On Opting Out of Public Services

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by

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Summary

We analyze a two-tier institution in which government provides public services, but individuals are allowed to opt out of public provision. Public provision is financed through a redistributive income tax whose tax parameter is endogenously chosen by majority vote. Even though preferences are not single peaked under a system with possible exit, we provide necessary and sufficient conditions for a majority vote outcome. Specifically, equilibrium outcomes are characterized for the case where individual preferences over tax rates in a one-tier benchmark model are increasing in incomes. The equilibrium tax rate is found to be strictly below that in a one-tier system, a result which confirms the slippery-slope argument. Moreover, a majority of the population but not the middle class prefers a transition from the one-tier to an opting-out system. Our results are similar to those from the 'topping up' literature but contrast with results in the earlier exit models, in which allowing opting out yielded a Pareto improvement.

Keywords: Publicly provided goods, Two-tier system, Exit Option.

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1 Introduction

While individuals can not opt out of their taxes they often have the possibility of opting out of publicly provided services. In some jurisdictions this is the case when it comes to publicly provided services such as schooling and public health care. However, the principle more broadly applies to many other functions of government. For example, an individual who never uses a roadway will not be successful in demanding a tax refund. Less obvious; individuals who commute by car do not directly gain from the heavy subsidization of public transit.

Despite the apparent significance of opting out in reality, this issue has received surprisingly little attention in the theoretical literature. Two early contributions were Stiglitz (1974) and Usher (1977). Besley and Coate (1991) analyzed the economic implications of exit options. Their paper analyzes the welfare implications of opting out in a model in which rich individuals resort to private alternatives to public services. By doing so, they impose positive spillovers on poor individuals who stay in the system and now benefit from a higher quality of the public service, caused by the reduced demand for these goods. Besley and Coate's analysis illuminates the fact that even with non-redistributive financing of public services, the ability to opt out can have severe redistributive consequences. On the other hand, the Besley and Coate analysis takes the provision level of the public good as given and therefore, does not speak to the question of how the provision level of public services will be chosen in a democratic setting.[1] Likewise, their paper does not address the issue whether introducing exit options changes the supply of public services in equilibrium.

The present paper is part of a growing literature on dual public-private regimes of public good provision, and endogenizes the political decisions in this

[1] While the public good funding level is exogenous, it can immediately be endogenized in the context of their simple binary-type model where agents are either 'rich' or 'poor'. If poor agents are in the majority, they would choose the funding level that balances their desire for a certain quality level, and their interest to induce rich individuals to exit the system.

regard using majority voting.

This literature can be divided into two main branches. In the 'topping up' literature, the consumption of publicly and privately provided services of the good in question is not mutually exclusive. Every individual in fact consumes the public provided level of the good, but is allowed to spend private resources to enhance her private consumption of the good. Examples of this scenario include public education augmented by private tutoring, mandatory insurance such as car liability insurance, which can be extended by private choice, and federally mandated emission ceilings in federal systems. In many contributions (Epple and Romano, 1996, 2003; Fernandez and Rogerson, 2003, Gouveia, 1997) society uses a redistributive income tax to finance public-good supply. They also assume that individually preferred tax rates to be increasing in the income of the individual. The equilibrium tax rate is found to be lower than in the standard political economy model where topping up is not allowed. Intuitively, rich individuals prefer being allowed to top up over a pure public system as the latter forces them to subsidize poorer agents. This drives the equilibrium tax rate down and in effect, rich individuals and poor individuals form a coalition against the middle class. While a majority of the population prefers a system which allows topping up, some will be hurt—possibly a sizable share of the middle class.[2]

Topping up models in which the public service is perfectly rival have single-peaked individual preferences, so that the median voter theorem can immediately be applied. In contrast, single peakedness is generally violated when the good in question causes positive externalities across members of society. Intuitively, different tax rates selected at the collective choice stage trigger different

[2]If alternatively, political choice determines a non-redistributive head tax instead, the tax rate is identical to the one in a pure public system, and a Pareto improvement over the pure public system is achieved. Conversely, a majority of the overall population may resist the transition from a pure private-supply system (Cremer and Palfrey, 2000).

topping up choices by private individuals at the second stage. In a setting with spillovers where these private choices affect the wellbeing of other agents in society, they in general lead to a loss of preference single peakedness and, presumably, to the non-existence of majority vote equilibrium. As a response to these problems, the small literature that allows for externalities has assumed myopic voter behavior (Epple-Romano, 2003), or resorted to the weaker equilibrium concept of 'local majority equilibrium' instead (Cremer-Palfrey, 2006).[3]

The second branch of the literature on dual public-private provision is the so-called 'exit option" or "opting out' framework, in which the consumption of publicly and privately provided services are mutually exclusive. Individuals may choose public rather than private schooling, or private rather than publicly provided health care. As emphasized by Besley and Coate there are many examples of public services where the opting out model is appropriate; if you need surgery you get it in the public sector *or* the private sector but not both. The comparison of a system where opting out is allowed with one where opting out is prohibited is also important because there are public health systems where opting out (two-tier) is not allowed. The Canadian Health Act prohibits the private for-profit provision of many medically necessary services. One might wonder about the possible defences of such a prohibition (against mutually beneficial gains from trade) but there are at least two. The first is that allowing a private system will draw the best resources (e.g. surgeons/teachers) out of the public system. The second is political and is what might be called a "slippery slope" argument–if the politically powerful rich are allowed to opt out of public system, the political process will generate a lower quality public service–our public schools and public health systems will suffer. The slippery slope argument will be the focus of this paper.

[3]Recently, Lülfesmann (2008) shows that under the symmetry assumption usually imposed in the literature, a majority voting equilibrium in presence of externalities exists even though preferences are not single peaked.

To our knowledge, Glomm and Ravikumar (henceforth GR, 1998) is the only paper that provides a systematic analysis of this opting out scenario. As was early recognized (Stiglitz, 1974), the opting out model always displays non single peaked preferences, regardless of the nature of the publicly provided good. Specifically, each agent in society exhibits a certain tax rate below which he would like to exit the public system and consume the privately optimal amount. Up to this tax rate, his utility falls in the tax rate, while it is increasing over some range for larger taxes. Notwithstanding these problems, GR find a set of sufficient conditions under which a majority voting equilibrium exists. Their setup posits a majority vote on a linear income tax, and agents in society who differ in incomes but are homogenous with respect to their preferences. In this framework, their paper establishes a majority vote outcome under two assumptions:

1. The threshold tax rate at which individuals exit the public system increases in income;

2. The preferred tax rate for an individual within the public system is weakly decreasing in income.

Under these conditions, the equilibrium tax rate in a public system when opting out is prohibited, remains the equilibrium tax when it is allowed. Hence, the opting out framework in the GR formulation is a Pareto improvement over a system which prohibits opting out. In fact, the only agents who are affected are those who leave the public system.[4] By revealed preferences, these individuals are clearly better off. In contrast to the topping up model, the opting out model in the GR version thus provides an unambiguous answer to the question of whether a parallel private system (two tier) should be allowed. One goal of the present paper is to find the underlying forces and assumptions that drive

[4]This is true if the service has public good characteristics. If not, exit as in Besley and Coate reduces queuing and therefore, enhances the quality of the public good for stayers as well.

these differences in results. In order to do so, our analysis will extend the scope of the GR analysis.

First and most importantly, we notice that while the first condition seems natural in a setting with identical preferences, the second condition does not. For a setting with income taxation, condition 2 is the exact *opposite* of a critical assumption made in the topping up literature, whereby individuals' preferred tax rates are *increasing* rather than decreasing in their incomes. An individual's preferred tax increasing in the individual's income is also implicit and critical in the slippery slope argument. If the rich prefer a lower tax than the median voter, even when opting out is prohibited as in GR, then their exit does not change the qualitative nature of their vote. Clearly this buys tractability, but it also assumes away the slippery slope argument. That condition 2 is not a necessary characteristic of reality can immediately be verified for a setting where the public good is not financed through income taxation, but via head taxation instead. As long as the public service is a normal good, richer people would prefer a larger tax contribution than poorer individuals in a pure public system.[5]

When condition 2 is violated, in contrast to GR, majority vote equilibrium does not always exist. Our main contribution is to provide a necessary condition for the majority voting tax rate t^* when preferred tax rates are increasing in income. Our procedure has the advantage that t^* can easily be identified, despite the fact that preferences are non single peaked. In addition, we also provide a sufficient condition that can can easily be computed.

If it exists, a majority voting equilibrium is characterized by the following features. First, the equilibrium tax rate t^* is no larger than the equilibrium

[5] A similar issue arises with sales taxes: since poor people consume a larger part of their incomes and save less, sales taxes are usually thought to be regressive: rich people pay more taxes, but less than proportionally to their incomes. Hence, even with homothetic preferences they would prefer larger tax rates then their less affluent counterparts.

tax rate under pure public provision (i.e. opting out prohibited) and is strictly smaller if a positive fraction of individuals exit the system. Not surprisingly, only agents with either high preferences and/or large incomes opt out of public provision in equilibrium. However, in contrast to GR and in line with the models from the topping up literature, a minority of agents are hurt by not prohibiting opting out (two-tier). As in the topping up literature, the majority in favor of the allowing opting out is comprised of poor individuals, and very rich individuals. Perhaps interestingly, we find that some individuals who exit the system prefer the prohibition of opting out (and the larger equilibrium tax rate). These are individuals who would not opt out (even if they could) if no one else opted out. Richer individuals opting out leads to a fall in the public system's quality which then induces these individuals to opt out (slippery slope).[6]

Our analysis also shows that in contrast to the topping up model, considering a service with public good characteristics (non-rivalry) considerably simplifies the analysis relative to the private goods case. With a non-rival publicly provided good, queuing is not an issue or more generally, the quality of the public service does not depend on the number of users of the system. This eliminates an additional source of possible non-regularity of preferences, making the opting out model significantly more tractable.[7]

Section 2 introduces the model. Section 3 provides context by looking at benchmark models, while Section 4 solves the general framework. Section 5 looks at a welfare comparison of allowing opting out or not and Section 6 looks at limitations and extensions while the appendix provides a numerical example.

[6] Although it should be noted that we do not have a dynamic model.

[7] For this reason, our formal model assumes the good to be a non-rival. But see the discussion in Section 6 below.

2 Analysis

We consider a setup in which N society members decide on the provision of a public good in democratic fashion. Each agent's preferences are represented by a utility function $U^i(q_i, c_i)$ where q_i indicates public good consumption, and c_i private goods consumption. We will further assume that preferences are well behaved (they are convex and monotonic), both goods are normal goods, and that standard Inada conditions hold. For simplicity, the unit prices of both goods are set to one. Individuals differ in incomes y_i, and (possibly) in their preferences for public and private goods consumption. The continuous function $F(y)$ on the closed interval $[y_l, y_h]$ describes the income distribution of all members of society.

Public goods provision is decided upon in a majority voting process where society members vote on a linear income tax rate t, so that financing entails a redistributive element. Total tax revenue is $Nt\overline{y}$ where $\overline{y}$ denotes the average income in society. In general the individual consumption of public services, q_i, will depend on total tax revenue and the size of the consuming population.[8] For reasons we will explain below, we assume that the publicly provided service is perfectly non-rival in which case $q_i = Nt\overline{y}\ \forall i$.[9] Further, we normalize the population size to unity so that $q_i = t\overline{y}$.[10]

Each agent can choose to not consume the public service and instead to buy a privately desired level q_i on a private market at unit costs of one. By

[8] If the public service was a private good (perfectly rival), for example, $q_i = Nt\overline{y}/n$ where n is the consuming population.

[9] This is then not a model which could directly address the argument that allowing exit is beneficial by reducing the queue in the public sector (e.g. waiting times for surgery). On the other hand we also do not focus on the common argument that allowing exit is detrimental by reducing the quality of available productive resources in the public sector (quality of doctors or teachers). We will focus on politics.

[10] To help with interpretation at various points we will consider the model, but with non-redistributive public financing or a uniform head tax T. This is not an uncommon assumption in the literature. Total tax revenue and public good consumption in this case are $q_i = T$ with non-rivalry and $N = 1$.

doing so the individual forgoes the public goods aspects of public provision while she is still required to pay her taxes (ty_i).[11] On the upside, opting out allows the individual to perfectly adjust public good consumption to her private preferences.

The game proceeds in two stages. In a first stage, society decides on a tax rate by majority vote. In a majority vote equilibrium, the equilibrium tax beats all other feasible taxes by more than half of all votes. Taxes are then collected. In a second stage, each individual decides whether to consume the publicly provided amount q_i, or to buy his preferred quantity q_i^* on a private market. The appropriate equilibrium concept is subgame perfect equilibrium. Note that while the tax paying population base, N is fixed (i.e. you pay taxes whether you exit or not) in general, q_i would depend on exit and thus t through the dependence of the size of the consuming population n on exit. This why we assumed non-rivalry; this is the one case where q_i does not depend on n. If we did not make this assumption we would have to assume myopic behaviour and/or unexploited profitable deviations to avoid non-single-peaked preferences even over the tax range where the individual does not exit.[12]

3 Preliminary Benchmarks

3.1 Pure public provision

Suppose first that individuals have no exit option and therefore, have to rely on public provision. In this setting, each individual's preferred tax rate maximizes $U^i(q(t), c_i(t))$ with respect to t. For a linear income tax this gives,

$$\frac{U_q^i(q, c_i)}{U_c^i(q, c_i)} = \frac{y_i}{\bar{y}}. \tag{1}$$

[11] Remember this is not a model in which private consumption simply tops up the consumption level derived from public provision. Also given the non-rivalry assumption the proper interpretation of the private alternatives are private uncongested clubs.

[12] We will explain fully in section 6 below, and argue that assuming some rivalry while drastically reducing tractability would not drastically increase insight.

Since preferences are well behaved, these conditions correspond to unique preferred taxes t_i^*[13]. Since all utility functions are single peaked, we can now invoke the median voter theorem. Under majority voting, the tax variable most preferred by the median person in society becomes the policy outcome. In what follows and with a slight abuse of notation, we will call this equilibrium choice t_m^*. Notice that under the Inada conditions, t_m^* leads to strictly positive taxation.[14]

3.2 Breakdown of the Public System

Suppose that individuals can leave the public system altogether, that is, also cease their participation in public financing of the public service. After quitting, individuals choose a consumption bundle (q_i^*, c_i^*) as determined by the tangency condition $U_q^i/U_c^i = 1$. We now show that the opportunity to exit triggers a total breakdown of the public system. This is true whether or not the tax system involves redistribution. With income taxes, as in our model, it is impossible to sustain a public system where some members have above-average incomes and therefore, subsidize other citizens. With uniform head taxes, collective uniform choice forces (almost) every agent to consume an undesired amount of the public service, which each agent can avoid by leaving the system and resorting to private markets for the optimal consumption level instead.

[13] With the uniform head tax the most preferred T_i^* for an individual would be given by $\frac{U_q^i(q,c_i)}{U_c^i(q,c_i)} = 1$ given $N = 1$. In particular, for individuals with incomes above (below) the average, the preferred public goods consumption is smaller (larger) under the income tax rather than uniform taxation system, reflecting the fact that the former system involves redistribution which is detrimental for the rich and beneficial for the poor.

[14] In our example below, we will employ a slight generalization of the tax structure above as we will assume total tax revenue equals $t\overline{y} + T$ where T is an exogenous uniform head tax. Nothing of significance changes: equation (1) still characterizes t_i^*, and the Inada conditions still imply $q > 0$ or positive taxation which is now $t_i^* > -T/\overline{y}$ rather than $t_i^* > 0$.

3.3 The Topping up Framework

Next, consider a model in which individuals do not abandon public provision but instead, can privately top up the consumption levels they receive within the system. This literature finds that a majority of the population always prefers the two-tier institution over the prohibition to top up, but the specifics depend on the underlying financing mode. With uniform head taxes, *all* individuals prefer the topping up model. This is because equilibrium taxes remain identical and agents who top up must be better off by revealed preferences (Cremer and Palfrey, 2000; Alesina et al., 2005).

With linear income taxes, results can be shown to crucially depend on the specifics of individual preferences. The literature (e.g. Epple and Romano (1996), Fernandez and Rogerson (2003)) exclusively focuses on the case in which each agent's preferred tax rate in a pure public system increases in his income, that is, where agents do not display 'too much' interest in redistribution.[15] In this scenario, topping up yields a coalition of the richest and poorest individuals in society, who all have an interest to lower taxes compared to the setting without topping up. This coalition drives the equilibrium tax rate below t_m^*. In welfare terms, introducing a topping-up system thus benefits the extremes of the income distribution, while hurting the middle class. Conversely, for the opposite case in which individually preferred tax rates are a negative function of income, one can easily show that adopting the two-tier institution does not affect the equilibrium tax. Clearly, the two-tier system then yields a Pareto improvement because individuals who decide to top up are better off.[16] Note that the latter result is in fact analogous to the one obtained in GR for the exit

[15] Remember that the exit model of GR employs exactly the opposite assumption: they assume that t_i^* is decreasing rather than increasing in income.

[16] In this latter scenario, allowing for topping up does not change the preferences of any individual relative to the median voter: rich individuals utilities peak at zero tax rates, but their peaks in the public system were below t_m^* any way.

model, and it employs the same assumption on underlying preferences.

More generally, equilibrium existence is unproblematic when the good in question has pure-private good characteristics. Otherwise, equilibrium existence is not guaranteed because similar to the exit option model, the median voter theorem cannot easily be applied (Cremer and Palfrey, 2006; Lülfesmann, 2008). Intuitively, when voting over the public goods supply, each individual has to consider the implication of different stage-1 tax policies for the stage-2 contribution vector. When individuals are rational, this additional concern causes a breakdown of preference single peakedness in the two tier institution, even if preferences are well behaved in the standard pure public provision model.[17] For these reasons, an analysis of topping models is simplified significantly when the public service is a private good.

4 The Opting-Out Model

We start our analysis of this scenario with stage 2 and ask which individuals will opt out for given tax rates chosen in stage 1. As a first important observation, note that each agent with $y_i > 0$ and the Inada conditions operative is characterized by a threshold tax rate $\hat{t}_i$ below which she exits. To see that this must be the case; if taxation becomes sufficiently low that q in the public system approaches zero, then even the poorest individual would be better off paying the (small) tax and devoting more of their resources to privately supplied q. Unsurprisingly richer individuals would require higher public provision levels (taxes) to prevent their opting out.

We impose the following natural assumption, also employed in GR:

Assumption: An individual's threshold tax rate $\hat{t}_i$ is increasing in her

[17] Lülfesmann (2008) shows that despite the non-single peakedness of preferences, a majority voting equilibrium can be shown to exist if the public service is a pure public good, or if (for intermediate cases) some standard symmetry assumptions are satisfied.

private income, y_i.

Next, let us consider an individual's preferred tax rate in the public system, t_i^*. Provided he does not want to exit, this tax rate is clearly described by (1). However, the associated utility level must be compared with the utility the individual would achieve when $t = 0$ is chosen, and hence, a purely private system is implemented. In a world with an exit option the individual's most preferred tax rate in the public system is either 0 or t_i^*.

Consider some special cases. With uniform head taxes and thus without redistribution, *each* individual is indifferent between $T = 0$ and $T = T_i^*$, and strictly prefers the zero tax over any positive $T \neq T_i^*$. With income taxation, any higher-than-average income person strictly prefers $t = 0$, while any agent with below-average income strictly prefers tax rates in some interval around t_i^* over $t = 0$ (see our figure 1).[18] Hence, if the public system can be sustained a necessary condition is that the majority of people have below-average incomes. This is, of course, a feature of most income distributions.

We also know that for each individual, $\widehat{T}_i < T_i^*$ under uniform head taxes. Suppose not. This would mean that for a non-empty range of tax contributions, the individual opts out and privately chooses a public goods consumption level smaller than the one she would receive in the public system, an obvious contradiction.[19]

We can now tackle the ensuing public choice problem. As mentioned in our previous discussion and illustrated in Figure 1, democratic decision making is complicated by the fact that in presence of an exit possibility, individual preferences over tax rates are *necessarily* not single peaked.[20] Starting at a tax

[18] In the example which the figure illustrates the taxation system is $t\overline{y}+T$ and T exogenous so that public system disappears $(q = 0)$ at $t = -T/\overline{y}$ rather than at $t = 0$.

[19] The same is not necessarily true with redistributive income taxation as in our model: if average income is low, very rich individuals may exit even at tax rates higher than t_i^*, in order to privately buy a larger amount of the public good. Our example provides a condition required to avoid $\hat{t}_i > t_i^*$

[20] With the possible exception of very rich individuals under income taxation financing –

rate sufficiently low that $q = 0$ where essentially, each individual exits and a fully private system prevails, individual utilities first fall until reaching the tax rate (and public provision level) at which an individual opts in, $\hat{t}_i$, then rises until they reach the agent's preferred ' public system' tax rate t_i^*, then falls again.

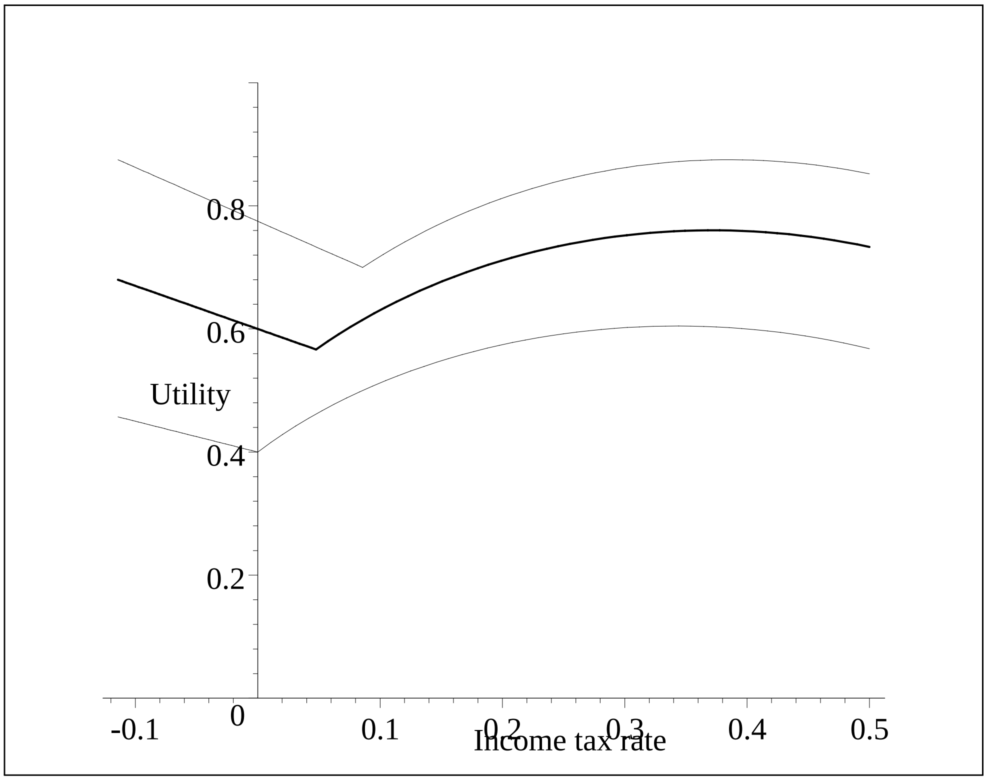


Figure 1: Utility against the tax rate (for low income, y_m (bold), and $\overline{y} > y_m$) for our appendix example.

As is well known, the lack of single peakedness is a serious obstacle for achieving a majority voting equilibrium. As noted in the introduction, Glomm and Ravikumar (1998) have found sufficient conditions under which an majority vote equilibrium nevertheless exists. The Proposition below gives an account of

see the previous footnote.

their findings.

Proposition (Glomm-Ravikumar) *Suppose individuals have identical preferences but different incomes and choose a linear income tax rate t by majority vote. If*

(1) $\hat{t}_i$ (weakly) increases in y_i and

(2) t_i^* (weakly) decreases in y_i,

the equilibrium tax rate t^* is identical to the tax rate t_m^* in a public system. Moreover, a (possibly empty) set of agents comprising the highest-income individuals options exits the public system.

Proof: Consider tax rate t_m^*. A majority of the population comprising all higher-than median income agents prefers this tax rate over any larger tax rate (see condition (2) in the Proposition). Note that this is true whether or not the agent exits at any such tax rate. At the same time, a majority of the population comprising all below-median income agents prefers t_m^* over any smaller tax rate. This is true for two reasons. First, the preferences of any such agent peak at a tax rate beyond t_m^* so that conditional on staying in the system, utility falls for tax rates below t_m^*: utility of all poor individuals is increasing in the range $t \in [\hat{t}_m, t_m^*]$ because of condition (1) and condition (2). Finally, consider tax rates below $\hat{t}_m$ at which a subset of poor individuals exit. If they do, each of them privately choose a consumption level of the public good $\tilde{q}_i$. This level is below the level they would obtain in the public system at tax rate t_m^*because goods are normal and they are poorer than m (who in addition benefits from redistribution in the public system). But since public-good consumption after exit is lower, every below-average income agent must find t_m^* preferable over any $t \in [0, \hat{t}_m]$ and in particular, over $t = 0$. This claim is proven by the following line of arguments. Suppose the public system provided the amount $\hat{q}_i$ that a poor individual i privately prefers after exit. Because of redistribution, the individual then clearly prefers the public system because it is associated with lower personal costs of consuming the public good. Moreover, our previous argument shows that the tax rate associated with amount of consumption $\hat{q}_i$ is below t_m^*. Hence, the agent's utility is increasing further when t is increased towards t_m^*, because by condition (2) his within-system utility is increasing over this range. The result follows. □

The GR result provides a set of circumstances under which a majority vote equilibrium always exist, as well as a characterization of this equilibrium. As we have argued before, the first of the two main assumptions that underlie the statement in the Proposition will usually be satisfied as long as individuals have identical preferences.[21] The second assumption is more problematic. When goods are normal and issues of redistribution are left aside, richer individuals prefer larger public goods contribution levels than poorer individuals. In contrast, a rich agent's attitude towards tax rates in a redistributive income tax setting is driven by an interplay of two opposing forces: preference normality leads them to prefer larger consumption levels of the public good. On the other hand, rich agents suffer from larger tax payments than poorer individuals. Their overall stance towards higher taxes must balance these two effects.

Condition (2) imposed in Glomm and Ravikumar can hold for a variety of utilities, in particular, for Cobb Douglas preferences.[22] With C-D preferences and linear income taxes preferred tax rates are actually *invariant* in income, which means that every individual in society prefers exactly he same tax rate. The requirement does not hold, for example, when public and private goods are perfect complements: in this case, t_i^* is strictly increasing in y_i because

[21] For heterogenous preferences, this assumption will not hold.

[22] Remember also that the topping-up models analyzed in the literature always make the opposite assumption: In these models [Epple-Romano, 1996, 2003; Fernandez-Rogerson, 2003], an individual's preferred linear income tax rate in a public system is assumed to be *increasing* in income. One can easily show that with the GR assumption, equilibrium tax rates in the topping up model would again be identical to those under pure public provision.

saving money for private consumption purposes has no value for an individual. It is also important to notice that Assumption (2) is *generically* violated if the public good is financed through uniform contributions, for example, for the case of school funding in homogenous neighborhoods.[23]

An important implication of the GR result, which is not discussed in their paper, is that under the assumptions stated in the Proposition, allowing opting out (two-tier) Pareto dominates the pure public provision model: nobody is made worse off, but individuals who leave the public system do strictly better by revealed preference. It is never in anyone's interest to prohibit opting out. This is in contrast to the topping-up literature where allowing topping up hurts the middle class. On the other hand, as noted above, an individual's preferred tax increasing in the individual's income is also implicit and critical in the slippery slope argument. So condition 2 assumes away that argument.

In what follows, we will concentrate on filling the gap left by the GR analysis. While for the most part we will retain the assumption that $\hat{t}_i$ increases in y_i (essentially ruling out preference heterogeneity), we will allow t_i^* to increase in y. Unfortunately, for these cases equilibrium existence cannot be taken for granted.

From now on, we make the following assumption.

Assumption: Each agent's preferred tax rate t_i^* is strictly increasing in income y_i.

The analysis proceeds with a series of preliminary lemmas.

Lemma 1 No tax rate $t \in]0, \hat{t}_m]$ can be an equilibrium tax rate.[24]

Proof: By definition of $\hat{t}_m$, utilities of more than half of the individuals (individuals with $y_i \geq y_m$) fall over t in the range $[0, \hat{t}_m]$. Hence, the result follows.

[23] It also does not necessarily hold when preferences are Cobb-Douglas but the tax system is generalized to $t\overline{y} + T$ with the exogenous $T \neq 0$ as in our example.

[24] In our example it is $t \in] - T/\overline{y}, \hat{t}_m]$

Lemma 2 No tax rate $t > t^*_m$ can be an equilibrium tax rate.

Proof: By the definition of t^*_m and because $\hat{t}_m < t^*_m$, utility is falling in t for any $t > t^*_m$ for a majority of the population (individuals with $y_i \leq y_m$). Hence, no such tax rate can beat t^*_m in majority vote.

At this point it becomes useful to partition the total population into two groups of identical size. The high-demand H group comprises all individuals with a threshold tax $\hat{t}_i$ larger than $\hat{t}_m$. By assumption, individuals from this group also exhibit optimal tax rates $t^*_i > t^*_m$. Second, L individuals are those with $\hat{t}_i < \hat{t}_m$ and $t^*_i < t^*_m$.[25] Let

• $F_H(t)$ be the proportion of H individuals with threshold tax rates $\hat{t}_i$ weakly smaller than t,

and

• $F_L(t)$ be the proportion of L individuals with preferred tax rate t^*_i weakly smaller than t.

Notice that since $\hat{t}(y)$ is strictly increasing in y, the inverse function $\hat{y}(t) = \hat{t}^{-1}(y)$ exists and is increasing in y. Hence, the function $F_H(t) = F_H(\hat{y}(t))$ exists. Likewise, since $t^*(y)$ is increasing in y, there exists an inverse function $y^*(t)$ that is increasing in t and can be used to generate $F_L(t) = F_L(y^*(t))$.

Consider the tax rate interval $[\hat{t}_m, t^*_m]$. By construction, $F_L(t)$ is either zero (if the peak of the lowest-income individual is beyond $\hat{t}_m$) or positive at the lower bound $\hat{t}_m$, and reaches $F_L(t) = 1$ at the interval's upper bound. Similarly, $F_H(\hat{t}_m) = 0$ by construction; at the upper boundary t^*_m, $F_H(t)$ is below unity if some rich individuals still exit at this tax rate, and $F_H(t) = 1$ if this is not the case.

Using these definitions, we now have the following result:

[25] With preference homogeneity and unequal incomes the H group would be the richer half of the population.

Proposition 1: Suppose that an equilibrium with positive tax rate t^* exists. Then, t^* is within the interval $[\hat{t}_m, t_m^*]$ and described by the solution to $F_H(t^*) = F_L(t^*)$. Moreover, $t^* = t_m^*$ only if $F_H(t_m^*) = 1$, i.e., if no agent exits at this tax rate.

<u>Proof:</u> Note that $F_H(t)$ and $F_L(t)$ are both continuous and increasing in t. Let $t_i^* > \hat{t}_m$ hold for all members of society (otherwise, no equilibrium can exist – show later). By construction, $F_H(t)$ is strictly increasing at $t = \hat{t}_m$ and $F_H(t \to \hat{t}_m) \to 0$. At the same time, $F_L(t)$ is increasing but $F_L(t) = 0$ in some nonempty interval $[\hat{t}_m, t_l^*]$, with agent l being the poorest agent in society.[26] Hence, $F_H(t) > F_L(t)$ for t slightly larger than t. In addition, $F_L(t_m^*) = 1$ and $F_H(t_m^*) \leq 1$, with inequality in the latter case unless no individual exits at tax rate t_m^*, i.e., $\hat{t}_i < t_m^*$ for the highest-preference agent. Accordingly, $F_L(t_m^*) \geq F_H(t_m^*)$ and by continuity, there exists (at least one) tax rate $t = t^*$ that satisfies $F_H(t) = F_L(t)$. Suppose that tax rate t^* is unique, and note that at t^* a majority of the population opposes a change of the tax rate in either direction.[27] To show that tax rate t^* is the only potential equilibrium tax rate, consider some $t \neq t^*$ within the interval: for any such t, a majority of the population prefers a slightly smaller (if $t > t^*$) or slightly larger (if $t < t^*$) over t. At $t > t^*$ so that $F_H(t) < F_L(t)$, the fraction of L individuals beyond their preference peak exceeds the fraction of H individuals with threshold tax before t. Since L and H individuals have the same mass, a majority of the population thus prefers a reduction of the tax rate below t. For $t < t^*$ so that $F_L(t) < F_H(t)$, an analogous argument applies. Only at tax rate t^*, a local

[26] Alternatively, we can assume that a fraction z of lowest-income earners pay no taxes. If these individuals are assigned to the H group, we obtain $F_H(\hat{t}_m) > 0$. In this case, $F_H(\hat{t}_m + \epsilon) > F_L(\hat{t}_m + \epsilon)$ for some small positive ϵ) does not require the assumption that $t_i^* > \hat{t}_m$ for each agent i.

[27] A sufficient condition for uniqueness is $dF_H/dt < dF_L/dt$. This condition will be satisfied in all our numerical examples below. Otherwise, if multiple values t^* exist, one of those tax rates will generically beat any other in pairwise comparison. Only this t^* then is the potential equilibrium tax rate. For convenience, we disregard these complications in what follows.

deviation from the tax rate in either direction is opposed by a majority. □

This result is illustrated in Figure 2 in the appendix.

Proposition 1 provides an appealing characterization of the potential equilibrium tax rate. In most applications, the functions F_H and F_L can often be solved explicitly. We must be aware, however, that while necessary for an equilibrium, the condition given in Proposition 1 is not sufficient. To see why, notice that the condition considers only local conditions in the neighborhood of a given tax rate. However, since preferences are not-single peaked, we must make sure that the identified t^* beats not only its immediate neighbors, but *any* other tax rate from the relevant interval.

To provide a sufficient condition, it is useful to define the following two distribution functions, again defined over the interval $t \in [\hat{t}_m, t_m^*]$. Let

- $\hat{F}_H(t)$ be the proportion of H agents who prefer t^* over t. Note that $\hat{F}_H(t)$ is positive at the lower bound: agents with a preference kink close enough to this lower bound prefer t^* over t. $\hat{F}_H$ is also increasing in t for $t \leq t^*$ because as t increases, more and more H agents feature kinks at a tax rate smaller than t.

- $\hat{F}_L(t)$ is the proportion of L agents who prefer t over t^*. At the lower bound, this fraction will (usually) be positive because agents with peak close to $\hat{t}_m$ prefer t. Moreover, for increasing $t < t^*$, the function $\hat{F}_L(t)$ increases because more and more L individuals peaked at a tax rate smaller than t.

Notice that each H individual with a preference kink smaller than t prefers t^* over t if $t < t^*$.[28] At given t, the proportion of these individuals within the H population is $F_H(t)$. But in addition, even some H individuals with kink beyond t prefer t^* over t, so that $\hat{F}_H(t) \geq F_H(t)$ for any $t < t^*$. For t approaching t^* from below, the two functions converge and become identical at $t = t^*$. For any

[28] Remember that for any such individual i, the preferred public-system tax rate is larger than t_m^*, the upper bound of the interval under consideration.

$t > t^*$, the picture changes and function $\hat{F}_H(t)$ displays a discontinuity at t^*: now, all $F_H(t)$ individuals with kink prior to t prefer t over t^*, which means that for taxes t slightly larger than t^*, the $\hat{F}_H(t)$ moves discontinuously and $\hat{F}_H(t)$ converges to $1 - F_H(t)$ from above. Moreover, as t grows, $\hat{F}_H(t)$ falls but always remains above $1 - F_H(t)$ because even some individuals with kink prior to t prefer t^* over t.

For $\hat{F}_L(t)$, the picture is similar: at any $t < t^*$, function $\hat{F}_L(t)$ is positioned above the function $F_L(t)$.[29] For $t = t^*$, both functions become identical but $\hat{F}_L(t)$ becomes discontinuous at this tax rate. Figures 3a and 3b in the appendix illustrates $\hat{F}_H(t)$ and $\hat{F}_L(t)$.

Using this notation, we can now state

Proposition 2: Suppose $\hat{F}_H(t) \geq \hat{F}_L(t)$ for any $t \in [\hat{t}_m, t_m^*]$. Then, t^* is the equilibrium tax rate if t^* beats $t = 0$ in majority vote.

Remember that for $t = t^*$, the identities $F_L = \hat{F}_L$ and $F_H = \hat{F}_H$ apply, and $\hat{F}_H(t^*) = \hat{F}_L(t^*)$. This property helps to establish the following result:

Corollary: Let $\hat{F}_H(t) > \hat{F}_L(t)$ at $t = \hat{t}_m$. Further, let $d\hat{F}_H(t) < d\hat{F}_L(t)$ for any $t < t^*$ and $d\hat{F}_H(t) > d\hat{F}_L(t)$ for $t > t^*$. Then, t^* is the equilibrium tax rate if t^* beats $t = 0$ in majority vote.

Proof: Under the conditions given, $\hat{F}_H(t)$ lies above $\hat{F}_L(t)$ for any $t \in [\hat{t}_m, t_m^*]$. The result follows. $\square$

Remark: in special cases, there are simpler ways to check the sufficiency condition. For example: suppose $t^* = t_m^*$ (implies: no H individual exits in equilibrium). Then: if all H people prefer t^* over $\hat{t}_m$ they also prefer t^* over any

[29] This is because not only the subset of L individuals with kink before t prefer t over t^*, but some of the other L individuals share these relative preferences.

$t \in [\hat{t}_m, t^*]$. Why? All H individuals have threshold tax rates in this interval. Hence, they prefer either t^* or $\hat{t}_m$ over any other tax rate from the interval.

5 Welfare Analysis

Proposition 3: Suppose the equilibrium tax rate in the exit model is $t^* < t^*_m$ (i.e. some rich opt out at t^*_m). Then, a majority of the population prefers the system where opting out is allowed (two-tier) over a system where it is not. Like the topping up literature but, unlike Glomm and Ravikumar, the 'middle class' is hurt and thus which system is "better" depends on one's theory of justice.

Proof: In the opting-out model, a majority of the population prefers t^* over t^*_m by the definition of majority vote equilibrium. Notice now that *for any tax rate*, *each* agent's utility in the two-tier model with exit is at least as large as his utility in the standard one-tier model of public provision: if the individual does not opt out, he is doing equally well while if he opts out, he is doing strictly better by revealed preference.[30] Hence, when an individual prefers t^* over t^*_m in the exit model because the former tax rate makes him better off, he must also be better off compared to the system where exit is not allowed, and where the equilibrium tax rate t^*_m is chosen. □

The result conveys that there is unambiguous majority support in favor of the exit system. This is true even though introducing a two tier system affects the equilibrium tax rate, and the voting behavior of economic agents is rational. The finding is based on a simple revealed preference argument. A majority of people including all poor and very rich individuals prefers t^* over t^*_m, while implementing t^*_m instead would already provide them with a utility at least weakly larger than in the model without exit. Hence, they clearly prefer the

[30] If the public service was not a pure public good, the argument is even stronger. Each agent's preferences are biased towards the exit system because individuals who opt out raise the quality of the public service for stayers, so exit makes everybody *strictly* better off.

two tier over the one tier institution.

In contrast to the main result in GR, though, a minority comprising the middle class in society will oppose the exit system. This is potentially true even for individuals from this group who take advantage of opting out in equilibrium.[31] Since system switches usually requires the consent of sizable super majorities of the population, our result casts some doubt on whether society will be willing to make a transition from a one-tier to a two-tier institution. Note also that our result is completely analogous to a similar result in the topping up literature: introducing a two tier system is majority but not Pareto preferred, and results confirm the slippery slope argument in that taxes and public service quality fall relative to the one tier system.

6 Limitations and Extensions

Our technical analysis treats the publicly provided good in question as perfectly non-rival. With this assumption, the provision level (or quality) of the public service is independent of the number of individuals who potentially leave the public system. While certainly restrictive, this scenario is adopted to avoid additional complications regarding single peaked preferences. To understand why, suppose the good in question has some private-good characteristics, in which case $q(t, n(t), \alpha) = t\bar{y}/n(t)^{\alpha}$, $\alpha \in [0, 1]$, with $\alpha = 1(0)$ representing the extreme cases of a perfectly rival (perfectly non-rival) good, and $n(t)$ indicates the number of individuals who stay in the public system for given t and thus consume the public service. Since $\hat{t}_i(y_i)$ is increasing in y_i the number of system users $n(t)$ positively depends on t. One immediately verifies that $q(\cdot)$ is not

[31] Consider an individual who prefers t_m^* over t^* in the one-tier system. Suppose this individual opts out at tax rate t^* in the two tier system but would still prefer t_m^* over t^*. Thus, she is worse off compared to the one-tier system even though she exits. (Note: the argument requires the individual to find exit dominated at the larger tax rate t_m^*. Otherwise, her payoff under tax rate t^* is in the two-tier system is necessarily larger than under tax rate t_m^*, and by our previous arguments she must prefer the two-tier over the one-tier system).

necessarily increasing in t – an increase in funding may be offset by an increase in the number of additional users, leading to more queuing and lower quality of service. More generally, it is far from clear that any agent's stage 1 preferences are single-peaked in t even in the tax range where she does not consider exit: the propensity at which increases in t lead more high-preference individuals to stay in the public system depends on the income distribution.[32] As a consequence, an individual's optimization program with respect to t will often be non-convex.

While this may be seen as unfortunate, let us now argue that restricting attention to purely public goods ($\alpha = 0$) where this problem does not arise already reveals all interesting insights of the exit model. Suppose $\alpha > 0$ instead and assume preferences *are* well behaved.[33] Then, the equilibrium tax rate $t^*(\alpha)$ (if it exists) will always be *smaller* than the tax rate for the case $\alpha = 0$. To understand this, consider $t^*(0)$. At this tax rate, a majority individual in a setting where $\alpha > 0$ has an incentive to lower the tax rate because a lower tax rate triggers a positive effect that was *not* present in the pure public goods case. In other words, allowing for private goods characteristics does not have a qualitative impact on the results of the present paper. In particular, the equilibrium tax rate is still below t^*_m, and welfare comparisons across regimes would remain valid even in a more general formulation.

This is a preliminary paper and we will be extending it along a number of dimensions. We will explore examples with non-homothetic preferences where progressive income tax systems will be consistent with t^*_i increasing in income. This would allow the study of more progressive systems where the poor do not pay taxes which would change results in potentially interesting directions. If the very poor do not pay taxes they become the highest demanders. So

[32] GR allow for publicly provided goods with private goods characteristics, but their general equilibrium formulation suppresses this issue: individuals at the voting stage act shortsightedly because they take the number of people that exit as given when deciding on the tax rate.

[33] Certainly, this requires assumptions on the underlying income distributions.

this additional bit of reality (absent in the literature) will possibly simplify the analysis as they have single peaked preferences and will change welfare conclusions as these very poor are never made better off by allowing opting out and thus will never form a coalition with the rich in support of allowing opting out (two-tier). Finally we will explore conditions (if they exist) under which allowing opting out would lead to a complete collapse of the public system.

7 Appendix: An Illustrative Example

7.1 Solving for t^*

We will employ a slight generalization of the tax structure above; total tax revenue equals $t\overline{y} + T$ where T is an exogenous uniform head tax, and $\bar{y}$ the average income in society. Nothing of significance changes: equation (1) still characterizes t_i^*. We will also assume Cobb-Douglas preferences or

$$\text{Assumption: } U = q^{\alpha}c^{1-\alpha}$$

We restrict the exogenous T to

$$T < y_i \quad \forall i.$$

Let us define t^L and t^U as the lower and upper bound of allowable income tax rates, respectively. Given the Inada conditions, for $c > 0$ necessary and sufficient is $t^U < \frac{y_i - T}{y_i}$. For $q > 0$ necessary and sufficient is $t^L > -T/\overline{y}$. Note that for $T = 0$ this reduces to $t^L > 0$ and $t^U < 1$.

>From (1) in main text the most preferred tax rate for individual i is,

$$t_i^* = \alpha - T(\frac{\alpha}{y_i} + \frac{1-\alpha}{\overline{y}})$$

Notice it is independent of income for $T = 0$ as in Glomm and Ravikumar. It is increasing in income for $T > 0$ or taxation which is less redistributive than proportional (but more redistributive than uniform head taxation). Thus, Assumption: $T > 0$

Note that $t_i^* - t^U < 0$ for $(y_i - T) \geq 0$ or every individual's most preferred tax is less than t^U as required for $c > 0$.

In solving for $\widehat{t}$ there are two roots one of which implies $c = 0$ and is thus eliminated. The other is potentially a higher order polynomial for $0 \leq \alpha \leq 1$. So,

Assumption: $\alpha = 1/2$

Then,

$$\widehat{t}_i = \frac{y_i - 5T}{y_i + 4\overline{y}}$$

Notice for $5T - y_i > 0$ the threshold is negative in which case individual i would not opt out for any positive $t < t^U$. Also the threshold is increasing in y, decreasing in T, and when positive, decreasing in $\overline{y}$.

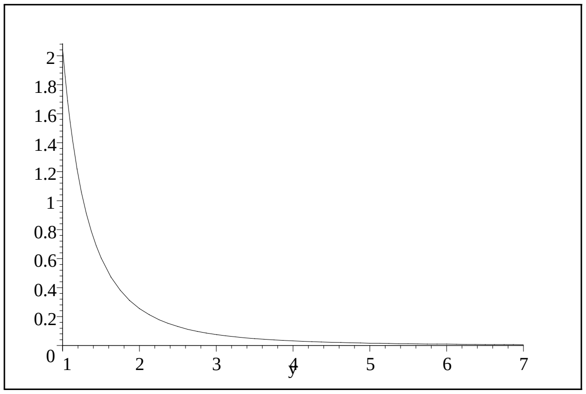
We have also assumed in our work that $t_i^* - \widehat{t}_i \geq 0$, here

$$t_i^* - \widehat{t}_i > 0 \Leftrightarrow (4\overline{y} - y_i) > 0$$

In what follows we will assume the Bounded Pareto Distribution for our income distribution; with shape parameter equal to two and a lower bound income equal to one and an upper bound income equal to seven or

Assumption: $f(y) = \dfrac{k y_L^k y^{-k-1}}{1 - (\frac{y_L}{yH})^k}$ with $k = 2,\ y_l = 1,$ and $y_H = 7$ or $f(y) = \dfrac{49}{24y^3}$

The graph is,



In this case the median and mean incomes are

$$y_m = \frac{7}{5} \text{ and } \overline{y} = \frac{7}{4}$$

Note that $4\overline{y} = y_H$ so $t_i^* - \widehat{t_i} \geq 0$ with equality only for the richest individual. In the absence of the possibility of opting out, the voting equilibrium tax rate is

$$t_m^* = \alpha - T(\frac{\alpha}{y_m} + \frac{1-\alpha}{\overline{y}}) = \frac{1}{2} - \frac{9}{14}T$$

Also

$$\widehat{t}_m = \frac{1}{6} - \frac{25}{42}T$$

In generating $F_H(t)$ and $F_H(t)$ we require the inverse functions:

$$\widehat{y}(t) = \frac{7t + 5T}{1 - t}$$

$$y^*(t) = \frac{7T}{7 - 14t - 4T}$$

These are applicable for the income range $1 \leq y_i \leq 7$.

$$
\begin{aligned}
F_H(t) &= 2\int_{7/5}^{\frac{7t+5T}{1-t}} \left(\frac{49}{24y^3}\right) = \frac{1}{24}\frac{(28t+7+25T)(42t+25T-7)}{(7t+5T)^2} \\
\text{so } F_H(\widehat{t}_m) &= 0 \text{ and } F_H(t_m^*) = \frac{7(T+3)(7-T)}{3(7+T)^2}
\end{aligned}
$$

Notice t_m^* is less than the required $t < 1/2 - \frac{5}{14}T$ for $\frac{7t+5T}{1-t} \leq 7$ for the inverse functions to be applicable. Also notice that $F_H(t_m^*) = \frac{238}{243}$ at $T = 1/5$ which implies that aproximately the richest 1% of the population (2% of the H population) opts out of the public system at t_m^*.

$$
\begin{aligned}
F_L(t) &= 2\left(1/2 - \int_{\frac{7T}{7-14t-4T}}^{7/5} \frac{49}{24y^3}\right) \text{ for } t \geq \frac{1}{2} - \frac{11}{14}T \text{ or} \\
F_L(t) &= 1 - \frac{1}{24}\frac{49-196t-56T+196t^2+112tT-9T^2}{T^2} \text{ for } t \geq \frac{1}{2} - \frac{11}{14}T \\
F_L(t) &= 0 \text{ for } t < \frac{1}{2} - \frac{11}{14}T \\
\text{with } F_L(\widehat{t}_m) &= 0,\ F_L\left(\frac{1}{2} - \frac{11}{14}T\right) = 0, \text{ and } F_L(t_m^*) = 1
\end{aligned}
$$

That $F_L(t)$ is zero over the range $t = \widehat{t}_m$ to $t = \frac{1}{2} - \frac{11}{14}T$ and is consistent with the lowest income individual's most preferred tax rate being larger than $\widehat{t}_m$ as in this example.

Now to solve for t^* set $F_L(t) = F_H(t)$. This is a solution to a higher order polynomial so we will choose $T = 1/5$ which yields a $\widehat{t}_L = 0$.

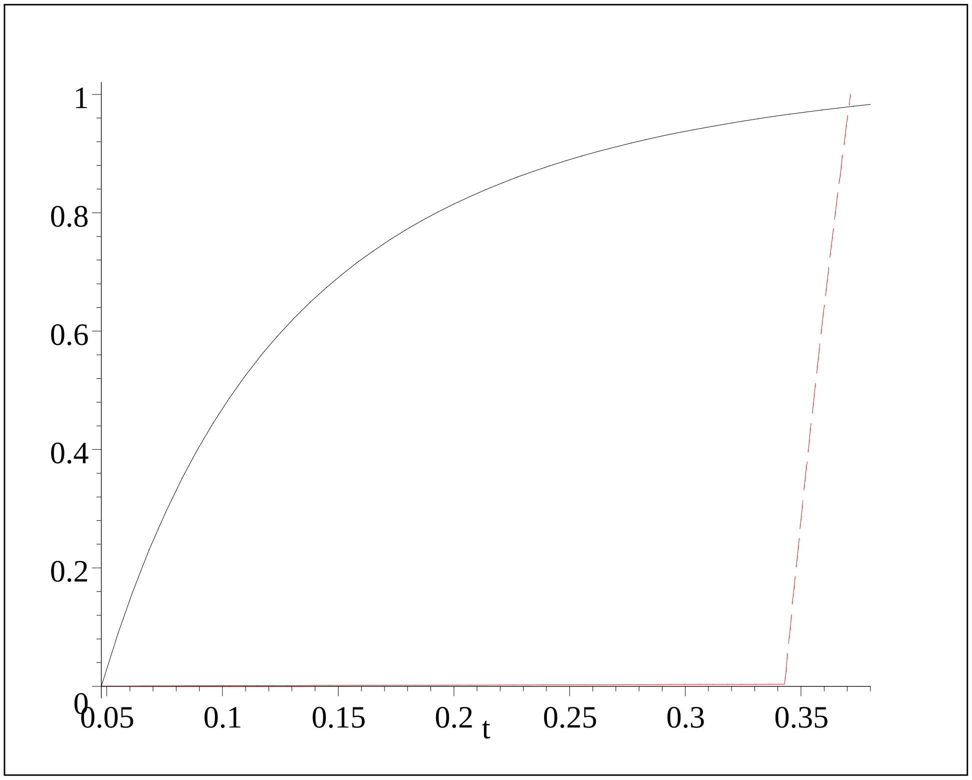


Figure 2: $F_L(t)$ (dashed) and $F_H(t)$ against $t \in (\widehat{t}_m, t_m^*)$ with t^* at the intesection.

Further,

$$t^* = 0.37072$$
$$\widehat{t}_m = 0.04762$$
$$t_m^* = 0.371\,43$$

At t^* an additional 0.2% of the population opts out

7.2 Sufficient conditions

Consider the range $\widehat{t}_m$ to t^*

Defining $\widehat{F}_H(t)$: Which of the H types prefer t^* to t in this range? All those with their threshold tax at or below t prefer t^* to t and for those with their threshold tax between t^* and t, some with their threshold tax rate (kink) sufficiently close to t will prefer t^* to t. These are the relatively poor, rich.

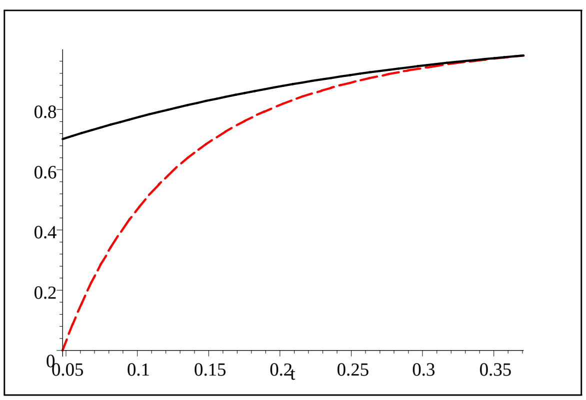
Define $U^0(t) = (\alpha(1-t)y_i - \alpha T)^\alpha((1-\alpha)(1-t)y_i - (1-\alpha)T)^{1-\alpha}$ as utility for i when out of the public system and $U^I(t) = (t\overline{y} + T)^\alpha((1-t)y_i - T)^{1-\alpha}$ when in. Define $y_{HL}(t)$ as the $y(t) > 7/5$ such that $U^0(t) = U^I(t^*)$, then all high types with $7/5 < y < y_{HL}$ will prefer t^*. Using $t^* = 0.370\,72$, $\alpha = 1/2$, and $T = 1/5$. Solving for $y_{HL}(t)$ leads to two roots, but only one which has $y > 7/5$ for $t \in (\widehat{t}_m, t^*)$.

$$y_{HL}(t) = -0.1\frac{-12.\,682 + 2.0t - 1.0\sqrt{(-67.901\,(t + 0.648\,62)\,(t - 2.019\,3))}}{(t - 1.0)^2}$$

Then,

$$\widehat{F}_H(t) = 2\int_{7/5}^{y_{HL}(t)} \frac{kL^k y^{-k-1}}{1 - (\frac{L}{H})^k} = \frac{25}{24} - \frac{49}{24(y_{HL}(t))^2}$$

Graphically, $\widehat{F}_H(t)$ is upward sloping and above $F_H(t)$ (dashed) until t^* is approached where they converge.



Defining $\widehat{F}_L(t)$: Which of low types prefer t to t^*in the range?

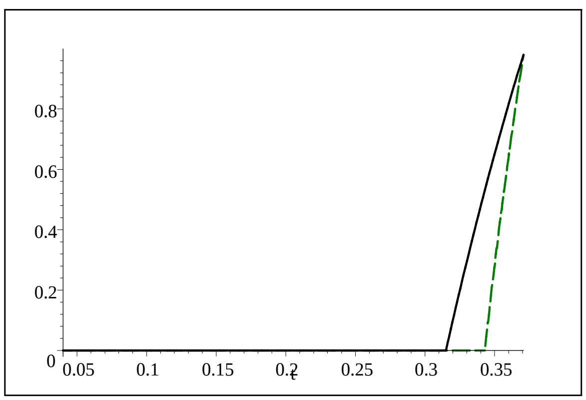
All those with their peak at or below t prefer t to t^* and for those with their peak between t and t^*, those with their peak sufficiently close to t will prefer

t to t^*. These are the relatively poor, poor individuals. Using $t^* = 0.370\,72$, $\alpha = 1/2$, and $T = 1/5$. Define $y_{LL}(t)$ as the $1 < y_{LL}(t) < 7/5$ such that $U^I(t) = U^I(t^*)$ then all lows with $y < y_{LL}$ will prefer t to t^*. This yields one root,

$$y_{LL}(t) = \frac{7}{18.\,025 - 35t} \text{ for } y_{LL}(t) \geq 1 \text{ or } t \geq 0.315$$

$$\begin{aligned} \widehat{F}_L(t) &= 51.042\,(t - 0.315)\,(0.715\,00 - t) \;\text{ for } t \geq 0.315 \\ \widehat{F}_L(t) &= 0 \text{ for } t < 0.315 \end{aligned}$$

Graphically, $\widehat{F}_L(t)$ is upward sloping for $t > 0.315$ and above $F_L(t)$ (dashed) until t^* is approached where they converge.



Finally, as required $\widehat{F}_H(t) > \widehat{F}_L(t)$

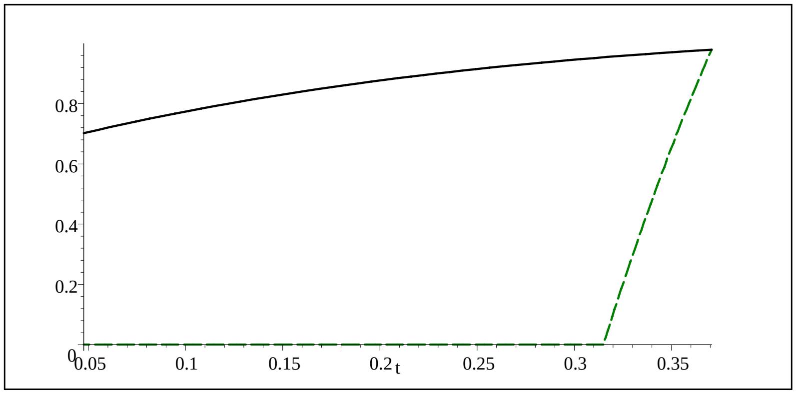


Figure 3a: $\widehat{F}_H(t)$ and $\widehat{F}_L(t)$ (dashed) against $t \in (\widehat{t}_m, t^*)$

Consider the range t^* to t_m^*

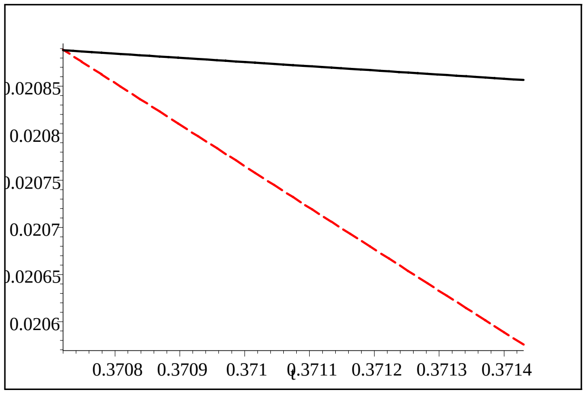
Defining $\widehat{F}_H(t)$: Which of high's prefer t^* to t in the range? All those with their threshold tax at or above t prefer t^* to any t in the range (including those who exit). For those with their threshold tax between t^* and t some with their threshold tax sufficiently close to t will prefer t^* to t. These are the relatively rich, rich individuals. Define $y_{HH}(t)$ as the $y(t) > 7/5$ such that $U^0(t^*) = U^I(t)$ then all high types with $y > y_{HH}$ will prefer t^* over t. The solution to the problem is two roots, but only one which has $y \geq 7/5$ for $t \in (\widehat{t}_M$ to $t^*)$.

$$y_{HH}(t) = 1.3279 - 8.8385t^2 + 7.8284t + 8.8385\sqrt{((t + 0.14256)(t + 0.11429)(t - 0.81462)(t - 1.2137))}$$

Then,

$$\widehat{F}_H(t) = 2\int_{y_{HL}(t)}^{7} \frac{kL^k y^{-k-1}}{1 - (\frac{L}{H})^k} = \frac{1}{24}\frac{49 - (y_{HH}(t))^2}{(y_{HH}(t))^2}$$

Graphically it is downward sloping and diverging away from $1 - F_H(t)$ (dashed).

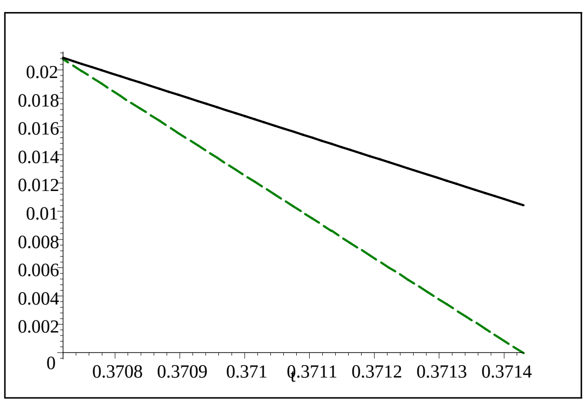


Defining $\widehat{F}_L(t)$: Which of L types prefer t to t^* in the range? All those with peak at or below t^* prefer t^* to any t. For those with peak between t^* and t, some with their peak sufficiently close to t will prefer t to t^*. These are the relatively rich, poor individuals. Define $y_{LH}(t)$ as the $1 < y(t) < 7/5$ such that $U^I(t^*) = U^I(t)$ then all low types with $y > y_{LH}$ will prefer t to t^*. This yields one root which satisfies $1 < y(t) < 7/5$ for $t \in (\widehat{t}_M, t^*)$.

$$y_{LH}(t) = \frac{7}{18.025 - 35t}$$

$$\widehat{F}_L(t) = \int_{y_{LH}(t)}^{7/5} = \frac{1}{48} \frac{49 - 25(y_{LH}(t))^2}{(y_{LH}(t))^2}$$

Graphically, $\widehat{F}_L(t)$ is downward sloping diverging away from $1 - F_L(t)$ (dashed).



As required $\widehat{F}_H(t) > \widehat{F}_L(t)$

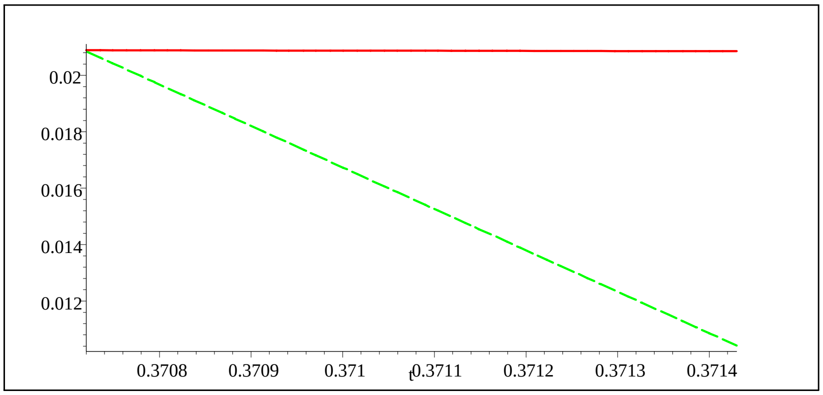


Figure 3b: $\widehat{F}_H(t)$ and $\widehat{F}_L(t)$ (dashed) against $t \in (t^*, t^*_m)$

The final check is whether a majority prefer t^* over $t = -T/\overline{y}$ where the public sector effectively shuts down. To evaluate this possibility we consider

$$U^I(t^*) - U^O(-T/\overline{y}) = \sqrt{\left((0.37072)7/4 + \frac{1}{5}\right)}\sqrt{\left((1-0.37072)\,y - \frac{1}{5}\right)} - \sqrt{y/2}\sqrt{y/2} \text{ over } y \in (1,7)$$

This is positive at $y_i = 1$, downward sloping in the income range and zero at $y_i = 1.7480 > 7/5$. therefore a majority of the population do not prefer the shutdown of the public sector and t^* is the equilibrium outcome.

8 References

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