhispanic.org/files/reports/107.pdf.

[2] See the article on *Immigration Policy* at http://www.thecanadianencyclopedia.com.

[3] The evidence concerning tax evasion by firms is very limited.

As for Canada, its informal sector ranged at 10-13.5% of its economy.

The theoretical literature on each of the above phenomena is large but somewhat segmented in that it tends to address each of them separately. For example, the tax evasion literature is not really concerned with the informal sector or illegal immigration. Initiated by Allingham and Sandmo (1972) and surveyed by Slemrod and Yitzhaki (2002), the tax evasion literature has mainly focused on the decision by individuals, otherwise perfectly honest, to conceal a portion of their income from the tax authorities. Following Reinganum and Wilde (1985), an important secondary strand of that literature developed attempting to characterize the optimal auditing policies of a tax authority facing individuals behaving à la Allingham and Sandmo (1972). As for the literature on illegal immigration, Ethier (1986) initiated it by studying the impact of illegal immigration on the host country and the best policies to account for it, while Bond and Chen (1987) enriched Ethier's model by adding a second country and capital mobility. It is probably fair to say that a significant portion of the literature that followed these papers has focused on the welfare impact of illegal immigrants on the well-being of domestic workers,[4] and that tax evasion was not really a concern for those working in this area. Finally, there is a theoretical literature on the informal sector. For example, Rauch (1991), Fortin *et al.* (1997), Fugazza and Jacques (2003), or de Paula and Scheinkman (2007), all model the choice by entrepreneurs to operate in the legal or the informal sector, based on factors like scale economies, wage regulations or taxes. However, this literature is not concerned with the presence of illegal immigrants despite the fact that by their very presence, they may affect this choice.

Few models have integrated the above three phenomena despite the fact that there is an obvious connection between them. In this paper, we study the informal sector of an economy populated by entrepreneurs, domestic workers, and undocumented immigrants. Entrepreneurs may choose to operate legally, in which case they have to pay taxes and the legal wage to any worker they hire, this legal wage being possibly a regulated minimum wage. Entrepreneurs may instead choose to operate informally, thereby evading taxes, paying the workers they hire the informal wage, and possibly hiring undocumented immigrants. Finally, entrepreneurs may choose to remain idle. Output is produced using a single unit of labour, but entrepreneurs have heterogenous managerial ability and those with a higher ability produce more output with this single

[4]Note that to this day, there is no consensus in the debate on the empirical impact of immigration (legal/illegal) on the native population (employment, wages). See Borjas (1999) and Borjas, Grogger and Hanson (2008).

unit of labour — ability simply equals output. The managerial ability of a given entrepreneur is the same whether production takes place in the legal or the informal sector. Domestic workers supply a single unit of labour, and choose to work for the entrepreneur that pays the highest wage, so they may work in the legal or the informal sector. A government levies taxes to finance the provision of a pure public good and it may invest in costly enforcement to reduce tax evasion. It is assumed that firms producing more output are more likely to be detected. If detected, an entrepreneur has to bear an exogenously fixed non-monetary sanction, but the worker hired by this entrepreneur is nonetheless paid his wage. In the basic version of the model, there are a number of undocumented immigrants who can only work in the informal sector. We characterize the optimal level of taxes, public good, and enforcement in this economy.

Our characterization hinges on whether the equilbrium is segmented or non-segmented. By this we mean, whether the informal sector contains only undocumented workers. In a non-segmented equilbrium, domestic workers will work in both sectors. This implies that wages will be equalized across these sectors, and in a very direct sense, by a wage arbitrage condition, legal wages will be pinned down by the mass of undocumented workers. In this case, taxes and enforcement are substitute policies since they both operate on the margin for firms between producing in the legal and the illegal sector. However, even in a segmented equilibrium, equilibrium wages are determined by the combination of firm decisions and optimal policy. We show that this will lead to net wages being equalized across these sectors in this case as well. For this reason, in our model, domestic workers always prefer to have fewer undocumented workers. However, total welfare is increasing in the number of undocumented workers. We find that the public good is under-provided relative to the first best. This is due to the fact that the enforcement is costly, that it using tax revenue, and that it distorting the provision of the public good. Enforcement and taxes interact in somewhat subtle ways. However, optimal enforement may not be always be decreasing in its cost. This is due to the fact that enforcement is costly in terms of the public good. When the marginal value of the public good is high, increasing the cost of the enforcement may actually lead the government to want to increase enforcement in order to maintain public good provision. When the marginal benefit is small, increasing the cost of enforcement leads to a reduction in optimal enforcement. We also find that increasing the number of undocument workers increases the cost of public good provision. This is because more undocumented workers reduce informal wages, making it more difficult to raise tax revenue in the legal sector. As a consequence, if the society does place sufficient weight on these undocumented workers' consumption of the public good, optimal public good provision will fall as the total population increases. We consider as well the question of whether a government would optimally choose policies that generate a segmented or non-segmented equilibrium. We find that when enforcement is too costly, segmentation is not optimal.

We extend the model in several important ways. We consider what happens when the number of undocumented workers or the level of illegal immigration is endogenous. We find similar results, but also an additional interesting feature, that the government can use the level of public provision directly as a way of altering the number of illegal migrants and as a lever on the illegal wage. This is due to the fact that the migration decision results from comparing source country utility with the destination country utility. Increasing the public good provided makes a destination country more attractive, which encourages migration. The arbitrage condition then implies that informal sector wages must fall.

We also consider what happens when firms in the formal sector are obligated to pay a minimum wage to their worker. A minimum wage breaks the arbitrage condition linking formal and informal sector wages. It also increases the cost of operating in the formal sector, and so can reduce the ability of the government to collect taxes. Consequently, the presence of a minimum wage strengthens the need for enforcement, and makes a segmented equilibrium less likely to be optimal.

We also consider amnesties for undocumented workers, and find that at the margin, they are socially beneficial. We also endogenize output price to ensure that our findings are robust.

Note that to our knowledge, there are only two papers similar to ours in the literature. In Djajic (1997), the incentive of firms to hire illegal immigrants arises because of a wage differential between the legal and the informal sector, but not from the obligation to pay taxes when hiring domestic workers in the legal sector. In our model, firms may be tempted by the informal sector because of a wage differential but also because they want to evade taxes.[5] Also, Djajic provides a positive analysis of the impact of government policies (enforcement, increase in the stock of illegal workers) but he does not attempt to characterize optimal policies as we do. In Epstein and Heizler (2007),

[5]There are taxes in Djajic (1997), but they are paid by employees.

a partial equilibrium model is constructed in which some representative firm may hire domestic workers and/or illegal immigrants. This is in contrast with our general equilibrium model in which firms cannot simultaneously hire both types of workers. Epstein and Heizler (2007) also do without taxes on firms while in our model, they play a key role. Like us, they perform an analysis of optimal enforcement policies.

The paper is organized as follows. Section 2 presents the basic version of the model, and Section 3 considers the extensions by incorporating the minimum wage, endogenous migration, amnesties, and endogenous price. Lastly, Section 4 concludes.

2 Basic Model

We model a simple economy in which firms may choose to operate in either an informal sector to evade taxes and potentially minimum wage restrictions, or to operate in a formal and regulated sector. We assume there are M domestic workers who can work either in the formal (legal) sector (M_L) or the informal sector (M_I), $M_L + M_I = M$; and U undocumented workers who can only work in the informal sector. Each of these workers, if hired, supplies one unit of labour inelastically. Domestic workers choose to work in the sector offering the highest wage.

The economy has N entrepreneurs with varying productivity θ. For simplicity, we assume that productivity is uniformly distributed on $[0, 1]$. Each entrepreneur has an endowment k that can be consumed or invested to start-up a firm. Entrepreneurs can choose not to operate a firm (N_0), to start-up a firm in the formal sector (N_L) or to start-up a firm in the informal sector (N_I). $N_0 + N_L + N_I = N$. Both informal and formal sector firms produce the same good X, which is sold at an exogenous price P.[6] To produce output, entrepreneurs need to hire one worker.[7] We also assume that $N > M + U$ in order to guarantee full employment since the level of wages, and not the level of employment, is the focus of our paper.

All $M + N + U$ individuals have the identical utility function $x + v(G)$, with $v' > 0 > v''$ and $v'(0) \to \infty$, where x is consumption of the domestically produced good X and G is the amount of public good provided by the government.

[6] Later, we endogenize this price.

[7] Alternatively, we could interpret this as a group of workers, however for simplicity we assume each firm hires only one worker.

An entrepreneur who does not start a firm can consume his endowment and obtain $\pi_0(\theta) = k$. Operating in the formal sector requires k to be invested. A formal firm produces θ units of the domestic good X, and pays the formal wage w_L to its worker and the tax t to the government.[8] This yields profits in the formal sector of

$$\pi_L(\theta) = P\theta - w_L - t. \tag{1}$$

An informal firm has the same investment costs and output as a legal firm;[9] however, the firm must also pay the informal market wage w_I and any sanction imposed in expectation $e\theta S$. This expected sanction can be decomposed into two parts, the probability of detection $e\theta$, where we assume that larger firms are more likely to be detected, and a non-monetary sanction S.[10] This yields profits in the informal sector of

$$\pi_I(\theta) = P\theta - w_I - e\theta S. \tag{2}$$

The government levies taxes on the formal sector to finance the provision of a pure public good G available to all residents and it may invest in costly enforcement to reduce tax evasion. The cost of enforcement is simply given by $C(e) = ce$ and the cost of a unit of the public good is unity. Therefore, the government budget constraint is

$$G + ce = N_L t. \tag{3}$$

[8] In this paper, we restrict attention to taxing formal firms. However, given imperfect informal sector monitoring, this is without much loss. In particular, if the government could tax workers, but only formal workers, this would result in exactly the same qualitative insights. Informal workers would face some probability of detection, and wages would have to equated between the formal and informal sectors for domestic workers to be willing to work and to be hired in both. Of course, the combination of taxes and penalties would never foreclose the formal sector entirely given the preferences for the public good. Similarly, we could allow the government to chose to tax consumption, but only consumption from formal sector firms. Since these two sources of consumption are perfect substitutes to consumers, they would have to be available at the same price. In this way, enforcement would have to be sufficiently high to prevent the legal sector from being foreclosed. This model would have a subtle difference since the benefit of avoiding tax would be proportional to productivity which it is not the case in our basic model. But regardless, our qualitative results would obtain.

[9] We could allow for informal firms to be less efficient producers. This will affect the marginal decision between the sectors as you would expect.

[10] The sanction could also be monetary, but fully dissipated by administrative costs. We assume this in order to remove the perverse outcome where governments want to encourage tax evasion to increase sanction revenue. Djajic (1997) makes a similar assumption.

where N_L denotes the number of firms (entrepreneurs) operating in the legal sector and paying taxes (endogenized below). The government picks $\{t, e, G\}$ to maximize a utilitarian welfare function, where we allow the government to weigh undocumented workers less heavily, subject to its budget constraint.

Now, consider the optimal decisions of entrepreneurs. Given wages and government policies, entrepreneurs will decide whether or not to start a firm, and if they start a firm, which type. We will restrict attention to the case where at least some entrepreneurs want to start formal firms.[11] Note that the slope of the profit function (with respect to entrepreneur's ability) is P in the formal sector, and $P - eS < P$ in the informal sector. The ability level $\hat{\theta}$ that makes an entrepreneur indifferent between starting a firm in the formal sector and starting a firm in the informal sector is determined from the intersection of the two profit functions. Because the relative cost of operating in the informal sector is increasing in θ, all entrepreneurs with $\theta > \hat{\theta}$ will prefer to operate in the formal sector, while all entrepreneurs with $\theta < \hat{\theta}$ prefer operating in the informal sector where

$$\hat{\theta} = \frac{w_L - w_I + t}{eS}. \tag{4}$$

Analagously, we define $\bar{\theta}$ as the ability that makes an entrepreneur indifferent between starting a firm in the informal sector and not starting a firm at all. Since profits are increasing in productivity, all entrepreneurs with $\theta > \bar{\theta}$ prefer starting a firm, while entrepreneurs with $\theta < \bar{\theta}$ prefer not starting a firm where

$$\bar{\theta} = \frac{w_I + k}{P - eS}. \tag{5}$$

Since $k > 0$, the least productive entrepreneur ($\theta = 0$) never starts a firm. Consequently, $\bar{\theta} > 0$.

With $\hat{\theta} > \bar{\theta}$ (as in Figure 1), there will be an informal sector. Entrepreneurs below $\bar{\theta}$ do not start-up a firm, those between $\bar{\theta}$ and $\hat{\theta}$ start a firm in the informal sector, and those above $\hat{\theta}$ start a firm in the formal sector. In this situation, formal sector labour demand will be given by $N_L = N(1 - \hat{\theta})$ and informal sector labour demand will be given by $N_I = N(\hat{\theta} - \bar{\theta})$.

To close the model, we need labour demand to equal labour supply in both sectors, and for a wage arbitrage condition equating wages across the sectors to hold when

[11]In other words, we will not consider the case where k is so high that no firms operate, and given our restrictions on $v(G)$, it will never be optimal for government policy to foreclose this sector completely.

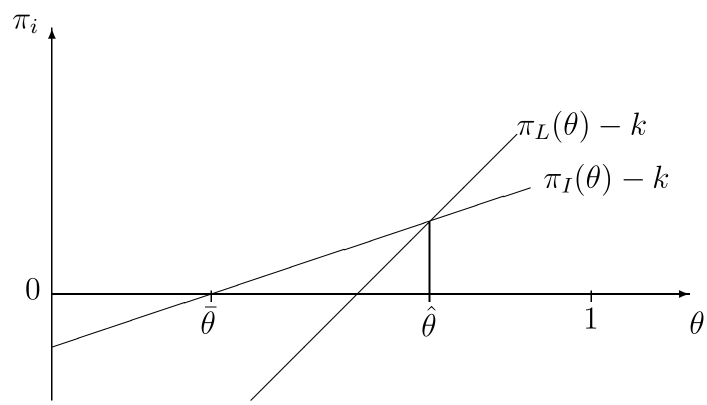

Figure 1: Profits in the Formal and Informal Sectors ($\hat{\theta} > \bar{\theta}$).

some domestic workers work in the informal sector. When only undocumented workers work informally such a condition does not need to be satisfied, as undocumented workers cannot work in the formal sector. The market will clear in this case as long as the formal wage is at least as large as the informal wage so that all domestic workers prefer to work in the formal sector. Consequently, the equilibrium will take one of two forms.

In the first type of equilibrium, labour markets are segmented and no domestic workers work in the informal sector. In this segmented equilibrium, $M_L = M$ domestic workers choose the formal sector and $M_I = 0$ domestic workers choose the informal sector. In the second type of equilibrium, labour markets are not segmented and $M_I > 0$ domestic workers choose the informal sector. The type of equilibrium obtained depends on government policy and so will be endogenous.

With a flexible informal wage, the supply of workers in the informal sector must equal the demand for workers by informal firms:

$$M_I + U = N(\hat{\theta} - \bar{\theta}). \tag{6}$$

With a flexible formal wage, the supply of domestic workers must equal the demand for workers by formal firms:

$$M_L = N(1 - \hat{\theta}). \tag{7}$$

From (6) and (7), and using $M = M_I + M_L$ we find that $\bar{\theta}^* = 1 - m - u$, where $m = M/N$ and $u = U/N$, and we have the following result.

Lemma 1 *In any equilibrium, there is full employment of undocumented and domestic workers and* $\bar{\theta}^* = 1 - m - u$.

Lemma 1 implies that all entrepreneurs with $\theta \geq 1 - m - u$ start up a firm in either the formal or informal sector, and produce θ. Therefore, we can calculate total output in the economy.

Lemma 2 *Total output in the economy is given by* $N \int_{1-m-u}^{1} \theta d\theta$ *and is independent of taxes and enforcement.*

Using Lemma 1 and the definition of $\bar{\theta}$ given by (5), we can solve for the wage in the informal sector as a function of government policies.

Lemma 3 *In any equilibrium, the wage in the informal sector is given by* $w_I(e) = (P - eS)(1 - m - u) - k$, *and is decreasing in the amount of enforcement.*

The government has a utilitarian objective and cares about all individuals in the economy, including, to a perhaps lesser degree, undocumented immigrants assigning them a welfare weight $\alpha \in [0, 1]$. For the time being, we will restrict government policy to be of only three dimensions: a tax on firms, a level of public good, and a level of enforcement. One could also imagine that the government may have some choice over U perhaps through a choice of border policy. In the extension section, we consider in a very simple way how optimal policy changes when U is endogenous. We will also brie?y discuss effects of other government instruments. We begin by defining the government's objective function.[12]

Definition 1 *Total weighted welfare* $\Omega(t, e, G; \alpha)$ *is given by:*

$$\Omega(t, e, G; \alpha) = N \int_{\bar{\theta}}^{1} \theta d\theta + [N - M - U]\frac{k}{P} - N \int_{\hat{\theta}}^{1} \frac{t}{P} d\theta - N \int_{\bar{\theta}}^{\hat{\theta}} \frac{e\theta S}{P} d\theta$$
$$- (1 - \alpha)U \left[\frac{P - eS}{P}(1 - m - u) - k\right] + [M + N + \alpha U]v(G).$$

[12]Derivation of this objective function is given in the Appendix.

Any wage paid is received by workers. Consequently, terms involving wages have no net effect on total welfare if all workers are counted equally. If the welfare of undocumented workers is discounted ($\alpha < 1$) then the welfare loss to entrepreneurs having to pay the informal wage is greater than the welfare gain to undocumented workers from receiving the informal wages. Consequently, there will be a welfare effect of enforcement policies through changes in the informal wage if $\alpha \neq 1$.

We first characterize the segmented equilibrium, and then characterize the non-segmented equilibrium.

2.1 Characterizing the Segmented Equilibrium

In a segmented equilibrium, all domestic workers choose to work in the formal sector, so $M_L = M$ and from (7), we obtain $\hat{\theta}^* = 1 - m$. From the definition of $\hat{\theta}$ given by (4), we can solve for the equilibrium wage in the formal sector.

Lemma 4 *In a segmented equilibrium, the formal wage is given by* $w_L(t, e) = P(1 - m - u) + ueS - k - t$ *and is increasing in enforcement and decreasing in the tax rate.*

To guarantee this equilibrium sorting of domestic workers, we need that $w_L(t, e) \geq w_I(e)$, or using Lemmas 3 and 4,

$$e \geq \frac{t}{(1 - m)S}. \tag{8}$$

Obviously the optimal policies will vary with the government objective. But, under either objective the government will choose tax and enforcement polices to ensure the constraint (8) is binding.

The government maximizes the weighted utilitarian welfare function $\Omega(t, e, G; \alpha)$ subject to its budget constraint (3) and the constraint (8) where δ is the Lagrange multiplier on this latter constraint. Using Lemmas 1–4, the first order conditions on t and e are given by the following, respectively:

$$[M + N + \alpha U]v'(G)M - \frac{M}{P} - \delta \frac{1}{(1 - m)S} = 0, \tag{9}$$

$$-[M + N + \alpha U]v'(G)c - \frac{S}{P}N \int_{1-m-u}^{1-m} \theta d\theta + (1 - \alpha)U\frac{S}{P}(1 - m - u) + \delta = 0, \tag{10}$$

where $G = Mt - ce$ since in a segmented equilibrium, $N_L = M$.

Enforcement is socially costly for two reasons. First, monitoring firms uses up government resources and reduces the amount of public good that can be provided when $c > 0$. Second, firms that are operating in the informal sector face some expected sanction. There is also a social benefit of enforcement when $\alpha < 1$. Increased enforcement reduces the informal wage which, when $\alpha < 1$, results in a net social benefit since the social gain to informal firms having to pay lower wages is greater than the social loss of lower wages to the undocumented workers. It turns out that for any α, the social cost of informal firms having to incur expected sanctions will always be greater than the potential social benefit through changes in the informal wage.[13]

Lemma 5 *The sum of the second and third term in (10) is always negative for any value of α.*

Lemma 5 implies that the constraint (8) will be binding and $\delta^* > 0$. Optimal policies will be chosen such that $t = (1 - m)eS$. From Lemmas 3 and 4 which define the informal and formal wages as a function of policies, we obtain the following result.

Lemma 6 *In a segmented equilibrium, wages will be equalized across the two sectors, $w_L^* = w_I^* = (P - e^*S)(1 - m - u) - k$ where e^* is the optimal level of enforcement.*

Note that in Djajic (1997) a segmented equilibrium necessarily entails a strictly positive net wage differential between the two sectors. Thus, firms may be attracted to the informal sector hoping to reduce their wage bill. In the current analysis, the government, by its choice of enforcement, removes this incentive and ensures that wages are equated. Consequently, firms who switch to the informal sector would do so solely to evade taxes.

Given the constraint is binding, we can now examine the optimal policies.

Proposition 1 *In a segmented equilibrium, optimal government enforcement is determined by*

$$[N + M + \alpha U]v'(G) = \left[\frac{1 + \frac{1}{M(1-m)} \left(N \int_{1-m-u}^{1-m} \theta d\theta - (1-\alpha)U(1-m-u) \right)}{1 - \frac{1}{M(1-m)} \frac{c}{S}} \right] \frac{1}{P}$$

where $G = Mt - ce$ and $t = (1 - m)eS$.

[13] See Appendix for proof.

Proposition 1 tells us that the optimal public good provision is determined from a modified Samuelson condition.

Corollary 1 *Relative to the first-best outcome, the public good is under-provided in a segmented equilibrium.*

In the first-best, the public good is set as if lump-sum taxes were available and enforcement is costless. Therefore, the sum of the marginal benefits from consumption of the public good will be equal to marginal cost of providing the public good, $1/P$, in the first-best. With costly enforcement, $c > 0$, the denominator is less than one and given enforcement imposes a net social cost, the numerator is greater than one. Therefore, the marginal social cost of providing the public good will be greater than $1/P$ and so, public good provision in the segmented equilibrium will be lower than in the first-best.

When monitoring becomes more costly less resourses can be allocated to the provision of the public good, and for the condition in Proposition 1 to continue to hold, the optimal amount of G being provided must go down.

Corollary 2 *Optimal public good provision is decreasing in the marginal enforcement cost.*

This result, however, does not necessarily imply that optimal enforcement decreases with c. The optimal amount of public good must match the resources available, so $G^* = t^*M - ce^* = [M(1-m)S - c]e^*$ and consequently,

$$\frac{dG^*}{dc} = [M(1-m)S - c]\frac{\partial e^*}{\partial c} - e^*. \tag{11}$$

As we know dG^*/dc is negative, but $\partial e^*/\partial c$ could be positive, and (11) could be satisfied. Intuitively, the government could compensate for diminution of available resources, by increasing tax and enforcement, but overall the direct strain on resources must dominate. We can show the following (see Appendix):

Corollary 3 *Optimal enforcement is increasing (decreasing) in the marginal enforcement cost c when the absolute value of the elasticity of the marginal benefit of the public good is greater than (less than) unity.*

When the marginal cost of enforcement goes up, enforcement uses up more resources and the government has an incentive to reduce the amount of monitoring. The increase in monitoring cost also reduces the amount of public good provision. If the absolute value of the elasticity of the marginal benefit is large, then a small decrease in G (via an increase in c) results in a large reduction in the marginal benefit of the public good. Consequently, the government will optimally want to increase enforcement to push up the amount of public good provision. The converse is then true.

Obviously, an increase in the number of undocumented workers increases the sum of marginal benefit for the public good provided $\alpha > 0$. Regardless of how undocumented workers are weighted, the marginal cost of providing the public good is increasing in U as shown in the Appendix.

Lemma 7 *For any value of α, an increase in U increases the social cost of providing the public good.*

When there is zero welfare weight on undocumented workers, there is no increase in the sum of the marginal benefit for the public good when U goes up. From Lemma 7, we know that the cost of providing the public goes up. Consequently, the amount of public good being provided will go down as U increases when $\alpha = 0$ and we have the following result.

Corollary 4 *Optimal public good provision is decreasing in U when $\alpha = 0$.*

When $\alpha > 0$, then there is also a social benefit to an increase in U and it is not clear what will happen to public good provision in this case. Likewise, an increase in α increases the social cost of providing the public good but also increases the marginal benefit of the public good. Again, the effect of α on optimal policies will be ambiguous. We are, however, able to say something about what happens when there is no undocumented workers.

Absence of Undocumented Workers

As a benchmark, it is useful to consider what happens when $U = 0$. In a segmented equilibrium, all domestic workers are in the formal sector so if there are no undocumented workers then there will be no informal sector. An entrepreneur now decides whether to operate a firm in the formal sector or not to operate a firm at all. We define $\tilde{\theta}$ as the cutoff ability that makes an entrepreneur indifferent between these two choices. Again, since profits are increasing in productivity, all entrepreneurs with $\theta > \tilde{\theta}$ prefer starting a firm in the formal sector, while entrepreneurs with $\theta < \tilde{\theta}$ prefer not starting a firm in the formal sector where

$$\tilde{\theta} = \frac{w_L + t + k}{P}. \tag{12}$$

There is only one labour market-clearing condition given by

$$M = N(1 - \tilde{\theta}). \tag{13}$$

From (13), we can solve for $\tilde{\theta}^* = 1 - m$ and together with the definition of $\tilde{\theta}$ given by (12) we can solve for the equilibrium formal wage as a function of the tax rate, $w_L(t) = P(1-m) - t - k$. The government, however, must ensure that any firm operating in the formal sector does not have an incentive to evade taxes. A firm will never evade taxes provided

$$P\theta - w_L(t) - t \geq P\theta - w_L(t) - e\theta S \quad \text{or} \quad \frac{t}{eS} \leq \theta$$

which must hold for all $\theta \geq \hat{\theta}^* = 1 - m$. Therefore, constraint (8) satisfied ensures that this will hold for all firms in the formal sector.

Definition 2 *Total weighted welfare* $\Omega(t, e, G)$ *is given by:*

$$\Omega(t, e, G) = N \int_{1-m}^{1} \theta d\theta + [N - M]\frac{k}{P} - N \int_{1-m}^{1} \frac{t}{P} d\theta + [M + N]v(G),$$

From the definition of the government's objective function, it is clear that an increase in enforcement costs uses up government resources and does not create any other social costs or benefits. Therefore, the government will want to reduce e for any given tax rate. Consequently, the constraint will bind and enforcement will be set such that $e = t/[(1-m)S]$. The entrepreneur with the lowest θ will be indifferent between not starting up a firm at all, operating a firm in the formal sector, and operating a firm, hiring a worker at the equilibrium wage and evading taxes. This is shown in Figure 2.

We have the following proposition:

Proposition 2 *Under a segmented equilibrium when there are no undocumented workers, optimal government enforcement is determined by the following condition:*

$$[N + M]v'(G) = \left[\frac{1}{1 - \frac{1}{M(1-m)}\frac{c}{S}}\right]\frac{1}{P},$$

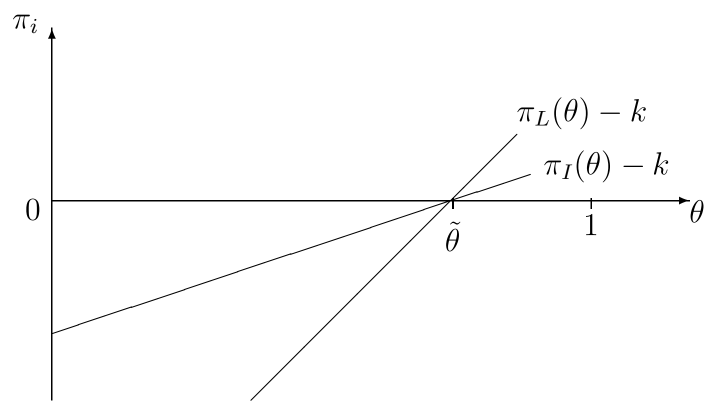

Figure 2: No undocumented workers.

where $G = Mt - ce$ *and* $t = (1 - m)eS$.

The condition in Proposition 2 is identical to the condition in Proposition 1 with $U = 0$. It follows then that the results (Corollary 1-3) established above also hold with $U = 0$.

The analysis of a non-segmented equilibrium is unchanged when $U = 0$ and we turn to this case next.

2.2 Characterizing the Non-Segmented Equilibrium

In a non-segmented labour market, some domestic workers choose to work in the informal sector, $M_I = M - M_L > 0$. Formal and informal wages must be equated in order to guarantee domestic workers are willing to take jobs in the informal sector. They are pinned down by the least profitable firm in the informal sector. From Lemma 3, equilibrium wages as a function of policies are given by:

$$w_I = w_L = [P - eS](1 - m - u) - k. \quad (14)$$

Wages in both sector will be independent of t, but will be a decreasing function of e.

Given the equality of wages in both sectors, it follows from the definition of $\hat{\theta}$

given by (4), that

$$\hat{\theta}^* = \frac{t}{eS}. \tag{15}$$

Differentiating (15), we obtain

$$\frac{\partial \hat{\theta}^*}{\partial t} = \frac{1}{eS} > 0, \quad \frac{\partial \hat{\theta}^*}{\partial e} = -\frac{t}{e^2 S} < 0. \tag{16}$$

The government maximizes the weighted utilitarian welfare function $\Omega(t, e, G; \alpha)$ subject to its budget constraint (3) and to a constraint that ensures some domestic workers are in the informal sector, that is, $\hat{\theta} \geq 1 - m$ or using (15)

$$e \leq \frac{t}{(1-m)S}. \tag{17}$$

The above constraint will provide a simple way to differentiate between the two types of equilbria. When it is binding, the equilibrium is segmented as can be seen from the discussion above, and if it is slack, the equilibrium is non-segmented. Let ϕ be the Lagrange multiplier on the constraint, the first order conditions on t and e are respectively given by:

$$[M + N + \alpha U]v'(G)\left(N(1-\hat{\theta}^*) - Nt\frac{\partial \hat{\theta}}{\partial t}\right) - \frac{N(1-\hat{\theta}^*)}{P} - \phi\frac{1}{(1-m)S} = 0, \tag{18}$$

$$[M+N+\alpha U]v'(G)\left[-Nt\frac{\partial \hat{\theta}}{\partial e} - c\right] - \frac{S}{P}N\int_{\bar{\theta}^*}^{\hat{\theta}^*} \theta d\theta + (1-\alpha)\frac{US}{P}(1-m-u) + \phi = 0, \tag{19}$$

$$\phi\left[e - \frac{t}{(1-m)S}\right] = 0. \tag{20}$$

To properly analyze these first order conditions, two separate issues need to be investigated. First we need to know if the constraint is binding or not. Intuitively, we need to know if the government prefers a segmented on non segmented type of equilibrium. The second issue is to describe what the tax and enforcement policies look like given we are in a non-segmented case. We begin with this latter issue and assume that $\phi = 0$.[14]

[14]Note that we assume that the term $(N(1-\hat{\theta}^*) - Nt\partial\hat{\theta}/\partial t)$ in equation (18) is strictly positive, reflecting the fact that the government chooses a tax rate on the left-hand side of the Laffer curve.

Proposition 3 *Under a non-segmented equilibrium, optimal government public good provision is determined by*

$$[M + N + \alpha U]v'(G) = \frac{1}{P}\left(\frac{(1-\hat{\theta}^*) + S\int_{\bar{\theta}^*}^{\hat{\theta}^*}\theta d\theta - (1-\alpha)uS(1-m-u)}{(1-\hat{\theta}^*) - t\frac{\partial\hat{\theta}}{\partial t} - t\frac{\partial\hat{\theta}}{\partial e} - \frac{c}{N}}\right).$$

To give some interpretation to optimal policies, we can eliminate the expressions for $v'(G)$ from the first-order conditions (18) and (19), substitute in the expressions from (16), and obtain:

$$\frac{(1-2\hat{\theta})}{\hat{\theta}^2 S - c/N} = \frac{(1-\hat{\theta})}{\frac{S}{2}(\hat{\theta}^2 - \bar{\theta}^2) - (1-\alpha)uS(1-m-u)}, \tag{21}$$

which gives $\hat{\theta}$ as a function of the parameters. Totally differentiating (21) we obtain the following result:

Corollary 5 *Under a non-segmented equilibrium, the size of the formal sector is decreasing in the number of undocumented workers when $a = 1$, and decreasing in the marginal cost of enforcement when $a = 1$ or $U = 0$.*

There are two ways the government can keep firms in the formal sector; depress taxes or increase enforcement. To induce firms to stay in the formal sector, the ratio of t/e must be kept low. As U increases, the total social cost of using enforcement increases. There are more firms in the informal sector and they are all paying the expected sanction. There is a social gain via the reduction in the informal wage but this gain is zero when undocumented workers are weighted equally in the government's objective. Therefore, in this case an increase in U gives the government an incentive to reduce enforcement (relative to taxes) so $\hat{\theta}$ moves up and the size of the formal sector shrinks. An increase in c increases the resource cost of using enforcement to keep firms in the formal sector. Again the potential social gain from a reduction in the informal wage is zero when either $\alpha = 1$ or when there are no undocumented workers in the economy. Consequently, the government will optimally reduce enforcement relative to taxes when the marginal cost of enforcement increases.

For $a < 1$, the social marginal cost of using enforcement relative to taxes is lower. Therefore, how G or t/e changes with enforcement costs and number of undocumented workers is unclear.

Lemma 8 *In a non-segmented equilibrium for any value of U and α, we have that $\hat{\theta}^* > \left[\frac{c}{NS}\right]^{1/2}$.*

Proof: From (19) and Lemma 5, and using the expression in (16) we have that

$$[M+N+U]v'(G)\left[-t\frac{\partial\hat{\theta}^*}{\partial e} - \frac{c}{N}\right] = \frac{S}{P}\int_{1-m-u}^{\hat{\theta}^*}\theta d\theta - (1-\alpha)\frac{uS}{P}(1-m-u) > 0,$$

which implies $\hat{\theta}^* > \left[\frac{c}{NS}\right]^{1/2}$. □

Recall, our condition for a non-segmented equilibrium requires that $\hat{\theta} > 1-m$. Therefore, Lemma 8 shows that it is possible to have a non-segmented equilibrium with some $c < N(1-m)^2S$. Further, a non-segmented equilibrium will necessarily be obtained when $c > N(1-m)^2S$.

2.3 Optimality of a Segmented Equilibrium

We now assess the condition under which it is optimal for the government to choose a segmented equilibrium. To do so we will examine the left hand side of first order condition on e *just* as it about to become binding, i.e. as $t/(eS) \to 1-m$. Since the constraint is not yet binding $\phi = 0$, but $\hat{\theta}^* \to 1-m$. The left hand side of first order condition (19) becomes

$$[M+N+\alpha U]v'(G)\left[N(1-m)^2S - c\right] - \frac{SN}{P}\int_{1-m-u}^{1-m}\theta d\theta + (1-\alpha)\frac{US}{P}(1-m-u). \quad (22)$$

Proposition 4 *When there are no undocumented workers, $U = 0$, the government will optimally enforce a fully segmented labour market if and only if $c < N(1-m)^2S$.*

Proof: When $U = 0$, the second and third term of (22) are both zero. If the first term is positive or zero, it implies that the government wants to increase e for any given tax rate when the constraint does not bind. Therefore, the government will want to increase enforcement until the constraint binds. However, if the first term is negative so $c > N(1-m)^2S$ then an interior solution will exist. □

Not surprisingly, if enforcement is too costly it is preferable to leave some domestic worker in the informal sector. The higher the sanction, the more effective enforcement is for a given e, and so a larger range of marginal enforcement costs can support a segmented equilibrium.

Proposition 5 *When there are some undocumented workers the range of marginal enforcement costs that can support the optimality of a segmented equilibrium is smaller.*

Proof: For any $U > 0$, by Lemma 5 the sum of the second and third terms in (22) is negative. Therefore, evaluating (22) at $c = N(1-m)^2S$ implies the government would want to reduce e for any given tax rate and therefore, a segmented equilibrium is not optimal. The minimal value of c that is required to make the segmented equilibrium optimal is strictly smaller than $N(1-m)^2S$. $\square$

Without undocumented workers, all firms operate in the formal sector in a segmented equilibrium. Obviously, to support this type of equilibrium the government needs to spend resources on monitoring and auditing firms. Because there are no firms are operating in the informal sector, no sanctioning is imposed and therefore no social loss from enforcement arises. With undocumented workers, some firms operate in the informal sector even in the segmented equilibrium, and so costly sanctions are imposed. This reduces the attractiveness of implementing a segmented equilibrium.

Unfortunately, we cannot say anything definite about how α changes the range of costs supporting the optimality of a segmented equilibrium since α both increases the first term and reduces the third term in (22).

3 Minimum Wage

We now introduce a minimum wage denoted by $\bar{w}$, that must be paid to all individuals working in the formal sector.[15] We treat this minimum wage as exogenous, but it could be justified on equity grounds or the result of a political equilibrium . Obviously, firms in the informal sector do not need to follow this regulation. We briefly discuss extending the model to generate an optimal minimum wage in the conclusion. With a minimum wage in place, the formal labour market no longer clears; some workers who want a minimum wage job may not be able to get one. However, relative to a situation with no informal sector, this excess supply will not translate into unemployment. In

[15] We assume that the penalty for evading the minimum wage and/or taxes are equivalent. So, no firm would ever choose to evade only one of these regulations. All firms in the formal sector will respect both, and all firms in the informal sector will not.

our framework, workers who cannot find a job in the formal sector can (and will) work in the informal sector instead.[16]

The cutoffs $\bar{\theta}$ and $\hat{\theta}$ are defined as before, but with $w_L = \bar{w}$. With a flexible informal wage, the supply of workers in this sector must equal the demand for workers by informal firms, so $U + M_I = N(\hat{\theta} - \bar{\theta})$, which can be re-written as:

$$U + M - N(1-\hat{\theta}) - N(\hat{\theta} - \bar{\theta}) = u + m - (1 - \bar{\theta}) = 0. \tag{23}$$

Equation (23) indicates that Lemmas 1, 2 and 3 all remain valid with the presence of a minimum wage. First, full employment prevails as $\bar{\theta}^* = 1 - m - u$. Second, total production is independent of all government policies including the minimum wage, and is given by

$$X^* = N \int_{\bar{\theta}^*}^{1} \theta d\theta = N[\frac{1-(1-m-u)^2}{2}].$$

This is due to the fact that the two sectors are equally productive, so the margin between the informal and formal sector does not impact output. The margin for entrepreneurs between the informal sector and being inactive does have consequences for output, but is not affected by the minimum wage. The informal wage is again given by $w_I^* = [P - eS](1 - m - u) - k$.

The formal market cutoff will be given by $\hat{\theta}^* = \frac{\bar{w} - w_I^* + t}{eS}$. With a minimum wage, the labour market will remain non-segmented as long as $e < (\bar{w} - w_I^* + t)/(1-m)S$. The reason is that firms in the informal sector benefit not only from avoiding taxes, but also from paying lower wages. We now define the objective function of the government still using $\alpha \in [0, 1]$ as the weight put on undocumented workers.

Definition 3 *Total weighted welfare* $\Omega^*(t, e, G, ; \alpha, \bar{w})$ *is given by :*

$$\begin{aligned} \Omega^*(t, e, G; \alpha, \bar{w}) = \quad & [N + M + \alpha U]v(G) + N \textstyle\int_{\bar{\theta}^*}^{1} \theta d\theta + N\bar{\theta}^* \frac{k}{P} \\ & - N \textstyle\int_{\hat{\theta}^*}^{1} \frac{t}{P} d\theta - N \int_{\bar{\theta}^*}^{\hat{\theta}^*} \frac{e\theta S}{P} d\theta - (1-\alpha) U \frac{w_I^*}{P}. \end{aligned}$$

[16] We have worked out the equilibria and optimal policies for the case in which workers who would like to work in the legal sector and are unlucky cannot switch *ex post* to an informal sector job. Thus, this is the case in which an excess supply in the legal sector corresponds to involuntary unemployment. It turns out that the algebra for that case is more involved but that the results are fairly similar.

3.1 Segmented Equilibrium

In a segmented equilibrium, all domestic workers are able to find work in the formal sector. In general, the introduction of a minimum wage creates excess supply. An important question to ask here, is how it is possible for all domestic workers to be employed legally despite the presence of a minimum wage? The answer resides in the fact that in our analysis, we include additional policies that can affect legal labour demand. Obviously, the minimum wage discourages firms to operate legally, but at the same time the threat of punishment discourages firms from operating in the informal sector. If e is sufficiently large relative to $\bar{w}$ and t, excess supply can be eliminated so that $M_L = M$, and $\hat{\theta}^* = 1 - m$. A government desiring to keep all its domestic workers in the formal sector has to ensure that

$$e \geq \frac{\bar{w} - w_I^* + t}{(1-m)S}. \tag{24}$$

We consider a government maximizing the weighted utilitarian welfare function $\Omega(t, e; \alpha, \bar{w})$ subject the government's budget constraint (3) and the constraint (24) where ψ is the Lagrange multiplier on this latter constraint. Using Lemmas 1, and 3, the first order conditions on t and e are given by the following, where $G = Mt - ce$.

$$M[N + M + \alpha U]v'(G) - \frac{M}{P} = \psi \frac{1}{(1-m)S}; \tag{25}$$

$$[N + M + \alpha U]v'(G)c + N\frac{S}{P}\left[\int_{1-m-u}^{1-m} \theta d\theta - (1-\alpha)u(1-m-u)\right] = -\psi \tag{26}$$

Notice that the first order conditions 25 and 26 are identical to the first order conditions 9 and 10 derived in the absence of a minimum wage. This implies that Lemma 5, and more importantly Proposition 1, describing the optimal level of the public good without a minimum wage, applies when minimum wage is present. As was stated in Lemma 5, the second term on the left hand side of (26) is always negative for any value of α. Intuitively, this means that the social cost of punishing additional firms who hire undocumented workers is larger than the reduction in wages paid to those workers. The public good provision G is given by the same modified Samuelson rule.

$$[N+M+\alpha U]v'(G) = \left[\frac{1+\frac{N}{M(1-m)}\left(\int_{1-m-u}^{1-m}\theta d\theta-(1-\alpha)u(1-m-u)\right)}{1-\frac{1}{M(1-m)}\frac{c}{S}}\right]\frac{1}{P}.$$

Relative to the first-best outcome, the public good is under-provided. Interestingly however, providing this same amount of public good requires higher taxes, and more enforcement as condition **??** is more restrictive in the presence of a minimum wage. It follows that the informal wage is lower in the current case than in the one without a minimum wage. Contingent on being in a segmented equilibrium, the level of the public good is the same, but as we will see later, it is less likely that a segmented equilibrium is desirable. The optimal public good provision is again decreasing in the marginal enforcement cost as stated in Corollary 2.[17] The effect of changes in the number of undocumented workers as well as the effect of α on optimal enforcement and public good provision are also the same as before. In particular, the optimal public good provision is decreasing in U when $\alpha = 0$.

We refrain from having a long discussion regarding the segmented equilibrium when $U = 0$ because it is essentially the same as in the last section without a minimum wage. In particular, public good provision is simply given by

$$[N+M]v'(G) = \left[\frac{1}{1-\frac{1}{M(1-m)}\frac{c}{S}}\right]\frac{1}{P}.$$

3.2 Non-Segmented Equilibrium

With a minimum wage, all domestic workers strictly prefer working in the formal sector. However, in a non-segmented labour market, some domestic workers will be forced to work in the the informal sector, because of the excess supply in the legal sector. From Lemma 3, the informal wage is given by $w_I^* = [P-eS](1-m-u)-k$, and it follows from the definition of $\hat{\theta}$ given by (4), that

$$\hat{\theta}^* = \frac{\bar{w}-w_I^*+t}{eS}. \tag{27}$$

[17]Note that Corollary 3 also applies, so enforcement is increasing (decreasing) in the marginal enforcement cost c when the absolute value of the elasticity of the marginal benefit of the public good is greater than (less than) unity.

Differentiating (27), we obtain

$$\frac{\partial\hat{\theta}^*}{\partial t} = \frac{1}{eS} > 0, \quad \frac{\partial\hat{\theta}^*}{\partial e} = \frac{-1}{e}\left[\hat{\theta}^* - (1-m-u)\right] < 0, \quad \frac{\partial\hat{\theta}^*}{\partial\bar{w}} = \frac{1}{eS} > 0. \tag{28}$$

Taxes and the minimum wage affect the size of the formal sector in the same manner. As taxes (or the minimum wage) increase, more entrepreneurs choose the informal sector. As enforcement increases, the formal sector becomes more attractive by fear of higher sanctions. Although at at the same time the informal wage falls, we can show that the direct effect dominates.

The government maximizes the weighted welfare function $\Omega(t, e, G; \alpha, \bar{w})$ given by Definition 3 subject to its budget constraint (3) and to a constraint that ensures some domestic workers are in the informal sector, that is, $\hat{\theta}^* \geq 1-m$ or using (15)

$$e \leq \frac{\bar{w} - w_I^* + t}{(1-m)S}. \tag{29}$$

The above constraint will again provide a simple way to differentiate between the two types of equilbria. When it is binding, $e = (\bar{w} - w_I^* + t)/(1-m)S$, the equilibrium is segmented as can be seen from the discussion above, and if it is slack, $e < (\bar{w} - w_I^* + t)/(1-m)S$, the equilibrium is non-segmented. With the Lagrange multiplier ϕ on the constraint, the first order conditions on t and e are respectively given by:

$$[N+M+\alpha U]v'(G)\left[(1-\hat{\theta}^*) - t\frac{\partial\hat{\theta}^*}{\partial t}\right] - \frac{(1-\hat{\theta}^*)}{P} - \frac{\bar{w}-w_I^*}{P}\frac{\partial\hat{\theta}^*}{\partial t} - \phi\frac{1}{N(1-m)S} = 0; \tag{30}$$

$$\begin{aligned}[N+M+\alpha U]v'(G)\left[-t\frac{\partial\hat{\theta}^*}{\partial e} - \frac{c}{N}\right] &- \frac{\bar{w}-w_I^*}{P}\frac{\partial\hat{\theta}^*}{\partial e} \\ -\frac{S}{P}\left[\int_{\bar{\theta}^*}^{\hat{\theta}^*}\theta d\theta + (1-\alpha)u(1-m-u)\right] &+ \phi\frac{1}{N} \quad \geq 0;\end{aligned} \tag{31}$$

$$\phi\left[e - \frac{\bar{w}-w_I^*+t}{(1-m)S}\right] = 0. \tag{32}$$

These first order conditions are similar to the ones obtained without a minimum wage. In particular, a non segmented equilibrium obtains only when $\phi = 0$, and equation 31 can be satisfied with equality. However, an important difference is worth highlighting. Equations 30 and 31 contain the additional terms $-(\bar{w} - w_I^*)/P$ multiplied by the derivative of $\hat{\theta}^*$ with respect to t and e, respectively. Without a minimum wage, the decision to operate legally or not is based solely on the difference between taxes and expected punishment, implying that for the marginal entrepreneur, the gain from evading taxes must equal the expected punishments. Any marginal changes in $\hat{\theta}^*$ will have an impact on the provision of the public good through tax revenues, but it will not affect total welfare otherwise. The presence of a minimum wage gives entrepreneurs an additional reason to participate in the informal sector, implying that expected punishment is strictly larger than the gain from evading taxes for the marginal entrepreneur.

We now describe the optimal tax, provision of the public good, and enforcement level in a non-segmented equilibrium.

Proposition 6 *In a non-segmented equilibrium, the optimal public good provision is determined by*

$$[M+N+\alpha U]v'(G) = \frac{1}{P}\left[\frac{(1-\hat{\theta}^*) + S\int_{\bar{\theta}^*}^{\hat{\theta}^*}\theta d\theta - (1-\alpha)uS(1-m-u) + [\bar{w} - w_I^*]\left(\frac{\partial\hat{\theta}^*}{\partial t} + \frac{\partial\hat{\theta}^*}{\partial e}\right)}{(1-\hat{\theta}^*) - t\frac{\partial\hat{\theta}}{\partial t} - t\frac{\partial\hat{\theta}}{\partial e} - \frac{c}{N}}\right].$$

To give some interpretation to optimal policies, we can eliminate the expressions for $v'(G)$ from the first-order conditions (**??**) and (**??**) to obtain the following condition:

$$\frac{(1-\hat{\theta}^*) - t\frac{\partial\hat{\theta}}{\partial t}}{-t\frac{\partial\hat{\theta}}{\partial e} - \frac{c}{N}} = \frac{(1-\hat{\theta}^*) + (\bar{w} - w_I^*)\frac{\partial\hat{\theta}^*}{\partial t}}{S\int_{\bar{\theta}^*}^{\hat{\theta}^*}\theta d\theta - (1-\alpha)uS(1-m-u) + (\bar{w} - w_I^*)\frac{\partial\hat{\theta}^*}{\partial e}}, \tag{33}$$

where $G = N(1-\hat{\theta}^*)t - ce$.

When comparing the Samuelson rule described in Proposition 6 with the one without a minimum wage in Proposition 3, one thing stands out. A minimum wage affects the provision of the public good because it affects the cost of using both instruments. On the one hand, a higher level of public good can be achieved by increasing taxes. An increase in taxes pushes more firms to operate informally, and this is costly because more firms are exposed to punishment instead of paying taxes. As we explained above, with a minimum wage, expected punishment is strictly larger than taxes paid. On the other hand, a higher level of the public good can be achieved by increasing enforcement instead. With higher enforcement, fewer firm operate informally. Consequently, we cannot say if the presence of a minimum wage implies higher or lower levels of the public good. However, if we compare the optimal policy ratios [i.e. (33) for the case with a minimum wage, and (21) for the case without, we can argue that the presence of a minimum wage favours the use of enforcement versus taxes. Intuitively, a minimum wage makes the informal sector more attractive, so the government reacts by monitoring more and taxing less.

3.3 Optimality of a Segmented Equilibrium

We now assess the condition under which it is optimal for the government to choose a segmented equilibrium. To do so, we examine the left hand side of the first order condition on e *just* as it is about to become binding. Since the constraint is not yet binding $\phi = 0$, but $\hat{\theta}^* \to 1 - m$. The left hand side of first order condition (31) becomes

$$[M + N + \alpha U]v'(G)\left[N(1-m)uS\frac{t}{t+\bar{w}-w_I^*} - c\right]$$
$$-\frac{SN}{P}\left[\int_{1-m-u}^{1-m}\theta d\theta - (1-\alpha)u(1-m-u)\right] - \frac{\bar{w}-w_I^*}{P}\frac{\partial\hat{\theta}^*}{\partial e}. \qquad (34)$$

Proposition 7 *When $U = 0$, a government maximizing total welfare will never enforce a fully segmented equilibrium.*

Because of the additional benefit of being in the informal sector, enforcement is not as effective. In fact, without undocumented workers, as $\hat{\theta}^*$ approaches $(1 - m)$, it becomes totally unresponsive to changes in e. Note that the equivalent to Proposition 5 does not apply anymore. The presence of undocumented workers may in fact increase the range of marginal enforcement costs that can support a segmented equilibrium. The reason is that it increases the responsiveness of $\hat{\theta}^*$ to changes in monitoring.

4 Extensions

4.1 Amnesties

Following Djajic (1997), we consider the consequences of legalizing the status of some of the undocumented workers. To do this, we conduct the following exercise: increase M and reduce U by the same amount.

First, what happens to total welfare as U increases marginally. Differentiating the maximized total welfare $\Omega^*(t, e, G; \alpha)$ with respect to U, we obtain

$$\frac{d\Omega}{dU} = \alpha \left(\frac{w_I^*}{P} + v(G^*) \right) + \left(\frac{P - e^*S}{P} \right) (1 - \alpha)u > 0. \tag{35}$$

The first term in (35) is the gain in welfare from one more undocumented worker which will be zero if $\alpha = 0$. The second term reflects the negative wage effect an increase in the supply of undocumented workers has on the economy. But, because undocumented workers are weighted less than entrepreneurs the benefit of the lower informal wage to firms is greater than the loss to the undocumented workers of having a lower wage. This provides an additional incentive for increasing the supply of undocumented workers. The above expression holds regardless of whether the economy is in a non-segmented or segmented equilibrium but of course the values of the expressions will depend on the optimal policies.

Consider now the effect on maximized total welfare of an increase in M. Differentiating maximized total welfare in the case of a non-segmented equilibrium, we obtain

$$\frac{d\Omega}{dM} = \left(\frac{w_L^*}{P} + v(G^*) \right) + \frac{P - e^*S}{P}(1 - \alpha)u. \tag{36}$$

The interpretation of this expression is similar to the one given to above. The first term is the gain in welfare from one more domestic worker and the second term reflects the negative effect on the informal wage of an increase in the number of domestic workers which is a social gain provided $\alpha \neq 1$. In the case of a segmented equilibrium, there will be an additional term reflecting the fact that domestic workers are fully employed in the legal sector so public good provision necessarily increases as a result of changing the status of an undocumented worker employed in the informal sector to a domestic worker employed in the formal sector. This additional positive term is given by $[N + M + \alpha U]v'(G^*)t^*$.

To determine whether an amnesty is optimal in either type of equilibrium, we simply look at the difference between this two expressions given $dU = -dM < 0$ and noting that in either type of equilibrium, the informal and formal wages will be the same. In the case of a non-segmented equilibrium, we find

$$\frac{d\Omega}{dM} - \frac{d\Omega}{dU} = (1-\alpha)\left(\frac{w_L^*}{P} + v(G^*)\right). \tag{37}$$

Therefore, for $\alpha \neq 1$ legalizing the status of an undocumented worker will be welfare-improving in a non-segmented equilibrium and will always be welfare-improving in a segmented equilibrium.[18] This exercise, of course, is only valid for marginal changes. Legalizing the status of a large number of undocumented workers would necessarily change the optimal policies and possibly the type of equilibrium in which the economy rests.

4.2 Endogenous Price

Until now we have assumed that the price of the good produced in the economy is fixed. With a fixed price, regardless of how the government weights the welfare of these undocumented workers, an increase in the number of these workers will unambiguously increase total welfare, and reduce the welfare of domestic workers as discussed above. By holding price fixed, we are ignoring the potential impact of an increase in the supply of undocumented workers on price. The effect of an increase in the number of undocumented workers on domestic workers' welfare (or, purchasing power) will depend not just on the wage effects but also on the price effects of an increase in number of undocumented workers.

Recent empirical work has identified two mechanisms for these price effects. First, the increase in the size of the low-skilled immigrant population puts downward pressure on low-skilled wages and therefore puts downward pressure on the prices of the goods and services produced by this labour (Cortes, 2008). Second, an increase in the low-skilled population who are more price sensitive can have a moderating effect on the price of the goods and services demanded by this population (Lach, 2007). This

[18] In either type of equilibrium, an increase in U puts downward pressure on the informal wage and consequently downward pressure on the formal wage. Consequently, the welfare of domestic workers is decreasing in the number of undocumented workers and would always prefer not to allow for such an amnesty.

former effect is clearly absent in this model because all individuals have the same demand for the good but the latter effect can occur.

To show this assume that there are now two goods, X and Y.[19] Good X is defined as before. Good Y is an imported good and the price of Y is normalized to unity. Individuals preferences over the two goods and public good provision are $X^{\gamma}Y^{1-\gamma} + v(G)$ with $\gamma \in (0,1)$. Let R be the resources of a given individual. The individual's demand for good X will be $\gamma R/P$ and for good Y will be $(1-\gamma)R$. Therefore, the indirect utility of the individual is $V = \Gamma \frac{R}{P^{\gamma}} + v(G)$ where $\Gamma = \gamma^{\gamma}(1-\gamma)^{1-\gamma}$. For the remainder of this discussion, we assume that the expected sanction is monetary but that all revenues are equal to administrative costs. In this case, the total amount of resources, denoted by TR, is the sum of the value of total production in the economy plus the total endowment of entrepreneurs not operating firms, less the taxes paid by firms in the formal sector and less the expected sanctions paid by firms in the informal sector. We also assume that $\alpha = 1$.

Total demand for good X is given by the total resources of all the individuals in the economy times γ divided by the price where total resources was defined above. Recall, total supply of good X is $N\int_{1-m-u}^{1}\theta d\theta$. Equating demand to supply yields the equilibrium price,

$$P^{*} = \frac{\gamma}{1-\gamma}\frac{N(1-m-u)k - N\int_{1-m-u}^{\hat{\theta}^{*}} e\theta S d\theta - N(1-\hat{\theta}^{*})t}{N\int_{1-m-u}^{1}\theta d\theta}. \tag{38}$$

which is decreasing in U. Supply of good X is independent of price and an increase in U shifts outwards this vertical supply curve. An increase in U also shifts outwards the demand curve since for a given price total resources goes up. But, the increase in demand is less than the increase in supply and consequently the equilibrium price falls.[20] Using (38) and the definition of total resources in the economy, we obtain

$$TR^{*} = \frac{1}{1-\gamma}\left(N(1-m-u)k - N\int_{1-m-u}^{\hat{\theta}^{*}} e\theta S d\theta - N(1-\hat{\theta}^{*})t\right).$$

Total welfare can be written as

$$\Omega = \Gamma\frac{TR^{*}}{P^{*\gamma}} + [M+N+U]v(G). \tag{39}$$

[19] A second good is required for a meaningful discussion of relative prices. The full analysis of this extension is provided in the Appenix.

[20] The change in demand is $\gamma((P-es)(1-m-u)-k)/P$ and the change in supply is $(1-m-u)$.

Proposition 8 *With an endogenous price, total welfare is increasing in the number of undocumented workers U.*

Proof: Substituting in for P^*, and differentiating with respect to U yields

$$\frac{d\Omega}{dU} = \Gamma \frac{1}{P^{\gamma}} w_I^* + v(G) > 0. \quad \square$$

With a fixed price, an increase in U has two effects. First, it attracts an entrepreneur into the informal sector who was previously not operating a firm. This entrepreneur is now better off and consequently, total welfare increases. Note also that total production increases. Second, there will be more individuals benefiting from the public good which also increases total welfare. These two effects are present when the price is endogenous but there is also an additional effect. The increased production puts downward pressure on the price which reduces the value of output produced by all firms in the economy and consequently, the welfare of all firms. At the same time, everyone benefits from the lower price. We have shown that the net effect of an increase in U on total welfare remains positive even when the price is endogenous.[21] It is also the case that having price endogenous does not change the relative trade-offs between the two polices.

4.3 Endogenous Undocumented Immigrants

The level of undocumented immigrants or workers may in some ways be a direct choice variable as above, however it is perhaps more natural to assume that the level of undocumented migration is a function of the opportunities available to them in the host country. We now assume that undocumented immigrants enter the country as long as the return to moving exceeds some fixed reservation wage w_R. This reservation wage can be thought of as simply the wage in their home country, or as that wage augmented by any moving or migration costs. In this way, we could imagine that captured within these costs is some border control policy. In this way, varying this reservation wage can be thought of as a very reduced form way of capturing the impact of border policy. This reservation wage can also incorporates utility of any public good provision in the source country. Given that we assume that every undocumented immigrants share the same reservation wage, there is a perfectly elastic

[21] With $\alpha < 1$, we have the additional net social benefit of increasing U by lowering the equilibrium wage in the informal sector as discussed earlier.

supply of undocumented immigrants (the other extreme to perfectly inelastic supply of undocumented immigrants considered in our base model). Any wage satisfying $w_I + v(G) > w_R$ will induce undocumented immigrants to migrate until these two wages are equated. The domestic market equilibrium will still take one of two forms: it will either be non-segmented in which case, the formal wage will also be driven down to this reservation utility, or it will be segmented, allowing a legal wage which is higher than the informal one. We consider these two cases in turn.

Non-segmented Equilibrium

In a non-segmented equilibrium the labour market clearing condition will be given by $w_L = w_I = w_R - v(G)$.[22] Using this we have our two critical cutoffs

$$\bar{\theta}^* = \frac{w_R - v(G) + k}{P - eS}, \qquad \hat{\theta}^* = \frac{t}{eS},$$

which together with the condition $\bar{\theta}^* = 1 - m - u$, and recalling that the level of the public good is given by $G = N(1 - \hat{\theta}^*)t - ce$, imply that the equilibrium number of undocumented immigrant workers is given by

$$U^* = N\left(1 - m - \frac{w_R - v(N(1 - \hat{\theta}^*)t - C(e)) + k}{P - eS}\right).$$

This equilibrium level of undocumented workers is quite intuitive. It is decreasing in the reservation wage and the number of documented workers. Increasing enforcement e shrinks the informal sector and reduces the number of undocumented immigrants employed in that sector through two effects. It makes setting informal firms less attractive by increasing enforcement. It also reduces the public good which increases the informal wage in equilibrium. This second channel is new here. If the reservation wage is loosely taken to proxy for some amount border policy then public goods behave in exactly the opposite way of encouraging entry. Note that in this environment, entrepreneurs will have a direct benefit from the public good as well as an indirect benefit from the effect of reduced wages. Workers will have the opposite.

A government who cares about weighted total welfare maximizes the following objective:

$$\max_{\{t,e\}} N \int_{\bar{\theta}^*}^{1} \theta d\theta - N\frac{eS}{P} \int_{\bar{\theta}^*}^{\hat{\theta}^*} \theta d\theta - N \int_{\hat{\theta}^*}^{1} \frac{t}{P} - (1-\alpha)U^* \frac{w_R - v(G)}{P} + N\bar{\theta}^* \frac{k}{P} +$$

[22]Note that this condition implicitly limits the total quantity of the public good, since workers will never work for negative wages.

$$(N + M + \alpha U^*)v(tN(1 - \hat{\theta}^*) - ce) - \phi \left[1 - m - \frac{t}{eS}\right].$$

The government chooses taxes using exactly the same tradeoffs as before, except now there is an additional benefit of higher taxes. By raising taxes, the government lowers the informal wage by increasing public good provision, expands the informal sector by raising U and lower $\bar{\theta}^*$. This redistributes some income from undocumented workers to entrepeneurs. If the welfare weight on undocumented workers is less than that on entrepreneurs, this is socially beneficial. If $\alpha = 1$ this additional benefit goes to zero.

When considering the optimal level of e, the costs and benefits are similar to the fixed U case. However, now there are several new effects. When e increases, it causes the size of the informal sector to shrink (U to fall and $\bar{\theta}^*$ to increase) and production to fall. This also reduces the total number of undocumented workers who consume the public good. Both of these reduce welfare. If undocumented workers are equally weighted $\alpha = 1$, these are the only two effects which both suggest that e should be lower than in the exogenous U case. If however undocumented workers are less heavily weighted $\alpha < 1$, then there is another important effect. By increasing e the government reduces the public good since e is costly. This puts upward pressure on the level of informal wage. This increase hurts entrepeneurs but helps workers. Given their unequal weighting in the welfare function, this is an additional cost of raising e.

This suggests another role for public good spending - it reduces the informal sector wage, which redistributes income toward entrepreneurs and away from undocumented workers. Notice however that in this type of non-segmented equilibrium, it also redistributes income away from documented workers since their wages are tied to those in the informal sector.

Segmented Equilibrium

If instead, we are in a segmented equilibrium the labour market clearing condition is given by $w_L > w_I = w_R - v(G)$. Using this, we can solve for the equilibrium $\bar{\theta}$ and number of undocumented immigrant workers.

$$\bar{\theta}^* = \frac{w_R - v(G) + k}{P - eS}; \quad U^* = N\left(1 - m - \frac{w_R - v(G) + k}{P - eS}\right)$$

With a flexible formal wage, the supply of workers in this sector must equal the demand for workers by formal firms:

$$M = N(1 - \hat{\theta}).$$

This yields $\hat{\theta}^{**} = 1 - m$ and the equilibrium formal wage: $w_L^{**} = (1-m)eS + w_R - t$. Note that for $w_L^{**} \geq w_I^{**} = w_R$ we need (8) to be satisfied. This implies that there is some relationship that must be respected between the levels of t and e chosen. This relationship says that these are complementary policies as you increase t you must eventually also increase e to remain in a segmented equilibrium, and if you reduce t you can reduce e. However now, we are introducing some some slightly different costs and benefits. In particular, increasing t now has two effects on firm decisions. As before, it directly makes the formal sector more expensive and less appealing. Now, this effect is reinforced, since higher taxes imply a higher level of public goods, and a lower informal sector wage. Similarly, increasing e makes it directly more costly to enter into the informal sector with its implied higher sanctions (a supply effect) and indirectly more costly with the reduced public good and the implied higher wages (a redistribution toward informal workers effect).

When considering the optimal policy in this case, much of the same intution will carry through as discussed in the previous section, except now the government has two additional margins. It can expand production by reducing the cost of setting up a firm in the informal sector by decreasing e or increasing G. And it can strategically manipulate the amount of the public good to reduce informal sector wages and redistribute surplus toward informal entrepreneurs.

5 Conclusion

In this paper, we construct a simple model of tax evasion with an informal sector, and consider the role of illegal immigration and undocumented workers on optimal tax and enforcement policy. There are natural ways in which to extend this work. We could consider the role of differential productivities across the formal and informal sectors. And perhaps, more importantly, we could attempt to fully characterize the optimal minimum wage. In the current paper, we have taken the minimum wage as fixed, however, if we allowed a welfare function with a heavier weight on workers' utility, we could derive optimal policies and characterize when a binding minimum wage would be socially desirable.

References

[1] Allingham, M.G., and A. Sandmo (1972), 'Income Tax Evasion: A Theoretical Analysis', *Journal of Public Economics* 1(3-4), pp. 323–338.

[2] Borjas, G. (1999), 'The Economic Analysis of Immigration,' in O.C. Ashenfelter and D. Card (eds.) *Handbook of Labor Economics*, (North-Holland, Amsterdam), pp. 1697–1760.

[3] Borjas, G., J. Grogger, and G. Hanson (2008), 'Imperfect Substitution Between Immigrants and Natives: A Reappraisal', NBER Working Paper 13887, March 2008.

[4] Cortes, P. (2008), 'The Effect of Low-Skilled Immigration on U.S. Prices: Evidence from CPI Data', *Journal of Political Economy* 116(3), pp. 381–422.

[5] Djajic, S. (1997), 'Illegal Immigration and Resource Allocation', *International Economic Review* 38(1), pp. 97–117.

[6] Epstein G., and O. Heizler (2007), 'Illegal Migration, Enforcement and Minimum Wage', IZA Discussion Paper Series, No. 2830.

[7] Ethier, W. (1986), 'Illegal Immigration: The Host-Country Problem', *American Economic Review* 76(1), pp. 56–71.

[8] Fortin, B., N. Marceau, and L. Savard (1997), 'Taxation, Wage Control and the Informal Sector', *Journal of Public Economics* 66(2), pp. 293–312.

[9] Fugazza M., and J.-F. Jacques (2003), 'Labor Market Institutions, Taxation and the Underground Economy', *Journal of Public Economics* 88(1-2), pp. 395–418.

[10] Lach, S. (2007), 'Immigration and Prices,' *Journal of Political Economy* 115(4), pp. 548–587.

[11] de Paula, Á., and J.A. Scheinkman (2007), 'The Informal Sector', NBER Working Paper 13486, October 2007.

[12] Rauch, J.E. (1991), 'Modeling the Informal Sector Formally', *Journal of Development Economics* 35(1), pp. 33–47.

[13] Reinganum, J., and L. Wilde (1985), 'Income Tax Compliance in a Principal Agent Framework', *Journal of Public Economics* 26(1), pp. 1–18.

[14] Schneider, F., and D.H. Enste (2000), 'Shadow Economies: Size, Causes, and Consequences', *Journal of Economic Literature* 38(1), pp. 77–114.

[15] Slemrod, J., and S. Yitzhaki (2002), 'Tax Avoidance, Evasion, and Administration', in A.J. Auerbach and M. Feldstein (eds.) *Handbook of Public Economics* 3, (North-Holland, Amsterdam), pp. 1423–1470.

6 Appendix

Definitions 2.1: Derivation of Government Objective Function

Welfare for each type of entrepreneurs are as follows. The $N - M - U$ entrepreneurs who don't start a firm collectively receive

$$(N - M - U)[\frac{k}{P} + v(G)].$$

The $N(\hat{\theta} - \bar{\theta})$ entrepreneurs who start a firm in the informal sector collectively receive

$$N\left[\frac{P - eS}{P}\int_{\bar{\theta}}^{\hat{\theta}} \theta d\theta - [\hat{\theta} - \bar{\theta}]\frac{w_I}{P} + v(G)\right],$$

while the $N(1 - \hat{\theta})$ of those who start a firm in the formal sector collectively get

$$N\left[\int_{\hat{\theta}}^{1} \theta d\theta + [1 - \hat{\theta}]\frac{w_L + t}{P} + v(G)\right].$$

Summing up the above expressions, total welfare for all entrepreneurs is given by:

$$\Omega_E = N\left[\int_{1-m-u}^{1} \theta d\theta - \int_{1-m-u}^{\hat{\theta}} \frac{e\theta S + w_I}{P} d\theta - [1 - \hat{\theta}]\frac{w_L + t}{P} + v(G)\right].$$

Welfare for each type of worker are as follows. The M_L domestics workers in the formal sector collectively receive

$$M_L\left(\frac{w_L}{P} + v(G)\right),$$

while the $M_I + U$ workers in the informal sector collectively receive

$$(M_I + U)\left(\frac{w_I}{P} + v(G)\right).$$

Total welfare for all workers with weight $\alpha \in [0,1]$ on the welfare of undocumented workers is given by:

$$M_L \frac{w_L}{P} + (M_I + \alpha U)\frac{w_I}{P} + (M + \alpha U)v(G).$$

Using the labour market clearing conditions (6) and (7), and Lemma 3, total weighted welfare is:

$$\Omega^*(t,e,G;\alpha) = N\int_{\bar{\theta}}^{1} \theta d\theta + [N - M - U]\frac{k}{P} - N\int_{\hat{\theta}}^{1} \frac{t}{P} d\theta - N\int_{\bar{\theta}}^{\hat{\theta}} \frac{e\theta S}{P} d\theta$$
$$- (1-\alpha)U\left[\frac{P - eS}{P}(1 - m - u) - k\right] + [M + N + \alpha U]v(G).$$

Proof of Lemma 5

From (10), we have

$$-\frac{S}{P}N\int_{1-m-u}^{1-m} \theta d\theta + (1-\alpha)U\frac{S}{P}(1-m-u) = \frac{SN}{P}\left(-\int_{1-m-u}^{1-m} \theta d\theta + (1-\alpha)u(1-m-u)\right),$$
$$= \frac{SN}{P}\left(-\frac{(1-m)^2}{2} + \frac{(1-m-u)^2}{2} + (1-\alpha)u(1-m-u)\right),$$
$$= \frac{SN}{P}\left(-\frac{(1-m)^2}{2} + \frac{(1-m)(1-m-u) - u(1-m-u)}{2} + (1-\alpha)u(1-m-u)\right),$$
$$= \frac{SN}{P}\left(\frac{-u(1-m) + u(1-m-u)}{2} - \alpha u(1-m-u)\right),$$
$$= \frac{SN}{P}\left(\frac{-u^2}{2} - \alpha u(1-m-u)\right) < 0,$$

and Lemma 5 follows.

Proof of Corollary 3

Let

$$F(e,c) = [N+M+\alpha U]v'(G) - \left[\frac{1 + \frac{1}{M(1-m)}\left(N\int_{1-m-u}^{1-m} \theta d\theta - (1-\alpha)U(1-m-u)\right)}{1 - \frac{1}{M(1-m)}\frac{c}{S}}\right]\frac{1}{P}$$

where $G = M(1-m)Se - ce$. By Proposition 2, $F(e,c) = 0$. This condition yields equilibrium enforcement e^* as a function of c. Totally differentiating, we obtain

$$\frac{de^*}{dc} = -\frac{F_c}{F_e}$$

where

$$F_e = [N + M + \alpha U]v''(G)(M(1-m)S - c) < 0,$$

$$\begin{aligned}F_c &= [N + M + \alpha U]v''(G)(-e) - [N + M + \alpha U]v'(G)\frac{1}{M(1-m)S - c} \\ &= -[N + M + \alpha U]v'(G)\frac{1}{M(1-m)S - c}\left[1 + \frac{v''(G)}{v'(G)}G\right]\end{aligned}$$

Optimal enforcement is increasing in c if $v''(G)G/v'(G) < -1$, is decreasing in c if $v''(G)G/v'(G) > -1$ and is independent of c if $v''(G)G/v'(G) = -1$ as stated in Corollary 3.

Proof of Lemma 7

Differentiating the right-hand side of the condition in Proposition 1 with respect to U, we obtain

$$\frac{1}{P}\frac{1}{(M(1-m)S - c)}\left[-N(1-m-u)(-1/N) - (1-\alpha)(1-m-u) + (1-\alpha)u\right] > 0.$$

Proof of Corollary 5

Eliminating the expressions for $v'(G)$ from the first-order conditions (18) and (19), we obtain:

$$\frac{(1-\hat{\theta}^*) - t\frac{\partial\hat{\theta}}{\partial t}}{-t\frac{\partial\hat{\theta}}{\partial e} - \frac{c}{N}} = \frac{(1-\hat{\theta}^*)}{S\int_{\bar{\theta}^*}^{\hat{\theta}^*}\theta d\theta - (1-\alpha)uS(1-m-u)}, \tag{40}$$

where $G = N(1-\hat{\theta}^*) - ce$.

Substituting in the expressions from (16), condition (40) can be written as

$$\frac{(1-2\hat{\theta})}{\hat{\theta}^2 S - c/N} = \frac{(1-\hat{\theta})}{\frac{S}{2}(\hat{\theta}^2 - \bar{\theta}^2) - (1-\alpha)uS(1-m-u)}$$

which is equation (21). Manipulating yields;

$$(1-2\hat{\theta})\left(\frac{S}{2}(\hat{\theta}^2 - \bar{\theta}^2) - (1-\alpha)uS\bar{\theta}\right) = \left(\hat{\theta}^2 S - \frac{c}{N}\right)(1-\hat{\theta})$$

$$\left(\frac{S}{2}(\hat{\theta}^2 - \bar{\theta}^2) - (1-2\hat{\theta}^*)(1-\alpha)uS\bar{\theta}\right) - S\hat{\theta}^3 + S\hat{\theta}\bar{\theta}^2 = \hat{\theta}^2 S - \hat{\theta}^3 S - (1-\hat{\theta})\frac{c}{N}$$

Let

$$G(\hat{\theta}, c, \alpha, U) = \hat{\theta}^2 \frac{S}{2} - (1-\hat{\theta})\frac{c}{N} + \frac{S}{2}\bar{\theta}^2 + (1-2\hat{\theta})(1-\alpha)uS\bar{\theta} - S\hat{\theta}\bar{\theta}^2 = 0$$

Differentiating G, we obtain

$$G_{\hat{\theta}} = \hat{\theta}S + \frac{c}{N} - 2(1-\alpha)uS\bar{\theta} - S\bar{\theta}^2$$

$$G_c = -\frac{1-\hat{\theta}}{N} < 0$$

$$G_\alpha = -(1-2\hat{\theta})uS\bar{\theta} < 0$$

$$G_U = \frac{(1-2\hat{\theta})(1-\alpha)S\bar{\theta}}{N} + \left(S\bar{\theta}(1-2\hat{\theta}) + (1-2\hat{\theta})(1-\alpha)uS\right)\frac{-1}{N}$$

When $\alpha = 1$, we have

$$G_{\hat{\theta}} = S(\hat{\theta} - \bar{\theta}^2) + \frac{c}{N} > 0$$

$$G_U = \left(S\bar{\theta}(1-2\hat{\theta})\right)\frac{-1}{N} < 0$$

and Corollary 5 follows.

Extensions: Effect on $\Omega(t, e, G; \alpha)$ of Changes in U and M

Non-segmented Equilbrium

Recall, $\bar{\theta}^* = 1 - m - u$ is decreasing in M and U, $\hat{\theta}^* = \frac{t}{eS}$ is unaffected by change in U or M, and the equilibrium wages are $w_L^* = w_I^* = (P - eS)(1-m-u) - k > 0$ evaluated at optimal enforcement.

$$\begin{aligned}\frac{d\Omega}{dU} &= (1-m-u) - \frac{P-eS}{P}(1-m-u)(1-\alpha) + \frac{P-eS}{P}(1-\alpha)u - \frac{eS}{P}(1-m-u) - \alpha\frac{k}{P} + \alpha v(G) \\ &= \alpha\left(\frac{w_I^*}{P} + v(G)\right) + \frac{P-eS}{P}(1-\alpha)u.\end{aligned}$$

$$\begin{aligned}\frac{d\Omega}{dM} &= (1-m-u) + \frac{P-eS}{P}(1-\alpha)u - \frac{eS}{P}(1-m-u) - \frac{k}{P} + v(G) \\ &= \frac{w_L^*}{P} + v(G) + \frac{P-eS}{P}(1-\alpha)u.\end{aligned}$$

$$\frac{d\Omega}{dM} - \frac{d\Omega}{dU} = (1-\alpha)\left(\frac{w_I^*}{P} - \frac{k}{P} + v(G)\right) \geq 0.$$

At $\alpha = 1$, there is no change in maximized total welfare. Otherwise, the difference is positive.

Segmented Equilbrium

Recall, $\bar{\theta}^* = 1 - m - u$, $\hat{\theta}^* = 1 - m$ so $N(1 - \hat{\theta}^*) = M$, and $w_L^* = w_I^* = (P - eS)(1 - m - u) - k > 0$ evaluated at optimal enforcement.

$$\frac{d\Omega}{dU} = \alpha \left(\frac{w_I^*}{P} + v(G) \right) + \frac{P - eS}{P}(1 - \alpha)u.$$

$$\begin{aligned} \frac{d\Omega}{dM} &= (1 - m - u) + \frac{P - eS}{P}(1 - \alpha)u - \frac{eS}{P}(1 - m - u) - \frac{k}{P} + v(G) + (M + N + \alpha U)v'(G)t \\ &= \frac{w_L^*}{P} + v(G) + \frac{P - eS}{P}(1 - \alpha)u + (M + N + \alpha U)v'(G)t. \end{aligned}$$

$$\frac{d\Omega}{dM} - \frac{d\Omega}{dU} = (1 - \alpha) \left(\frac{w_I^*}{P} - \frac{k}{P} + v(G) \right) + (M + N + \alpha U)v'(G)t > 0.$$

For any α, the difference in total maximized welfare is positive.

Derivations with Endogenous Price

In this section, we make the additional assumptions discussed in the text. Most notably, we introduce an imported numeraire good. All individuals (Domestic Workers, Undocumented Workers and Entrepreneurs) share the same utility $X^\gamma Y^{1-\gamma} + v(G)$ with $\gamma \in (0, 1)$. Define R_i^j as the resources of individual of type $j \in \{E, W\}$ (E for entrepreneurs and W for workers documented or not) in state of the world $i \in \{\emptyset, L, I\}$, where L and I stand for formal and informal sector applying to both workers and entrepreneurs, and where $\emptyset$ stands for entrepreneur who don't start a firm. Note that indirect utility is linear in resources, so all individuals are risk neutral and try to maximize expected income. We also assume that N, D and U are sufficiently large, so people have their expected resources on average. In this model, an entrepreneur who does not start a firm has resources: $R_\emptyset^E = k$, an entrepreneur who starts a firm with a formal worker has resources $R_L^E = P\theta - w_L - t$, one who starts a firm with an informal worker has resources $R_I^E = P\theta - w_I$, and a formal sector worker has resources $R_L^W = w_L$, and an informal sector worker has resources $R_I^W = w_I$.

Consider first a non-segmented equilibrium. The analysis determining $\bar{\theta}$, $\hat{\theta}$ and wages is as in the model with fixed price. We now need to determine the equilibrium price as a function of policies, and then turn to the government's policies.

Recall total demand for good X is given by $\gamma \sum_{i,j} R_i^j / P$. There are M_L legal workers earning $w_L^* = [P - eS](1 - m - u) - k$, $M_I + U$ illegal workers earning $w_I^* = [P - eS](1 - m - u) - k$, $N\bar{\theta}^*$ entrepreneurs not operating a firm, $N(\hat{\theta}^* - \bar{\theta}^*)$ illegal entrepreneurs earning profits $PE(\theta | \hat{\theta}^* > \theta > \bar{\theta}^*) - w_I^*$, and there are $N(1 - \hat{\theta}^*)$ legal entrepreneurs earning profits $PE(\theta | \theta > \hat{\theta}^*) - w_L^* - t$. Again recall sanctions are non-monetary in the base model. For now, assume monetary but any sanctions paid are used up for administrative purposes.

Note that wages paid out by firms exactly equal wages received by workers. Consequently, total resources are given by the sum of the endowment of all entrepreneurs who don't start firms plus the value of output produced by entrepreneurs who started a firm minus the total taxes paid

$$TR = \sum_{i,j} R_i^j = N(1-m-u)k + NP\int_{1-m-u}^{1} \theta d\theta - N\int_{1-m-u}^{\hat{\theta}^*} e\theta S d\theta - N(1-\hat{\theta}^*)t.$$

For a given price, total resources are increasing in U. Consider each term in turn. First, an increase in U means there are fewer entrepreneurs not operating firms so total resources of this group falls by k. Second, an increase in U increases production and for a fixed P the value of production and therefore the profits of firms all go up. Third, an increase in U means there is one more marginal firm in the informal sector and faces the (non-monetary) expected sanction which is costly. The last term is not affected by an increase in U. The change in total resources (given P) is

$$-k + P(1-m-u) - eS(1-m-u) = w_L^* > 0.$$

Total supply of good X is given by:

$$X^S = N\int_{1-m-u}^{1} \theta d\theta.$$

Setting total demand to total supply yields the equilibrium price:

$$P^* = \frac{\gamma}{1-\gamma}\frac{N(1-m-u)k - N\int_{1-m-u}^{\hat{\theta}^*} e\theta S d\theta - N(1-\hat{\theta}^*)t}{N\int_{1-m-u}^{1} \theta d\theta}$$

Consider the effect of an increase in U on the price level:

$$\frac{\partial P^*}{\partial U} = \frac{\gamma}{1-\gamma}\frac{-k - eS(1-m-u) - P(1-m-u)}{N\int_{1-m-u}^{1} \theta d\theta} < 0.$$

Substituting the price back into total resources (TR), we have:

$$TR^* = \frac{1}{1-\gamma}\left(N(1-m-u)k - N\int_{1-m-u}^{\hat{\theta}^*} e\theta S d\theta - N(1-\hat{\theta}^*)t\right).$$

An increase in U decreases total resources: $\partial TR^*/\partial U = \frac{1}{1-\gamma}\left[-k - eS(1-m-u)\right] < 0$. Increasing U increases production but it also pushes down the price. It turns out that the price effect dominates and consequently, the value of total production goes down. With an endogenous price, an increase in U reduces total resources. Note also that $P^* = \gamma TR^*/\left(N\int_{1-m-u}^{1} \theta d\theta\right)$.

Total welfare (assuming that undocumented workers are weighted equally) is

$$\Omega = \Gamma \frac{TR^*}{{P^*}^\gamma} + [M+N+U]v(N(1-\hat{\theta}^*)t - ce).$$

or equivalently,

$$\Omega = \Gamma \frac{1}{\gamma^\gamma} {TR^*}^{1-\gamma} \left(N \int_{1-m-u}^{1} \theta d\theta \right)^\gamma + [M+N+U]v(N(1-\hat{\theta}^*)t - ce).$$

Effect of an increase in U on welfare is given by

$$\begin{aligned}
\frac{d\Omega}{dU} &= \Gamma \frac{1}{\gamma^\gamma} \left((1-\gamma) {TR^*}^{-\gamma} \left(N \int_{1-m-u}^{1} \theta d\theta \right)^\gamma \frac{\partial TR^*}{\partial U} + \gamma {TR^*}^{1-\gamma} \left(N \int_{1-m-u}^{1} \theta d\theta \right)^{\gamma-1} \frac{\partial X^S}{\partial U} \right) + v(G) \\
&= \Gamma \frac{1}{P^\gamma} \left((1-\gamma) \frac{\partial TR^*}{\partial U} + \gamma TR^* \left(N \int_{1-m-u}^{1} \theta d\theta \right)^{-1} \frac{\partial X^S}{\partial U} \right) + v(G) \\
&= \Gamma \frac{1}{P^\gamma} \left((1-\gamma) \frac{\partial TR^*}{\partial U} + P^* \frac{\partial X^S}{\partial U} \right) + v(G) \\
&= \Gamma \frac{1}{P^\gamma} \left(-k - eS(1-m-u) + P^*(1-m-u) \right) + v(G) \\
&= \Gamma \frac{1}{P^\gamma} w_L^* + v(G) > 0
\end{aligned}$$

The government's first-order conditions on (t, e):

$$\Gamma \frac{1}{\gamma^\gamma} \left((1-\gamma) {TR^*}^{-\gamma} \left(N \int_{1-m-u}^{1} \theta d\theta \right)^\gamma \frac{\partial TR^*}{\partial t} \right) + [M+N+U]v'(G) \left(N(1-\hat{\theta}^*) - Nt \frac{\partial \hat{\theta}^*}{\partial t} \right) = 0$$

$$\Gamma \frac{1}{\gamma^\gamma} \left((1-\gamma) {TR^*}^{-\gamma} \left(N \int_{1-m-u}^{1} \theta d\theta \right)^\gamma \frac{\partial TR^*}{\partial e} \right) + [M+N+U]v'(G) \left(-Nt \frac{\partial \hat{\theta}^*}{\partial e} - c \right) = 0$$

Using the above expressions, theses can be written as:

$$\Gamma \frac{1}{P^\gamma} \left(-N(1-\hat{\theta}^*) \right) + [M+N+U]v'(G) \left(N(1-\hat{\theta}^*) - Nt \frac{\partial \hat{\theta}^*}{\partial t} \right) = 0$$

$$\Gamma \frac{1}{P^\gamma} \left(-N \int_{1-m-u}^{\hat{\theta}^*} \theta S d\theta \right) + [M+N+U]v'(G) \left(-Nt \frac{\partial \hat{\theta}^*}{\partial e} - c \right) = 0$$

The above conditions are identical to the case with one good and fixed price P when $\gamma = \Gamma = 1$. Note, having price determined in the economy does not change the relative trade-offs between the two policies.

In a segmented equilibrium, given $\alpha = 1$, the expression for total welfare is unchanged except that $\hat{\theta}^* = 1 - m$ and does not depend on policies. Enforcement is costly so the government would want to reduce e until $t/(eS) = 1 - m$. At this point, the wages in the two sectors are the same and given by the wage expression in the non-segmented equilibrium (although policies and prices will differ). Subsequently the expressions for the impact of a marginal increase in U on total welfare and the welfare of domestic workers will be the same as above.