# Social and Private Long Term Care Insurance with Variable Altruism

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**Abstract**

This paper studies the design of an optimal public scheme for long term care (LTC) in a setting where LTC services are also provided by the market and the family. Private insurance offers fixed reimbursement that is conditioned on the loss of autonomy but not on the size of the actual loss. Family solidarity is not uniform. Some parents can count on their children's altruism; some others cannot. In most cases asymmetric information may induce some families to hide their altruism and their resources and this sizeably affects the structure of social insurance. We use two non linear instruments: a tax on children's earnings and a subsidy on parents' purchase of private insurance.

Keywords: long term care, altruism, optimal taxation

JEL classification: D64, H55, I18.

## 1 Introduction

The ongoing demographic ageing process represents a major challenge both from a social and an economic point of view. This is because ageing can be felt across the board. It touches all age groups from the very young to the oldest old. One often cited example is the provision of long-term care (LTC) insurance to the oldest old, be it under the form of a private or a public system. Only a handful of countries have set up such long-term care insurance systems also sometimes called dependency insurance. The relative scarcity of such systems, and the difficulties of organizing them are linked to a number of conceptual problems that have not been yet fully explored.

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This paper analyzes the design of an optimal social insurance aimed at LTC in a setting where both the market and the family also intervene. To do so it studies a society consisting of a number of families represented by parent-child pairs. Families differ in several respects. First some families are linked by altruism and thus share their resources when the parent loses his autonomy. Second, families can be rich or poor. In the absence of a government policy, dependent parents can be helped in two ways: either their children can give them some financial aid, or for those who can afford it they benefit from private insurance compensations or from selfinsurance. Private insurance presents some limits besides the fact that not everyone can buy it: it does not reimburse all the expenses incurred following a loss of autonomy; it only provides a average amount regardless of the severity of the loss. [1] In contrast family solidarity and self-insurance can cover the full cost of dependence. But not everyone benefits from family solidarity and self-insurance depends on the parent's resources.

In a world of perfect information and individualized lump sum taxes, the first best would be implementable under one of the following two scenarios: (i) social insurance is financed without distortion and can cover the true costs of dependence, (ii) private insurance covers actual dependency-related expenses. However neither of these scenarios is realistic; further, lump sum taxes are also not realistic. Even if all famlies were altruistic, family solidarity cannot lead to the first best. Like self-insurance it does not pool risks. As it appears the three ways of coping with dependency are imperfect and furthermore interpersonal redistribution is distortionary. In this paper we study the design of an optimal LTC social insurance in a setting of asymmetric information, namely in a world in which children's productivity, parents' wealth and the severity of dependence are not observable but their purchase of private insurance, if any, is commun knowledge. Whether the family is altrusitic or not is also private information. To keep the analysis relatively simple we distinguish 4 types of families according to two characteristics: rich and poor, altruistic and not.

Ultimately what we are interested in is the optimal policy chosen by a utilitarian government given a revenue constraint and a number of self selection constraints. Quite clearly our model does not include all the aspects of long term care, and it rests on a number of assumptions. Some are pretty realistic; others are made to keep the analysis within reasonable limits.

Surprisingly there is little theoretical work devoted to optimal long term care policy. Jousten *et al.* (2005) focus on the moral hazard problem arising when children's altruism is not observable. There is naturally the seminal

[1] see Kessler (2008). See also Eeckhoudt et al. (2003).

paper by Pauly (1990), who argues that the demand for long-term care insurance suffers from a particular moral hazard in that children may decide to diminish their caregiving in favor of low-cost care provided by third parties, such as a public long-term care program. More recently, Zweifel and Struwe (1998) have shown in a principal-agent setting that the existence of long-term care insurance coverage diminishes the amount of care provided by the major "natural" caregivers. The rational for this result is rather straightforward: in the face of long-term care coverage, children earning low wages are less constrained to spare wealth by providing care themselves. Anticipating this moral hazard, parents demand low levels of long-term care insurance. Pauly (1996) puts forward a provocative argument: long-term care insurance would largely protects bequests for non poor, non needy heirs. This is an interesting point that cannot be addressed to here as we assume away bequest motive. In earlier papers [Pestieau and Sato, 2006, 2008] we only considered linear instruments. The present paper extends this work in allowing for non linear instruments.

The rest of the paper is organized as follows. The next section presents the basic model along with the *laissez-faire* solution with private insurance and the policy tools, namely a non linear tax on children's earnings and a non linear subsidy on the purchase of private insurance. Section 3 presents the first-best solution and shows how it can be decentralized. In section 4 we look at the general second best problem and in section 5 we analyze some special cases to get better insights on a rather difficult problem. A final section concludes.

Before proceeding a remark is in order. In LTC one generally includes two different types of care: health care and nursing care. In most countries social insurance cover the former and quite often the coverage is rather generous. See the Medicare program in the USA. In contrast nursing costs are only covered partially by social and private insurance. Nursing needs tend to be more subjective and more open to moral hazard problems than health care needs[2]. Nobody goes to doctors or hospitals by pleasure; having some one nursing you or taking care of your housework is a much more tempting proposion. In this paper, we mainly deal with this aspect of LTC.

[2] This explains why in a number of European countries both private and public LTC schemes do not reimburse actual expenses but provide a flat rate benefit.

# 2 Basic model and *laissez-faire*

## 2.1 Family types

We consider a society consisting of two-person families: a parent P who may become dependent and require some LTC and his child C. Families are divided in altruistic families wherein resources are shared between the parent and the child and selfish families where there is no such link for whatever reasons. Besides this distinction, we distinguish families according to the wealth of the parent and the wage of the child but to keep things simple, we assume that these are correlated. Basically we have 4 types of families: rich and poor, altruistic and not. More concretely we have:

- families (HS) where the two generations share resources and where the child is skilled $(w_H)$ and the parent is wealthy $(I_H)$,
- families (LS) where both parent and child share resources but the child is unskilled $(w_L)$ and the parent is poor $(I_L)$,
- families (HN) where the parent and the child are totally separate, the child is skilled $(w_H)$and the parent has a high level of resources $(I_H)$.
- families (LN) where the parent and the child are totally separate; the child is unskilled $(w_L)$ and the parent has a low level of resources $(I_L)$.

We will denote those types by subscripts $ij$ with i = H or L and j = S or N. The frequency of type ij is denoted $n_{ij}$.

## 2.2 Health status

Dependence occurs with a probability $\pi$ and can be severe or not. Severe dependence implies a loss of $D_2$ and light dependency a loss of $D_1$ with $D_2 > D_1$.We assume that the probability of severe or light dependence is identical, that is, equal to $\frac{\pi}{2}$. Assume that private insurance only covers average dependency, namely $\bar{D}$. The difference between the actual cost of dependence and the insured cost is covered by family solidarity, if any, or by self-insurance. Following a loss of $D_k$ (k=1, 2) the level of autonomy is equal to

$$m_k = m_0 - D_k + x_k \tag{1}$$

where $m_0$ is the initial level of autonomy and $x_k$ denotes spending on LTC.

We can now write the utility of health (or of the level of autonomy) as $H(m_k)$ with k=1,2. We normalize it so that $H(m_0) = 0$.

## 2.3 Utility functions

We now write the utility functions of child and parent as:

$$u(c) - h(y/w) \tag{2}$$

$$u(d) + H(m) \tag{3}$$

where c is child's consumption, d, parent's consumption, y, gross earning, w, wage level and $h(.)$ is labor disutility. Both u(.) and H(.) are strictly concave and h(.) strictly convex.

## 2.4 Health spending

Parents can ex ante purchase some private LTC insurance at an actuarially fair premium. We denote A the LTC insurance premium that is invariant to the severity of dependence. Insurers cannot (or do not want to) observe the level of D, which makes private insurance inefficient in spite of the fact that the premium is $\pi A$. In other words we assume away any loading factor.

In addition to private LTC insurance benefit, parents may use part of their resources when they know their true health status as self-insurance. Furthermore, in altruistic families, children may also contribute to self-insurance. We write:

$$x_k = A + q_k \qquad k = 1, 2, \tag{4}$$

where $q_k$ is the level of self-insurance.

We now look at the individual choices in the laissez faire for the two types of families.

## 2.5 Non altruistic families

In the non altruistic families, parents and children act separately. Parents' choice is limited to the *ex ante* purchase of private insurance (that is subsidized) and the *ex post* choice of self-insurance. Children's choice is restricted to labor supply, given an income tax function $T(y)$. Their problem is to maximize:

$$U_{iN} = u(y_{iN} - T(y_{iN})) - h(y_{iN}/w_i) = U_{iN}(z, y) \tag{5}$$

where $z = y - T(y)$ is disposable income. Turning to the parent, in case of dependence of level k, he chooses $x_k (= A + q_k)$ to solve:

$$Max_{x_k} u(I_i - R + A - x_k) + H(m_0 - D_k + x_k) \tag{6}$$

where $R = \pi A - s$, s being an insurance subsidy. R is the net insurance premium, namely what the parent has to pay. The insurance premium is paid in cash and we allow the solution of the above problem $x_k^*$ to be negative. As an example, assume that both u(.) and H(.) are logarithmic. Then we have:

$$\begin{aligned} x_k^* &= 0.5(I_i - R + A + D_k - m_0) \\ d_k^* &= 0.5(I_i - R + A - D_k + m_0) \end{aligned}$$

The expected utility of parents in non altruistic families is given by:

$$\begin{aligned} V_{iN} = \frac{\pi}{2} \sum_k & [u(I_i - R + A - x_k^*) + H(m_0 - D_k + x_k^*)] \\ & + (1 - \pi) u(I_i - R) = V_{iN}(R, A), \end{aligned} \tag{7}$$

with

$$\begin{aligned} \frac{\partial V_{iN}}{\partial I} &= E\left[u'(d_{iN})\right] \\ \frac{\partial V_{iN}}{\partial A} &= \frac{\pi}{2} \sum_k \left[u'(d_{iN}^k)\right] = \pi E_2 \left[u'(d_{iN}\right] \end{aligned}$$

Note that the operator $E$ is the expected value defined on the three states of nature indexed k=0 with probability $1 - \pi$, and k=1,2 with probability $\pi/2$. The operator $E_2$ is the expected utility in case of dependence. The LF solution is straightforward. The child consumes his income: $c_{iN} = y_{iN}$ and supplies an amount of labor $\ell_{iN}$ given by:

$$u'(c_{iN}) \left(w_i - T'(y_{iN})\right) = h'(\ell_{iN})$$

As to the parent to the extent he can afford it he buys some insurance given by:

$$2u'\left(d_{iN}\right) = H'\left(m_{iN}^1\right) + H'\left(m_{iN}^2\right).$$

## 2.6 Altruistic families

In the altruistic families, both parent and child pool their resources to maximize their overall expected utility. Their consumption will thus depend on the state of the world they are in. After $D_k$. is known the family solves:

$$Max_{x_k} u(c) + u(d) + H(m_0 - D_k + x_k)$$

subject to the family's budget constraint :

$$c + d + x_k = z + I_i - R + A$$

This yields:

$$c_k^* = d_k^* = \frac{1}{2}\left[z + I_i - R + A - x_k^*\right]$$

We thus assume that child and parent share equally both their resources and the cost of LTC. With the log functions, we have:

$$c_k^* = d_k^* = \frac{1}{2}\left[z + I_i - R + A + m - D_k\right]$$

The dynastic utility of type iS can be written as:

$$W_i = \sum_k \frac{\pi}{2}\left[2u(\frac{z + I_i + A - x_k^* - R}{2}) + H(m_0 - D_k + x_k^*)\right] \qquad (8)$$
$$-h\left(y/w_i\right) + (1-\pi)u(\frac{z + I_i - R}{2}) = W_i(R, A, z, y)$$

with

$$\begin{aligned}
\frac{\partial W_i}{\partial R} &= -E\left[u'(d_{iS}\right] = -E\left[u'(c_{iS}\right] \\
\frac{\partial W_i}{\partial A} &= \pi E_2\left[u'(d_{iS}\right] \\
\frac{\partial W_i}{\partial z} &= E\left[u'(c_{iS}\right] \\
\frac{\partial W_i}{\partial y} &= -h'(y_{iS}/w_i)/w_i
\end{aligned}$$

## 2.7 Timing and ex ante optimization

We can distinguish three stages in decision making:

- the government chooses its policy instruments

- P purchases private LTC insurance and C chooses his labor supply.
- Health status is revealed and $x_k$ is selected.

We can now look at the second stage, namely the *ex ante* choices : labor supply by children and purchase of private insurance by parents. We have thus:

$$\begin{aligned}
u'(c_{iN}))(1 - T'(y_{iN})) &= h'(y_{iN}/w_i)/w_i \\
\pi E_2 \left[u'(d_{iN}\right] &= R'(A_{iN})E(u'(d_{iN}) \\
E\left[u'(c_{iS}))(1 - T'(y_{iS}))\right] &= h'(y_{iS}/w_i)/w_i \\
\pi E_2 \left[u'(d_{iS}\right] &= R'(A_{iS})E(u'(d_{iS})
\end{aligned}$$

For future use we can relate those marginal taxes to the relevant marginal rates of substitution (MRS). We can thus rewrite the above first and third equations as:

$$MRS_{yz}^{ij} = \frac{h'(y_{ij}/w_i)}{wu'(z_{ij})} = 1 - T'(y_{ij})$$

We see that the MRS between y and z is equal to 1 when there is no tax distortion. Similarly we can relate the MRS between A and R to the marginal subsidy on private LTC insurance.

$$MRS_{AR}^{ij} = \frac{\pi E_2 \left[u'(d_{ij}\right]}{Eu'(d_{ij})} = R'(A_{ij}) = \pi - s'(A_{ij})$$

## 2.8 Government policy instruments and revenue constraint

The optimal policy will consist of non linear tax instruments: a tax on earnings $T(y)$ and a subsidy on private insurance $R(A) = \pi A - s(A)$. As usual in optimal taxation we select the optimal values of $(y_{ij}, z_{ij}, R_{ij}, A_{ij})$ instead of these tax functions. In doing that, we have to take into account the revenue constraint of the government:

$$\sum n_{ij}\left[y_{ij} - z_{ij} - s_{ij}\right] = 0$$

or

$$\sum n_{ij}\left[y_{ij} - z_{ij} + R_{ij}\right] = \pi \sum n_{ij} A_{ij}$$

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# 3 First-best and decentralization

We now turn to the first-best and its decentralization. We assume an utilitarist government that maximizes the utilities of both parents and children. This problem can be expressed by the following Lagrangean expression[3]:

$$\begin{aligned}\mathcal{L}_1 &= \sum n_{ij}p_k \left\{\left[u\left(c_{ij}^k\right) + u\left(d_{ij}^k\right) - h(y_{ij}/w_i) + H\left(m_0 - D_k + x_{ij}^k\right)\right]\right. \\ &\quad \left. -\mu\left[c_{ij}^k + d_{ij}^k + x_{ij}^k - I_i - y_{ij}\right]\right\}\end{aligned}$$

where $k = 0, 1, 2$ denotes the 3 states of the world: no dependency, severe and light one with $p_0 = 1 - \pi$ and $p_1 = p_2 = \pi/2$; $x_{ij}^k$ is the amount of LTC and $D_0 = x_{ij}^0 = 0$.

We easily obtain from the FOCs:

$$\begin{aligned}u'\left(c_{ij}^k\right) &= u'\left(d_{ij}^k\right) = h'(\ell_{ij})/w_i = \mu \\ H'\left(m_{ij}^k\right) &= \mu \quad \text{for } k = 1, 2\end{aligned}$$

where $m_{ij}^k = m_0 - D_k + x_{ij}^k < m_0$ by assumption.

In other words, in the first-best consumption levels are the same for all and LTC adjust to the two levels of severity to obtain the same level of autonomy (health).

Can we decentralize such an optimum? This requires lump sum transfers to equate consumption and an insurance device to cover the two types of dependency. If private insurance could cover the two levels of severity in an actuarial way or alternatively if $D_1 = D_2 = \bar{D}$, then decentralization would just require lump sum transfers and an actuarially neutral private insurance.

In our model private insurance is restricted to a fixed benefit that is invariant to the severity of the dependence and social insurance is related to private insurance premium. In other words, even with perfect information, this restriction yields a suboptimal solution. To see that let us consider the following Lagrangean:

$$\begin{aligned}\mathcal{L}_2 &= \sum n_{iN}\left[U_{iN}(z, y) + V_{iN}(R, A)\right] + \sum n_{iS}\left[W_i\left(z, y, R, A\right)\right] \\ &\quad +\lambda \sum n_{ij}\left[y_{ij} - z_{ij} + R_{ij} - \pi A_{ij}\right]\end{aligned}$$

Maximising $\mathcal{L}_2$ with respect to $z, y, R$ and $A$ yields the following FOCs:

$$\frac{\partial \mathcal{L}_2}{\partial z_{iN}} = n_{iN}\left(u'(z_{iN}) - \lambda\right) = 0;$$

[3] As one sees we avoid double counting in the social utility of altruistic families. See on this Hammond (1987), Cremer and Pestieau (2003).

$$\frac{\partial \mathcal{L}_2}{\partial y_{iN}} = -n_{iN}\left(h'(y_{iN}/w_i)/w_i - \lambda\right) = 0;$$

$$\frac{\partial \mathcal{L}_2}{\partial z_{iS}} = n_{iS}\left(Eu'(c_{iS}) - \lambda\right) = 0;$$

$$\frac{\partial \mathcal{L}_2}{\partial y_{iS}} = -n_{iS}\left(h'(y_{iS}/w_i)/w_i - \lambda\right) = 0;$$

$$\frac{\partial \mathcal{L}_2}{\partial R_{iN}} = -n_{iN}\left(Eu'(d_{iN}) - \lambda\right) = 0;$$

$$\frac{\partial \mathcal{L}_2}{\partial A_{iN}} = n_{iN}\pi\left(E_2u'(d_{iN}) - \lambda\right) = 0;$$

$$\frac{\partial \mathcal{L}_2}{\partial R_{iS}} = -n_{iS}\left(Eu'(d_{iS}) - \lambda\right) = 0;$$

$$\frac{\partial \mathcal{L}_2}{\partial A_{iS}} = n_{iS}\pi\left(E_2u'(d_{iS}) - \lambda\right) = 0$$

It appears at once that in this setting our two marginal taxes $T'(y_{ij})$ and $s'(A_{ij})$ are equal to 0. Furthermore the labor suply is first best efficient. What is missing is a full insurance scheme. In our model, individuals can smooth their resources between consumption and health: $d$ and $m$ for the non altruistic parent and $d, c$ and $m$ for the altruistic family but not across the three states of the world. The best they can do is to equate $u'(d_{ij}^0)$ to $E_2u(d_{ij})$ where $d_{ij}^0$ is parent's consumption in the state of full autonomy.

To sum up, with the available instruments, the first-best could not be decentralized even if lump sum transfers were not an issue. In the absence of lump sum transfers redistribution between rich and poor families and between selfish children and selfish parents will imply further departure from the first-best. In the next section we consider the general second best problem and then we study particular cases that allow us to reach intuitive results, keeping in mind that our reference is not the full first-best but the constrained optimum we have just analysed.

# 4 Second best problem

We start by considering the self-selection constraints that prevent non altruistic family members from mimicking other individuals. In the non altruistic family parent and child act separately. Their self-selection constraints along with the Lagrange multipliers ($\mu$) that will be used in the optimisation problem can be written as:

$$U_{iN}(z_{iN}, y_{iN}) \geq U_{iN}(z_{\ell h}, y_{\ell h}) : \mu_i^C(\ell h) \tag{9}$$

$$V_{iN}(R_{iN}, A_{iN}) \geq V_{iN}(R_{\ell h}, A_{\ell h}) : \mu_i^P(\ell h) \tag{10}$$

When the relevant self-selection constraint is binding, we have either $\mu_i^C(\ell h) > 0$ or $\mu_i^P(\ell h) > 0$.Note that $lh$ denotes the type of P or C who is mimicked.

In altruistic families, the issue of mimicking is more difficult. Parents and children can mimick parents and children who belong to different families. We thus write the self-selection constraint of members of type iS family as:

$$W_i(R_{iS}, A_{iS}, z_{iS}, y_{iS}) \geq W_i(R_{\ell h}, A_{\ell h}, z_{\ell' h'}, y_{\ell' h'}) \tag{11}$$

which admits the possibility of $(\ell h) \neq (\ell' h')$.The Lagrange multiplier associated with this self-selection constraint is written $\gamma_i(\ell h, \ell' h')$ where $\ell h$ pertains to the child and $\ell' h'$ to the parent. This multiplier takes a positive value when the associated constraint is binding. Naturally $\gamma_i(iS, iS) = 0$.We allow for the case when either the parent or the child is not mimicking. In other words,(11) includes:

$$W_i(R_{iS}, A_{iS}, z_{iS}, y_{iS}) \geq W_i(R_{iS}, A_{iS}, z_{\ell' h'}, y_{\ell' h'})$$

and

$$W_i(R_{iS}, A_{iS}, z_{iS}, y_{iS}) \geq W_i(R_{\ell h}, A_{\ell h}, z_{iS}, y_{iS})$$

With this new notation we can now write down the problem of the central planner under the form of the following Lagrangean:

$$\begin{aligned}
\mathcal{L}_3 \quad = \quad & \sum n_{iN} \left[ U_{iN}(z, y) + V_{iN}(R, A) \right] + \sum n_{iS} \left[ W_i \left( z, y, R, A \right) \right] \\
& + \lambda \sum n_{ij} \left[ y_{ij} - z_{ij} + R_{ij} - \pi A_{ij} \right] \\
& + \sum_i \sum_{lh} \mu_i^C(\ell h) \left[ U_{iN} - U_{iN}(z_{\ell h}, y_{\ell h}) \right] \\
& + \sum_i \sum_{lh} \mu_i^P(\ell h) \left[ V_{iN} - V_{iN}(R_{\ell h}, A_{\ell h}) \right] \\
& + \sum_{lh} \sum_{l'h'} \gamma_i(\ell h, \ell' h') \left[ W_i - W_i(R_{\ell h}, A_{\ell h}, z_{\ell' h'}, y_{\ell' h'}) \right]
\end{aligned}$$

We obtain the following first-order conditions:

$$\frac{\partial \mathcal{L}_3}{\partial z_{iN}} = \left(n_{iN} + \sum_{\ell h} \mu_i^C\right) u'(z_{iN}) - \sum_{\ell \neq i} \mu_\ell^C (iN) u'(z_{iN})$$
$$- \sum_{\ell} \sum_{\ell' h'} \gamma_\ell (iN, \ell' h') E \left[u'(\hat{c}_{\ell S}) / (iN, \ell' h')\right] - \lambda n_{iN} = 0;$$

$$\begin{aligned} \frac{\partial \mathcal{L}_3}{\partial y_{iN}} &= -\left(n_{iN} + \sum_{\ell h} \mu_i^C\right) \frac{h'(y_{iN}/w_i)}{w_i} + \sum_{\ell \neq i} \mu_\ell^C (iN) u'(z_{iN}) \frac{h'(y_{iN}/w_\ell)}{w_\ell} \\ &\quad + \sum_{\ell} \sum_{\ell' h'} \gamma_\ell (iN, \ell' h') \frac{h'(y_{iN}/w_\ell)}{w_\ell} + \lambda n_{iN} = 0; \end{aligned}$$

$$\frac{\partial \mathcal{L}_3}{\partial z_{iS}} = \left(n_{iS} + \sum \gamma_i\right) E u'(c_{iS}) - \sum_{\ell \neq i} \mu_\ell^C (iS) u'(z_{iS})$$
$$- \sum_{\ell} \sum_{\ell' h'} \gamma_\ell (iS, \ell' h') E \left[u'(\hat{c}_{\ell S}) / (iS, \ell' h')\right] - \lambda n_{iS} = 0;$$

$$\begin{aligned} \frac{\partial \mathcal{L}_3}{\partial y_{iS}} &= -\left(n_{iS} + \sum \gamma_i\right) \frac{h'(y_{iS}/w_i)}{w_i} + \sum_{\ell} \mu_\ell^C (iS) u'(z_{iS}) \frac{h'(y_{iS}/w_\ell)}{w_\ell} \\ &\quad + \sum_{\ell} \sum_{\ell' h'} \gamma_\ell (iS, \ell' h') \frac{h'(y_{iS}/w_\ell)}{w_\ell} + \lambda n_{iS} = 0; \end{aligned}$$

$$\frac{\partial \mathcal{L}_3}{\partial R_{iN}} = \left(n_{iN} + \sum \mu_i^P\right) E \left[u'(d_{iN})\right]$$
$$+ \sum_{\ell \neq i} \mu_\ell^P (iN) E \left[u'(\hat{d}_{\ell N}) / (iN)\right]$$
$$+ \sum_{\ell'} \sum_{\ell h} \gamma_{\ell'} (\ell h\ , iN) E \left[u' \left(\hat{d}_{\ell' s}\right) / (\ell h,\ iN)\right]$$
$$+ \lambda n_{iN} = 0$$

$$\frac{\partial \mathcal{L}_3}{\partial R_{iS}} = -\left(n_{iS} + \sum \gamma_i\right) E \left[u'\left(d_{is}\right)\right]$$
$$+ \sum_{\ell} \mu_\ell^P (iS) E \left[u'(\hat{d}_{\ell N}) / (iS)\right]$$
$$+ \sum_{\ell'} \sum_{\ell h} \gamma_{\ell'} (\ell h,\ iS) E \left[u'(\hat{d}_{\ell' s}) / (\ell h\ , iS)\right]$$
$$+ \gamma n_{iS} = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}_3}{\partial A_{iN}} &= \left(n_{iN} + \sum \mu_i^P\right) \pi E_2 \left[u'\left(d_{iN}\right)\right] \\ &- \sum_{\ell \neq i} \mu_\ell^P \left(iN\right) \pi E_2 \left[u'(\hat{d}_{\ell N}) / \left(iN\right)\right] \\ &- \sum_{\ell'} \sum_{\ell h} \gamma_{\ell'} \left(\ell h,\ iS\right) \pi E_2 \left[u'\left(\hat{d}_{\ell' s}\right) / \left(\ell h,\ iS\right)\right] \\ &- \pi \lambda n_{iN} = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_3}{\partial A_{iS}} &= \left(n_{iS} + \sum \gamma_i\right) \pi E_2 \left[u'\left(d_{iS}\right)\right] - \sum_{\ell} \mu_\ell^P \left(iS\right) \pi E_2 \left[u'(\hat{d}_{\ell N}) / \left(iS\right)\right] \\ &- \sum_{\ell'} \sum_{\ell h} \gamma_{\ell'} \left(\ell h\ , iS\right) \pi E_2 \left[u'(\hat{d}_{\ell' s}) / \left(\ell h\ , iS\right)\right] \\ &- \pi \lambda n_{iS} = 0\end{aligned}$$

where we denote the utility of type $iN$ mimicking type $\ell' h$ as

$$\begin{aligned} Eu'\left(\hat{c}_{\ell N} / \left(iN\ , \ell' h'\right)\right) &= \left(1 - \pi\right) u' \left(\frac{z_{iN} + I_\ell - R_{\ell' h'}}{2}\right) \\ &+ \frac{\pi}{2} \sum_{k=1,2} u' \left(\frac{z_{iN} + I_\ell - R_{\ell' h'} + A_{\ell' h'} - x_k^*}{2}\right)\end{aligned}$$

From the FOCs, we can now write the expressions for the optimal tax on income and subsidy on private insurance.

**Optimal wage tax:**

$$\begin{aligned}\lambda n_{iN}\ T'\left(Y_{iN}\right) &= \sum_{\ell \neq i} \mu_\ell^C(iN) u'(z_{iN}) \left[MRS_{yz}^{iN} - MRS_{yz}^{\ell N}\left(z_{iN} y_{iN}\right)\right] \\ &+ \sum_{\ell} \sum_{\ell' h'} \gamma_\ell(iN, \ell' h') E\left[u'\left(\hat{c}_{\ell s}\right) / \left(iN, \ell' h'\right)\right] \\ &* \left\{MRS_{yz}^{iN} - MRS_{yz}^{\ell S}\left(R_{\ell' h'}, A_{\ell' h'}, z_{iN}, y_{iN}\right)\right\}\end{aligned}$$

$$\begin{aligned}\lambda n_{iS} T'(Y_{iS}) &= \sum_{\ell} \mu_\ell^C(iS) u'(z_{iS}) \left[MRS_{yz}^{iS} - MRS_{yz}^{\ell N}\left(z_{iS} y_{iS}\right)\right] \\ &+ \sum_{\ell} \sum_{\ell' h'} \gamma_\ell(iS, \ell' h') E\left[u'(\hat{c}_{\ell S}) / \left(iS, \ell' h'\right)\right] \\ &* \left\{MRS_{yz}^{is} - MRS_{yz}^{\ell S}\left(R_{\ell' h'}, A_{\ell' h'}, z_{iS}, y_{iS}\right)\right\}\end{aligned}$$

**Insurance subsidy:**

$$\begin{aligned}\lambda n_{iN}\ s'\left(A_{iN}\right) &= \sum_{\ell\neq i}\mu_\ell^P\left(iN\right)\pi E_2\left[u'(\hat{d}_{\ell N})/\left(iN\right)\right]\left\{MRS_{AR}^{iN}-MRS_{AR}^{\ell N}\left(R_{iN}A_{iN}\right)\right\}\\ &\quad+\sum_{\ell'}\sum_{\ell h}\gamma_{\ell'}\left(\ell h\ ,iS\right)E\left[u'\left(\hat{d}_{\ell' s}\right)/\left(\ell h\ ,iS\right)\right]\\ &\quad*\left\{MRS_{AR}^{iN}-MRS_{AR}^{\ell' S}\left(R_{iN}A_{iN}z_{ih}y_{ih}\right)\right\}\end{aligned}$$

$$\begin{aligned}\lambda n_{iS}\ s'\left(A_{iS}\right) &= \sum_{\ell}\mu_\ell^P\left(iS\right)\pi E_2\left[u'(\hat{d}_{\ell N})/\left(iS\right)\right]\left\{MRS_{AR}^{is}-MRS_{AR}^{\ell N}\left(R_{iS}A_{iS}\right)\right\}\\ &\quad+\sum_{\ell'}\sum_{\ell h}\gamma_{\ell'}\left(\ell h,\ iS\right)E\left[u'(\hat{d}_{\ell' S})/\left(\ell h,\ iS\right)\right]\\ &\quad*\left\{MRS_{AR}^{iS}-MRS_{AR}^{\ell' S}\left(R_{iS},A_{iS},z_{\ell h},y_{\ell h}\right)\right\}\end{aligned}$$

At this level of generality, these formulas cannot be easily interpreted. In the next section, we look at some special cases that bring some insights to the main result.

# 5 Special cases

Given the complexity of the problem at hand we now consider two particular cases. In the first, we only look at a society comprising altruistic families $(HS, LS)$. In the second, we have a society with altruistic and non altruistic families having both the same resources. (i.e., $HS$ and $HN$ or $LS$ and $LN$).

## 5.1 Altruistic families

In the appendix, we show that the only relevant (binding) self selection constraint is:

$$W_H\left(R_{HS},A_{HS},z_{HS},y_{HS}\right)=W_H\left(R_{LS},A_{LS},z_{LS},y_{LS}\right).$$

For there on, we obtain the following optimal marginal tax rates:

$$\begin{gathered}T'\left(y_{HS}\right)=0\\ T'\left(y_{LS}\right)=\frac{\gamma_H\left(LS,LS\right)}{\lambda n_{LS}}E\left[u'\left(\hat{c}_{HS}\right)\left(LS,LS\right)\right]\\ \left[MRS_{yz}^{LS}-MRS_{yz}^{HS}\left(R_{LS},A_{LS},z_{LS},y_{LS}\right)\right]>0\end{gathered}$$

$$s'(A_{HS}) = 0$$

$$s'(A_{LS}) = \frac{\gamma_H(LS, LS)}{\lambda n_{LS}} E\left[u'\left(\hat{d}_{HS}/(LS, LS)\right)\right]$$
$$\left[MRS_{AR}^{LS} - MRS_{AR}^{HS}(R_{LS}, A_{LS}, z_{LS}, y_{LS})\right] > 0.$$

These results are pretty intuitive and expected. First, we have the traditional no distortion at top for $HS$. Second, to relax the above self-selection constraint, namely to prevent the rich families from mimicking the poor families one needs an earning tax and a tax on private insurance for type $LS$'s children and parents. By downwardly distorting the choice of labor supply and the purchase of private insurance of the $LS$ the government makes it difficult for both the child and the parent of $HS$ not to reveal their type.

## 5.2 One income class. Altruists and non altruists.

We now consider the altruistic and non altruistic families but both having the same resources. Furthermore, we focus on the case when the resources of the elderly are very different from those of the young.

### 5.2.1 $HS$ and $HN$ with $I_H$ being much higher than $w_H$

Assume that $I_H$ is large enough relative to $w_H$. In other words, in the Laissez Faire, the best off individual is the parent of the non altruistic type, followed by the members of the altruistic family, the worse off being the child of the non altruistic family. This implies that the government may want to redistribute from $P$ of type $HN$ to type $HS$ family and to $C$ of type $HN$. Therefore in choosing $(RA)$ $P$ of type $HN$ has the incentive to mimick $P$ of type $HS$, while $C$ of type $HS$ family may mimick $C$ of type $HN$: $\mu_H^P(HS) > 0$ and $\gamma_H(HN, HS) > 0$.

With $I_H$ large enough we can thus expect:

$$MRS_{AR}^{HN} > MRS_{AR}^{HS} \quad ; \quad MRS_{yz}^{HS} > MRS_{yz}^{HN}.$$

The second inequality is due to the income effect on $C$ of $HS$.

We thus find the following values for the tax functions:

$$T'(y_{HS}) = 0$$

$$\lambda n_{HN} T'(y_{HN}) = \gamma_H(HN\ , HS)\, E\left[u'(\hat{c}_{HS})/(HN,\ HS)\right]$$
$$* \left(MRS_{yz}^{HN} - MRS_{yz}^{HS}(R_{HS}, A_{HS}, z_{HN}, y_{HN})\right) < 0$$

$$
\begin{gathered}
\lambda n_{HS} s'(A_{HS}) = \mu_H^P (HS) \left[ Eu'\left(\hat{d}_{HN}\right) / (HS) \right] \\
* \left( MRS_{AR}^{HS} - MRS_{AR}^{HN} (R_{HS}, A_{HS}) \right) \\
+\gamma_H (HN, HS) E \left[ u'\left(\hat{d}_{HS}\right) / (HN, HS) \right] \\
* \left( MRS_{AR}^{HS} - MRS_{AR}^{HS} (R_{HS}, A_{HS}, z_{HN}, y_{HN}) \right) < 0
\end{gathered}
$$

$$
s'(A_{HN}) = 0
$$

The child of HS and the parent of HN being the mimickers, they are subject to a zero marginal tax. The child of HN faces a upward distortion on his labor supply and the parent of HS a tax on his purchase of private insurance.

#### 5.2.2 $LN$ and $LS$ with $I_L$ being very low relative to $w_L$

In this particular case, the parent of $LN$ is the worst off individual and his child the best off.

The government will transfer from C of $LN$ to family of type $LS$ and to $P$ of $LN$ leading to $\mu_L^C (LS) > 0$ and $\gamma_L (LS\ LN) > 0$.

We let

$$
MRS_{AR}^{LN} < MRS_{AR}^{LS} \quad ; \quad MRS_{yz}^{LS} < MRS_{yz}^{LN}
$$

and thus:

$$
\begin{gathered}
\lambda n_{HS} T'(y_{LS}) = \mu_L^C (LS) u'(z_{LS}) \\
* \left\{ MRS_{yz}^{LS} - MRS_{yz}^{LN} (z_{LS}\ y_{LS}) \right\} \\
+\gamma_L (LS\ LN) E \left[ u'(\hat{c}_{LS}) / (LS\ LN) \right] \\
* \left( MRS_{YZ}^{LS} - MRS_{YZ}^{LS} \right) (R_{LN}, A_{LN}, z_{LS}, y_{LS}) < 0
\end{gathered}
$$

$$
\begin{gathered}
T'(y_{LN}) = 0 \\
s'(A_{LS}) = 0
\end{gathered}
$$

$$
\begin{gathered}
\lambda n_{LN}\ s'(A_{LN}) = \gamma_L (LS\ , LN) E \left[ U'\left(\hat{d}_{LS}\right) / (LS,\ LN) \right] \\
* \left( MRS_{AR}^{LN} - MRS_{AR}^{LS} \right) (R_{LN}, A_{LN}, z_{LS}, y_{LS}) < 0.
\end{gathered}
$$

We can now summarize the main results of this subsection. They have been obtained by making some assumptions on the relative size of $I$ and $w$.A relatively high $I$ is to the advantage of the selfish parent and a relatively high $w$ is favorable to the selfish child.

**Summary**

| | Type H | | Type L | | |
|---|---|---|---|---|---|
| | ***P*** | ***C*** | | ***P*** | ***C*** |
| $j = N$ | $s'(A_{HN}) = 0$ | $T'(y_{HN}) < 0$ | $j = N$ | $s'(A_{LN}) < 0$ | $T'(y_{LN}) = 0$ |
| $j = S$ | $s'(A_{HS}) < 0$ | $T'(y_{HS}) = 0$ | $j = S$ | $s'(A_{LS}) = 0$ | $T'(y_{LS}) < 0$ |

## 5.3 General case

We now turn to the general case but by assuming specific directions for the incentive compatibility constraints. If we want to consider the general case with the 4 types, we have to restrict the number of binding incentive constraints. We will adopt a pattern that corresponds to the idea that $I_H$ is much higher than $w_H$ and conversely for $I_L$ and $w_L$. For the parent, we assume the following sequence of mimicking:

$$HN \rightarrow HS \rightarrow LS \rightarrow LN,$$

and for the child, we have:

$$\begin{array}{ccc} HS & \rightarrow & HN \\ & \searrow & \\ LN & \rightarrow & LS \end{array}$$

With such a pattern, we get the following signs for the tax instruments:

$$s'(A_{LN}) < 0,\ s'(A_{LS}) < 0,\ s'(A_{HS}) < 0,\ s'(A_{HN}) = 0$$

$$T'(y_{LN}) = 0,\ T'(y_{LS}) \lesseqgtr 0,\ T'(y_{HS}) = 0,\ T'(y_{HN}) < 0.$$

The expression for $T'(y_{LS})$ can be given explicitly:

$$\begin{aligned} \lambda n_{LS} T'(y_{LS}) = {} & \mu_L^C(LS)\, u'(z_{LS}) \left[MRS_{yz}^{LS} - MRS_{yz}^{LN}(z_{LS}, y_{LS})\right] \\ & + \gamma_H(LS, LS)\, E\left[u'(\hat{c}_{HS}) / (LS, LS)\right] \\ & * \left[MRS_{yz}^{LS} - MRS_{yz}^{HS}(R_{LS}, A_{LS}, z_{LS}, y_{LS})\right] \\ & + \gamma_L(LS, LN)\, E\left[u'(\hat{c}_{LS}) / (LS, LN)\right] \\ & \star \left[MRS_{YZ}^{LS} - MRS_{YZ}^{LS}(R_{LN}, A_{LN}, z_{LS}, y_{LS})\right] \gtreqless 0 \end{aligned}$$

We use in the expression the effect of the earning tax for type LS on the self-selection constraints pertaining to parents of type LS.The first and the third term of the RHS are negative. The second term is positive.

## 6 Concluding remarks

The purpose of this paper was to design an optimal non linear tax transfer policy for long term care. The setting is relatively simple. Each elderly person may have or not an altruistic child who will help him in case of loss of autonomy. Besides child's assistance, LTC can be obtained from private insurance, self-insurance and public insurance. Each of these ways has its own shortcomings. Both family solidarity and self-insurance provide only partial insurance. Private insurance only covers a lump sum amount in case of dependency and financing public insurance is distortionary. Public insurance plays two roles: redistribution (between parents and children and across generations) and insurance.

LTC social insurance in this paper is providing cash benefits that are a non linear function of private insurance premiums. Thus both private and public schemes are invariant to the severity of dependence and are imperfect insurance devices. Non linear taxes imply that the government controls the purchase of private insurance by all individuals, but that in case of dependence the actuarially fair benefits are paid by the private insurance. In other words, the public sector is not a provider of LTC benefits[4]. An alternative approach that we plan to examine in future work is to introduce in kind LTC benefits such as nursing facilities that would allow for self-selection of dependent parents according to the severity of their dependence.

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