Incentive Equivalence with Fixed Migration Costs

S. Bucovetsky

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Abstract

If migration between communities is costless, and if policy makers in each community anticipate the migration response to policy changes, then the interests of the two communities are perfectly aligned. Decentralization is efficient. Here, the consequences of positive migration costs for this incentive equivalence are considered. In contrast with most of the literature, migration costs are assumed the same for everyone. This provides a more simple (if less realistic) model than the usual "attachment to home" assumption of heterogeneous migration costs. Conditional on the direction of migration, interests of different communities are still perfectly aligned. But natives of different communities may prefer different directions of migration, weakening incentive equivalence if local policy makers have the power to induce large changes in migration flows.

1 Introduction

Boadway (1982) showed that mobility across jurisdictions aligns the incentives of self–interested local policy makers, if these policy makers recognize the impact of their policies on migration. This observation has given rise to an extensive literature. Following Wellisch (2000), the theme of this literature will be described here as "incentive equivalence".

This theme is the relation between outcomes of two allocation mechanisms, in a nation consisting of two communities. In the first, a benevolent central planner chooses policies for both communities, so as to maximize some measure of national welfare. In the second, agents in each community pick policies for that community, seeking to maximize some parochial measure of welfare of natives of their own community. The outcomes under this second mechanism are the Nash equilibria, when the 2 communities' policy makers act simultaneously and non–cooperatively. Much of the literature shows that the two sets of outcomes are the same, if people are mobile between communities.

In Boadway (1982), the policies being chosen are the level of expenditure in each community of some publicly provided consumption good. There are no costs to migration between communities. Transfers are not made between communities.

When the publicly provided good is not perfectly rivalrous, free mobility is actually inefficient, as noted by Flatters, Henderson and Mieszkowski (1974), and Boadway and Flatters (1982). The central planner in the first allocation mechanism would like to make non–zero transfers between communities if these were possible, and if migration could not be controlled directly. Myers (1990) extends the set of government instruments, by allowing for such transfers. He shows that incentive equivalence still holds, if the two local decision makers have — between them — the same set of policy instruments as does the central planner.

Mansoorian and Myers (1993) relax the assumption of costless mobility, by assuming that migration costs vary among people. They show that a form of incentive equivalence still holds : all Nash equilibria are efficient. However, the introduction of migration costs introduces some conflict : there is a trade–off between the net income of natives of the two communities.

Much of the literature has used the Mansoorian–Myers "attachment to home" formulation of migration costs.[1] The key feature of this formulation is that the cost of moving from community 1 to community 2 is distributed over some interval on both sides of zero.

Here a slightly different imperfection to mobility is considered. All residents, regardless of their place of origin, face an identical, fixed, non–negative cost μ of moving to the other community. Hercowitz and Pines (1991) use this specification, in a dynamic, stochastic model, in which productivity shocks occur over time.

[1]For example, Wellisch (1994), Mansoorian and Myers (1997), and Caplan, Cornes and Silva (2000). The specification of the distribution of transport costs in Kanbur and Keen (1993) is formally equivalent to this formulation.

There are two issues which are examined here. The first is the potential conflict between equity and efficiency which is introduced by non–zero migration costs. This conflict has been examined elsewhere [2], but arises here in a fairly stark and simple manner. The second is the implications of non–zero mobility costs for incentive equivalence.

The basic model, presented in the following section, is quite standard, and represents something of a special case of those used in the very large literature on incentive equivalence which has developed in the last quarter–century or so. A distinguishing feature of the analysis is an emphasis on large policy deviations. A subsidiary message of this paper is that calculus is not necessary for many of the results on incentive equivalence — and that calculus is not sufficient for many of the results unless strong convexity assumptions are imposed.

2 The Model

There are two communities. The number of people born in community i is $\bar{N}_i$. The cost of migration between communities is $\mu \geq 0$; this cost is the same for all people. The population of community i will be denoted by N_i, with $N_1 + N_2 = \bar{N}_1 + \bar{N}_2$.

The fiscal variables chosen in community i will be divided. There is a vector $\mathbf{z}^i$ of "policies" : the obvious example here would be the quantity provided of a pure public good. Distinct from that is a special head tax t_i devoted to financing transfers to the other community. So if $t_1 > 0$ is chosen for community 1, then every resident of community 1 has to pay a special tax of t_1, and every resident of community 2 gets a transfer of $\frac{N_1}{N_2}t_1$. The net transfer to each resident of community i therefore is

$$g_i \equiv \frac{N_j}{N_i}t_j - t_i$$

where j indexes the other community.

So the tax t_i pays only for transfers to the other community's residents. The cost of domestic policies $\mathbf{z}^i$ is divided equally among all residents of the community.

The utility anyone gets from living in community i will be denoted

$$U^i(N_i, g_i, \mathbf{z}^{\mathbf{i}})$$

[2] It is a major concern in Mansoorian and Myers (1993)

with $\frac{\partial U^i}{\partial g_i} > 0$.

Several aspects of this specification of preferences should be noted :

- The form of the utility function may vary between communities : one community may have a more attractive climate, or distinctive features which affect the net effect of public expenditure.
- The utility function does not vary among people : immigrants and natives have the same $U^i(N_i, g_i, \mathbf{z}^i)$
- This measure U^i is gross of migration costs : a native of community 1 would get a payoff of $U^1(N_1, g_1, \mathbf{z}^1)$ if she stayed at home, and a payoff of $U^2(N_2, g_2, \mathbf{z}^2) - \mu$ if she migrated to community 2.
- The cost of public expenditure is included in the impact of each component in $\mathbf{z}^i$: so $\frac{\partial U^i}{\partial z^i_k}$ would be the benefit of increases in the quantity of the public good (if z^i_k is the quantity of the public good), minus the harm from the increased costs paid by each resident.
- There are no spillovers.

Costs of migration are measured in utility units. This assumption that preferences are separable in migration costs simplifies the analysis considerably. If migration costs were measured in dollars, and if preferences were not quasi–linear, then income effects would imply that immigrants to community i would prefer a different policy $\mathbf{z}^i$ than natives of the community.

The above specification includes as a special case a more specific model, in which : (i) a homogeneous output is produced in community i using a concave production function $F(N_i)$; (ii) residents of community i are paid a wage equal to their marginal product ; (iii) all residents of the nation share equally in the rents earned from production ; (iv) the fiscal policy is the quantity Z of a single pure public good ; (v) people have quasi–linear preferences, getting utility $x + v_i(Z)$ if they consume x units of the private good and Z units of the public good. In this special case

$$U^i(N_i, g_i, \mathbf{z}^i) \equiv F_i'(N_i) + r + v_i(Z_i) - \frac{Z_i}{N_i} \tag{1}$$

where the rent share r is defined by

$$r = \frac{F_1(N_1) + F_2(N_2) - F'(N_1)N_1 - F'(N_2)N_2}{\bar{N}_1 + \bar{N}_2}$$

In fact, the generality is quite spurious. All of the counter–examples constructed in subsequent sections will involve this special case. The special case described by equation (1) will be denoted below as the "standard example".

3 Public Policies and Migration Responses

For ease of comparison, the same set of policy instruments will be available, whether a national central planner is in charge, or whether policies are decentralized to local authorities.

In the first case, the national central planner chooses the policy vector $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$ to maximize some national welfare function, taking into account the migration response of people to the policies chosen. In the second case, the authority in community i chooses the fiscal variables $(t_i, \mathbf{z}^i)$ for her own community, taken as given the fiscal variables chosen in the other community, as well as the migration response of people to the policies chosen. As in Myers (1990), I assume that communities can give, but cannot take unilaterally : under decentralization, each t_i is constrained to be non–negative.[3]

No redistribution among residents of a given community is allowed. There are no direct impediments to mobility. These restrictions imply that all natives of a given community will get the same payoff, whether authority is centralized or decentralized. If u^i denotes the utility achieved by a native of community i, then the central planner's preferences can be represented by some welfare function $W(u^1, u^2)$. For the most part, all that will be assumed about the welfare function is that it is strictly monotonic. But for some results, it will be further assumed that the central planner has "equalizing" preferences.

The welfare function $W(u^1, u^2)$ is **equalizing** if :

1. $W(u^1, u^2)$ is a quasi–concave function.
2. $\frac{W_1(u,u)}{W_2(u,u)} = \frac{\bar{N}_1}{N_2}$ whenever utility is the same for natives of each community

These assumptions ensure an equalizing central planner would choose complete equality of utility if the utility possibility frontier were a line, $N_1 u^1 + N_2 u^2 = \bar{U}$.

[3]For symmetry, the central planner as well can choose t_i for each community, and must have $t_i \geq 0$. This non–negativity restriction is not needed for the central planner, since one of the tax instruments is redundant.

It also should be emphasized what is implied by not allowing redistribution among residents of a given community. Any transfer system based on people's place of birth is being ruled out.[4] In a first–best world, in which productivity differences between communities are large, and migration costs are fairly high, an egalitarian central planner might want to induce migration from the less–productive to the more–productive community, but then to redistribute these productivity gains by paying transfers to the less–well–off natives of the less–productive community. By resdistributing based on people's place of origin, rather than their place of residence, the planner would avoid distorting migration choices. In this paper, such redistribution based on place of origin is not allowed. This restriction raises the possibility of a conflict between equity and efficiency. As in most of the literature on incentive equivalence, the issue is whether mobility between communities eliminates this conflict.

The migration response to a pair of fiscal variables is defined in the usual manner. People are assumed numerous enough that they ignore their own impact on utility attainable in each community. Then a "migration outcome" is any pair (N_1, N_2), with $N_1 + N_2 = \bar{N}_1 + \bar{N}_2$, such that no–one wants to move. Given that migration can go in either direction, and that corner solutions may occur, 5 possible types of migration outcome can arise :

Given fiscal variables $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$, a **migration outcome** is any population pair (N_1, N_2), with $N_1 + N_2 = \bar{N}_1 + \bar{N}_2$, such that one of the following 5 conditions holds :

i. $N_1 = 0$ and $U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) < -\mu$

ii. $0 \leq N_1 \leq \bar{N}_1$ and $U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) = -\mu$

iii. $N_1 = \bar{N}_1$ and $\mu > U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) > -\mu$

iv. $\bar{N}_1 \leq N_1 \leq \bar{N}_1 + \bar{N}_2$ and $U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) = \mu$

v. $N_1 = \bar{N}_1 + \bar{N}_2$ and $U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) > \mu$

For any set of fiscal variables $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$, some migration outcome must exist, provided only that the U^i's are continuous in the population levels. But the problem is that there may be multiple migration outcomes. This problem

[4] except when there is no migration

was outlined clearly by Stiglitz (1977). In the standard example described in the previous section, population increases in a community have two effects, one "stabilizing" and one destabilizing. With a concave production function, migration lowers the wage in a community, making the community less attractive. But migration lowers the per capita cost of the public sector, making the community more attractive. If the latter effects, the consequences of non–rivalry in consumption of the public good, are strong, then $U^1 - U^2$ may increase as N_1 increases and N_2 decreases, leading to the possibility that there are multiple migration outcomes corresponding to a given set of fiscal variables.

With a single central planner, the multiplicity of migration outcomes is usually not taken as a great difficulty. As is standard, it is assumed here that the central planner can somehow pick the migration outcome she wants, if there are multiple migration outcomes for the fiscal variables chosen. So the set of utility pairs (u^1, u^2) which can be achieved by a central planner are those pairs corresponding to some vector $(N_1, N_2, t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$ such that

a. $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$ is feasible

b. (N_1, N_2) is a migration outcome for $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$

c. $u^i = \max(U^i(N_i, g_i, \mathbf{z}^i), U^j(N_j, g_j, \mathbf{z}^j) - \mu) \quad j \neq i$, where $g_1 = \frac{N_2}{N_1}t_2 - t_1$ and $g_2 = \frac{N_1}{N_2}t_1 - t_2$

Under decentralization, the selection of a migration outcome may be more problematic. If authorities in some community are somehow allowed to pick and choose among migration outcomes, then deviation from a given outcome becomes more attractive. But is it sensible for a community to expect to attract population by making the community less attractive to immigrants? If this sort of selection appears unreasonable, then the following selection rule may be a useful restriction on the consequences of changes in fiscal variables. Here a unilateral change of fiscal variables in one community can affect the utility from living in the other community, since an increase in t_1 must increase $g_2 = \frac{N_1}{N_2}t_1 - t_2$.

Selection Rule : Suppose that (N_1, N_2) is a migration outcome for fiscal variables $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$, and community 1 is considering a unilateral deviation to some set of other, less attractive fiscal variables $(t_1', \mathbf{z}^{1'})$, for which $U^1(N_1, g_1', \mathbf{z}^{1'}) - U^2(N_2, g_2', \mathbf{z}^{1'}, \mathbf{z}^2) < U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^1, \mathbf{z}^2)$

(where $g_1' \equiv \frac{N_2}{N_1}t_2 - t_1'$ and $g_2' \equiv \frac{N_1}{N_2}t_1' - t_2$). Then the new migration outcome (N_1', N_2') corresponding to $(t_1', t_2, \mathbf{z}^{1'}, \mathbf{z}^2)$ obeys the selection rule if $N_1' \leq N_1$. An analogous rule applies for unilateral changes in community 2's fiscal variables.

In other words, the selection rule says that authorities expect to lose population if they make their policies less attractive.

For any unilateral policy deviation, there always must be some migration outcome satisfying the selection rule, so that this rule is a fairly convenient way of making more precise the consequence of policy changes in a decentralized environment.

Lemma 1 *If* (N_1, N_2) *is a migration outcome for fiscal variables* $(t_1, t_2, \mathbf{z}^1, \mathbf{z}^2)$, *then for any feasible change in fiscal variables by community* i, *which lowers* $U^i - U^j$, *there exists a new migration outcome* (N_1', N_2') *satisfying the selection rule. If* $N_1 \neq \bar{N}_1$ *initially, and* $0 < N_1 < \bar{N}_1 + \bar{N}_2$ *[that is, if there is some migration, and if both communities still are populated], then any new migration outcome satisfying the selection rule does so with strict inequality. [That is, if* $N_1 \neq \bar{N}_1$ *initially, for any change in community 1's fiscal variables which lowers* $U^1 - U^2$, *holding population constant, there is a new migration outcome for which* $N_1' < N_1$.*]*

Proof Without loss of generality, only changes in fiscal variables by community 1 will be considered here. Five cases are possible for the initial migration outcome.

In each case, the proof consists of considering the effects of changes in fiscal variables on the function defining $U^1 - U^2$ as a function of N_1. This function is continuous. By construction, the change in fiscal variables must lower this curve, at the initial migration outcome N_1.

case 1 , $N_1 = 0$ *:* Initially, before the change in fiscal variables, $U^1 - U^2 < -\mu$, from the definition of migration outcome. The change in fiscal variables in community 1 lowers $U^1 - U^2$ at $N_1 = 0$, so that $N_1 = 0, N_2 = \bar{N}_1 + \bar{N}_2$ is still a migration outcome after the change in fiscal variables.

case 2, $0 < N_1 < \bar{N}_1$ *:* Initially, before the change in fiscal variables, $U^1 - U^2 = -\mu$, from the definition of migration outcome. The change in fiscal variables lowers $U^1 - U^2$ below $-\mu$ at the initial population level N_1. Continuity then implies that either $U^1 - U^2 < -\mu$ for all $0 \leq N_1' \leq N_1$, in which case there is a migration outcome at $N_1 = 0$, or that there is some N_1'

in $(0, N_1)$ for which $U^1 - U^2 = -\mu$; such an $N_1' < N_1$ would be a migration outcome for the new fiscal variables.

case 3, $N_1 = \bar{N}_1$ *:* Initially, before the change in fiscal variables, $\mu \geq U^1 - U^2 \geq -\mu$, from the definition of migration outcome. If the change in fiscal variables lowers the $U^1 - U^2$ curve by a small amount at $N_1 = \bar{N}_1$, so that $U^1 - U^2$ still is greater than or equal to $-\mu$, then $N_1 = \bar{N}_1$ still is a migration outcome. On the other hand, if $U^1 - U^2 < -\mu$ at $N_1 = \bar{N}_1$ after the change, then the analysis of case 2 above applies.

case 4, $\bar{N}_1 + \bar{N}_2 > N_1 > \bar{N}_1$ *:* Initially, before the change in fiscal variables, $U^1 - U^2 = \mu$, from the definition of migration outcome. After the change, $U^1 - U^2 < \mu$ at the original N_1. If there is some N_1' in $[\bar{N}_1, N_1)$ for which $U^1 - U^2 = \mu$, then there is a new migration outcome, with $\bar{N}_1 \leq N_1' < N_1$. If not, then $U^1 - U^2 < \mu$ at $N_1 = \bar{N}_1$ after the change, and the analysis of case 3 above applies.

case 5, $N_1 = \bar{N}_1 + \bar{N}_2$*:* Since N_1 is as large as it can be initially, no change in fiscal variables can increase N_1. If $U^1 - U^2 \geq \mu$ at $N_1 = \bar{N}_1 + \bar{N}_2$ after the change in fiscal variables, then the new migration outcome is still $N_1 = \bar{N}_1 + \bar{N}_2$. Otherwise, the analysis of case 4 applies, and $N_1' < N_1 = \bar{N}_1 + \bar{N}_2$ after the change.

In cases 2 and 4, the new migration outcome must be a strictly smaller population for community 1, so that the second part of the lemma has also been proved. •

For a stable migration outcome, in which $U^1 - U^2$ is a decreasing function of N_1, this rule is consistent with using the implicit function theorem to calculate the migration response to a policy change. (But if the initial migration outcome were unstable, then using the selection rule means expecting big changes in migration in response to arbitrarily small policy changes.) Of course, this selection rule does not guarantee a unique migration outcome : there might be several different migration outcomes for which $N_1' < N_1$.

It also should be noted that the stability of migration response here is sensitive to the specification of the authorities' fiscal variables. Here the tax rate t_1 which finances any transfers to community 2 has been taken as the strategic variable. That means that the actual transfer paid to each resident of community 2, $\frac{N_1}{N_2} t_1$, will vary with the population distribution. An alternative formulation would be to assume that the transfer was held constant, and that the tax varied to satisfy the government budget constraint. This latter formulation would lead to less stability, which is why it was not chosen. Bucovetsky (2003) provides more detail on the sensitivity of migration responses to the specification of fiscal variables.

4 Zero Migration Costs

Nothing new is presented in this section. It is included to emphasize that instability, non–convexity and selection rules really do not complicate matters much if migration is costless.

Lemma 2 *When $\mu = 0$, the central planner's utility possibility frontier consists of a single point.*

Proof Perfect mobility implies $u^1 = u^2$ in any migration outcome. Any policy which increases u^1 must also increase u^2. •

Proposition 1 *When $\mu = 0$, any optimum to the central planner's problem can be sustained as a Nash equilibrium under decentralization.*

Proof Suppose the fiscal variables $(t_1^*, t_2^*, \mathbf{z}^{1*}, \mathbf{z}^{2*})$, and the resulting migration outcome (N_1^*, N_2^*), maximize $W(u^1, u^2)$. Under decentralization, would authorities in community 1 have any incentive to deviate from $(t_1^*, \mathbf{z}^*)$ if community 2's fiscal variables were $(t_2^*, \mathbf{z}^*)$? They would want to change their fiscal variables unilaterally only if this change yielded a higher value of u^1, for some migration outcome to the new policies. But $u^1 = u^2$ in any migration outcome, so any change which raises u^1 must raise u^2, which would contradict the optimality of the original fiscal variables. Analogously, community 2's authorities have no incentive to deviate unilaterally. •

Without imposing further strong restrictions on the utility functions, it will not be true that every Nash equilibrium under decentralization is efficient. But the possibility of inefficient Nash equilibria is no more surprising nor significant than the fact that both (t, L) and (b, R) are Nash equilibria to the coordination game below, even though both players are better off at (t, L).

$$\begin{pmatrix} 1\backslash 2 & L & R \\ & & \\ t & (5,5) & (0,0) \\ b & (0,0) & (2,2) \end{pmatrix}$$

In situations depicted by pure coordination games, any pre–play communication will solve the problems of players picking the wrong equilibrium ; presumably policy makers in different communities can communicate, and cheap talk here would enable them to coordinate on the best outcome.

As will be shown below (for the case in which $\mu > 0$), at any Nash equilibrium the fiscal variables chosen must satisfy the first–order conditions for optimality. So the possibility of multiple equilibria, which can be Pareto–ranked (as in the game depicted above) arises only if the central planner's problem has multiple local extrema. But the discussion below also indicates that if there are multiple local extrema, the possibility of a Nash equilibrium which is not a global optimum cannot be ruled out.

5 Positive Migration Costs with No Public Good

Suppose that the only influence of population on utility was through the productivity of workers. That is, take the standard example defined by equation (1), and set $v_1(G) = v_2(G) = 0$ for all G. To further simplify, assume that the initial population each community is the same. The production function $F_i(N)$ in each community is assumed increasing and strictly concave. For specificity, and to give a possible reason for migration, assume that community 1 has the more productive technology

$$F_1'(\bar{N}) > F_2'(\bar{N})$$

where $\bar{N}$ is the identical initial population of each community.

Under these simplifications, there is a natural measure of net national income, as a function of the distribution of population,

$$B(N_1) \equiv F_1(N_1) + F_2(2\bar{N} - N_1) - \mu(N_1 - \bar{N}) \tag{2}$$

Expression (2) is only valid when $N_1 \geq \bar{N}$, but population distributions in which there is reverse migration ($N_1 < \bar{N}$) will never be Pareto optimal under the assumption that $F_1'(\bar{N}) > F_2'(\bar{N})$.

The function $B(N_1)$ is concave ; it is still concave when extended to the "inefficient" population distributions by setting $B(N_1) = F_1(N_1) + F_2(2\bar{N} - N_1) + \mu(N_1 - \bar{N})$ for $N_1 < \bar{N}$. The derivative $B'(N)$ falls discontinuously at $N_1 = \bar{N}$, and $B(N_1)$ has a unique local (and global) maximum.

The population level N_1^* which maximizes $B(N_1)$ will be $N_1 = \bar{N}$ if and only if $B'_+(N_1) \leq 0$ at $N_1 = \bar{N}$. Otherwise, N_1^* is the unique population level for community 1 for which

$$F'_1(N_1^*) - F'_2(2\bar{N} - N_1^*) - \mu = 0 \tag{3}$$

Governments do not have much to do here. There is no public sector, and the only fiscal variables are transfers between communities. If there were no transfers, then migration would be efficient : the unique migration outcome when $g_1 = g_2 = 0$ is the population distribution which maximizes $B(N_1)$.

But when lump–sum transfers (based on place of origin) are impossible, the Pareto optimum may not be unique. If $N_1^* > \bar{N}$, then the utility combination

$$(u^{1*}, u^{2*}) \equiv (\frac{F_1(N_1^*) + F_2(2\bar{N} - N_1^*)}{2\bar{N}} + \frac{\mu}{2}, \frac{F_1(N_1^*) + F_2(2\bar{N} - N_1^*)}{2\bar{N}} - \frac{\mu}{2})$$

must be on the utility possibility frontier. This is the outcome under complete laissez–faire.

But any utility combination (u^1, u^2) with $(u^1 + u^2)\bar{N} = F_1(\bar{N}) + F_2(\bar{N})$ and $\mu \geq u^1 - u^2 \geq -\mu$ is also feasible for a central planner : all that is needed is a transfer from community 1 to community 2 which is large enough to eliminate the incentive to migrate, and not so large as to induce reverse migration. The best of these combinations for the natives of community 2 is the utility combination

$$(\tilde{u}^1, \tilde{u}^2) \equiv (\frac{F_1(\bar{N}) + F_2(\bar{N})}{2\bar{N}} - \frac{\mu}{2}, \frac{F_1(\bar{N}) + F_2(\bar{N})}{2\bar{N}} + \frac{\mu}{2})$$

If this latter combination is better for natives of community 2, if $\tilde{u}^2 > u^{2*}$, then the utility possibility frontier has two parts : the downward–sloping line (with slope -1) connecting $(\tilde{u}^1, \tilde{u}^2)$ with $(\frac{F_1(\bar{N})+F_2(\bar{N})}{2\bar{N}} - u^{2*}, u^{2*})$ and the isolated point (u^{1*}, u^{2*}).

Given the definition of u^{2*} and $\tilde{u}^2$, the condition that $\tilde{u}^2 > u^{2*}$ is equivalent to the gain in total output from moving to the optimal population distribution $(F_1(N_1^*) + F_2(2\bar{N} - N_1^*) - F_1(\bar{N}) - F_2(\bar{N}))$ be less than $2\mu(\bar{N})$.

The shape of the utility possibility frontier means that a central planner may not prefer the population distribution N_1^*. Certainly a Benthamite planner, with welfare function $W(u^1, u^2) = \bar{N}(u^1 + u^2)$ will always want to induce the outcome which maximizes $B(N_1)$. But if the social welfare function is quasi–concave, then the planner may prefer an outcome with no migration, even though it involves lower net national income. Without lump–sum taxation, the only way to transfer income to natives of the poorer community 2 is to make residence–based payments. And these payments will alter the distribution of income only if they are large enough to prevent any migration.

If the planner's welfare function is equalizing, then there are at most two points the planner might choose on the utility possibility frontier. One is the Benthamite optimum (u^{1*}, u^{2*}). The other is the point on the utility possibility frontier at which

$$u^1 = u^2 = \frac{F_1(\bar{N}) + F_2(\bar{N})}{2\bar{N}}$$

This latter point will be on the utility possibility frontier only if productivity differences are relatively small, or migration costs are relatively high. But if $\frac{F_1(\bar{N})+F_2(\bar{N})}{2\bar{N}} > u^{2*}$, then the planner will prefer either big transfers, or none at all. For example, suppose the planner had an iso–elastic welfare measure

$$W(u^1, u^2) = \frac{1}{1-\omega}[(u^1)^{1-\omega} + (u^2)^{1-\omega}] \quad \omega \geq 0$$

Then there is some threshold value for the inequality aversion parameter ω which determines the planner's policy. For lower values of ω, the planner makes no transfers between communities, and the migration outcome is the population level N_1^* which maximizes net national income, and which leads to community 1's natives being better off. For higher values of ω, the optimal policy is a transfer from community 1 to community 2, which equalizes income and which cuts off all migration.

Put otherwise : starting from laissez–faire, a small increase in transfers to community 2 actually makes everyone worse off, as the migration outcome moves to some inefficient N_1 in $(\bar{N}, N_1^*)$. Only when the transfer is large enough to cut off all migration do increases in the transfer begin to benefit natives of community 2.

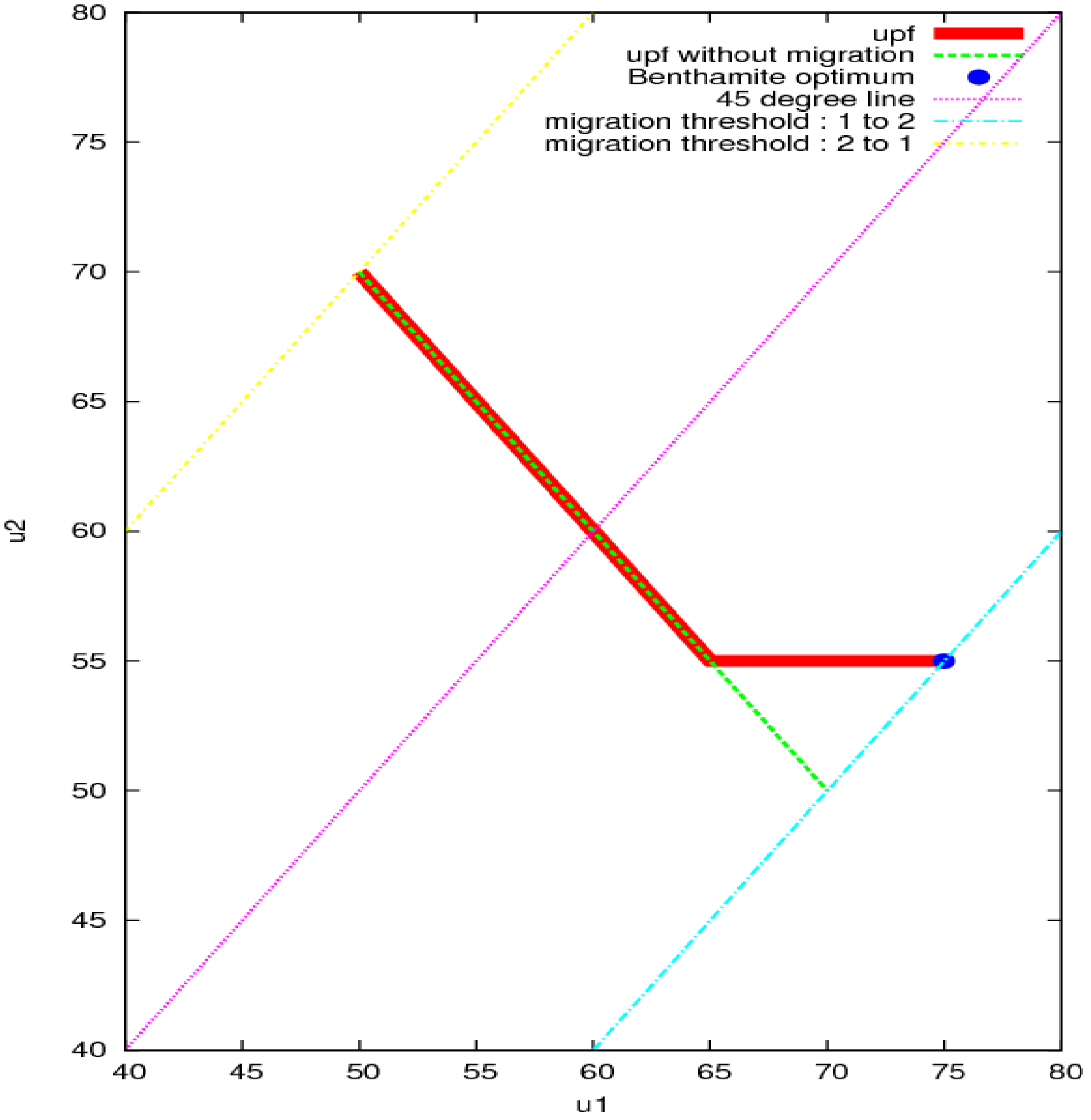

Figure 1 : the central planner's utility possibility frontier with costly migration

Figure 1 illustrates the utility possibility frontier, for a case in which migration costs are low enough that the Benthamite optimum requires migration, but the costs are high enough that the Benthamite optimum is not the only Pareto optimum.[5]

Consider the total level of welfare $W(u^1, u^2)$ as a function of the central planner's only policy instrument, the transfer between communities. Even when $W(u^1, u^2)$ is concave, and even though net national output is a concave function of the transfer, welfare is not a concave function of the transfer. As long as the downward–sloping portion of the utility possibility frontier crosses the 45–degree line (as in Figure 1), there must be two local maxima for welfare, as a function of the transfer, if the welfare measure is equalizing. The planner's iso–welfare curve must be tangent to the upf where it crosses the 45–degree line. This outcome can be achieved by net payments g_i to residents of community i, where

$$g_2 = \frac{F_1'(\bar{N}) - F_2'(\bar{N})}{2} = -g_1$$

Since $u^1 = u^2$ whenever the transfer to community 2 is too small to prevent emigration, welfare attains a local maximum as a function of the transfer at $g_1 = g_2 = 0$, whenever $F_1'(\bar{N}) - F_2'(\bar{N}) > \mu$.

Figure 2 illustrates welfare as a function of the transfer g_1 paid to residents of community 1, for the technology underlying figure 1, when the central planner's welfare measure is

$$W(u^1, u^2) = \bar{N}_1\sqrt{u^1} + \bar{N}_2\sqrt{u^2}$$

Although the presence of positive migration costs does introduce some heterogeneity among people, and thus may lead to more than one Pareto optimal outcome, incentive equivalence still holds.

Proposition 2 *In the standard example without public goods, there is a unique pair of Nash equilibrium payoffs* [6] *when communities each choose their t_i's (the head tax which funds transfers to the other region) non–cooperatively, and these payoffs are Pareto optimal.*

[5] The upf in figure 1 would arise if $N_1 = N_2 = 100$, $\mu = 20$, $F_1(N) \equiv 7925\sqrt{N} - 250N - 53000$ and $F_2(N) \equiv 7925\sqrt{N} - 310N - 47000$. In this case net output per capita is 120 if there is no migration, and is maximized at 130 when $N_1 = 110$.

[6] which may actually be achieved by many different pairs of equilibrium strategies ; the outcome is unique, not the Nash equilibrium

Proof It suffices to show that $t_1 = t_2 = 0$ is a Nash equilibrium, and that all other Nash equilibria achieve the same payoffs, since complete laissez–faire will always maximize the sum of utilities $B(N_1)$ in the standard example when there is free migration, and when there are no public goods.

case 1 : $B'_+(\bar{N}_1) > 0$ In this case, the Benthamite optimum involves migration. Any deviation from $t_1 = 0, t_2 = 0$ must lower u^1, so community 1 would not want to change its strategic variable from $t_1 = 0$. Any increase in t_2 above 0 must lead to more migration from community 2 to community 1, lowering both u^1 and u^2, so that community 2 would not want to deviate from $t_2 = 0$, since it is constrained to choose non–negative tax rates. So $(0, 0)$ is a Nash equilibrium.

Consider any other pair of strategic variables (t_1, t_2), for which the migration outcome is not the N_1^* which maximizes $B(N_1)$. If the resulting N_1 is greater than N_1, then it must be the case that $t_2 > 0$. Then community 2 would want to lower t_2 slightly, thereby lowering N_1, and increasing both u^1 and u^2. If the resulting N_1 is less than N_1^*, then it must be the case that $t_1 > 0$, so that community 1 would gain by lowering t_1 slightly, thereby raising N_1 and increasing both u^1 and u^2.

case 2 : $B'_+(\bar{N}_1) \leq 0$ In this case, the Benthamite optimum is no migration at all. For either community, an increase in t_i must lower u^i if it does not induce migration ; and this increase must lower it even more if it does induce migration, since migration lowers $B(N_1)$.

Any choice of taxes which leads to migration (to community 1) must have $t_2 > t_1 \geq 0$; community 2 would want to deviate by lowering t_2, thereby reducing migration and increasing u^2. If (t_1, t_2) led to reverse migration, then it would have to be true that $t_1 > t_2 \geq 0$, so that community 1 would want to deviate.

And if $(t_1, t_2) \neq (0, 0)$ and there were no migration, then the community making a positive transfer would want to reduce it. •

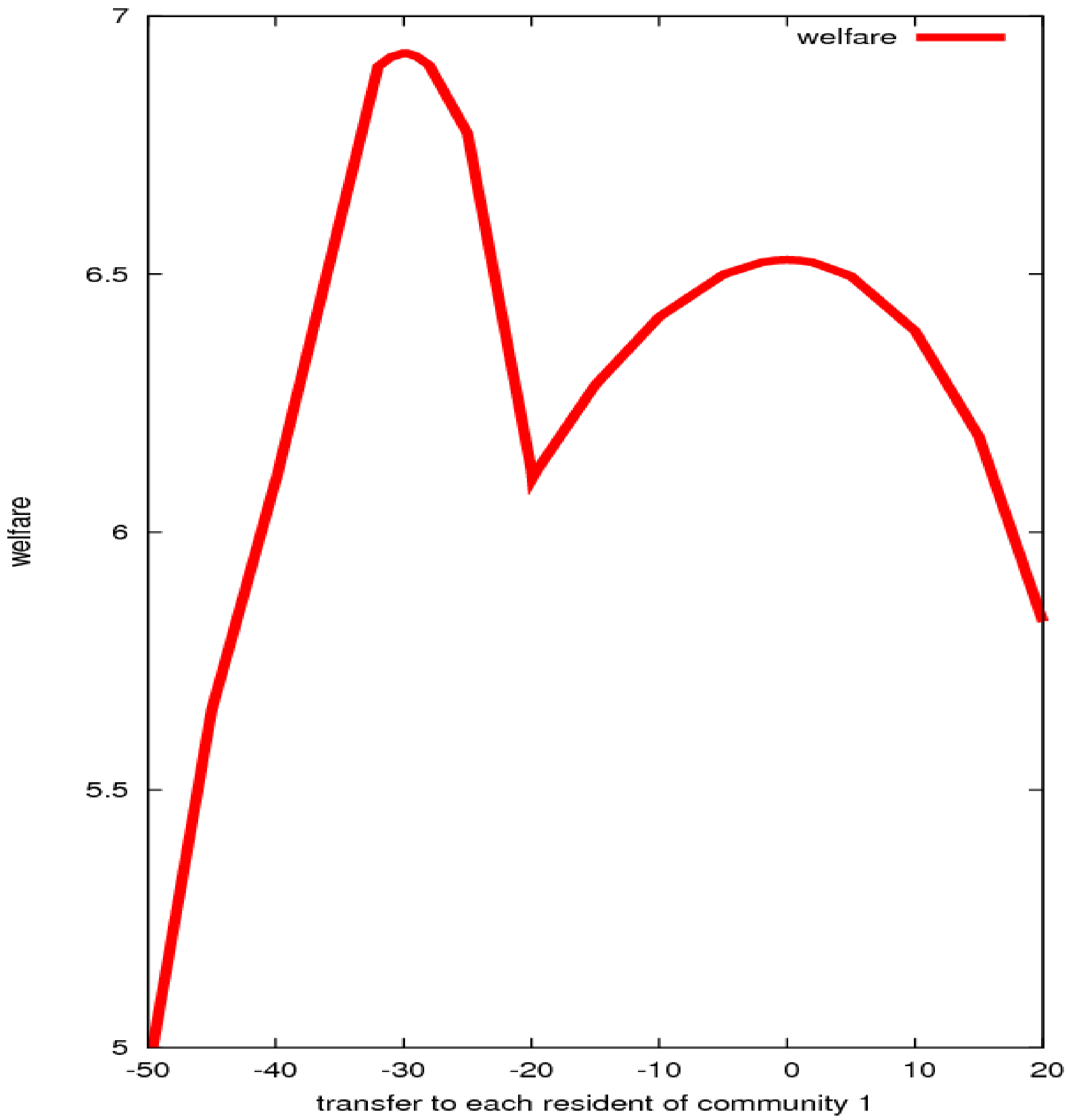

Figure 2 : social welfare as a function of the inter-jurisdictional transfer

6 Incentive Equivalence in the General Case

In the fairly special case of section 5, despite the possible multiplicity of Pareto optima, a fairly strong form of incentive equivalence must hold in both directions. A particular Pareto optimum – the Benthamite optimum — must be achievable as a Nash equilibrium. And any Nash equilibrium must be Pareto optimal.

Without the convexity of the $B(N_1)$ function, neither direction of incentive equivalence need hold.

The transfers, which may be necessary to attain the Benthamite optimum when there are public goods, are a cause of the problems with the first direction of incentive equivalence. If the transfers go in one direction, the Benthamite optimum can still be decentralized, as the following proposition indicates. If they go in the other direction, then this may not be possible, as the examples in section 8 indicate.

Proposition 3 *If the Benthamite optimum involves some migration, and if the Benthamite optimum requires non–negative transfers from the destination community for the migration, to the origin community, then this optimum can be achieved as a Nash equilibrium if communities choose their fiscal variables non–cooperatively, and if each community's decision makers use the selection rule to forecast the impact of any changes in fiscal variables.*

Proof Suppose that the Benthamite optimum requires migration from community 2 to community 1. The hypothesis of the Proposition then requires that $g_1 \leq 0 \leq g_2$. Let $(g_1^*, g_2^*, \mathbf{z}^{1*}, \mathbf{z}^{2*})$ be the policies which implement this optimum for the central planner, and (N_1^*, N_2^*) the resulting population distribution.[7] This optimum can be decentralized if each community chooses fiscal variables $(t_i^*, \mathbf{z}^{i*})$, where

$$t_1^* = -g_1^* \quad ; \quad t_2^* = 0$$

Does either community have an incentive to change its fiscal variables? Since the initial situation is the Benthamite optimum, and since migration equilibrium requires $U^1 = U^2 + \mu$ in this initial situation, the utility of natives of community 1 is maximized (over all feasible outcomes) in the initial situation. Policy makers in community 1 have no incentive to deviate.

[7] the fiscal variables may not be unique ; the population distribution may not be unique

Possible deviations by community 2 consist of some combination of an increase in t_2 above 0, and changing some of the policies $\mathbf{z}^2$ away from their optimal level. But any increase in t_2 above 0, or any change in $\mathbf{z}^2$ away from $\mathbf{z}^{2*}$ must lower $U^2(N_2^*, g_2, \mathbf{z}^2) - U^1(N_1^*, g_1, \mathbf{z}^{1*})$. The selection rule then implies that any feasible unilateral change in community 2's fiscal variables must lead to more emigration to community 1 in the new migration outcome. The new migration outcome, after any changes in community 2's fiscal variables, therefore must have $U^1 - U^2 = \mu$ (or $U^1 - U^2 > \mu$ if the change leads to the complete depopulation of community 2). The fact that the initial situation maximizes U^1 therefore implies that no feasible unilateral change in fiscal variables by community 2 can increase U^2. The policies $(t_1^*, \mathbf{z}^{1*}), (t_2^*, \mathbf{z}^{2*})$ must be a Nash equilibrium.

If the Benthamite optimum involved migration in the reverse direction, then an analogous argument applies : the optimum is a global maximum for U^2, and any feasible change in fiscal variables by community 1 must lead to more emigration to community 2. •

The reverse direction of incentive equivalence — that any Nash equilibrium is Pareto optimal — can be demonstrated easily only when the Benthamite measure of net national income $B(N)$ is is quasi–concave. Any "interior" Nash equilibrium will satisfy a central planner's first–order conditions for optimality.

Proposition 4 *If the strategic variables $(t_1^N, \mathbf{z}^{1N}), (t_2^N, \mathbf{z}^{2N})$ are a Nash equilibrium, with migration outcome (N_1^N, N_2^N) such that $0 < N_1^N < \bar{N}_1$, or $\bar{N}_1 < N_1 < \bar{N}_1 + \bar{N}_2$, then the corresponding central planner's policies $(g_1^N, g_2^N, \mathbf{z}^{1N}, \mathbf{z}^{2N})$, with*

$$g_1^N = \frac{N_2}{N_1} - t_1^N \quad ; \quad g_2^N = \frac{N_1}{N_2} t_1 - t_2^N$$

satisfy the first–order conditions for the maximization of

$$\bar{N}_1 U^1(N_1, g_1, \mathbf{z}^1) + \bar{N}_2 U^2(N_2, -\frac{N_1}{N_2} g_1, \mathbf{z}^2) - \mu |N_1 - \bar{N}_1|$$

with respect to $N_1, N_2, g_1, \mathbf{z}^1$ and $\mathbf{z}^2$ subject to the constraints $N_1 + N_2 = \bar{N}_1 + \bar{N}_2$ and

$$U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) = \begin{cases} -\mu & \text{if } N_1 < \bar{N}_1, \\ \mu & \text{if } N_1 > \bar{N}_1. \end{cases}$$

Proof Without loss of generality, suppose that the Nash equilibrium led to an outcome where migration flowed from community 2 to community 1.

Suppose that the central planner's first–order conditions for optimality were **not** satisfied at the Nash equilibrium. Then a small change in one of the planner's choice variables would increase the planner's payoff. Since small changes in the choice variables do not change the direction of migration, therefore a small change in one of the planner's variables would lead to an increase in both u^1 and in u^2, since $u^1 = u^2 + \mu$ whenever migration occurs from community 2 to community 1.

The planner can also be viewed as choosing $(g_1, \mathbf{z}^1, \mathbf{z}^2)$ to maximize the sum of utilities, taking into account the migration response to these variables (and the budget constraint $N_1 g_1 + N_2 g_2 = 0$). So if the planner's first–order conditions for optimality were not satisfied at the Nash equilibrium, then both u^1 and u^2 could be increased by some small change in g_1, or in one of the components of $\mathbf{z}^1$ or $\mathbf{z}^2$.

Under decentralization, decision makers in community i control $\mathbf{z}^i$. So if a small change in $\mathbf{z}^i$ from $\mathbf{z}^{iN}$ could increase u^i (and u^j), then the original choice of strategic variables could not be a Nash equilibrium.

Community 2's decision makers can increase g_1 unilaterally (and decrease $g_2 = -\frac{N_1}{N_2} g_1$) by raising t_1. Community 1's decision makers can decrease g_1 unilaterally. So if a small change – in either direction — in g_1 were to increase u^1 and u^2, then the original situation cannot be a Nash equilibrium, since either community 1 will want to lower g_1 or community 2 will want to raise g_1.

Therefore, if the central planner's first–order conditions for optimality were **not** satisfied at a Nash equilibrium in which $N_1 > \bar{N}_1$, then decision makers in one of the two communities would want to change one of their strategic variables : the original choice of fiscal variables cannot be a Nash equilibrium.

An analogous argument applies if the migration went in the opposite direction. Not satisfying the planner's first–order conditions means not being a Nash equilibrium, so that the contrapositive must hold : if $(t_1^N, \mathbf{z}^{1N}), (t_2^N, \mathbf{z}^{2N})$ are Nash equilibrium choices (and lead to migration) then the first–order condition for the central planner's optimum must hold. •

Since utilities in the two communities move together whenever there is migration, the form of the central planner's maximand did not matter in the above proof. The result can be strengthened to

Corollary : If the strategic variables $(t_1^N, \mathbf{z}^{1N}), (t_2^N, \mathbf{z}^{2N})$ are a Nash equilibrium, with migration outcome (N_1^N, N_2^N) such that $0 < N_1^N < \bar{N}_1$, or $\bar{N}_1 < N_1 < \bar{N}_1 + \bar{N}_2$, then the corresponding central planner's policies $(g_1^N, g_2^N, \mathbf{z}^{1N}, \mathbf{z}^{2N})$, with

$$g_1^N = \frac{N_2}{N_1} - t_1^N \quad ; \quad g_2^N = \frac{N_1}{N_2} t_1 - t_2^N$$

satisfy the first–order conditions for the maximization of $W(u^1, u^2)$ with respect to $N_1, N_2, g_1, \mathbf{z}^1$ and $\mathbf{z}^2$ subject to the constraints $N_1 + N_2 = \bar{N}_1 + \bar{N}_2$ and

$$U^1(N_1, g_1, \mathbf{z}^1) - U^2(N_2, g_2, \mathbf{z}^2) = \begin{cases} -\mu & \text{if } N_1 < \bar{N}_1, \\ \mu & \text{if } N_1 > \bar{N}_1. \end{cases}$$

where $W(u^1, u^2)$ is any strictly monotonic welfare function, and where u^i is the maximum of U^i and $U^j - \mu$.

Even with zero migration costs, there may be inefficient Nash equilibria. The presence of pure public goods (or anything else that leads to non–convexities) means that $B(N_1)$ may have multiple local extrema in the interval $(\bar{N}_1, \bar{N}_1 + \bar{N}_2)$. All of these local extrema involve positive migration from community 2 to community 1. So $u^1 = u^2 + \mu$ for each of them, and the local extrema can be Pareto ranked. But it still may be the case that some local optimum which is not a global optimum may be sustained as a Nash equilibrium. It is even possible that a local minimum for $B(N_1)$ might be sustained as a Nash equilibrium. Figure 3 illustrates. In the figure, the solid line represents what a central planner can achieve. The dotted lines represent what a single community planner can achieve unilaterally, starting from policies which achieve sub–optimal outcome. The solid line is the upper envelope of the dotted lines.

The problem is that moving from a sub–optimal local optimum of $B(N_1)$ to a global optimum involves changing policies in both communities. If the move involves increasing N_1, it typically will require an increase in Z_1 and a decrease in Z_2, if Z_i is the quantity of a pure public good provided in community i. Given the perfect mobility, community i cannot increase utility as much as the central planner, since it conjectures that community j is not cooperating.

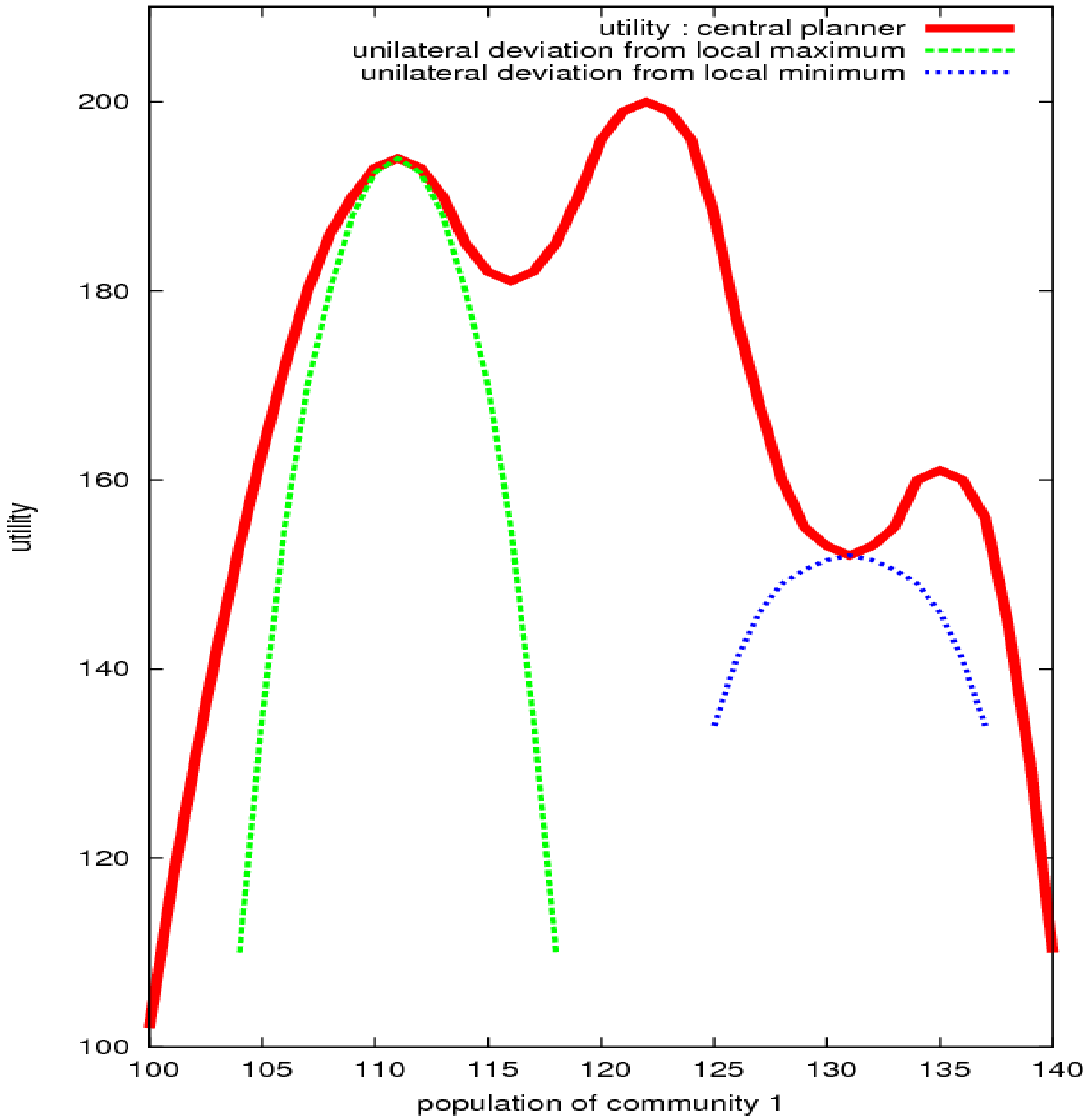

Figure 3 : inefficient Nash equilibria

Positive migration costs exacerbate the problem of multiple equilibria in two ways. First, they may make it more likely that the planner's optimum has multiple extrema. When $\mu > 0$, $B'(N)$ falls discontinuously at $N_1 = \bar{N}_1$, increasing the likelihood that there is local maximum with no migration. Figure 4 illustrates this sort of problem. Unlike figure 3, the values for the payoffs here are computed from an explicit example of decreasing–return technology and public goods provision. In the figure, the sum of payoffs has a local maximum at $N_1 = \bar{N}_1$, but also at a positive level of N_1.

This figure shows a standard example, with utility a quasi–linear function of private consumption and public good consumption. Each resident of each community is paid the value of her marginal product, and the remaining rents are shared equally by all residents of all communities. In this case, $\bar{N}_1 = \bar{N}_2 = 100$, $\mu = 5.4$, the production functions are

$$F_i(N_i) = a_i N_i^{0.5} \quad ; \quad a_1 = 200 \quad ; \quad a_2 = 100$$

the unit cost of the public good is 1, and the valuation function (in each community) for the public good is

$$v(Z) = A + \frac{\alpha}{1-\beta} Z^{1-\beta}$$

For $Z \leq 358.964$,

$$A = 0 \quad ; \quad \alpha = 0.011 \quad ; \quad \beta = 0.05$$

but for $Z > 358.964$,

$$A = 6.03952 \quad ; \quad \alpha = 1056.19 \quad ; \quad \beta = 2$$

so that both $v(Z)$ and $v'(Z)$ are continuous at the break point.

In this example, natives of country 1 certainly do better at the Benthamite global optimum (at $N_1 \approx 138$). But they cannot unilaterally improve their payoff from a Nash equilibrium with no migration : the outcome with mobility is only more attractive if community 2 lowers its public good provision (and pays a transfer to residents of community 1), and community 1 cannot achieve this unilaterally.

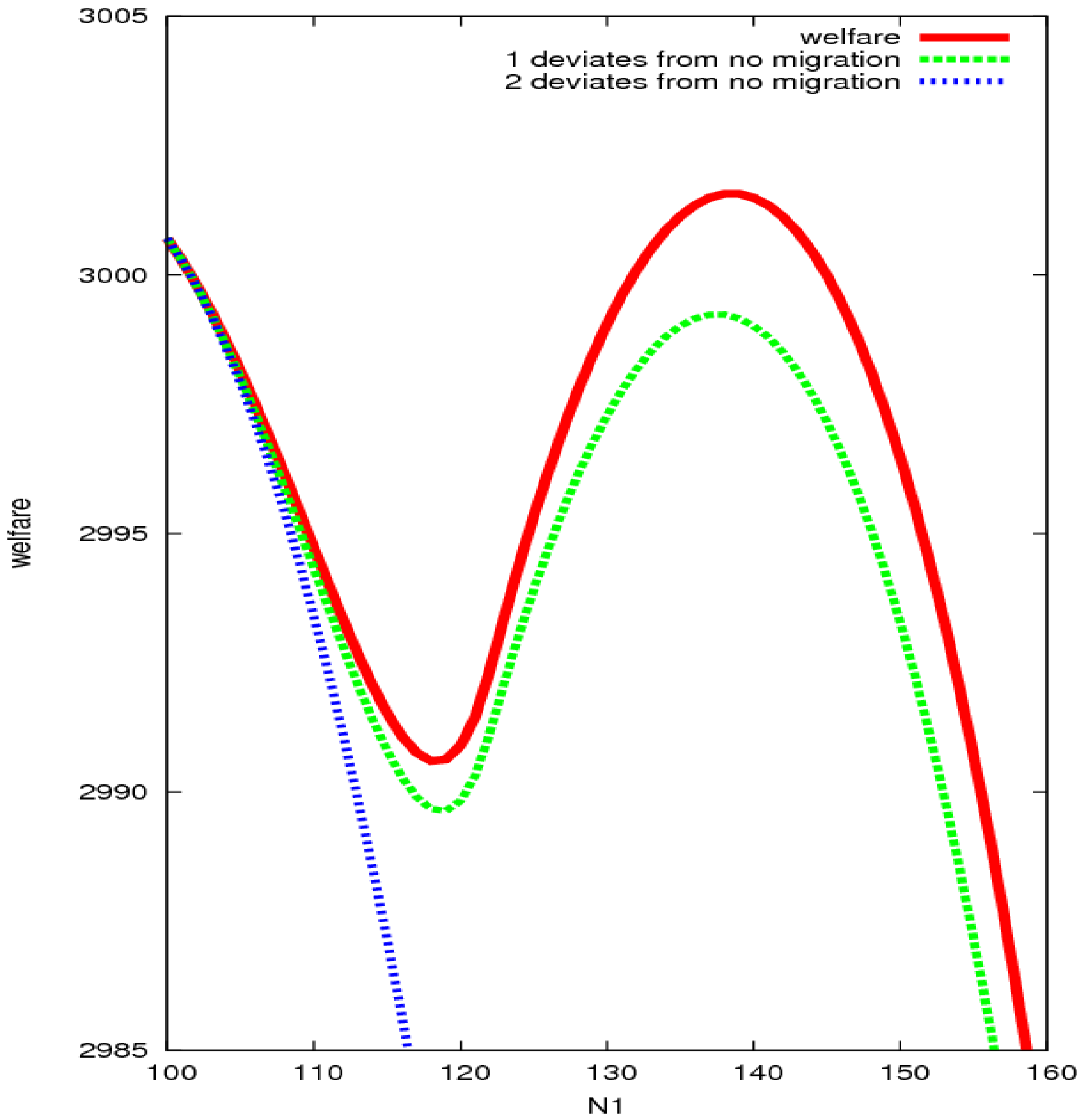

Figure 4 : A “low level no migration trap”

The second way in which migration costs exacerbate the problem of multiple equilibria probably is more serious. The complete commonality of interest is broken. In a game of complete coordination, such as the game in strategic form depicted in section 4, pre–play communication can plausibly enable players to avoid the inferior Nash equilibrium, even when talk is cheap. It is not nearly so plausible that cheap talk can enable players to coordinate on an outcome which maximizes the sum of their payoffs when payoffs are not perfectly coordinated. In the game below there is an obvious conflict between players, and both will try and bluff their way to their preferred Nash equilibrium if there is pre–play communication.

$$\begin{pmatrix} 1\backslash 2 & L & R \\ & & \\ t & (10,5) & (0,0) \\ b & (0,0) & (2,7) \end{pmatrix}$$

Local politicians do communicate with each other. But without a central government to enforce agreements between them, they may not easily be able to commit to actions.

7 Local Incentive Equivalence

As long as there is some migration between communities, the fixed migration costs ensure that the interests of the jurisdictions are perfectly aligned. Consider, for example, the policies which result in a migration outcome in which $N_1 > \bar{N}_1$. Any policies which maximize u^1 — subject to there being some positive migration into community 1 — must also maximize $u^2 = u^1 - \mu$ subject to this constraint.

So suppose that there is some Pareto optimum in which there is some migration from community 2 to community 1. Suppose that the policies $(t_1, \mathbf{z}^1), (t_2, \mathbf{z}^2)$ achieve this optimum (through some migration outcome (N_1, N_2) in which $N_1 > \bar{N}_1$). Then neither community's policy maker would wish to deviate from this policy, if the deviation did not cut off or reverse the migration flow.

It seems reasonable that if policy changes are small enough, then they will not have a large effect on migration. So a migration outcome will be described as **stable** if migration flows are continuous in the policy variables.

Stable Migration Outcome : The migration outcome (N_1^0, N_2^0) resulting from community policies $(t_1^0, \mathbf{z}^{10}), (t_2^0, \mathbf{z}^{20})$ is stable if N_1 and N_2 are continuous functions of the community policies at $(t_1, \mathbf{z}^1), (t_2, \mathbf{z}^2) = (t_1^0, \mathbf{z}^{10}), (t_2^0, \mathbf{z}^{20})$.

A local Nash equilibrium is defined in the usual way :

The policies $(t_1^0, \mathbf{z}^{10}), (t_2^0, \mathbf{z}^{20})$ are a **local Nash equilibrium** if there is some $\epsilon > 0$ such that it is impossible for community i to increase its payoff u^i by any unilateral deviation to any policy $(t_i, \mathbf{z}^i)$ within a distance ϵ or less of $(t_i^0, \mathbf{z}^{i0})$.

The following result then follows, virtually by definition :

Proposition 5 *If the policies $(t_1^*, \mathbf{z}^1*), (t_2, \mathbf{z}^2*)$ and the resulting stable migration outcome (N_1^*, N_2^*) are Pareto optimal, and $N_1^* \neq \bar{N}_1$, then the policies $(t_1^*, \mathbf{z}^1*), (t_2, \mathbf{z}^2*)$ are a local Nash equilibrium.*

Proof Without loss of generality, assume that $N_1^* > \bar{N}_1$. Then stability of the migration outcome implies that, if ϵ is small enough, any unilateral deviation by community i to some $(t_i, \mathbf{z}^i)$ within ϵ of $(t_i^*, \mathbf{z}^i*)$ will still result in a migration outcome for which $N_1 > \bar{N}_1$. If the deviation increased one community's payoff u^i, then it must increase the other community's payoff, since $u^1 = u^2 + \mu$ when $N_1 > \bar{N}_1$. So if the deviation increased one community's payoff, then the original outcome could not have been Pareto optimal, contradicting the hypothesis of the proposition. •

Although the stability of the migration outcome may seem a relatively weak assumption, the possibility of pure public goods (or other forms of increasing returns to population) does imply that making a community more attractive might set in motion an unstable process, if the increased immigration into the community itself also made it more attractive.

But the definition of stability of a migration outcome here is **stronger** than the usual stability notion, that $U^1 - U^2$ be a decreasing function of N_1, for given policies. The selection rule does not guarantee the stability of a migration outcome, as defined in this section.

Suppose that the utility difference $U^1 - U^2$ is decreasing in N_1 (when the policies are $(t_1^*, \mathbf{z}^1*), (t_2, \mathbf{z}^2*)$). This weaker notion of stability ensures that there must be some migration outcome (N_1, N_2) near (N_1^*, N_2^*), if the policies $(t_1, \mathbf{z}^1), (t_2, \mathbf{z}^2)$ are near $(t_1^*, \mathbf{z}^1*), (t_2, \mathbf{z}^2*)$. But there may also be other migration outcomes corresponding to $(t_1, \mathbf{z}^1), (t_2, \mathbf{z}^2)$, some of them far away, and some of them involving no migration, or migration in the opposite direction. The definition of stability here requires agents not to decide to jump to another migration outcome, in response to a small change in policy. In other words, it requires not only that the implicit function theorem holds, but that all agents choose to use the implicit function in predicting the migration response to small changes in policies.

8 Counter–examples

Perhaps the most important incentive equivalence result is that any Pareto optimum can be sustained as a Nash equilibrium, when communities set policies non–cooperatively.

Proposition 3 indicates that this incentive equivalence will hold when migration costs are positive (and identical for all residents) — provided that transfers go in the right direction. The proof of that Proposition used the fact that any transfers flowed from the destination community for migration, to the origin community.

In this section, a few simple numerical examples are presented, in order to show that this incentive equivalence need not hold without the assumptions of the Proposition. In each example, the Benthamite optimum requires migration from community 2 to community 1, but also requires transfers from residents of community 2 to community 1. But policy makers in community 2 will not want to make these transfers ; they can gain by cutting the transfers to zero and eliminating (or reversing the direction of) the migration.

The examples are quite contrived ; the assumptions of Proposition 3 certainly are not necessary for incentive equivalence. But they do show that positive migration costs can drive a wedge between communities' interests.

In the standard example, Flatters, Henderson and Mieszkowski (1974) show that transfers should be paid from community 2 to community 1 if and only if $Z_1/N_1 > Z_2/N_2$ at the optimum. In the first two counter–examples, this condition on public good provision is forced on the communities, by assuming that no benefit can be obtained from the public good in community 2 under any circumstances, so that $Z_2 = 0$.

8.1 Counter–Example 1

In this first counter–example, community 2 does not provide a public good at all. That means that the only policy to be chosen there, if decision–making is decentralized, is how much of a grant to pay to residents of community 1.

The total absence of public good provision in community 2, and its presence in community 1, may seem artificial and adhoc. But it might arise if the two communities had different physical settings, and only community 1's was suitable for some leisure activity which was non–rivalrous but which required some public expenditure.

Here $\bar{N}_1 = \bar{N}_2 = 100$. The cost of migration μ is 2. The production technology is

$$F_1(N) = 10N + 100\sqrt{N} \quad ; \quad F_2(N) = 30N + 100\sqrt{N}$$

The valuation function for the public good in community 1 is

$$v_1(G_1) = \frac{(G_1)^{0.3}}{0.3}$$

The cost of a unit of the public good is 1.

Each community has its advantage here. Community 1 provides better consumption opportunities, but community 2 has a better production technology. If there were no migration, and no transfers, and if community 1 provided the optimal quantity of the public good corresponding to a population of $\bar{N}_1$, then here $u^2 - u^1 \approx 3.2 > \mu$. To prevent migration to community 2, transfers would have to be made to residents of community 1.

But the sum of utilities is increased by shifting population from community 2 to community 1. Here $B(N_1)$ is a strictly concave function, and it reaches a global maximum at $N_1^* = 155.63 > 100 = \bar{N}_1$.

The planner's policy $(g_1^*, Z_1^*, g_2^*, Z_2^*)$ which yields the Benthamite optimum, with a level of migration $N_1^* - \bar{N}_1 > 0$, is

$$g_1^* = 1.930, g_2^* = -6.769, Z_1^* = 1353.823, Z_2^* = 0$$

Transfers must flow from the "worse off" community 1 to the better off community 1 here, since $Z_1 > Z_2 = 0$. Consistent with Flatters, Henderson and Mieszkowski (1974), the net transfer which supports this efficient outcome is

$$g_1^* - g_2^* = \frac{Z_1^*}{N_1^*} - \frac{Z_2^*}{N_2^*} \approx 8.7$$

The payoffs to natives of the two communities at this outcome is

$$u^1 = 49.8116 \quad ; \quad u^2 = 47.8116$$

The sum of everyone's utilities here is 9762.312 ; that exceeds the sum of people's utilities if there were no migration (and if community 1 provided the optimal level of the public good for a population of $\bar{N}_1$, $G_1 = 719.686$), which equals 9679.267.

The outcome which maximizes $B(N_1)$ requires a high transfer from residents of community 2 to residents of community 1, $t_2^* = 6.769$, because of the importance of the public good in community 1. But natives of community 2 would not want to see a small decrease in this transfer ; as long as the transfer results in net migration from community 2 to community 1, everyone's interests are in harmony and everyone wants to stay at the sum–of–utility–maximizing policies. But if a change in policy is large enough to eliminate, or reverse, migration, the harmony disappears.

In response to community 1's choice of the (efficient) policies $Z_1 = 1353.823$, $t_1 = 0$, community 2's policy makers do not actually wish to eliminate all transfers to community 1. If they did so, then "reverse migration" would result, and this immigration makes natives of community 2 worse off (in this example). So their best response to $Z_1 = 1353.823, t_1 = 0$ is to set $t_2 = 1.271$, a much smaller transfer than is required to sustain the Benthamite optimum. This lower transfer leaves natives of community 1 on the margin of wanting to migrate to community 2. Given community 1's policy, setting $t_2 = 1.271$ results in no migration at all, and payoffs of $u^1 = 46.7292, u^2 = 48.7292$.

Natives of community 2 gain from deviation from the transfer which sustains the Benthamite optimum. But they only gain by making such a large deviation that all migration is eliminated. If the payoff to natives of community 1 is graphed as a function of the transfer t_2 they choose, this payoff has a local maximum at $t_2 = 6.769$. But (if $Z_1 = 1353.823, t_1 = 0$), the payoff is not a concave function of t_2, and it has a global maximum at $t_2 = 1.271$.

In this example there are two different migration patterns which a central planner might choose. $N_1 = N_1^* > 100$ maximizes the sum of utilities. But an outcome with no migration at all is better for natives of community 2. A policy of $Z_1 = 719.686$, and $0.604 \leq g_1 \leq 2.604$ will lead to a migration outcome with no migration, and the efficient supply of the public good in community 1. If $g_1 < 2.189$ then this outcome without migration is better than the Benthamite outcome for natives of community 2.

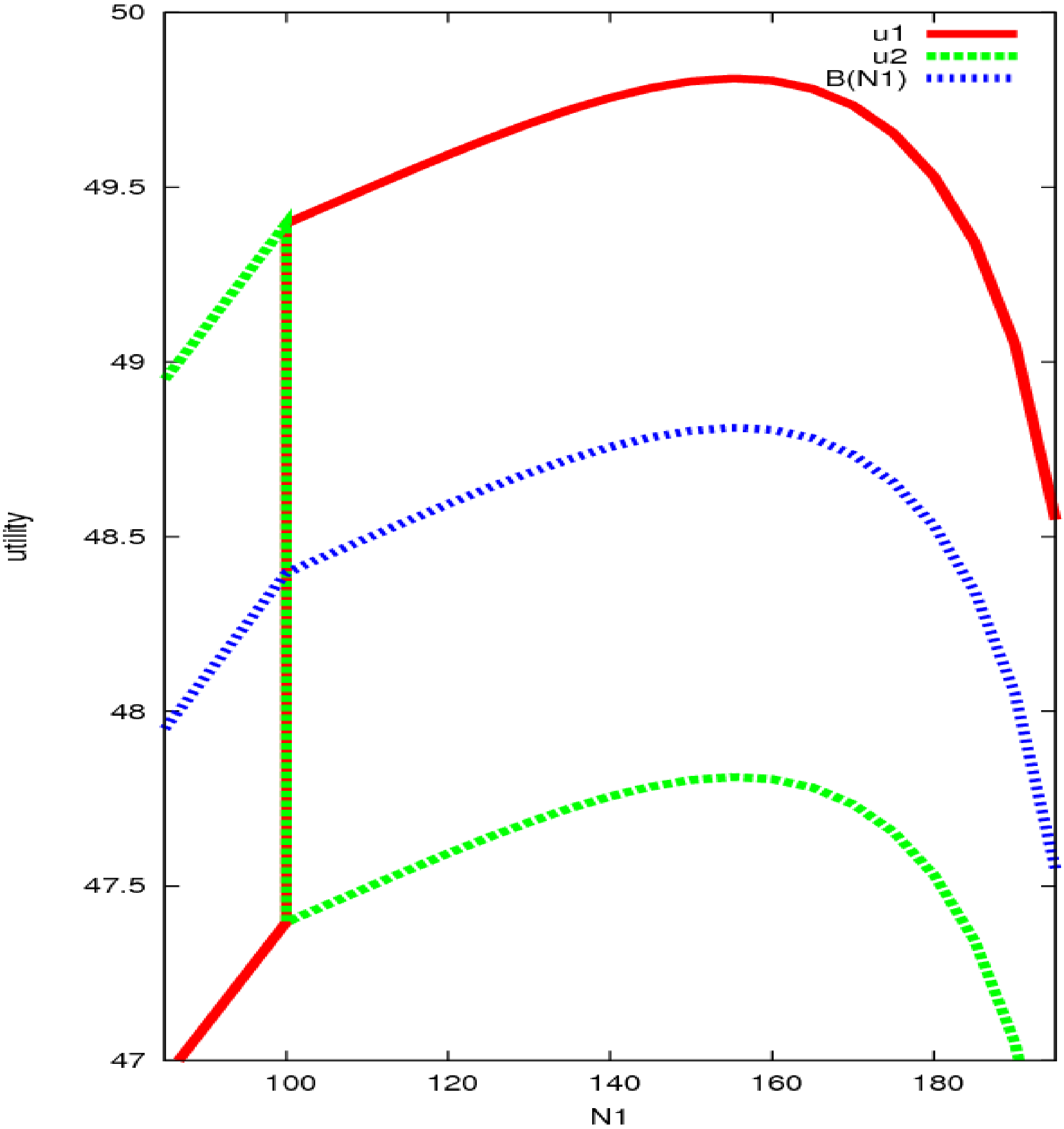

Figure 5 : payoffs in counter–example 1

There is an efficient — although not sum–of–utility–maximizing — allocation here which can be sustained as a Nash equilibrium. The most–preferred policy for natives of community 2 is to cut off all migration with the minimum possible transfer. If $(t_1, Z_1) = (0, 719.686)$, and $(t_2, Z_2) = (0.604, 0)$ here, then natives of community 1 are just on the margin of wanting to migrate to community 2. This allocation maximizes the utility of community 2 natives. Given community 2's policies, community 1 cannot induce migration, and cannot achieve any higher utility.

So in counter–example 1, there is an efficient outcome which can be sustained as a Nash equilibrium. But no efficient outcome involving migration can.

Figure 5 shows the payoffs to natives of each community in this example, and the average of their payoffs, as functions of the distribution of population.

8.2 Counter–example 2

This second counter–example is a slight modification of the first : productivity differences are smaller, and migration costs are higher. The one qualitative difference is that now transfers are not needed in order to sustain no migration as an outcome : when $N_1 = \bar{N}_1$ (and community 1 sets its public good supply optimally), the difference between utilities in the two communities is less than the migration cost.

Other than the migration cost, which is 5 instead of 2, and the production function in community 2, which is $25N + \sqrt{N}$ instead of $30N + 100\sqrt{N}$, technology and tastes are exactly as in the previous counter–example.

Now the population which maximizes $B(N_1)$ is $N_1^* \approx 168$. The policy $(g_1^*, g_2^*, Z_1^*, Z_2^*)$ which can implement this allocation is $(1.438, -7.550, 1510.126, 0)$, and the resulting payoffs are $(u^1, u^2) = (49.4365, 44.4365)$. As in the previous counter–example, community 2 can benefit by deviating unilaterally from its (t_2^*, Z_2^*) which would implement the Benthamite outcome. In (slight) contrast to the previous example, here cutting the transfer t_2 will not induce reverse migration. If community 1 chooses the policy $Z_1 = 1510.126, t_1 = 0$ consistent with the Benthamite optimum, community 2's best response is to choose $Z_2 = 0, t_2 = 0$, a policy which will eliminate all migration, and result in utility levels of $u^1 = 44.8616, u^2 = 45.00$.

8.3 Counter–example 3

The absence of any value from public good provision in community 1 is not necessary to generate counter–examples to incentive equivalence. What is needed is that the public good provision be higher in the destination community at the Benthamite optimum. So what is required, if the destination community has a larger population, is that the price elasticity of demand for the public good to be high, since the per capita price of the public good in community i is proportional to $1/N_i$.

In this third example, communities are identical in every respect, except for the initial population. The production function in each community is

$$F(N_i) = 10N_i + 100\sqrt{N_i}$$

the unit cost of the public good is 1 in each community, and the valuation function for consumption of the public good is

$$v(G_i) = \frac{(G_i)^{0.3}}{0.3}$$

The cost μ of migration is 3.

Now the initial population levels are $\bar{N}_1 = 120, \bar{N}_2 = 80$. Because of community 1's higher initial population (and because of the non–zero migration costs), the Benthamite optimum exploits scale economies in population in provision of the public good, and involves large–scale migration from community 2 to community 1. At this optimum [8]

$$N_1^* = 183, Z_1^* = 1706.369, Z_2^* = 57.251, g_1^* = 0.507, g_2^* = 5.456$$

The payoffs to natives of the two communities are $u^1 = 48.4814, u^2 = 45.4814$.

As in the previous example, decision makers in community 2 can gain if they reduce their transfer from the level ($t_2^* = 5.456$) which sustains the Benthamite optimum to zero. In this case, responding to $Z_1 = 1706.369, t_1 = 0$ by setting $Z_2 = 258, t_2 = 0$ leads to a migration outcome in which no–one wants to move : $u^1 = 45.5393, u^2 = 45.9397$. Since natives of community 2 gain from this deviation, the Benthamite optimum cannot be sustained as a Nash equilibrium.[9]

[8] The positive migration costs, and the declining marginal productivity of labour, imply that the Benthamite planner does not want a corner solution, in which all population moves to the larger community.

[9] The public output level $Z_2 = 258$ is **not** the optimal level for community 2, if there is

9 Subsidiarity

In all the counter–examples (to the sustainability of a Pareto optimum as a Nash equilibrium), reductions in transfers were used by a community to increase its natives' payoff. Proposition 3 indicates that a community will never want to deviate from a Pareto optimum by changing only "domestic" fiscal variables $\mathbf{z}^i$.

These results are consistent with a version of the subsidiarity principle : they suggest that transfers among regions are best left to higher levels of government, but public expenditure decisions should be devolved to the lowest possible level of government.

In particular, suppose that the policy–setting game is played by three policy makers. Now community i gets to choose $\mathbf{z}^i$, but the transfers (t_1, t_2) between communities are chosen by a central government. The central government's payoff is some welfare function $W(u\cdot u^2)$.

Assume that all three players move simultaneously. This is the simplest case to analyze. So the central government is not endowed with the commitment power sometimes ascribed to it in models of federalism in which it moves first. Neither can it respond to local government policies by adjusting marginal utilities, as in Wellisch's extension of the attachment–to–home model, in which the central government moves last.

Since the source community for migration cannot attract migrants by changing $\mathbf{z}^i$ if the original situation is Pareto optimal, it may not be surprising that Nash equilibria can be sustained once the power to adjust transfers is given to the central government.

Proposition 6 *Suppose that a central government chooses transfers (t_1, t_2), and community i chooses $\mathbf{z}^i$. The central government's payoff is some welfare measure $W(u^1, u^2)$ depending on communities' payoffs. If communities'*

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no migration. But the selection criterion is being imposed here. The policy which is best for community 2, should there be no migration, is actually $Z_2 = 523.259$. But such a high level of public output would be very expensive if community 2 only had 17 inhabitants, as it does at the Benthamite optimum. The selection criterion says that community 2 can expect to reverse migration flows only if it deviates to a policy (Z_2, t_2) which offers natives of community 2 a utility level greater than what they would get (net of migration costs) from moving to community 1, at the population distribution $N_1 = 183, N_2 = 17$ corresponding to the Benthamite optimum. If $Z_2 = 258$ and $t_2 = 0$, then $U^1 - U^2 < \mu = 3$ when $N_1 = 183, N_2 = 17$, so that the policy deviation does increase the relative attraction of remaining in community 2, as required by the selection criterion.

decision makers use the selection rule, then any policy which maximizes the central planner's welfare measure can be sustained as a Nash equilibrium in this 3–player game.

Proof Let $t_1^*, \mathbf{z}^1*, t_2^*, \mathbf{z}^2*$ be the policies which lead to the central government's preferred outcome. Clearly the central government has no incentive to change its transfer (t_1, t_2) from the transfers (t_1^*, t_2^*) which lead to its most–preferred outcome. If the outcome is Pareto optimal, and if it leads to migration to community 1, then the outcome must maximize u^1 over all feasible policies, so that community 1's planners have no incentive to deviate. If the outcome is Pareto optimal, then z^2* maximizes $U^2(N_2^*, t_2*, \mathbf{z}^2)$.So any deviation by community 2's policy makers would reduce $U^2(N_2^*, t_2*, \mathbf{z}^2)$, which, under the selection rule, would lead to a migration outcome in which $N_1 > \bar{N}_1$. Therefore the change in $\mathbf{z}^2$ cannot increase $u^2 = u^1 - \mu$.

An analogous argument shows that none of the three governments would want to change policies if the outcome involved migration to community 2 (and was Pareto optimal).

Finally, suppose there were no migration at the central government's preferred outcome. Again, $\mathbf{z}^i*$ must maximize $U^i(\bar{N}_i, t_i^*, \mathbf{z}^i)$. So a deviation by community i must lower $u^i = U^i(\bar{N}_i, t_i^*, \mathbf{z}^i)$ if it did not induce migration. By the selection rule, if it induces any migration, it would induce emigration from community i. So if the policy led to an increase in u^i to $\tilde{u}^i$, it would lead to a payoff of $\tilde{u}^i + \mu$ to natives of the other community. If such a deviation is beneficial for community i, then $\tilde{u}^i > u^i*$. But the fact that there was no migration originally implies that $u^j* \leq u^i * + \mu$, o that if the deviation increased u^i it would also increase u^j, which contradicts the fact that the original policy maximized $W(u^1, u^2)$. •

10 Spillovers

Community i's fiscal variables have been divided between the "domestic policies" $\mathbf{z}^i$ and the transfer t_i to the other region. Under the selection rule used here, it is only changes in the transfers which can induce unilateral deviation from a Pareto optimal outcome.[10].

The key distinction between the domestic policies and the transfers is not that the former are quantities of goods and the latter are changes in income

[10]as Propositions 3 and 6 suggest

: domestic policies typically consist of both quantities of locally consumed public goods, and the local taxes used to finance them.

The key distinction is that the utility U^i from residing in community i depends on the community's own $\mathbf{z}^i$ and t_i, and transfers t_j received from the other community, but **not** on the domestic policies $\mathbf{z}^j$ in the other community. If some policies in community i affected directly[11] the well–being of residents of community j, then Propositions 3 and 6 need not apply. In particular, the taxonomy used here rules out spillovers of the benefits of public goods provided in a community, and shifting of some of the burden of community i's own taxes onto residents of community j.

Why are deviations from the efficient policies $\mathbf{z}^i*$ unattractive for policy makers in community i, if there are no spillovers? Efficiency implies that each component of $\mathbf{z}^i*$ maximizes $U^i(N_i*, g_i^*, \mathbf{z}^i)$, given the efficient population N_i^* and the efficient net transfer g_i^*. So any deviation from $\mathbf{z}^i*$ must lower $U^i(N_i*, g_i^*, \mathbf{z}^i)$. With no spillovers, this deviation must therefore also lower $U^i(N_i*, g_i^*, \mathbf{z}^i) - U^j(N_j^*, g_j^*, \mathbf{z}^j*)$, since $\mathbf{z}^i$ does not affect U^j directly. And the selection rule implies that a fall in $U^i - U^j$ must lead to more emigration from i to j, which implies natives of community i won't want to change their domestic policies.

If there are spillovers associated with domestic policy k, then the efficient policy does not maximize U^i, it maximizes $U^i(N_i^*, g_i^*, \mathbf{z}^i, \mathbf{z}^j*) + U^j(N_j^*, g_j^*, \mathbf{z}^j$. And a change in a domestic policy in community i might increase $U^i - U^j$, even if it decreased U^i.

So suppose that that one of the public goods provided in community 2 produced strong positive spillovers for residents of community 1. Suppose as well that this public good is financed by a head tax on residents of community 2. Efficiency requires that community 2 provide a higher quantity of this public good than is optimal from a myopic, self–interested perspective. (That is, if community 2's policy makers ignored migration, they would provide an inefficiently low quantity of this public good.) So starting from an optimum, a reduction by community 2 in the quantity of this public good would raise U^2 and it would lower U^1.

If the spillovers are important enough, and if the non–convexities are significant enough, reducing the supply of this spillover–producing public good might reverse the direction of migration. Spillovers (or tax exporting) give a community the opportunity to reverse migration flows through a unilateral

[11]that is, holding constant the population N_j

policy change, just as positive transfers from source to destination made it possible for the source community to reverse migration by eliminating transfers (in all the examples in section 8).

So mobility alone may not be sufficient to internalize externalities among communities. As with voluntary transfers, incentive equivalence fails only when migration costs are large, and when scale economies in population are significant enough that there are multiple Pareto optima. As with voluntary transfers, externalities pose no threat to incentive equivalence if changes are small ; the only possible value to a community from deviating from efficiency is inducing an elimination or reversal of outbound migration.

11 Concluding Remarks

The basic incentive equivalence result is a powerful one. Uncoordinated, self–interested behaviour by lower–level governments is efficient. No policy coordination, or intervention by a higher level of government is needed.

The argument underlying this result is quite a simple one, which can be applied fairly generally — if migration costs are 0. If each player gets the same payoff as the other player, in any cell of a two–player game, then the strategies which maximize that common payoff must be a Nash equilibrium.

Here, two consequences of positive migration costs are emphasized. First, there may be a conflict between equity and efficiency, even if agents are identical except for place of birth. This conflict can arise even when multiple maxima are not a problem. World output may be a strictly convex function of the amount of migration. It may be maximized by inducing migration from community 2 to community 1. But if migration costs are positive, natives of community 2 may be better off in an inefficient outcome without any migration, if lump–sum transfers based on place of birth are not possible.

The second consequence is purely an efficiency issue, and arises only if non–convexities are important. The policy which maximizes the sum of people's utilities may not be sustainable as a Nash equilibrium under decentralization, if there are multiple local maxima to this sum.

Finally, it should be emphasized that this second consequence requires **both** positive migration costs and non–convexities. If migration costs are zero, any Pareto optimal outcome will be a Nash equilibrium, regardless of the extent of non–convexities. If the sum of people's payoffs is convex (as a function of population), then any Pareto optimal outcome will be a Nash equilibrium, regardless of migration costs.

References

[1] R. Boadway. On the method of taxation and the provision of local public goods : Comment. *American Economic Review*, 72:846–851, 1982.

[2] R. Boadway and F. Flatters. Efficiency and equalization payments in a federal system of government : A synthesis and extension of recent results. *Canadian Journal of Economics*, 15(4):613–633, November 1982.

[3] S. Bucovetsky. Public input competition. *Journal of Public Economics*, 89:1763–1787, 2005.

[4] A. Caplan, R Cornes, and E. Silva. Pure public goods and income distribution in a federation with decentralized leadership and imperfect labor mobility. *Journal of Public Economics*, 77:266–284, 2000.

[5] F. Flatters, J. V. Henderson, and P. Mieszkowski. Public goods, efficiency and regional fiscal equalization. *Journal of Public Economics*, 3:99 – 112, May 1974.

[6] Z. Hercowitz and D Pines. Migration with fiscal externalities. *Journal of Public Economics*, 46:163–180, 1991.

[7] R. Kanbur and M.Keen. Jeux sans frontieres : Tax competition and tax coordination when countries differ in size. *American Economic Review*, 83:877–892, 1993.

[8] R. Krelove. Efficient tax exporting. *Canadian Journal of Economics*, 25(1):145–155, February 1992.

[9] A. Mansoorian and G. Myers. Attachment to home and efficient purchases of population in a fiscal externality economy. *Journal of Public Economics*, 52:117–132, 1993.

[10] G. Myers. Optimality, free mobility, and the regional authority in a federation. *Journal of Public Economics*, 43:107–121, 1990.

[11] J. Stiglitz. The theory of local public goods. In M. Feldstein and R. Inman, editors, *The Economics of Public Services*, pages 274–333. Macmillan, 1977.

[12] D. Wellisch. *Theory of Public Finance in a Federal State*. Cambridge University Press, Cambridge, UK, 2000.