The design of pension plans with endogenously determined fertility, education, and ability types

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Abstract

This paper studies the design of pension schemes in an overlapping generations model with endogenous fertility and human capital accumulation. Every generation consists of high earners and low earners with the proportion of types being determined endogenously. The number of children is deterministically chosen but the children's future ability is in part stochastic, in part determined by the family background, and in part through education. In addition to the customary externality source associated with a change in average fertility rate, this setup highlights another externality source. This is due to the effect of a parent's choice of number and educational attainment of his children on the proportion of high-ability individuals in the steady state. Our results include: (i) Investments in education of high- and low-ability parents must be subsidized, (ii) direct child subsidies to one or both parent types can be negative; i.e., they can be taxes, (iii) net subsidies to children (direct child subsidies plus education subsidies) to high-ability parents are always positive, and to low-ability parents can be positive or negative, (iv) either education subsidies or child subsidies, when used alone, can dominate the other instrument, (v) using child subsidy instruments alone entails a higher fertility rate and a lower ratio of high- to low-ability children, as compared to using education subsidies alone.

JEL classification: H55; J13.

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1 Introduction

One of the most pressing problems facing the economies of the industrialized world is the fiscal solvency of their pay-as-you-go (PAYGO) social security systems.[1] An important contributing factor to this problem has been the recent drastic fertility declines in Western Europe and Japan. What truly determines fertility, and what accounts for the observed evolution in fertility behavior, are still open questions. What is clear, however, is that, faced with a PAYGO social security system, parents do not have the right incentives to choose a fertility rate that is optimal. In such systems, each person's fertility decision affects the economy's population growth rate and with it everybody's pension benefits. Specifically, an increase in the rate of population growth increases the number of future workers who will have to support a retired person. No individual, however, takes this impact into account and that leads to a decentralized equilibrium outcome with too few children.[2]

The above problem is exacerbated by another externality associated with the "quality" of children, and their human capital accumulation, through the education decisions of parents. The rate of return of a pay-as-you-go system depends not just on the fertility rate, but also on productivity growth. The more productive the children, the higher will be their ability to produce and to pay taxes. This reinforces the public good nature of a family's child-rearing activities.[3]

Most of the literature has thus far treated the quality and quantity issues separately; or else have lumped the investments in quantity and quality together as if one decision

[1] This has led to reforms in a number of countries. See Penner (2007) who surveys the recent reforms in Canada, Germany, Italy, Japan, Sweden, and the UK.

[2] In addition to this "intergenerational transfer" effect, the literature has also noted an offsetting force called "capital dilution" effect: A higher fertility rate, given the aggregate capital saved by the previous generation, implies a lower capital to labor ratio reducing per capita output; see Michel and Pestieau (1993) and Cigno (1993).

[3] To internalize the quantity and quality effects, some economists have advocated a policy of linking pension benefits (or contributions) to individuals' fertility choices. See, among others, Abio *et al.* (2004), Bental (1989), Cigno *et al.* (2003), Fenge and Meier (2004), Kolmar (1997), van Groezen *et al.* (2000, 2003).

determines both.[4] A basic shortcoming of this approach is that it cannot distinguish between child subsidies, which correct externalities emanating from fertility decisions, and education subsidies which correct for externalities due to investing in education. This lack of distinction becomes more of a serious problem when the two types of externalities interact as they often do.

To be sure, there are a number of studies in the literature that distinguish between quantity and quality decisions and study them both in one unified framework. Peters (1995) is an early example of this. In his model, both fertility and education choices are made deterministically. The main shortcomings of his approach are the deterministic nature of both quantity and quality decisions, and the lack of any heterogeneity among parents. Cigno *et al.* (2003) also allow for both fertility and quality. Fertility is fully deterministic, but children's quality, which Cigno *et al.* define in terms of "lifetime tax contributions", is in part random and in part determined through actions of parents. The limitations of their study come from the static nature of their model, in looking at the decisions of the initial parent only, and their not allowing for heterogeneity among parents.

Cigno and Luporini (2003), while building on Cigno *et al.* (2003), allow for parents' heterogeneity in terms of their ability to influence their children's probability of success in life.[5] However, their model remains static in nature as they too do not go beyond the decisions of the initial parents. In Meier and Wrede (2008) both fertility and types are partly stochastic and partly determined by investments. The limitation of their model comes from their ignoring the impact of fertility and education investments on the distribution of types in the economy. But this induced change in the distribution of types constitutes an important component of fertility and education externalities.[6]

[4] Cremer *et al.* (2003, 2008) are examples of this latter approach, while Cremer *et al.* (2006) is concerned only with quantity decisions.

[5] They also drop Cigno *et al.*'s (2003) assumption that fertility is fully deterministic.

[6] Sinn (2004) also considers a model that allows or both fertility and quality. In his setup fertility is fully random and quality fully deterministic. However, Sinn is interested more in examining the

The current paper addresses the quantity and quality questions in an overlapping generations model with high- and low-ability individuals. The unique feature of our study is its endogenous determination of the distribution of types. Specifically, we allow for this distribution to be affected by both education and fertility decisions. This framework gives rise to three sources of externality. First, there is the customary externality associated with the change in average fertility—the intergenerational transfer effect. It arises from the fertility decisions of parents. This source of externality disappears if the pension system is a pre-funded one. The second source of externality emanates from decisions that, while keeping average fertility constant, change the distribution of types. It arises from both education decisions and fertility decisions. Its unique feature is that it does not depend on the institution of social security and exists for pre-funded systems as well. The third source of externality is due to interaction between average fertility and the distribution of types. It too arises from both education decisions and fertility decisions. It is different from the second externality source in that it exists because of the PAYGO institution and disappears if one moves to a pre-funded system. It is also different from the first externality source because it will not exist if the distribution of types were immutable.

One distinguishing element between quantity and quality decisions is that of timing. One decides on the number of children quite early; the quality of children, i.e. their future earning capacity, is determined much later. We incorporate this timing sequence in our overlapping-generations model by assuming that the young decide on starting a family and having children first. Subsequently, as parents, they decide on the extent of their children's education. Nevertheless we take a short cut when it comes to specifying the number of periods in the model. To explicitly account for the timing, one needs a model that allows for children, adults, parents, and the retired (grand parents) to overlap, requiring a four-period overlapping generations model. However, analyzing a full-fledged

properties of a traditional PAYGO system rather than the design of an optimal pension plan.

four period model quickly becomes cumbersome and too detailed for developing insights. To avoid getting bogged down in details, we resort to the familiar two-period model while assuming the decisions of having children and educating them occur sequentially just prior to the beginning of one's retirement.

We assume that parents choose the number of their children deterministically. It is true that the actual number of children in a family does not necessarily coincide with the number that parents initially intended to have.[7] However, this choice is intrinsically more deterministic and less susceptible to random and other shocks than determining the quality of one's children. As to the quality, it is unrealistic to expect that one can determine the future earning abilities of one's children in a deterministic fashion simply by investing in their education and training. We assume that quality is determined by three factors. One is random; the second is due to education; and the third is pre-determined by one's "genes" and family background. The randomness implies that investing in education does not necessarily transform a child into a high-ability type; it only increases the probability of its occurrence. We also make the simplifying assumption that all children of a particular parent turn out to be either of high- or of low-ability; no mix of high- and low-ability children is possible.

Finally, we study the design of pension systems within the Samuelson's (1958) overlapping generations framework as opposed to Diamond's (1965). We thus assume that transfer of resources to the future can occur only through a storage technology with a fixed rate of return. This approach makes the choice of PAYGO or storage to be optimally mutually exclusive: One uses one mechanism or the other depending on whether the average fertility rate[8] or the interest rate is higher. This dichotomy yields a stark picture of the externality sources that remain even in the absence of PAYGO pension plans.

[7] Infertility, premature death, misplanning and multiple births are some of the reasons explaining this gap.

[8] What Samuelson~(1958) called the "biological" rate of interest.

Figure 1:

2 The model

2.1 Preliminaries

Consider, within an overlapping generations framework, the sequence of decisions a child has to face after he is born. First, upon reaching adulthood, he has to decide on starting a family and having children. Subsequently, as a parent, he has to decide on the extent of his children's education. Finally, the retirement period arrives. Such a rich model allows for children, adults, parents, and the retired (grand parents) to overlap, requiring a four-period overlapping generations model. Figure~1 depicts this sequence. However, analyzing a full-fledged four period model quickly becomes cumbersome and too detailed for developing insights. We thus take a short cut and transform the four-period setup we have in mind into a simple two-period overlapping generations model. To do this we assume the decisions of having children and educating them occur sequentially just prior to the beginning of one's retirement; see Figure 2. This saves us from having to distinguish between working as an adult and working as a parent.

Assume each generation consists of two types of people; they posses either a high or a low earning ability. Denote high- and low-ability types by subscripts h and l and

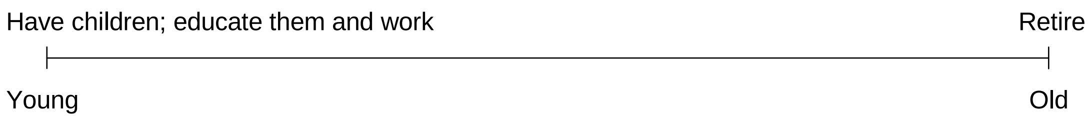

Figure 2:

let $j = h, l$. All children of a particular parent will turn out to be either of high- or of low-ability; no mix of high- and low-ability children is possible. There are three factors that determine if a child turns into a high- or a low-ability individual. One is random; the second is due to education; and the third is pre-determined by one's "genes" and family background. The randomness implies that investing in education does not necessarily transform a child into a high-ability type; it only increases the probability of its occurrence. Thus, leaving the family background aside, investing e "units" in educating a child leads to a *probability* of having a *high-ability* child equal to $\pi(e)$. Each j-type's family background changes this probability through a parameter θ_j according to $\pi_j = \pi(e) + \theta_j$.[9] Naturally, the probability that a j-type parent will have a low-ability child is $1 - \pi_j$. We assume that $\pi(\cdot)$ is an increasing and strictly concave function, $\pi(0) > 0$, and $\theta_l \leqslant \theta_h \leqslant 1 - \pi(e)$. Without any loss of generality, one can set $\theta_l = 0$ and $\theta_h = \theta \geqq 0$.

[9] This formulation assumes that there is no interaction between family background and education. That is, the marginal productivity of spending e dollars on educating one's children is the same regardless of the parent's type. An alternative assumption is that there is an interaction and that the marginal productivity of spending e dollars is higher for the more able parents. This assumption requires a multiplicative formulation for θ and e with $\pi_j = \theta_j \pi(e)$ rather than $\pi_j = \pi(e) + \theta_j$. We examine the implications of this alternative formulation below in Section .

Assume generation T consists of N_T people. Denote the proportion of high ability persons in generation T by δ_T $(0 < \delta_T < 1)$, so that the number of high ability persons in generation T is $\delta_T N_T$. Parents choose the number of the children they want to have and do so deterministically. Denote the number of children each j-type person will have by n_j. Thus $\delta_T N_T$ high-ability persons of generation T end up with $(\delta_T N_T)\, n_h \pi_h$ high-ability children and $(\delta_T N_T)\, n_h\, (1 - \pi_h)$ low-ability children. Similarly, $(1 - \delta_T)\, N_T$ low-ability persons of generation T end up with $(1 - \delta_T)\, N_T n_l \pi_l$ high-ability children and $(1 - \delta_T)\, N_T n_l\, (1 - \pi_l)$ low-ability children. Consequently, the proportion of high-ability children in the next generation will be

$$\delta_{T+1} = \frac{\delta_T N_T n_h \pi_h + (1-\delta_T) N_T n_l \pi_l}{\delta_T N_T n_h + (1-\delta_T) N_T n_l} = \frac{\delta_T n_h \pi_h + (1-\delta_T) n_l \pi_l}{\delta_T n_h + (1-\delta_T) n_l}. \tag{1}$$

2.2 Steady state

In the steady state, $\delta_{T+1} = \delta_T \equiv \delta$. It then follows from equation (1 relating δ_{T+1} to δ_T that

$$\frac{\delta n_h \pi_h + (1-\delta) n_l \pi_l}{\delta n_h + (1-\delta) n_l} = \delta. \tag{2}$$

Observe that δ is a weighted average of π_h and π_l and thus bracketed by them. Moreover, equation (2) indicates that δ is homogeneous of degree zero in (n_l, n_h). It follows from Euler's Theorem that

$$n_h \frac{\partial \delta}{\partial n_h} + n_l \frac{\partial \delta}{\partial n_l} = 0. \tag{3}$$

Let e_j denote the j-type's investment on the education of his children. Consequently, $\pi_j = \pi(e_j) + \theta_j$. One can then solve equation (2) for δ, writing the solution as $\delta = \delta(e_h, e_l, n_h, n_l)$. Introduce

$$Z \equiv 2\delta(n_h - n_l) + n_l(1 + \pi_l) - n_h \pi_h. \tag{4}$$

Differentiating (2) yields the following partial derivatives:

$$\frac{\partial\delta}{\partial e_h} = \frac{\delta n_h \pi'(e_h)}{Z}, \tag{5}$$

$$\frac{\partial\delta}{\partial e_l} = \frac{(1-\delta) n_l \pi'(e_l)}{Z}, \tag{6}$$

$$\frac{\partial\delta}{\partial n_h} = \frac{\delta(\pi_h - \delta)}{Z}, \tag{7}$$

$$\frac{\partial\delta}{\partial n_l} = \frac{(1-\delta)(\pi_l - \delta)}{Z}. \tag{8}$$

We prove in Appendix A that a sufficient condition for stability of the steady-state solution for δ, namely $|\partial\delta_{T+1}/\partial\delta_T| < 1$, is that $Z > 0$. We assume that this is the case so that

$$\frac{\partial\delta}{\partial e_h} > 0, \quad \frac{\partial\delta}{\partial e_l} > 0.$$

2.3 Laissez faire

Individuals have preferences over consumption when young, c, consumption when retired, d, and the number of children, n. Denote the non-education cost of raising a child by a and the "quantity" of education provided to a child by e. Choose the units of measurement for c, d, and e such that their producer prices are one. Preferences are represented by [10]

$$U = u(c) + v(d) + \varphi(n). \tag{9}$$

A j-type person earns an income equal to $\beta_j I$, where $\beta_h > \beta_l$, when young. Without any loss of generality, one can set $\beta_l = 1$ and $\beta_h = \beta > 1$. The young individual spends a portion of his income on his immediate consumption, c, a portion on raising his children, $a\ n$, and yet another portion on educating his children, $e\ n$. He saves the rest of his

[10] This formulation assumes one loves his children equally regardless of the child's ability. We will discuss the implications of this alternative assumption below in Section .

income in a storage technology with a rate of return equal to r. Upon retirement, the individual receives and spends all his savings plus interest, leaving no bequests.

The budget constraint for the j-type is given by

$$\beta_j I = c_j + \frac{d_j}{1+r} + e_j\ n_j + a\ n_j. \tag{10}$$

The j-type young individual then chooses c_j, d_j, n_j, and e_j to maximizes his utility (9) subject to his budget constraint (10). One can easily see that the solution for education expenditures requires $e = 0$. This is not surprising given that education is costly to the parent but bestows no utility on him.[11] Setting $e = 0$ and manipulating the first-order conditions with respect to c_j, d_j, and n_j, the laissez faire solutions for these variables are found from

$$\frac{v'(d_j)}{u'(c_j)} = \frac{1}{1+r}, \tag{11}$$

$$\frac{\varphi'(n_j)}{u'(c_j)} = a, \tag{12}$$

$$\beta_j I = c_j + \frac{d_j}{1+r} + n_j a. \tag{13}$$

Given strong separability and concavity of all subutility functions, c, d, and n are all normal goods so that $c_h > c_l, d_h > d_l$, and $n_h > n_l$. This result is summarized as

Proposition 1 *Consider an overlapping generations model in the steady state with two types of people in each generation: high and low ability. Each type receives an income commensurate with his ability when young and has preferences over the number of children he will have as well as consumption during working years and retirement. Each type can have children of either ability. Then*

(i) The probability of having a high-ability child depends positively on investment in education and is higher, ceteris paribus, for high-ability parents. The proportion of high-

[11] This result is due to the assumption that parents love children of the same ability equally. If parents prefer a high-ability child to a low-ability child, then their utility will be affected through educational attainment of their children. Under this circumstance, $e \neq 0$. See Section below.

ability persons in a given generation, δ, moves positively with investment in education, but may move in either direction with the number, of children by both types of parents.

(ii) In the laissez-faire, high-ability parents have a higher number of children and consume more during working years and retirement. Neither types invests in education.

3 Utilitarian First Best

3.1 The problem and its solution

Denote the savings of an individual of type j by $S_j \geqq 0$ and the population growth rate by

$$\overline{n} \equiv \delta n_h + (1-\delta) n_l. \tag{14}$$

The economy's resource constraint is then written as

$$\begin{aligned}&[\delta\beta + (1-\delta)]\, I + \frac{[\delta S_h + (1-\delta) S_h]\,(1+r)}{\overline{n}} \geqq \delta \big[c_h + \frac{d_h}{\overline{n}} + \\ &n_h\,(a+e_h) + S_h \big] + (1-\delta)\,\big[c_l + \frac{d_l}{\overline{n}} + n_l\,(a+e_l) + S_l \big].\end{aligned} \tag{15}$$

Given a fixed rate of return on savings, the consumption of the retired should be financed either through private savings or from taxes imposed on the young as in a pay-as-you-go retirement system. The mechanism with a higher rate of return, r or $\overline{n}$, Samuelson's (1958) biological rate of return, should be used exclusively. Expositionally, it will be simpler to consider the social planner's problem sequentially. First, one finds the optimum conditional on the use of storage and PAYGO; then one compares the associated welfare levels of the two conditional optima. We study the more interesting case of PAYGO in the text, and discuss the storage technology in Appendix B.

In the absence of private savings, the economy's resource constraint is simplified to

$$[1 + (\beta - 1)\delta]\, I \quad \geqq \quad \delta \left[c_h + n_h\,(a+e_h) + \frac{d_h}{\overline{n}} \right] + (1-\delta) \left[c_l + n_l\,(a+e_l) + \frac{d_l}{\overline{n}} \right]. \tag{16}$$

The government's optimization problem is then summarized by the Lagrangian

$$\begin{aligned}
\pounds &= \delta\left[u(c_h)+v(d_h)+\varphi(n_h)\right]+(1-\delta)\left[u(c_l)+v(d_l)+\varphi(n_l)\right] \\
&+\mu\left\{\left[1+(\beta-1)\delta\right]I-\delta\left[c_h+n_h\left(a+e_h\right)+\frac{d_h}{\overline{n}}\right]\right. \\
&\left.-(1-\delta)\left[c_l+n_l\left(a+e_l\right)+\frac{d_l}{\overline{n}}\right]\right\},
\end{aligned}$$

leading to the following first-order conditions with respect c_h, c_l, d_h and d_l: They are:

$$\frac{\partial \pounds}{\partial c_h} = \delta[u'(c_h)-\mu]=0, \tag{17}$$

$$\frac{\partial \pounds}{\partial c_l} = (1-\delta)[u'(c_l)-\mu]=0, \tag{18}$$

$$\frac{\partial \pounds}{\partial d_h} = \delta[v'(d_h)-\frac{\mu}{\overline{n}}]=0, \tag{19}$$

$$\frac{\partial \pounds}{\partial d_l} = (1-\delta)[v'(d_l)-\frac{\mu}{\overline{n}}]=0. \tag{20}$$

Manipulating these conditions yield

$$c_h=c_l=c, \text{ and } d_h=d_l=d.$$

3.2 Externalities and the optimal characterizations of e and n

Introduce

$$\begin{aligned}
D \equiv \frac{\partial \pounds}{\partial \delta} &= [\varphi(n_h)-\varphi(n_l)] \\
&+u'(c)\left\{(\beta-1)I-[n_h\left(a+e_h\right)-n_l\left(a+e_l\right)]+\frac{(n_h-n_l)\,d}{\overline{n}^2}\right\},
\end{aligned} \tag{21}$$

so that D shows the change in social welfare due to an increase in the proportion of high-ability persons in the population.[12] The first bracketed term on the right-hand side shows the net change in utilities, and the second bracketed expression shows the *net* change in resources (the increase in the available resources minus the extra resources

[12] Being a proportion,this is matched by a reduction in the proportion of low-ability persons.

required in consumption). Using the definition of D and the previous findings that $c_h = c_l = c$, $d_h = d_l = d$, and $\mu = u'(c)$, one can rewrite the first-order conditions for the maximization of social welfare with respect to n_h, n_l, e_h, and e_l as

$$\frac{\partial \pounds}{\partial e_h} = -\delta n_h u'(c) + D\frac{\partial \delta}{\partial e_h} = 0, \tag{22}$$

$$\frac{\partial \pounds}{\partial e_l} = -(1-\delta) n_l u'(c) + D\frac{\partial \delta}{\partial e_l} = 0, \tag{23}$$

$$\frac{\partial \pounds}{\partial n_h} = \delta\left[\varphi'(n_h) - \left(a + e_h - \frac{d}{\overline{n}^2}\right)u'(c)\right] + D\frac{\partial \delta}{\partial n_h} = 0, \tag{24}$$

$$\frac{\partial \pounds}{\partial n_l} = (1-\delta)\left[\varphi'(n_l) - \left(a + e_l - \frac{d}{\overline{n}^2}\right)u'(c)\right] + D\frac{\partial \delta}{\partial n_l} = 0. \tag{25}$$

Note that, with $\partial\delta/\partial e_h > 0$ and $\partial\delta/\partial e_l > 0$, either conditions (22) or (23) imply $D > 0$.

Recall that investing in education imposes only a cost on the individual but no benefit. Indeed, considering that the individual treats δ as given, this cost will be the only effect on him. Increasing it then will entail a cost measured by $-n_j$. Comparing this with the expressions in equations (22) and (23) reveals the existence of an externality represented by

$$\frac{D}{u'(c)}\frac{1}{\delta}\frac{\partial \delta}{\partial e_h} \quad \text{for increasing } e_h, \tag{26}$$

$$\frac{D}{u'(c)}\frac{1}{1-\delta}\frac{\partial \delta}{\partial e_l} \quad \text{for increasing } e_l. \tag{27}$$

This externality arises through the effect of e_j on δ. Moreover, given that $\partial\delta/\partial e_j > 0$ and $D > 0$, this is a positive externality.

The externality coming through δ may be divided into two parts. One is due to the direct change in δ as e_j changes. When there is an increase in the proportion of high-ability persons in the population, matched of course by a reduction in the proportion of low-ability persons, social welfare changes by the difference in the utilities of high- and low-ability persons *and* the change in the *net* resources (income minus consumption). This effect is present also in the absence of PAYGO pension plans when all second-period consumptions are financed by private savings. This is because it does not work through fertility. The second part, on the other hand, work through changing average fertility. Its existence depends on having a PAYGO pension plan in place.[13] It arises indirectly as the change in δ changes $\overline{n}$ as well. Remember that $\overline{n}$ depends on δ and δ depends on e_j (as well as n_j). This change in $\overline{n}$ is also neglected in private calculations. With $\overline{n} = n_l + \delta(n_h - n_l)$, this effect depends on the difference between n_h and n_l. The various terms in $D/u'(c)$ represent these two direct and indirect externalities. The latter is captured by the $(n_h - n_l)\, d/\overline{n}^2$ term that appears in the definition of $D/u'(c)$, and the former by the remaining expressions therein. Its existence depends also on

Similarly, increasing n_j has externalities of its own. When a j-type individual increases his fertility rate, he does not take the effect of his decision on $\overline{n}$ into consideration. He thus perceives the effect of increasing n_j in his *net* welfare.to consist of an increase in his utility, $\varphi'(n_j)/u'(c)$ when expressed in monetary units, minus an increase in his expenditures on n_j, measured by a. Comparing this with the expressions in equations (24) and (25) reveals the existence of externalities represented by

$$\frac{d}{\overline{n}^2} + \frac{D}{u'(c)} \frac{1}{\delta} \frac{\partial \delta}{\partial n_h} \qquad \text{for increasing } n_h, \tag{28}$$

$$\frac{d}{\overline{n}^2} + \frac{D}{u'(c)} \frac{1}{1-\delta} \frac{\partial \delta}{\partial n_l} \text{ for increasing } n_l. \tag{29}$$

The externalities associated with n_j consist of two distinct elements. The first element, represented by $d/\overline{n}^2$, captures the effect of increasing n_h or n_l on $\overline{n}$, and through it on the aggregate resources available for distribution between the young and the old under PAYGO. Specifically, this externality tells us that increasing fertility increases the number of future working people who support a retired person. This is the familiar positive "intergenerational transfer" effect that appears in the literature on growth with endogenous fertility; see Cigno (1993) and Michel and Pestieau (1993). The second externality source, represented by the second expressions in (28)–(29), is

[13] That only one component of the externality through δ exists for pre-funded systems is demonstrated in Appendix B.

new and due to the change in δ. It is the same type of externality discussed previously in relation to the effect of e_j on δ. The point is that the externalities that emanate from a change in δ can come about from a change in n_j in the same way as they do with a change in e_j. It is thus no surprise that these second expressions in (28)–(29) are identical to the expressions in (26)–(27) except for $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ replacing $\partial\delta/\partial e_h$ and $\partial\delta/\partial e_l$. Finally, observe that with $D > 0$, this externality source is positive if $\partial\delta/n_j > 0$ and is negative if $\partial\delta/n_j > 0$. Recall also that $\partial\delta/n_h$ and $\partial\delta/n_l$ are of opposite signs. Hence one type exerts a positive externality and the other a negative externality.

3.2.1 Optimal solutions for e_j and n_j

Turning to the characterization of the first-best solutions for e_j and n_j, substitute the expressions for $\partial\delta/\partial e_h$ and $\partial\delta/\partial e_l$ from (5)–(6) into equations (22)–(23), simplify, and subtract one equation from another to get

$$\frac{D}{u'(c)}\frac{\pi'(e_h) - \pi'(e_l)}{Z} = 0. \tag{30}$$

With $D > 0$, it follows from (30) that $\pi'(e_h) = \pi'(e_l)$. Consequently,

$$e_h = e_l \equiv e. \tag{31}$$

The intuition is that the effect of e_h and e_l on the *net* resources of the economy are the same. With no interaction between types and marginal productivity of education, e_h and e_l imply identical externalities in addition to their having identical private marginal utility (zero) and marginal costs.[14]

Next, substitute the expressions for $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ from (7)–(8) into equations (24)–(25) and simplify making use of $e_h = e_l \equiv e$. Then subtract one equation from another to get

$$\varphi'(n_h) - \varphi'(n_l) + D\frac{\theta}{Z} = 0.$$

[14] Section below discusses how this result will change if higher-ability parents have a higher marginal productivity in educating their children (for the same expenditures).

With $D\theta/Z > 0$, it follows from the above expression that

$$\varphi'(n_h) - \varphi'(n_l) < 0.$$

Then, from strict concavity of $\varphi(\cdot)$, we have

$$n_h > n_l.$$

To see the intuition for this result, note that $D\theta/Z$ shows the impact of externality on aggregate resources of the economy as a result of a concomitant increase in n_h and a reduction in n_l when $e_h = e_l$. Such a change also affects utilities of the two individual types. This is given by $\varphi'(n_h) - \varphi'(n_l)$ in our utilitarian framework. At the optimum, the two must just offset one another. Now, given that the externality effect is always positive (because $D > 0$), the change in aggregate utilities must be negative.at the optimum. This in turn implies that $n_h > n_l$.

Finally, we observed earlier that the externality due to δ is positive if $\partial\delta/n_j > 0$ and is negative if $\partial\delta/n_j > 0$. Now with $n_h > n_l$, equations (7)–(8) imply that $\partial\delta/\partial n_h > 0$ and $\partial\delta/\partial n_l < 0$. Consequently, it is n_h that has a positive externalities here with n_l having a negative externality. The results thus far are summarized as

Proposition 2 *(i) Under the utilitarian first-best solution with PAYGO, consumption when working, consumption when retired, and investment in education are equalized across types. However, the high-ability types have more children.*

(ii) Investing in education of children by either type of parents increases the proportion of high-ability children in the economy and bestows a positive externality on everybody else. This externality has two components, one of which exists only in the presence of PAYGO pension plans.

(iii) A parent's fertility choice imposes two kinds of externalities on everyone else. One is the familiar positive externality known as "intergenerational transfer" effect. The other emanates from a change in the proportion of high-ability children. This externality

too has two components, one of which exists only in the presence of PAYGO pension plans.

(iv) When high-ability parents increase their fertility rate, they increase the proportion of high-ability children in the economy and thus bestows a positive externality on everybody else. On the other hand, an increase in the fertility rate of low-ability parents, reduces the proportion of high-ability children and imposes a negative externality on everybody else.

3.3 Decentralization

As observed earlier, the choice of storage technology or PAYGO is mutually exclusive in our setup. Thus, assuming that PAYGO is preferable, one wants to ensure that all second-period consumptions are financed through pensions. This requires the government to impose a one-hundred percent tax on savings and their returns. Recall also that the optimum required equal consumption levels both during working years and retirement. Consequently, the government must provide everyone with the same pension $P = d_h = d_h = d$ where d is evaluated at its first-best value. Next, to induce the correct choice of fertility and education, introduce two types of subsidies. One is a subsidy on education at the rate τ_j to the j-type, the other is a direct child subsidy to the j-type equal to t_j dollars per child. Finally, first-period lump-sum taxes, T_j, are required to ensure that consumption levels during working years are the same for both types.

Give this setup, parents decide only on their first-period consumption and fertility rate. Let α_j denote the Lagrangian multiplier associated with the budget constraint of a j-type parent. The optimization problem of this parent is then summarized by the Lagrangian expression,

$$\mathcal{L}_j = u(c_j) + \varphi(n_j) + \alpha_j \left[\beta_j I - c_j - n_j(a - t_j) - (1 - \tau_j)e_j n_j - T_j\right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}_j}{\partial c_j} = u'(c_j) - \alpha_j = 0, \tag{32}$$

$$\frac{\partial \mathcal{L}_j}{\partial e_j} = -\alpha_j(1-\tau_j)n_j = 0, \tag{33}$$

$$\frac{\partial \mathcal{L}_j}{\partial n_j} = \varphi'(n_j) - \alpha_j\left[a - t_j + (1-\tau_j)e_j\right] = 0. \tag{34}$$

The question one needs to examine now is how to set the tax rates such that the solution to the individual's first-order conditions (32)–(34) above coincide with the first-best solution (c, e, n_h, n_l) from equations (17)–(25).

First, compare equation (33) with (22) or (23). This tells us that education costs must be subsidized at a rate equal to

$$\tau_h = \frac{D}{u'(c)}\frac{1}{\delta n_h}\frac{\partial \delta}{\partial e_h} = 1, \tag{35}$$

$$\tau_l = \frac{D}{u'(c)}\frac{1}{(1-\delta)\, n_l}\frac{\partial \delta}{\partial e_l} = 1, \tag{36}$$

where c is set at its first-best values. To understand the intuition behind equations (35)–(36), note that the algebraic expressions in these equations are precisely the externality terms that come into play through δ as e_h and e_l change. The equations then tell us that at the optimum the subsidy rates on education must equate their marginal externality benefits. Moreover, at the optimum, the values of these externalities is unity. This should not be surprising. With education investment generating no private benefits, its price must be subsidized at one-hundred percent. Otherwise, one never invests in education.

Second, substitute $\tau_h = \tau_l = 1$ in equation (34) to rewrite it as

$$\varphi'(n_j) - \mu\,(a - t_j) = 0.$$

Comparing this equation with (24) and (25) results in unit child subsidies equal to

$$t_h = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)} \frac{1}{\delta} \frac{\partial \delta}{\partial n_h} - e, \tag{37}$$

$$t_l = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)} \frac{1}{1-\delta} \frac{\partial \delta}{\partial n_l} - e, \tag{38}$$

where c and e are set at their first-best values.

To understand the intuition behind equations (37)–(38), it will be useful to rewrite them as

$$t_h + \tau_h e_h = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)} \frac{1}{\delta} \frac{\partial \delta}{\partial n_h}, \tag{39}$$

$$t_l + \tau_l e_l = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)} \frac{1}{1-\delta} \frac{\partial \delta}{\partial n_l}, \tag{40}$$

by moving e to the left-hand side and recalling that $\tau e = e$ because $\tau = 1$. The left-hand sides of (39) and (40), $t_h + \tau_h e_h$ and $t_l + \tau_l e_l$, show the *net* subsidy given to an h-type and to an l-type parent for each of his children. The right-hand sides of (39) and (40) consist of the two externality sources described previously; they both are present when n_h and n_l change. These equations thus tell us that, at the optimum, we should subsidize the net cost of having a child by an amount equal to its net externality benefit.

Recall that the cost of raising and educating a child is $a + e$. A child subsidy of t dollars per child, reduces this cost. Similarly, a subsidy to education reduces this cost but through lowering the price of one particular element of it, namely, education cost. Thus a subsidy to education is also a subsidy to children. The difference is that the education subsidy lowers the share of education cost in total cost. On the other hand, a subsidy to children is "neutral" between the two sources of costs.

With $\partial \delta / \partial n_h > 0$, equation (39) tells us that $t_h + \tau_h e_h > 0$, so that the *net* subsidy given to an h-type parent for each of his children must be positive. On the other hand, with $\partial \delta / \partial n_l < 0$, one cannot a priori determine the sign of the expression in the right-hand side of (40). Consequently, the sign of $t_l + \tau_l e_l$ remains indeterminate.

Equations (37)–(38) do not allow us to determine the signs of t_h and t_l. However, if we subtract equation (38) from equation (37), while substituting the expressions for $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ from (7)–(8) in them, we get

$$t_h - t_l = \frac{D}{u'(c)}\frac{\theta}{Z} > 0,$$

so that $t_h > t_l$. This makes sense. Recall that we have at the optimum $\partial\delta/\partial n_h > 0, \partial\delta/\partial n_l < 0$. Increasing n_h has positive externalities and increasing n_l has negative externalities emanating from δ. The net marginal benefits associated with increasing n_h thus exceeds the net marginal benefits associated with increasing n_l. Because of this, the net subsidy on n_h must exceed the net subsidy on n_l.

Finally, to ensure that the two types will have identical consumption levels during working years, one has to set first-period lump-sum taxes such that both individual types spend the same amount of money on c. Comparing equation (32) with (17), while setting $\tau_h = \tau_l = 1$, then tells that T_h and T_l must satisfy the following condition

$$T_h - T_l = (\beta - 1)I + n_l(a - t_l) - n_h(a - t_h), \tag{41}$$

where t_h and t_l are set according to equations (37)–(38).

To sum, we have shown that first-best education subsidies must be levied at one hundred percent and that higher ability parents should receive higher child subsidies. However, we have not been able to determine the signs of t_h and t_l. That is, we have not been able to rule out child taxes (as opposed to child subsidies). Nor have we been able to determine the size of the net subsidy for the children of the low-ability parents, $t_l + \tau_l e_l$. Although, we have established that $t_h + \tau_h e_h > 0$. To throw some light on this issue, we resort to a numerical example. With $t_h > t_l$ and $t_l + \tau_l e_l > t_l$, the strongest candidate is of course t_l.

3.4 A numerical example

Assume there preferences are logarithmic and presented by

$$u = \ln c + b \ln d + \ln n.$$

Further assume that when the probability of having a high-ability child is related to investment in education according to

$$\pi(e) = 0.75 - \frac{1}{e+2}.$$

Finally, assume that $\theta = 0.05$, $\beta = 8.5$, and $I = 10$. We then solve this problem following the steps taken in our study of the first best. The following solutions emerge under the assumptions:

(i) $b = 1.1$:

$$\begin{aligned} n_h &= 8.04;\ n_l = 5.65;\ \overline{n} = 6.81;\ \delta = 0.48;\ c = 11.04;\ d = 82.69; \\ e &= 1.40;\ t_h + \tau_h e_h = 2.03;\ t_l + \tau_l e_l = 1.45;\ t_h = 0.63;\ t_l = 0.05. \end{aligned}$$

(ii) $b = 1$:

$$\begin{aligned} n_h &= 7.98;\ n_l = 5.70;\ \overline{n} = 6.81;\ \delta = 0.48;\ c = 11.59;\ d = 78.88; \\ e &= 1.40;\ t_h + \tau_h e_h = 1.95;\ t_l + \tau_l e_l = 1.37;\ t_h = 0.55;\ t_l = -0.03. \end{aligned}$$

(iii) $b = 0.10$:

$$\begin{aligned} n_h &= 7.44;\ n_l = 6.16;\ \overline{n} = 6.78;\ \delta = 0.48;\ c = 20.95;\ d = 14.77; \\ e &= 1.41;\ t_h + \tau_h e_h = 0.59;\ t_l + \tau_l e_l = 0.01;\ t_h = -0.82;\ t_l = -1.40. \end{aligned}$$

(iv) $b = 0.09$:

$$\begin{aligned} n_h &= 7.43;\ n_l = 6.17;\ \overline{n} = 6.78;\ \delta = 0.48;\ c = 21.14;\ d = 13.47; \\ e &= 1.41;\ t_h + \tau_h e_h = 0.57;\ t_l + \tau_l e_l = -0.02;\ t_h = -0.85;\ t_l = -1.43. \end{aligned}$$

Thus as the weight of retirement consumption in the utility function decreases, d decreases and with it the intergenerational transfer effect. This reduces the size of the positive externality of the first type. As a result, we see that first t_l, then t_h, and finally $t_l + \tau_l e_l$, net subsidy for children of the low-ability type, turn into a tax. The following proposition summarizes our results on decentralization.

Proposition 3 *(i) Investments in education of high- and low-ability parents must be subsidized at one hundred percent and equal to the externalities they bestow to everyone as given by expressions (35)–(36).*

(ii) Let t_j denote the direct child subsidy to a j-type parent in dollars. Its value must be set according to (37)–(38). We have $t_h > t_l$ and both t_h and t_l can take positive as well as negative values.

(iii) Direct child subsidies and education subsidies both reduce the cost of raising children. Thus a subsidy to education is also a subsidy to fertility. The difference is that the education subsidy lowers the share of education cost in the fertility subsidy. On the other hand, a subsidy to children is "neutral" between the two sources of costs.

(iv) Denote the subsidy rate on education investment to the j-type by τ_j. Net subsidies to children are then equal to $t_j + \tau_j e_j$. They must be set equal to the net externalities associated with increasing n_j as shown by expressions (39) and (40). While $t_h + \tau_h e_h$ is always positive, $t_l + \tau_l e_l$ can take positive as well as negative values.

4 Limited instruments

This section studies the properties of optimal child subsidies versus optimal education subsidies. The underlying assumption is that either one or the other instrument is used so that we have a second-best solution.

4.1 Education subsidy

Without child subsidies, one cannot directly control the number of children. However, one can affect parents' fertility through education subsidies. Equations (22) or (23) continue to apply

$$\begin{aligned} -\delta n_h u'(c) + D\frac{\partial \delta}{\partial e_h} &= 0, \\ -(1-\delta) n_l u'(c) + D\frac{\partial \delta}{\partial e_l} &= 0, \end{aligned}$$

With $\partial\delta/\partial e_h > 0$ and $\partial\delta/\partial e_l > 0$, we continue to have $D > 0$. Again, substituting the expressions for $\partial\delta/\partial e_h$ and $\partial\delta/\partial e_l$ into the above equations and subtracting one equation from another yields

$$\frac{D}{u'(c)} \frac{\pi'(e_h) - \pi'(e_l)}{Z} = 0.$$

Consequently,

$$e_h = e_l \equiv e.$$

To decentralize this, one must again subsidize the price of education at one hundred percent. That is

$$\begin{aligned} \tau_h &= \frac{D}{u'(c)} \frac{1}{\delta n_h} \frac{\partial \delta}{\partial e_h} = 1, \\ \tau_l &= \frac{D}{u'(c)} \frac{1}{(1-\delta)\, n_l} \frac{\partial \delta}{\partial e_l} = 1. \end{aligned}$$

Finally, turning to the choice of n_j, individuals set the marginal rate of substitution between n_j and c_j equal to the cost of raising a child, $\varphi'(n_j)/u'(c_j) = a$ as in equation (12). Now since the solution requires $c_h = c_l$, it follows that $n_h = n_l$.

4.2 Child subsidy

Without education subsidies, and with parents not benefiting directly from educating their children, nobody invests in education so that $e_h = e_l = 0$. In this case, equations

(24) and (25) continue to apply albeit for the suboptimal value of $e_h = e_l = 0$. We have

$$\begin{aligned} \varphi'(n_h) - \left(a - \frac{d}{\overline{n}^2}\right) + \frac{D}{\delta}\frac{\partial\delta}{\partial n_h} &= 0, \\ \varphi'(n_l) - \left(a - \frac{d}{\overline{n}^2}\right) u'(c) + \frac{D}{(1-\delta)}\frac{\partial\delta}{\partial n_l} &= 0. \end{aligned}$$

Moreover, for education to be useful, one must have at $e_h = e_l = 0$, $\partial\pounds/\partial e_h > 0$ and $\partial\pounds/\partial e_l > 0$. It then follows from equations (22)–(23) that $D > 0$. Subtracting one equation from the other then yields

$$\varphi'(n_l) - \varphi'(n_h) = \frac{D\theta}{Z} > 0.$$

Consequently,

$$n_h > n_l.$$

For the purpose of decentralization, equation (34) so that

$$t_j = a - \frac{\varphi'(n_j)}{u'(c)}.$$

and from the above equations

$$\begin{aligned} t_h &= \frac{d}{\overline{n}^2} + \frac{D}{u'(c)}\frac{(\pi(0) + \theta - \delta)}{Z}, \\ t_l &= \frac{d}{\overline{n}^2} + \frac{D}{u'(c)}\frac{(\pi(0) - \delta)}{Z}. \end{aligned}$$

Observe that in this case, child subsidies and net subsidies to children are one and the same.

4.3 Education or child subsidy

The question we would like to address is which instrument should be used if one can use only one of the two. To answer this question, one has to compare the optimal solution when using the education subsidy with the optimal solution when using the child subsidy.

There does not seem to be a general answer to this question. One expects that education subsidies will be the better policy whenever productivity differential between high- and low-ability individuals is high, and whenever one's family background plays a minor role in determining the ability of a child. To shed some light on this issue, we again resort to a numerical question.

We use the same logarithmic specification for preferences as before, with the coefficient of $\ln d$ being equal to one, the same functional form for $\pi(e)$, and $I = 10$. The parameter values that we allow to change are those for θ and β. Below are three sets of solutions.

(i) $\theta = 0.05$, $\beta = 5$:

$$\text{Education subsidy} \;:\; \overline{n} = 2.57;\ \delta = 0.53;\ e = 2.05;\ \delta u_h + (1-\delta)\, u_h = 6.57.$$

$$\text{Child subsidy} \;:\; \overline{n} = 5.18;\ \delta = 0.27;\ e = 0.00;\ \delta u_h + (1-\delta)\, u_h = 6.56.$$

(ii) $\theta = 0.05$, $\beta = 4$:

$$\text{Education subsidy} \;:\; \overline{n} = 2.25;\ \delta = 0.51;\ e = 1.75;\ \delta u_h + (1-\delta)\, u_h = 5.88.$$

$$\text{Child subsidy} \;:\; \overline{n} = 4.51;\ \delta = 0.27;\ e = 0.00;\ \delta u_h + (1-\delta)\, u_h = 6.01.$$

(iii) $\theta = 0.10$, $\beta = 5$:

$$\text{Education subsidy} \;:\; \overline{n} = 2.64;\ \delta = 0.56;\ e = 2.10;\ \delta u_h + (1-\delta)\, u_h = 6.71.$$

$$\text{Child subsidy} \;:\; \overline{n} = 5.50;\ \delta = 0.30;\ e = 0.00;\ \delta u_h + (1-\delta)\, u_h = 6.74.$$

Case (i) illustrates a solution where an education subsidy dominates a child subsidy. In case (ii) we lower the value of β, leaving all other parameter values intact, and the child subsidy dominates. A lower β represents a smaller productivity differential between high- and low-ability individuals. Similarly, in case (iii) we increase the value of θ, leaving all other parameter values intact, and the child subsidy dominates. A higher θ represents signifies a more significant role for family background in determining the ability of a child. Rather unsurprisingly, the numbers also indicate that child subsidies generally entail higher fertility and a lower ratio of high- to low-ability children. The following proposition summarizes our results on decentralization.

Proposition 4 *(i) Assume education subsidies are feasible but not child subsidies. The optimal solution requires equalization of all objects of choice across the two types. education subsidies continue to be levied at one hundred percent and equal to the positive externalities bestowed on everyone through education.*

(ii) Assume child subsidies are feasible but not education subsidies. The optimal solution requires $c_h = c_l$*,* $d_h = d_l$*,* $e_h = e_l = 0$*, and* $n_h > n_l$*. Child subsidies are set equal to fertility externalities and equal to net subsidies on children.*

(iii) Either education subsidies or child subsidies can dominate the other instrument. In general, child subsidies become the more dominant policy if productivity differential between high- and low-ability individuals become smaller or family background assumes a more significant role in determining the ability of a child.

(iv) In general, child subsidies entail a higher fertility rate and a lower ratio of high- to low-ability children, as compared to education subsidies.

5 Extensions

5.1 Interaction between types and education

Assume that education and family background interact positively so that the same expenditures made on education leads to a higher probability for a high-ability parent to have a high-ability child. This changes the additive specification $\pi_j = \pi(e) + \theta_j$ to a multiplicative one

$$\pi_j = \theta_j \pi(e),$$

with $\theta_h = \theta$ and $\theta_l = 1$.This reformulation does not change any of the expressions we have derived previously except that whenever a substitution is required for $\partial\delta/\partial e_j$, one has to do this through

$$\begin{aligned}\frac{1}{\delta}\frac{\partial\delta}{\partial e_h} &= \frac{n_h\theta\pi'(e_h)}{Z},\\ \frac{1}{(1-\delta)}\frac{\partial\delta}{\partial e_l} &= \frac{n_l\pi'(e_l)}{Z},\end{aligned}$$

instead of previously derived equations (5)–(6).

Observe also that the $c_h = c_l$, $d_h = d_l$, results do not change. However, the results concerning e_h versus e_l and n_h versus n_l do. Specifically, subtracting equation (36) from (35) now yields

$$\frac{D}{Zu'(c)}\left[\theta\pi'(e_h) - \pi'(e_l)\right] = 0.$$

With $\theta > 1$, this equation implies $\pi'(e_h) < \pi'(e_l)$. Hence, given the concavity of $\pi(\cdot)$,

$$e_h > e_l.$$

This replaces $e_h = e_l$ result we had previously.

Turning to the comparison between n_h and n_l, subtracting (23) from (22) now yields, using the multiplicative formulation of θ in $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$,

$$\varphi'(n_h) - \varphi'(n_l) = (e_h - e_l)\,u'(c) + \frac{\pi_l - \pi_h}{Z}$$

Now observe that $\pi_l - \pi_h = \pi(e_l) - \theta\pi(e_h) < 0$. Consequently, the sign of the right-hand side of the above equation is indeterminate. This also implies that one no longer knows if n_h is larger or n_l.

5.2 Different utilities for children

Assume parents prefer higher ability children. One can then write the utility of a j-type having children of type i as

$$U_j = u(c_j) + v(d_j) + \gamma^i\varphi(n_j),$$

where $\gamma^h > \gamma^l$ with $\gamma^h - \gamma^l$ indicating the strength of preferences for higher ability children. Under this circumstance, given the partly stochastic nature of children ability, each j-type will have an ex-ante expected utility depending on the outcome of his investment in children. Setting $\gamma^l = 1$, and $\gamma^h = \gamma > 1$, we have

$$\begin{aligned} EU_j &= u(c) + v(d) + \pi_j \gamma \varphi(n) + (1 - \pi_j)\,\varphi(n) \\ &= u(c) + v(d) + [1 + (\gamma - 1)\,(\pi(e) + \theta_j)]\,\varphi(n). \end{aligned} \quad (42)$$

Thus the coefficient of $\varphi(n)$ in the utility function (9) is no longer one, and changes to $[1 + (\gamma - 1)\,(\pi(e) + \theta_j)]$. Observe also that if $\gamma = 1$, (42) simplifies to (9) and we are back to our previous formulation. Individuals' budget constraints and the economy's resource constraint, however, remain intact. education investments now become utility enhancing so that this formulation results in an interior value for e even in the laissez faire when parents maximize (42) subject to their budget constraint.

Turning to the first best, our formulation of the nature of externalities, the expressions that characterize these externalities, and the discussion of the decentralization do not change.[15] Similarly, the result that consumption levels are equalized across types, $c_h = c_l$ and $d_h = d_l$, do not change. What changes is the results that $e_h = e_l$ and $n_h > n_l$. The first-order conditions with respect to e_j and n_j that characterize first best now take the following forms

$$(\gamma - 1)\varphi(n_h)\frac{\pi'(e_h)}{u'(c)} - n_h + \left[\frac{(n_h - n_l)\,d}{\overline{n}^2} + \frac{D}{u'(c)}\right]\frac{1}{\delta}\frac{\partial \delta}{\partial e_h} = 0, \quad (43)$$

$$(\gamma - 1)\varphi(n_l)\frac{\pi'(e_l)}{u'(c)} - n_l + \left[\frac{(n_h - n_l)\,d}{\overline{n}^2} + \frac{D}{u'(c)}\right]\frac{1}{1-\delta}\frac{\partial \delta}{\partial e_l} = 0, \quad (44)$$

$$\begin{aligned} &[1 + (\gamma - 1)\,(\pi(e_h) + \theta)]\,\frac{\varphi'(n_h)}{u'(c)}\varphi'(n_h) - \left(a + e_h - \frac{d}{\overline{n}^2}\right) + \\ &\left[\frac{(n_h - n_l)\,d}{\overline{n}^2} + \frac{D}{u'(c)}\right]\frac{1}{\delta}\frac{\partial \delta}{\partial n_h} = 0, \end{aligned} \quad (45)$$

$$\begin{aligned} &[1 + (\gamma - 1)\pi(e_l)]\,\frac{\varphi'(n_l)}{u'(c)} - \left(a + e_l - \frac{d}{\overline{n}^2}\right) + \\ &\left[\frac{(n_h - n_l)\,d}{\overline{n}^2} + \frac{D}{u'(c)}\right]\frac{1}{1-\delta}\frac{\partial \delta}{\partial n_l} = 0. \end{aligned} \quad (46)$$

[15] However, τ_h and τ_l are no longer equal to one.

Divide equation (43) by (44) and substitute the expressions for $\partial\delta/\partial e_h$ and $\partial\delta/\partial e_l$ from (5)–(6) in the resulting equation. Simplifying yields

$$\frac{(\gamma-1)\varphi(n_h)\pi'(e_h)-\mu n_h}{(\gamma-1)\varphi(n_l)\pi'(e_l)-\mu n_l}=\frac{n_h\pi'(e_h)}{n_l\pi'(e_l)}.$$

Multiplying through and rearranging, one can rewrite this as

$$\mu n_h n_l\left[\pi'(e_h)-\pi'(e_l)\right]=(\gamma-1)\pi'(e_h)\pi'(e_l)\left[n_h\varphi(n_l)-n_l\varphi(n_h)\right]. \tag{47}$$

It follows from the concavity of $\pi(\cdot)$ that the left-hand side of (47) has the same sign as (e_l-e_h). Similarly, concavity of $\varphi(\cdot)$ implies that the right-hand side of (47) has the same sign as (n_h-n_l). Consequently, at the first-best, (e_h-e_l) and (n_h-n_l) are of opposite signs. One cannot go beyond this result in establishing the relationship between e_h and e_l or between n_h and n_l.

6 Conclusion

In discussing PAYGO pension plan, models with endogenous fertility have emphasized the positive externality that each person's fertility decision bestows on everybody by increasing everybody's pension benefits through a higher population growth rate. This type of externality, has been argued, may be internalized through child subsidies. Similarly, models with endogenous human capital formation have emphasized the positive externality of investing in education of one's children (because parents cannot expropriate the children's extra earnings due to parents' education expenditures). The same argument has been put forward in cases when parents build their own human capital which they subsequently pass on to their children. These types of externalities may be internalized through education subsidies.

In this paper, we have combined the different externality sources to learn what their interactions teach us about the combination of child and education subsidies one must use to internalize them both. We have also been concerned with the question of heterogeneity of parents and how this may come into play in connection with externality-correcting policies. This is particularly relevant when child and education subsidies change the distribution of parent types. To this end, the paper has modeled endogenous fertility and human capital formation in an overlapping-generations framework wherein every generation consists of high earners and low earners with the proportion of types being determined endogenously. We have found, among other results, that:

(1) Investing in education of children by either type of parents increases the proportion of high-ability children in the economy and bestows a positive externality on everybody else. This externality has two components, one of which exists only in the presence of PAYGO pension plans.

(2) When high-ability parents increase their fertility rate, they increase the proportion of high-ability children in the economy and thus bestows a positive externality on everybody else. On the other hand, an increase in the fertility rate of low-ability parents, reduces the proportion of high-ability children and imposes a negative externality on everybody else.

(3) Direct child subsidies and education subsidies both reduce the cost of raising children. Thus a subsidy to education is also a subsidy to fertility. The difference is that the education subsidy lowers the share of education cost in the fertility subsidy. On the other hand, a subsidy to children is "neutral" between the two sources of costs.

(4) Investments in education of high- and low-ability parents must be subsidized at one hundred percent and equal to the externalities they bestow to everyone.

(5) Direct child subsidies to one or both parent types can be negative; i.e., they can be taxes. However the high-ability type should always get a higher subsidy per child (or pay a lower tax).

(6) Net subsidies to children of a particular parent type (direct child subsidies plus education subsidies) must be set equal to the net externalities associated with increasing the fertility rate of that type. While net child subsidies to high-ability parents are always

positive, net child subsidies to low-ability parents can be positive or negative.

(7) Either education subsidies or child subsidies, when used alone, can dominate the other instrument. In general, child subsidies become the more dominant policy if productivity differential between high- and low-ability individuals become smaller or family background assumes a more significant role in determining the ability of a child.

(8) In general, using child subsidy instruments alone entails a higher fertility rate and a lower ratio of high- to low-ability children, as compared to using education subsidies alone.

As a final observation, we remind our readers that our study has been conducted primarily in a first-best environment. Many other interesting issues surface in a second-best environment when investments and/or type are not publicly observable. We have left the examination of these other issues to a subsequent paper.

Appendix A

Proof of $2\delta(n_h - n_l) + n_l(1+\pi_l) - n_h\pi_h > 0$**:** Rewrite equation(1) as

$$\delta_{T+1} = \frac{\delta_T n_h \pi_h + (1-\delta_T) n_l \pi_l}{\delta_T n_h + (1-\delta_T) n_l} \equiv f\left(\delta_T, n_h, \pi_h, n_l, \pi_l\right). \tag{A1}$$

The steady-state value of δ is found from

$$\begin{cases} \delta_{T+1} = & f\left(\delta_T, n_h, \pi_h, n_l, \pi_l\right), \\ \delta_{T+1} = & \delta_T = \delta. \end{cases} \tag{A2}$$

Differentiating δ totally with respect to π_h yields

$$\frac{d\delta}{d\pi_h} = \frac{\partial f}{\partial \delta_T}\frac{d\delta}{d\pi_h} + \frac{\partial f}{\partial \pi_h}. \tag{A3}$$

Then one finds $d\delta/d\pi_h$ from equation (A3) as

$$\frac{d\delta}{d\pi_h} = \frac{\partial f/\partial \pi_h}{1 - \partial f/\partial \delta_T}. \tag{A4}$$

Next, partially differentiate equation (A1) with respect to π_h to arrive at

$$\frac{\partial \delta_{T+1}}{\partial \pi_h} = \frac{\partial f}{\partial \pi_h} = \frac{\delta_T n_h}{\overline{n}}. \tag{A5}$$

Substituting from (A4) into (A5) yields

$$\frac{d\delta}{d\pi_h} = \frac{\delta_T n_h/\overline{n}}{1 - \partial f/\partial \delta_T},$$

or, alternatively,

$$\frac{d\delta}{de_h} = \frac{d\delta}{d\pi_h}\theta\pi'(e_h) = \frac{\delta_T n_h \theta \pi'(e_h)}{\overline{n}\left[1 - \partial f/\partial \delta_T\right]}. \tag{A6}$$

Comparing the expressions for $d\delta/de_h$ as given by equation (A6) above and equation (7) derived in the text tells us that the denominator in equations (7)–(8) is equal to the denominator of (A6). That is,

$$2\delta(n_h - n_l) + n_l(1+\pi_l) - n_h\pi_h = \overline{n}\left[1 - \partial f/\partial \delta_T\right].$$

Now if $\partial f/\partial\delta_T < 0$, then $1-\partial f/\partial\delta_T > 0$. On the other hand, if $\partial f/\partial\delta_T > 0$, the stability condition $|\partial\delta_{T+1}/\partial\delta_T| = |\partial f/\partial\delta_T| < 1$ ensures that $1 - \partial f/\partial\delta_T > 0$. Consequently, either way, we have

$$2\delta(n_h - n_l) + n_l(1 + \pi_l) - n_h\pi_h > 0.$$

Appendix B: Storage

When the use of storage technology is the better option, all second-period consumption is financed by private savings. We thus have

$$[\delta S_h + (1-\delta)S_l](1+r) = \delta d_h + (1-\delta)d_{l.}$$

This simplifies the resource constraint (15) to

$$[1+(\beta-1)\delta]\,I \quad \geqq \quad \delta\left[c_h + n_h(a+e_h) + \frac{d_h}{1+r}\right] + (1-\delta)\left[c_l + n_l(a+e_l) + \frac{d_l}{1+r}\right]. \tag{B1}$$

The social planner's problem is thus summarized by the Lagrangian

$$\begin{aligned} \pounds \quad = \quad & \delta\,[u(c_h) + v(d_h) + \varphi(n_h)] + (1-\delta)\,[u(c_l) + v(d_l) + \varphi(n_l)] + \mu\Bigg\{[1+(\beta-1)\delta]\,I \\ & -\delta\left[c_h + n_h(a+e_h) + \frac{d_h}{1+r}\right] - (1-\delta)\left[c_l + n_l(a+e_l) + \frac{d_l}{1+r}\right]\Bigg\}. \end{aligned} \tag{B2}$$

Observe that the difference of this expression with $\pounds$ under PAYGO is that $d_j/(1+r)$ has replaced $d_j/\overline{n}$. Start with the first-order conditions for this problem with respect c_h, c_l, d_h and d_l. They are identical to their counterparts under PAYGO except for $(1+r)$ replacing $\overline{n}$. Hence we again have $c_h = c_l = c$, and $d_h = d_l = d$.

Next introduce

$$D \equiv \frac{\partial \pounds}{\partial \delta} = \varphi(n_h) - \varphi(n_l) + u'(c)\,[(\beta-1)I - n_h(a+e_h) + n_l(a+e_l)]\,, \tag{B3}$$

and note that, unlike D under PAYGO given by equation (21), this expression does not contain the term $u'(c)\,(n_h - n_l)\,d/\overline{n}^2$; the other terms are identical. The first-order conditions with respect to n_h, n_l, e_h and e_l as

$$\begin{aligned}
\frac{\partial \pounds}{\partial e_h} &= -\delta n_h u'(c) + D\frac{\partial \delta}{\partial e_h} = 0, && \text{(B4)}\\
\frac{\partial \pounds}{\partial e_l} &= -(1-\delta) n_l u'(c) + D\frac{\partial \delta}{\partial e_l} = 0, && \text{(B5)}\\
\frac{\partial \pounds}{\partial n_h} &= \delta\left[\varphi'(n_h) - (a+e_h)\, u'(c)\right] + D\frac{\partial \delta}{\partial n_h} = 0, && \text{(B6)}\\
\frac{\partial \pounds}{\partial n_l} &= (1-\delta)\left[\varphi'(n_l) - (a+e_l)\, u'(c)\right] + D\frac{\partial \delta}{\partial n_l} = 0. && \text{(B7)}
\end{aligned}$$

Note that the expressions in (B4)–(B5) are the same as their counterparts under PAYGO, equations (22)–(23), except for the difference in D. Equations (B6)–(B7) differ with their PAYGO counterparts (24)–(25) not only in terms of D, but they do not contain $d/\overline{n}^2$ in their middle expression either. Manipulating these conditions in the same way as we did with PAYGO, yield $e_h = e_l = e$ and $n_h > n_l$.

Turning to the externality terms, they are now given by

$$\begin{aligned}
&\frac{D}{u'(c)}\frac{1}{\delta}\frac{\partial \delta}{\partial e_h} \quad \text{for increasing } e_h, && \text{(B8)}\\
&\frac{D}{u'(c)}\frac{1}{1-\delta}\frac{\partial \delta}{\partial e_l} \quad \text{for increasing } e_l, && \text{(B9)}\\
&\frac{D}{u'(c)}\frac{1}{\delta}\frac{\partial \delta}{\partial n_h} \quad \text{for increasing } n_h, && \text{(B10)}\\
&\frac{D}{u'(c)}\frac{1}{1-\delta}\frac{\partial \delta}{\partial n_l} \quad \text{for increasing } n_l. && \text{(B11)}
\end{aligned}$$

They all arise from a change in δ; there are no externalities associated with a change in $\overline{n}$ whether directly as in intergenerational transfer effect or indirectly through δ. That there is no indirect externality through interaction of $\overline{n}$ and δ also means that the extent of this externality depends on the type of pension plan in use. Observe also that if δ remains unchanged, we have only the usual private calculations of utility and cost changes; there will be no externality. It is thus the externality associated with the change in δ that distinguishes the storage story here as compared to Cremer *et al.* (2006, 2008) where the laissez faire solution under the storage technology was optimal.

Regarding decentralization, we will again have 100% education subsidies with τ_h and τ_l also being equal to their corresponding externality terms (B8)–(B9). Net subsidies to children, $t_h + \tau_h e$ and $t_l + \tau_l e$ are set equal to the externality terms associated with n_h and n_l as given by (B10)–(B11). With $D > 0$, from (B4) or (B5), and $\partial\delta/\partial n_h > 0$ and $\partial\delta/\partial e_l < 0$, we now have

$$t_h + \tau_h e > 0 \text{ and } t_l + \tau_l e < 0.$$

This also means that while the sign of t_h is indeterminate, $t_l < 0$. Finally, we continue to have $t_h > t_l$.

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