A comparison of optimal tax policies when compensation and responsibility matter*

Work in Progress.

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Abstract

This paper examines optimal redistribution in a model with high- and low-skilled individuals with heterogeneous tastes for labor, that either work or not. With such double heterogeneity, it is well known that traditional Utilitarian and Welfarist criteria suffer serious flaws. As a response, several other criteria have been proposed in the literature. We compare the extent to which optimal policies based on different normative criteria obey the principles of compensation (for differential skills) and responsibility (for preferences for labor). In full information, Roemer's proposal, the Conditional Equality and Egalitarian Equivalence criteria are the only ones that obey one of the principles. However, when preferences and skills are unobservable, the second best constraints forces even these criteria to perform poorly. The paper also shows that the well-established sufficient condition for advocating an Earned Income Tax Credit remains valid under Utilitarian, Welfarist, Boadway *et al.* (2002), Roemer (1993), Van de gaer (1993) and Egalitarian Equivalent objectives, and how this condition must be adjusted under a Non-welfarist and a Conditional Equality criterion.

Key Words: optimal income taxation, equality of opportunity, heterogeneous preferences for labor.
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1 Introduction

Since the seminal paper of Mirrlees (1971), the literature on optimal income taxation has traditionally assumed that agents differ only in one dimension, their skills. The government maximizes either the sum of individual utilities (the Utilitarian objective) or the sum of an increasing concave transformation of individual utilities (which we label the Welfarist objective). The former was used in, e.g., Ebert (1992) and Hellwig (2007), the latter by, e.g., Mirrlees (1971) and Diamond (1998). Sandmo (1992) studies optimal taxes when people only differ in tastes. He shows that the case for redistribution is weaker with taste heterogeneity and Utilitarianism if the rich, due to a lower preference for leisure, are more efficient at generating utility. He points at the fundamental philosophical issues that arise in a world with taste heterogeneity.

Recently, the optimal taxation literature derived income tax schedules when agents are heterogeneous in both skills and preferences.[1] In this context, the optimal tax has been derived under a Utilitarian criterion (e.g., Diamond, 1980; Choné and Laroque, 2008) and under a Welfarist criterion (e.g., Kaplow, 2008). However, preference heterogeneity poses ethical questions which challenge these standard objective functions, see, e.g., Rawls (1971), Sen (1980) and Dworkin (1981). Several responses to deal with these issues have been formulated.

Boadway *et al.* (2002) use a Utilitarian social welfare function where different weights can be assigned to individuals with different preferences for leisure. This amounts to using different cardinalizations of individual utility functions. Paternalistic criteria (which we label non-welfarist), in which the planner uses a reference value for the taste for work and maximizes the sum of these adjusted utilities have also been considered, by, e.g. Schokkaert *et al.* (2004). The use of such a reference preference is motivated from a concern with a person's objective well-being.

Other authors explicitly appeal to equality of opportunity in one way or another. Roemer (1993 and 1998) proposes that equality of opportunity for welfare holds when the utilities of all those who exercised a comparable degree of responsibility are equal, irrespective of their skills. Assuming that those that have the same preferences have exercised a comparable degree of responsibility, the ideal is to give the same utility to those with the same preferences, irrespective of their skills. Since utilities have to be equal for each preference, it will usually (except, as we will see in the first best) not be possible to achieve this. Roemer therefore suggests to maximize a weighted average of the minimal utilities across individuals having the same tastes. As a result, Fleurbaey (2008) calls this the mean of mins criterion. Van de gaer (1993) proposed a related criterion. For each level of skill, utility as a function of the taste parameter can be interpreted as the utilities to which someone with that skill level has access. The proposal is then to maximize the value of the smallest opportunity set, where the opportunity set is the surface under utilities to which he has access, weighted by the frequency with which the corresponding preference parameter occurs. Hence we should maximize the average utility of the skill group that has lowest average utility. For this reason, Fleurbaey calls this the min of means criterion.[2]

[1] Heterogeneous individuals in terms of skills and needs are also studied in the optimal income tax literature (e.g., Rowe and Woolley, 1999; Boadway and Pestieau, 2007). In this case, tagging is used to relax the self-selection constraints. Tagging improves the equity-efficiency tradeoff we would obtain with only a nonlinear income tax.

[2] Characterizations of both criteria have been provided by Ooghe et al (2007) and Fleurbaey (2008).The former maximatization contrasts the equality of opportunity idea present in the mean of mins and the equality of opportunity set idea present in the min of means criterion. The latter stresses that, contrary to liberal egalitarian theories of equality of opportunity, both share the idea of utilitarian compensation, which is the principle of zero inequality

The dominant branch of the equality of opportunity literature, liberal egalitarian theories of justice, argue that income or welfare inequalities arising from non-responsibility factors such as innate skills should be eliminated (the compensation principle) and inequalities arising from responsibility factors such as preferences should be respected (the responsibility principle)[3]. These two principles characterize the equality of opportunity approach, see Fleurbaey (1995a). However, as demonstrated by Fleurbaey (1994) and Bossert (1995), they appear to be incompatible even in the first best. To overcome this difficulty, one or both principles have been weakened to find suitable allocations in the first best. Fleurbaey (1995b) weakens one of the principles, maintaining the other. More in particular, he showed that weakening the compensation principle leads to the Conditional Equality allocation, where everyone has the same utility if his actual resource bundle is evaluated with reference preferences. And weakening the responsibility principle leads to the Egalitarian Equivalent allocation, where everyone is indifferent between his actual resource bundle and a reference bundle. If conditional equality cannot be achieved, or if there exists no allocation that renders everyone indifferent, the strategy is to determine a social ordering based on the ideas behind these allocation rules. This lead Bossert *et al.* (1999) to define the Conditional Equality social ordering. The latter maximizes the lowest value of the evaluation of individuals' resource bundles, using the reference tastes. Similarly, one can define an Egalitarian Equivalent social ordering that maximizes the smallest consumption bundle that makes an individual indifferent to this bundle and the reference bundle. It will turn out that, translated in the context of our model, this gives rise to the social ordering used and characterized by Fleurbaey and Maniquet (2006).

We assume that labor supply responses are along the extensive margin (labor force participation) and that individuals have two types of skills and heterogeneous tastes for labor. We derive the optimal tax policies, as function of the behavioral elasticities, under Utilitarianism, the Welfarist objective, the criterion of Boadway *et al.* (2000), the one of Schokkaert *et al.* (2004), the conditional equality and egalitarian equivalent social orderings. Under full information, we show that only the latter two have nice properties from the perspectives of equality of opportunity. Under asymmetric information, this paper shows that all the previous criteria fail to implement equality of opportunity.

Moreover, the optimal policies under the various criteria are close to each other. In particular, this paper characterizes who gets the largest transfer under the distinct objective functions. By definition, an Earned Income Tax Credit (EITC) provides the largest transfer to the low-skilled workers. On the contrary, with a Negative Income Tax (NIT), the inactive agents receive the largest transfer. As usual in the literature, let us define the ratio of the social marginal utility to the marginal value of public funds as the (average) marginal social welfare weight. The literature has well established that when labor supply responses are modeled along the extensive margin (i.e. the agent decides to participate or not in the labor force), a marginal social welfare weight lower (larger) than one on disabled workers implies a NIT (EITC) (Diamond, 1980; Saez, 2002). This paper shows that this result is valid under the Utilitarian, the Welfarist and the Boadway *et al.* criteria. However, we show how this sufficient condition is modified under a Non-welfarist, Conditional Equality and Egalitarian Equivalence criterion.

aversion for inequalities due to different preferences.

[3]For an overview of this literature, see Fleurbaey (2008) or Fleurbaey and Maniquet (2009).

The paper is organized as follows. In Section 2, we describe the model, provide the characterization of the individuals' behavior, and describe the decision variables of the government under full and asymmetric information. Section 3 states the axioms behind equality of opportunity and presents the distinct objective functions. Section 4 investigates the optimal tax policies under full information. In Section 5, we extend the analysis to the asymmetric information economy. Sufficient conditions for a NIT or a EITC are given. Section 6 concludes the paper. All proofs are gathered in appendix.

2 The model

2.1 Individual behavior

We consider an economy where agents differ along two dimensions. First, they differ by their skill (or productivity) levels which take two values, $w_H > w_L > 0$. Skill levels equal wages before taxation, since the production function exhibits constant returns to scale. The proportion of low-skilled agents (or w_L-type) in the population is given by γ, $1-\gamma$ is the proportion of high-skilled people (or w_H-type). Second, there is an heterogeneity parameter, α, that describes labor disutility (disutility of effort or preference for leisure). The α parameter is distributed on the interval $[0,+\infty)$, according to the cumulative distribution function $F(\alpha):[0,+\infty)\to[0,1]:\alpha\to F(\alpha)$ and the corresponding density function $f(\alpha)$. The latter is continuous and positive over its domain. These functions are common knowledge. We assume that productivity and labor disutility are independently distributed.

The agents' only decision is whether to work or not, i.e. labor supply is modeled along the extensive margin.[4] Utility is quasilinear and represented by:

$$v(x)-\alpha \text{ if they work,}$$
$$v(x) \text{ if they do not work,}$$

where x is consumption, $v(x):\mathbb{R}^+\to\mathbb{R}:x\to v(x)$ with $v'>0\geq v''$ and $\lim_{x\to\infty}v'(x)=0$.

2.2 The government's decisions

Under full information (so-called first-best), the government implements a tax policy depending on α and w_i, $i:=L,H$. Individuals are then assigned to low-skilled jobs (activity l, where the gross wage is w_L), to high-skilled jobs (activity h, where the gross wage is w_H) or to inactivity (activity u) depending on their α and w_i. Their consumption bundles also depend on their activity, their skills and taste parameter. Formally, the government determines the consumption functions $x_j(\alpha,w_i)$: $\{l,h,u\}\times[0,+\infty)\times\{w_L,w_H\}\to\mathbb{R}^+$. Low-skilled people cannot get access to high-skilled jobs, and, since efficiency matters, it will never be optimal that high-skilled people work in low-skilled jobs. By putting these people in high-skilled jobs instead of low-skilled jobs, they produce more which can be used to increase someone's consumption. Consequently, the consumption function $x_\ell(\alpha,w_H)$ and $x_h(\alpha,w_L)$ can be neglected in the sequel.

The government has to assign people of both types of skill to work or inactivity, depending on their taste for leisure α. Since the variable α is continuously distributed, we need to work with

[4] There is growing evidence that the extensive margin matters a lot, e.g. Meghir and Philips (2008).

functions defined over measurable subsets of the domain. Let $\mathcal{A}$ denote the set of the measurable subsets of $[0, +\infty)$.

$\delta_L(\alpha) : \mathcal{A} \to \{0, 1\} : \forall \alpha \in A : \delta_L(\alpha) = 1$ ($\delta_L(\alpha) = 0$) if all w_L-type agents with α in A are employed (inactive).

$\delta_H(\alpha) : \mathcal{A} \to \{0, 1\} : \forall \alpha \in A : \delta_H(\alpha) = 1$ ($\delta_H(\alpha) = 0$) if all w_H-type agents with α in A are employed (inactive).

As a consequence $n_L \overset{\text{def}}{\equiv} \int_0^\infty \delta_L(\alpha)\, dF(\alpha)$ is the fraction of low-skilled that are employed and $n_H \overset{\text{def}}{\equiv} \int_0^\infty \delta_H(\alpha)\, dF(\alpha)$ is the fraction of high-skilled that are employed. The Government budget constraint can now be formulated as follows:

$$\gamma \left[\int_0^\infty \left[\delta_L(\alpha)(w_L - x_\ell(\alpha, w_L)) - (1 - \delta_L(\alpha)) x_u(\alpha, w_L) \right] dF(\alpha) \right] \tag{1}$$
$$+(1-\gamma) \left[\int_0^\infty \left[\delta_H(\alpha)(w_H - x_h(\alpha, w_H)) - (1 - \delta_H(\alpha)) x_u(\alpha, w_H) \right] dF(\alpha) \right] \geq R,$$

where R is an exogenous revenue requirement, which can be positive or negative, the latter meaning that the economy has external resources at its disposal. The government's budget constraint must be binding at the optimum as all government objectives considered in the paper are increasing in individuals' consumption.

The problem for the government in the first best is to determine an allocation, that is a set of functions $x_\ell(\alpha, w_L), x_h(\alpha, w_H), x_u(\alpha, w_H), x_u(\alpha, w_L), \delta_L(\alpha), \delta_H(\alpha)$ that is normatively desirable and satisfies the government budget constraint (1).

In the second best, the government implements a tax schedule that depends only on income levels ($0, w_L$ or w_H) hence it is conditioned only on activity status (l, h or u). The government then defines three consumption levels x_j with $j = \ell, h, u$ denoting consumption levels respectively in low-skilled jobs, in high-skilled jobs and when not participating in the labor force. These consumption bundles have to meet the government budget constraint, the set of self-selection constraints (which will be stated in Section 5) and have to be normatively desirable. The next section discusses which normative principles or criteria the government can use.

3 Equality of opportunity

This paper studies whether the normative criteria usually assumed in the optimal tax literature succeed in reaching equality of opportunity. The next subsection formally defines equality of opportunity.

3.1 Two equality of opportunity principles

Let, for the case where $Y = L$ or $H, y = l$ if $Y = L$ and $y = h$ if $Y = H$,

$$u(x(\alpha, w_Y), \delta_Y(\alpha), \alpha) = \begin{cases} v(x_y(\alpha, w_Y)) - \alpha & \text{if } \delta_Y(\alpha) = 1, \\ v(x_u(\alpha, w_Y)) & \text{if } \delta_Y(\alpha) = 0. \end{cases}$$

We assume throughout that people are responsible for their tastes, but not for their skills[5]. We

[5] Two remarks can be made at this point. First, if people are not responsible for anything, from a perspective of equality of opportunity, the only possible objective is leximin which results in full equality of utility levels. Second, it is possible to follow the suggestion by Pestieau and Racionero (2009) to disentangle the parameter α in two components: $\alpha = \alpha_P + \alpha_D$, where people are responsible for α_P (a preference parameter), but not for α_D (a

can then apply Fleurbaey (1994) 's model to capture the intuitions of equality of opportunity in two axioms. The first equality of opportunity axiom expresses the idea of compensation:

EWEP (Equal Welfare for Equal Preferences):
$\forall \alpha \in [0, +\infty) : u\left(x\left(\alpha, w_L\right), \delta_L\left(\alpha\right), \alpha\right) = u\left(x\left(\alpha, w_H\right), \delta_H\left(\alpha\right), \alpha\right)$.

This axiom ensures that the allocation is such that differences in skills do not influence a person's welfare. Resources (consumption and activity status) have to be assigned such that they compensate completely for all inequalities in welfare due to differences in skills. The second axiom of equality of opportunity expresses the idea of responsibility:

ETES (Equal Transfers for Equal Skills):
$\forall \alpha, \alpha' : \delta_L\left(\alpha\right) = \delta_L\left(\alpha'\right) = 1$ and $\forall \alpha'' : \delta_L\left(\alpha''\right) = 0:$
$x_\ell\left(\alpha, w_L\right) - w_L = x_\ell\left(\alpha', w_L\right) - w_L = x_u\left(\alpha'', w_L\right) = x_u\left(w_L\right)$,
$\forall \alpha, \alpha' : \delta_H\left(\alpha\right) = \delta_H\left(\alpha'\right) = 1$ and $\forall \alpha'' : \delta_H\left(\alpha''\right) = 0:$
$x_h\left(\alpha, w_H\right) - w_H = x_h\left(\alpha', w_H\right) - w_H = x_u\left(\alpha'', w_H\right) = x_u\left(w_H\right)$,

with some abuse of notations for the last term in both expressions. The latter emphasizes that taxes only depend on w_i. This axiom has three implications. For each skill level all inactive get the same benefit, all workers pay the same tax, and the transfer received by the inactive is equal to minus the tax paid by the workers. As a result, people are hold responsible for their taste for leisure. The transfer that each individual obtains only depends on his skill level, not on his preference for leisure. Therefore, welfare differences that are caused by differential tastes are not compensated and fully respected.

We say that an allocation satisfies full equality of opportunity if it satisfies both EWEP and ETES. Formally

FEO (Full Equality of Opportunity):
An allocation satisfies full equality of opportunity if it satisfies both EWEP and ETES.

In the traditional framework, where the government only (re-)distributes consumption, even in the first best there does not exist a FEO allocation -see, e.g., Fleurbaey (1994) and Bossert (1995). For this reason, Fleurbaey (1995b) suggested weakening at least one of the axioms, while maintaining the other[6]. This allowed him to define two allocations, the first requires the identification of a reference value for the taste parameter, $\widetilde{\alpha}$, the second a reference value for the resource bundle, here taken to be the consumption level $\widetilde{x}$ and $\delta_Y = 1$, $y = L$ or H.

CE (Conditional Equality):
An allocation is the conditional equality allocation if and only if for all α and $Y = L$ or H it equalizes $u\left(x\left(\alpha, w_Y\right), \delta_Y\left(\alpha\right), \widetilde{\alpha}\right)$ at the highest feasible level.

EE (Egalitarian Equivalence):

disability parameter). The present framework can be adjusted to deal with this issue, without altering the main results of the paper.

[6] Of course, it is also possible to weaken both axioms simultaneously -see, e.g., Bossert and Fleurbaey (1996) or Fleurbaey and Maniquet (2009).

An allocation is egalitarian equivalent if and only if for all a, w_Y and $\delta_Y(\alpha) : u(x(\alpha, w_Y), \delta_Y(\alpha), \alpha) = u(\widetilde{x}, 1, \alpha)$ and $\widetilde{x}$ is at the highest feasible level.

The CE allocation ensures that all individuals are equally well of with their actual bundle of resources when this is evaluated using the reference preference $\widetilde{\alpha}$. The EE allocation makes all individuals indifferent between their actual resource bundle and the reference bundle which gives them $\widetilde{x}$ and where they have to work. In our definition here, we already incorporated that the allocation has to be efficient (no resources are wasted) by, in the CE allocation, equalizing at the highest possible level, and in the EE allocation pursuing indifference at the highest feasible level of $\widetilde{x}$. A CE or EE allocation need not exist. In particular, in the second best, it will not be possible to equalize the reference utilities as required by CE, and, even in the first best, indifference for all individuals with the reference bundle is not feasible in our model. We formulate maximin social orderings inspired by the CE and EE allocation in the next section. First we look at other social objective functions that have been proposed in the literature.

3.2 Different social objective functions

In the literature on optimal taxation several social objective functions have been introduced. We now introduce the alternatives that will be considered in the paper.

A Utilitarian social objective function maximizes average utility in the (given) population. Hence our Utilitarian planner maximizes

$$\begin{aligned} S^U &= \gamma \int_0^\infty \delta_L(\alpha) \left[v(x_\ell(\alpha, w_L)) - \alpha \right] dF(\alpha) \\ &+\gamma \int_0^\infty (1 - \delta_L(\alpha)) v(x_u(\alpha, w_L)) dF(\alpha) \\ &+(1-\gamma) \int_0^\infty \delta_H(\alpha) \left[v(x_h(\alpha, w_H)) - \alpha \right] dF(\alpha) \\ &+(1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) v(x_u(\alpha, w_H)) dF(\alpha). \end{aligned} \tag{2}$$

The Utilitarian criterion has been extensively used in the optimal income tax literature (a.o., Ebert (1992), Diamond and Sheshinski (1995), Boadway *et al.*(2000)).

Our Welfarist social objective maximizes the average of a concave transformation of individual utilities. The concave transformation allows the expression of inequality aversion with respect to the distribution of utilities. Let the function $\Psi : \mathbb{R} \to \mathbb{R} : a \to \Psi(a)$ be a strictly concave function. Our Welfarist objective function becomes

$$\begin{aligned} S^W &= \gamma \int_0^\infty \delta_L(\alpha) \Psi \left(v(x_\ell(\alpha, w_L)) - \alpha \right) dF(\alpha) \\ &+\gamma \int_0^\infty (1 - \delta_L(\alpha)) \Psi \left(v(x_u(\alpha, w_L)) \right) dF(\alpha) \\ &+(1-\gamma) \int_0^\infty \left(\delta_H(\alpha) \Psi \left(v(x_h(\alpha, w_H)) - \alpha \right) dF(\alpha) \right) \\ &+(1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) \Psi \left(v(x_u(\alpha, w_H)) \right) dF(\alpha). \end{aligned} \tag{3}$$

Assumed in the seminal article of Mirrlees (1971), this welfare function has been very popular since then (e.g., Atkinson and Stiglitz (1980), Diamond (1998), Choné and Laroque (2005)).

The Boadway *et al.* (2002)'s objective function allows to attach a weight to individuals' utilities that depends on their taste for leisure. Let $W(\alpha) : \mathbb{R}^+ \to \mathbb{R}^+ : \alpha \to W(\alpha)$ be the social welfare weight given to the utility of an individual with disutility of labor equal to α. The Boadway *et al.* objective function is given by

$$\begin{aligned}
S^B &= \gamma \int_0^\infty \delta_L(\alpha) W(\alpha) \left[v(x_\ell(\alpha, w_L)) - \alpha \right] dF(\alpha) \\
&+\gamma \int_0^\infty (1 - \delta_L(\alpha)) W(\alpha) v(x_u(\alpha, w_L)) dF(\alpha) \\
&+(1-\gamma) \int_0^\infty \delta_H(\alpha) W(\alpha) \left[v(x_h(\alpha, w_H)) - \alpha \right] dF(\alpha) \\
&+(1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) W(\alpha) v(x_u(\alpha, w_H)) dF(\alpha). \qquad (4)
\end{aligned}$$

This objective function was explicitly introduced to deal with individuals that are heterogeneous in skills and preferences. Also used in Cremer *et al.* (2004 and 2007) for instance, this criterion adopts distinct cardinalizations of individual utilities depending on the individual's taste parameter α.

Our Non-welfarist social objective function uses a paternalistic view for the valuation of labor disutility. We define the reference labor disutility as $\overline{\alpha} \geq 0$, which is the weight attached by the government to the labor disutility of every individual. The social objective becomes

$$\begin{aligned}
S^N &= \gamma \left[\int_0^\infty \delta_L(\alpha) \left[v(x_\ell(\alpha, w_L)) - \overline{\alpha} \right] dF(\alpha) \right] \\
&+\gamma \int_0^\infty (1 - \delta_L(\alpha)) v(x_u(\alpha, w_L)) dF(\alpha) \\
&+(1-\gamma) \int_0^\infty \delta_H(\alpha) \left[v(x_h(\alpha, w_H)) - \overline{\alpha} \right] dF(\alpha) \\
&+(1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) v(x_u(\alpha, w_H)) dF(\alpha). \qquad (5)
\end{aligned}$$

With this objective function, the social planner has a different idea than the individuals themselves about the 'correct' or reasonable disutility of work. There is then a clear paternalistic motive for taxation which arises from differences between social and private preference. Schokkaert *et al.* (2004) consider this social objective function. Marchand *et al.* (2003) and Pestieau and Racionero (2009) consider an alternative paternalistic approach in which the government attaches a larger weight to the labor disutility of disabled individuals.

To state the next two objective functions, we define an operator that takes the first element of a set with 2 elements if $\delta(\alpha)$ equals one, and the second element otherwise. Formally, we define the operator as

$$\underset{\delta(\alpha)}{\text{oper}} \{a, b\} = a \text{ if } \delta(\alpha) = 1 \text{ and } \underset{\delta(\alpha)}{\text{oper}} \{a, b\} = b \text{ if } \delta(\alpha) = 0.$$

Roemer' s (1998) objective function can be written as

$$S^R = \int_0^\infty \min\{\underset{\delta_L(\alpha)}{\text{oper}}\{v\left(x_\ell\left(\alpha, w_L\right)\right) - \alpha, v\left(x_u\left(\alpha, w_L\right)\right)\},$$
$$\underset{\delta_H(\alpha)}{\text{oper}}\left\{v\left(x_h\left(\alpha, w_H\right)\right) - \alpha, v\left(x_u\left(\alpha, w_H\right)\right)\right\}\}dF\left(\alpha\right). \tag{6}$$

For each α, the government assigns low and high skilled individuals to employment or unemployment. The min function in the integral term takes, for each α level, the smallest utility accross skill types. The Roemer rule maximizes the sum (over α) of these minimal utility levels. It has been used by Roemer *et al.* (2003) to empirically compare the extent to which fiscal policies manage to equalize opportunities for income acquisition in a set of countries.

While Roemer's proposal is well known, an obvious alternative was proposed by Van de gaer (1993). He suggests to specify the social objective function as

$$S^V = \min\{\int_0^\infty \underset{\delta_L(\alpha)}{\text{oper}}\left\{v\left(x_\ell\left(\alpha, w_L\right)\right) - \alpha, v\left(x_u\left(\alpha, w_L\right)\right)\right\}dF\left(\alpha\right),$$
$$\int_0^\infty \underset{\delta_H(\alpha)}{\text{oper}}\left\{v\left(x_h\left(\alpha, w_H\right)\right) - \alpha, v\left(x_u\left(\alpha, w_H\right)\right)\right\}dF\left(\alpha\right)\} \tag{7}$$

The basic intuition for this criterion is that we look at the smallest opportunity set across types, and try to make this opportunity set as big as possible (see also Schokkaert *et al.* (2004) and Ooghe *et al.* (2007)). Opportunity sets are measured by the surface below the utilities that a type can reach as a function of its value for α, weighted by $dF\left(\alpha\right)$.

We formulate the maximin objective function inspired by the Conditional Equality allocation:

$$S^C = \min_{\alpha, w_Y} u\left(x\left(\alpha, w_Y\right), \delta_Y\left(\alpha\right), \widetilde{\alpha}\right), \tag{8}$$

meaning that the optimal policy is determined such that the lowest level of utility that someone in the population gets with his actual allocation, evaluated at the reference preferences $\widetilde{\alpha}$, is as high as possible. This criterion was explicitly considered by Bossert *et al.* (1999).

Finally, we formulate a maximin objective function inspired by the Egalitarian Equivalent allocation. To be as well off as with their actual resource bundles, workers require this resource bundle. While inactive people, to be as well off working as with their actual consumption require a consumption equal to $v^{-1}\left(v\left(x_u\left(\alpha, w_Y\right)\right) + \alpha\right)$, where $x_u\left(\alpha, w_Y\right)$ is their actual consumption level. Hence, we can define an Egalitarian Equivalent ordering as maximizing

$$\begin{aligned} S^E \quad = \quad & \min_{\alpha, w_Y}\left\{x_\ell\left(\alpha, w_L\right), x_h\left(\alpha, w_H\right), v^{-1}\left(v\left(x_u\left(\alpha, w_L\right)\right) + \alpha\right),\right. \\ & \left. v^{-1}\left(v\left(x_u\left(\alpha, w_H\right)\right) + \alpha\right)\right\}. \end{aligned} \tag{9}$$

In our framework, this social ordering is the natural counterpart of the ordering proposed by Fleurbaey and Maniquet (2005 and 2006). In their papers, the equivalent wage for an individual is defined as the wage rate such that he is indifferent between his actual bundle and the bundle that he could reach if he had his equivalent wage. Their proposed social ordering is then to maximize the minimal equivalent wage. Fleurbaey and Maniquet work in an intensive labor supply choice model;

the computation of the equivalent wage involves a counterfactual labor supply choice lying between inactivity and full time employment. In our extensive labor supply model, such a choice in not available. However, we can adjust the concept by comparing the actual consumption bundle with the wage making the individual indifferent with full time employment. Formally, in our extensive margin model, the equivalent wage is defined for the employed as $x_y^E(\alpha, w_Y) = x_y(\alpha, w_Y)$ and for the inactive as $x_u^E(\alpha, w_Y) : v\left(x_u^E(\alpha, w_Y)\right) - \alpha = v\left(x_u(\alpha, w_Y)\right)$, which implies that $x_u^E(\alpha, w_Y) = v^{-1}\left(v\left(x_u(\alpha, w_Y)\right) + \alpha\right)$. Maximinning this equivalent wage leads to the social ordering defined in (9).

4 First best optima

The following theorem characterizes the optimal policies under full information. It allows us to verify whether these optima satisfy the EWEP or the ETES axiom. The superscripts U, W, B, N, R, V,C and E are used to characterize the variables at the optimum under the Utilitarian, Welfarist, Boadway *et al.*, Non-welfarist, Roemer, Van de gaer, Conditional Equality and Egalitarian Equivalent objectives, respectively. The first theorem studies the optimal policies under the criteria that are only loosely based on equality of opportunity principles. We state the analytical properties, interpret them and check whether the EWEP and ETES axioms are satisfied.

Theorem 1: With full information, the following configuration of policies is optimal:

(a) Utilitarian planner:

Consumption bundles:

$$\overline{x}^U = x_\ell^U(\alpha, w_L) = x_u^U(\alpha, w_L) = x_h^U(\alpha, w_H) = x_u^U(\alpha, w_H).$$

Activity assignment:

$$\begin{array}{l}\delta_L(\alpha) = 1 \text{ for all } \alpha \leq \alpha_L^{U*} \text{ and } \delta_H(\alpha) = 1 \text{ for all } \alpha \leq \alpha_H^{U*}.\\ \text{with } \alpha_i^{U*} = v'(\overline{x}^U)w_i > 0 \qquad i = L, H. \Rightarrow \alpha_H^{U*} > \alpha_L^{U*}.\end{array}$$

(b) Welfarist planner:

Consumption bundles:

$$\begin{array}{l}\overline{x}_u^W = x_u^W(\alpha, w_L) = x_u^W(\alpha, w_H) = x_\ell^W(0, w_L) = x_h^W(0, w_H).\\ x_\ell^W(\alpha, w_L) = x_h^W(\alpha, w_H).\\ \frac{\partial x_y^W(\alpha, w_Y)}{\partial \alpha} > 0 \text{ and } \frac{\partial\left[v\left(x_y^W(\alpha, w_Y)\right) - \alpha\right]}{\partial \alpha} < 0,\ y = l, h \text{ and } Y = L, H.\end{array}$$

Activity assignment:

$$\begin{array}{l}\delta_L(\alpha) = 1 \text{ for all } \alpha \leq \alpha_L^{W*} \text{ and } \delta_H(\alpha) = 1 \text{ for all } \alpha \leq \alpha_H^{W*}.\\ \alpha_L^{W*} = \Psi'\left(v(x_\ell^W(\alpha_L^{W*}, w_L)) - \alpha_L^{W*}\right) v'(x_\ell^W(\alpha_L^{W*}, w_L))w_L > 0,\\ \alpha_H^{W*} = \Psi'\left(v(x_h^W(\alpha_H^{W*}, w_H)) - \alpha_H^{U*}\right) v'(x_\ell^W(\alpha_H^{W*}, w_H))w_H > 0,\\ \alpha_H^{W*} > \alpha_L^{W*}\end{array}$$

(c) Boadway *et al.* planner:

Consumption bundles:

$$\begin{array}{l}x^B(\alpha) = x_\ell^B(\alpha, w_L) = x_u^B(\alpha, w_L) = x_h^B(\alpha, w_H) = x_u^B(\alpha, w_H).\\ \frac{\partial x^B(\alpha)}{\partial \alpha} \geq (\leq) 0 \text{ if } W'(.) \geq (\leq) 0.\end{array}$$

Activity assignment:

Case 1: $\frac{\partial W(\alpha)}{\partial\alpha}\frac{\alpha}{W(\alpha)} > -1$:
$\delta_L(\alpha) = 1$ for all $\alpha \le \alpha_L^{B*}$, $\delta_H(\alpha) = 1$ for all $\alpha \le \alpha_H^{B*}$ and $\alpha_H^{B*} \ge \alpha_L^{B*}$,

Case 2: $\frac{\partial W(\alpha)}{\partial\alpha}\frac{\alpha}{W(\alpha)} = -1$ (i.e. $W(\alpha)\alpha$ is constant):
$\lambda^B = \int_0^\infty W(\alpha) v'\left(x^B(\alpha)\right) dF(\alpha)$,
$w_H\lambda^B > w_L\lambda^B > W(\alpha)\alpha \Rightarrow n_H^B = n_L^B = 1.$
$w_H\lambda^B > w_L\lambda^B = W(\alpha)\alpha \Rightarrow n_H^B = 1, 0 < n_L^B < 1$
$w_H\lambda^B > W(\alpha)\alpha > w_L\lambda^B \Rightarrow n_H^B = 1, n_L^B = 0.$
$w_H\lambda^B = W(\alpha)\alpha > w_L\lambda^B \Rightarrow 0 < n_H^B < 1, n_L^B = 0.$
$W(\alpha)\alpha > w_H\lambda^B > w_L\lambda^B \Rightarrow n_H^B = n_L^B = 0,$

Case 3: $\frac{\partial W(\alpha)}{\partial\alpha}\frac{\alpha}{W(\alpha)} < -1$:
$\delta_L(\alpha) = 1$ for all $\alpha \ge \alpha_L^{B**}$, $\delta_H(\alpha) = 1$ for all $\alpha \ge \alpha_H^{B**}$ and $\alpha_L^{B**} \ge \alpha_H^{B**}$.

(d) Non-welfarist planner:

Consumption bundles:

$$\overline{x}^N = x_\ell^N(\alpha, w_L) = x_u^N(\alpha, w_L) = x_h^N(\alpha, w_H) = x_u^N(\alpha, w_H)$$

Activity assignment:

$\lambda^N = v'\left(\overline{x}^N\right)$
$w_H\lambda^N > w_L\lambda^N > \overline{\alpha} \Rightarrow n_H^N = n_L^N = 1.$
$w_H\lambda^N > w_L\lambda^N = \overline{\alpha} \Rightarrow n_H^N = 1, 0 < n_L^N < 1.$
$w_H\lambda^N > \overline{\alpha} > w_L\lambda^N \Rightarrow n_H^N = 1, n_L^N = 0.$
$w_H\lambda^N = \overline{\alpha} > w_L\lambda^N \Rightarrow 0 < n_H^N < 1, n_L^N = 0.$
$\overline{\alpha} > w_H\lambda^N > w_L\lambda^N \Rightarrow n_H^N = n_L^N = 0.$

(e) Roemer planner:

Consumption bundles:

$$\begin{aligned}
\forall\alpha &\in [0, \alpha_L^*) \cup [\alpha_H^*, \infty) : x_\ell^R(\alpha, w_L) = x_h^R(\alpha, w_H) = x_u^R(\alpha, w_L) = x_u^R(\alpha, w_H) = \overline{x}^R, \\
\forall\alpha &\in [\alpha_L^*, \alpha_H^{*R}) : x_u^R(\alpha, w_L) = v^{-1}\left(v\left(x_h(\alpha, w_H)\right) - \alpha\right) < \overline{x}^R. \qquad (10)
\end{aligned}$$

Activity assignment:

$\delta_L(\alpha) = 1$ for all $\alpha \le \alpha_L^{R*}$ and $\delta_H(\alpha) = 1$ for all $\alpha \le \alpha_H^{R*}$.
with $\alpha_H^{R*} \ge \alpha_L^{R*}$.

(f) Van de Gaer planner:

Consumption bundles:

$$x_\ell^V(\alpha, w_L) = x_u^V(\alpha, w_L) = \overline{x}^V \le x_h^V(\alpha, w_H) = x_u^V(\alpha, w_H) = \overline{\overline{x}}^V.$$

Activity assignment:

$\delta_L(\alpha) = 1$ for all $\alpha \le \alpha_L^{V*}$ and $\delta_H(\alpha) = 1$ for all $\alpha \le \alpha_H^{V*}$.
with $\alpha_H^{V*} \ge \alpha_L^{V*}$.

(a) Utilitarian planner

A Utilitarian planner gives the same consumption $\overline{x}^U$ to everyone, irrespective of his skill level and his taste parameter. More high-skilled than low-skilled workers have to work (i.e. $\alpha_H^{U*} > \alpha_L^{U*}$), since $w_H > w_L$. Therefore, there exist values for α for which high-skilled, contrary to low-skilled, have to work. This results in a lower welfare level for these high-skilled people. The EWEP axiom is then violated. Moreover, workers are clearly worse off than inactive people; the worst off will be the high-skilled workers with taste parameter α_H^*. As far as the ETES axiom is concerned, note that all low- (high-) skilled workers pay the same tax and all inactive agents get the same transfer. However, minus the tax paid by the low-skilled workers is not equal, in general, to the transfer received by the low- (high-) skilled inactive. Thus the ETES axiom is violated.

(b) Welfarist planner

The main difference between a Welfarist and a Utilitarian planner is that a Welfarist planner will give different consumption bundles to workers, depending on their disutility of labor. More precisely, $\partial x_y^W(\alpha, w_Y)/\partial\alpha > 0$ with $y = l, h$ and $Y = L, H$. A Welfarist planner tries to compensate workers with a higher disutility of labor by giving them additional consumption, but the compensation is insufficient to make utility independent of labor disutility: $\partial\left(v\left(x_y^W(\alpha, w_Y)\right) - \alpha\right)/\partial\alpha < 0$. As a result, the high-skilled worker with taste α_H^* remains the worst off, as under the Utilitarian criterion. Moreover, consumption of workers is equalized at each α level, i.e. $x_\ell^W(\alpha, w_L) = x_h^W(\alpha, w_H)$. EWEP and ETES are violated for the same reasons as with a Utilitarian planner.

(c) Boadway et al. planner

The Boadway *et al.* planner's consumption function depends on tastes only. If the weight given to individuals with a higher disutility of labor increases (decreases), those with a higher (lower) disutility of labor get more consumption, i.e. $\partial x^B(\alpha)/\partial\alpha \geq (\leq) 0$ if $W'(.) \geq (\leq) 0$. Activity assignment can take many forms, depending on the elasticity of the social welfare function with respect to the taste parameter, $(\partial W(\alpha)/\partial\alpha)(\alpha/W(\alpha))$. If this elasticity is larger than -1 (as in the Utilitarian case where $W(\alpha)$ is a constant and so the elasticity is zero), assignment to activities occurs as in the Utilitarian and Welfarist cases. However, if this elasticity is smaller than -1 (which requires that $W(\alpha)$ is sufficiently declining in α), the Boadway *et al.* planner wants to keep those with a high disutility of labor in work. If the elasticity is exactly -1, corner solutions prevail in which at least everyone in one skill group works or is inactive. If there exists a group for which no corner solution occurs, the planner is indifferent to who (i.e. which value for the taste parameter) is assigned to work. Which case occurs if $W(\alpha)\alpha$ is constant depends crucially on the level of this constant. Observe that in all solutions for the Boadway *et al.* planner, more high-skilled than low-skilled individuals have to work, i.e. $\alpha_H^{B*} \geq \alpha_L^{B*}$ and $\alpha_L^{B**} \geq \alpha_H^{B**}$. Again, therefore, EWEP is not satisfied. Unequal taxes are paid within the group of low- (high-) skilled workers, unequal transfers are received within the group of low- (high-) skilled inactive and taxes paid by low- (high-) skilled workers need not be equal to minus the transfer received by low- (high-) skilled inactive. As a consequence ETES is violated in all respects.

(d) Non-welfarist planner

Under the Non-welfarist criterion, everyone receives the same consumption, $\overline{x}^N$, irrespective of his skill and his taste parameter. In other words, the Non-welfarist consumption function has the same features as the Utilitarian one. The activity assignment crucially depends on the level of

$\overline{\alpha}$. Moreover, the Non-welfarist and Boadway *et al.* criterion, with elasticity of $W(\alpha)$ equal to -1, both lead to similar activity assignment, with the reference $\overline{\alpha}$ playing the role of the constant $W(\alpha)\alpha$. Again, EWEP is then not satisfied. All low- (high-) skilled workers pay the same tax and all inactive receive the same transfer. However, ETES is not satisfied since the tax paid by low- (high-) skilled workers need not be equal to minus the transfers received by the low- (high-) skilled inactive people.

(e) Roemer planner

Roemer planner's consumption function depends on tastes only. The Roemer planner wants to keep those with a low disutility of labor in work. More high-skilled than low-skilled individuals have to work, i.e. $\alpha_H^{R*} \geq \alpha_L^{R*}$. However, $x_u^R(\alpha, w_L) = v^{-1}(v(x_h(\alpha, w_H)) - \alpha)$ $\forall \alpha \in [\alpha_L^*, \alpha_H^{*R})$ equalizes utility levels of workers and inactive agents with the same α in this range. Moreover, $\forall \alpha \in [0, \alpha_L^*)$ ($\forall \alpha \in [\alpha_H^*, \infty)$), everyone works (is inactive) and receives the same consumption. It can then be concluded that the first-best allocation satisfies EWEP. As far as the ETES axiom is concerned, minus the tax paid by the workers is not equal to the transfer received by the low- and high- skilled inactive. Thus the ETES axiom is violated.

(f) Van de gaer planner

The main difference between Roemer and Van de gaer planners is that Van de gaer's criterion will give different consumption bundles to people with identical α and the same activity choice, when their skills differ. In particular, high-skilled people receive a larger consumption level than low-skilled people, $\overline{\overline{x}}^V \geq \overline{x}^V$. Hence this allocation violates EWEP. Under Van de gaer's criterion, at each skill level, minus the tax paid by the workers is not equal to the transfer received by inactive workers since they both have the same consumption bundle. ETES axiom is then violated.

We can summarize the performance of the criteria in theorem 1 from the perspective of the equality of opportunity principles in the following corollary.

Corollary 1: With full information, both the EWEP and ETES axioms are violated under the Utilitarian, Welfarist, Boadway *et al.,* Non-welfarist and Van de gaer criteria. Roemer's criterion satisfies EWEP but not ETES.

We next turn to the FEO, CE and EE criteria described in the previous section. These criteria are directly inspired from the equality of opportunity axioms.

Theorem 2: With full information, the following configuration of policies is optimal:

(a) FEO:

(i) $n_H = n_L = 1$ and $x_\ell = x_h = \gamma w_L + (1-\gamma) w_H - R$.

(ii) $n_H = n_L = 0$ and $x_u = -R$.

(b) CE:

There are five types of optimal allocations possible:

(i) $n_H = n_L = 1$ and $x_\ell = x_h = \gamma w_L + (1-\gamma) w_H - R$.

(ii) $n_H = 1, 0 < n_L < 1, -x_u = [w_L - x_\ell]$ and $x_\ell = x_h = v^{-1}(v(x_u) - \widetilde{\alpha})$.

(iii) $n_H = 1, n_L = 0$ and $x_u = [(1-\gamma)(w_H - x_h) - R]/\gamma$ and $x_h = v^{-1}(v(x_u) - \widetilde{\alpha})$.

(iv) $0 < n_H < 1, n_L = 0, -x_u = [w_H - x_h]$ and $x_h = v^{-1}(v(x_u) - \widetilde{\alpha})$.

(v) $n_H = n_L = 0$ and $x_u = -R$.

(c) EE:

$x_\ell = x_h = \gamma w_L + (1-\gamma) w_H - R$ and $x_u = 0$,

$\alpha_L^* = \alpha_H^* = v(\gamma w_L + (1-\gamma) w_H) - v(0)$.

(a) FEO allocation

By construction, the FEO allocations satisfy both EWEP and ETES, however they are quite trivial. FEO (i) assigns everyone to work while FEO (ii) implies that everyone is inactive. FEO (i) and (ii) give everyone the same consumption. Note that, contrary to FEO (i), FEO (ii) gives everyone the same utility.

(b) CE allocation

With the CE criterion, the two FEO allocations can be optimal as well as three others. The latter are denoted by CE (ii), (iii) and (iv). The CE allocation equalizes $u(x(\alpha, w_Y), \delta_Y(\alpha), \widetilde{\alpha})$ for all α and $Y = L, H$. Therefore, welfares are equalized when bundles are evaluated with references preferences, but not with actual preferences (see, (ii), (iii), (iv)). EWEP is thus not satisfied. We will now check the validity of ETES. In the CE allocation (ii), all high-skilled work and a fraction of the low-skilled work. All high-skilled people receive the same consumption bundle x_h and all low-skilled people receive the same transfer $-x_u = [w_L - x_\ell]$. This CE allocation thus satisfies ETES. CE allocation (iii) has all high-skilled and no low-skilled working. ETES is then satisfied. The CE allocation (iv) has only a fraction of the high-skilled working. Again the planner does not care which high-skilled. Since no low-skilled work, and for the high-skilled $-x_u = [w_H - x_h]$, this allocation too satisfies ETES.

Which of these CE allocations is the optimal one depends on the parameters of the model. For $\widetilde{\alpha}$ sufficiently low, the optimum will be of type (i). As α increases, we move over cases (ii), (iii) and (iv) to (v). The properties of the CE allocation clearly shows that it is possible to find allocations that have attractive properties from the perspective of equality of opportunity in the first best.

Moreover, note the qualitative similarity between the job assignment with the CE objective in theorem 2 and the Boadway *et al.* objective in case 2 of theorem 1 and the Non-welfarist objective in theorem 1. The crucial difference between these allocations in theorem 1 and the CE allocation is the determination of the consumption bundles: the Welfarist planner gives the same consumption to everyone, the Boadway *et al.* planner in case 2 gives lower consumption to the individuals with less deserving tastes (i.e. with a higher α), while the CE planner determines the consumption bundles such that they satisfy the ETES axiom.

(c) EE allocation

Under the EE allocation, all workers receive the same consumption bundle, irrespective of their skill level. The inactive get zero benefits. This looks harsh at first sight, but in terms of equivalent wages, the metric used by the planner in this case, these individuals are best-off, and, in the present framework people are responsible for their preference. Observe that this policy satisfies EWEP. All high skilled pay the same tax, all low skilled pay the same tax, and all inactive get the same zero transfer. The tax paid is not equal to minus the transfer received, however. Hence ETES is not satisfied.

The EE allocation assigns the same consumption bundle to workers as allocations FEO (i) and CE (i), but contrary to these allocations, those with high disutility of labor are not working. They are inactive, and are, actually better off (both in terms of utility and equivalent wages) than under allocations FEO (i) and CE (i).

We can summarize the performance of the criteria in theorem 2 from the perspective of the equality of opportunity principles in the following corollary.

Corollary 2: With full information, the EWEP and ETES axioms are satisfied by the FEO allocation. CE satisfies ETES, but not EWEP, and EE satisfies EWEP, but not ETES.

Finally, all the first best solutions listed in Theorem 1 and 2 depend on both α and w_i. Therefore, they are not implementable when the government only observes income (second best). The next section deals with this issue.

5 Second best optima

In second best, the government determines a set of consumption bundles $\{x_\ell, x_h, x_u\}$ taking into account that, given these consumption levels, people choose themselves their activity status.

This has immediate consequences for the equality of opportunity axioms discussed in section 3.1. First, since all individuals assigned to a particular task receive the same consumption bundle, EWEP cannot be satisfied as soon as there exist a measurable subset of the population for which the high skilled and low skilled have a different activity. Second, ETES now requires that $x_\ell - w_L = x_u = x_h - w_H$, which implies that low- and high-skilled have to pay the same tax. Hence no redistribution between high- and low-skilled is possible if we impose ETES. As a result, in the second best, the usefulness of these two principles, and especially the ETES axiom can be disputed.

In this second-best setting, the Government needs to take into account the set of self-selection or incentive compatibility constraints (hereafter ICC) in order to prevent individuals from a given type from mimicking (i.e. taking the tax-treatment designed for) individuals of other types.

Agents of w_L-type choose between $v(x_u)$ and $v(x_\ell) - \alpha$. Introducing the threshold value α_L^*, and dropping the superscripts U, W, B, N, R, V, C and E for notational simplicity, the ICC constraint on agents of type-w_L can be written as:

$$v(x_\ell) - \alpha_L^* = v(x_u), \tag{11}$$

such that a low skilled with taste parameter α chooses low skilled employment in stead of inactivity if and only if $\alpha < \alpha_L^*$.

Agents of w_H-type choose between $v(x_u)$, $v(x_\ell) - \alpha$ and $v(x_h) - \alpha$. Since all our objective functions are increasing in individuals' consumption, it will, just like in the first best, never be optimal that high-skilled people work in low-skilled jobs. By putting these people in high-skilled jobs instead of low-skilled jobs, they produce more which can be used to increase everyone's consumption in a way that respects the ICC and hence increases the social objective's value. Consequently, to induce high-skilled people to work in high-skilled jobs,

$$x_h \geq x_\ell, \tag{12}$$

and, introducing the threshold value α_H^*, the ICC on agents of w_H-type states

$$v(x_h) - \alpha_H^* = v(x_u) \ , \tag{13}$$

such that a high skilled agent with taste parameter α prefers high-skilled employment to inactivity if and only if $\alpha < \alpha_H^*$. Moreover, from (11), (12) and (13), we have that

$$\alpha_H^* \geq \alpha_L^*. \tag{14}$$

The second best framework has important consequences for the specification of the social objective functions. Combining the expressions for the social objective functions (2), (3), (4), (5), (6), (7), (8) with expression (11), (12), (13) and (14) results in the following corollary. Again, we skip the superscripts U, W, B, N, R, V, C and E for notation simplicity.

Corollary 3: social objective functions in the second best.

(a) Utilitarian

$$\widetilde{S}^U = \gamma \int_0^{\alpha_L^*} [v(x_\ell) - \alpha]\, dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} v(x_u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\alpha_H^*} [v(x_h) - \alpha]\, dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} v(x_u) dF(\alpha).$$

(b) Welfarist

$$\widetilde{S}^W = \gamma \int_0^{\alpha_L^*} \Psi\left(v(x_\ell) - \alpha\right) dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} \Psi\left(v(x_u)\right) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\alpha_H^*} \Psi\left(v(x_h) - \alpha\right) dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} \Psi\left(v(x_u)\right) dF(\alpha).$$

(c) Boadway *et al.*

$$\widetilde{S}^B = \gamma \int_0^{\alpha_L^*} W(\alpha) [v(x_\ell) - \alpha]\, dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} W(\alpha)\, v(x_u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\alpha_H^*} W(\alpha) [v(x_h) - \alpha]\, dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} W(\alpha)\, v(x_u) dF(\alpha).$$

(d) Non-welfarist

$$\widetilde{S}^N = \gamma \left[\int_0^{\alpha_L^*} [v(x_\ell) - \overline{\alpha}]\, dF(\alpha)\right] + \gamma \int_{\alpha_L^*}^{\infty} v(x_u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\alpha_H^*} [v(x_h) - \overline{\alpha}]\, dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} v(x_u) dF(\alpha).$$

(e) Roemer and (f) Van de gaer

$$\int_0^{\alpha_L^*} \left(v\left(x_\ell\right) - \alpha\right) dF\left(\alpha\right) + \int_{\alpha_L^*}^{\infty} v\left(x_u\right) dF\left(\alpha\right).$$

(g) Conditional Equality

$$\widetilde{S}^C = v\left(x_\ell\right) - \widetilde{\alpha}$$
$$\text{subject to } \widetilde{\alpha} \geq \alpha_L^*.$$

(h) Egalitarian Equivalent

$$\widetilde{S}^E = x_\ell.$$

Under asymmetric information, Roemer and Van de gaer's criterion are equal. Due to the second best constraint, utility as a function of the taste parameter of the low skilled will never be below utility as a function of the taste parameter for the high skilled. One implication of this is that the opportunity set for the lowly skilled is below the one for the highly skilled, hence, in the second best the mean of mins and min of means criterion will yield the same solution.

We can now state the following theorem.

Theorem 3: Under asymmetric information, the following configuration of policies is optimal:
(a) Utilitarian, Boadway et al. and Welfarist planner:

$$\infty > \alpha_H^{X*} > \alpha_L^{X*} > 0 \text{ and } x_h^X > x_\ell^X > x_u^X \text{ with } X = U, W, B. \tag{15}$$

(b) Non-welfarist planner:
(i) R sufficiently negative and $\overline{\alpha}$ high enough:

$$\infty > \alpha_H^{N*} = \alpha_L^{N*} = 0 \text{ and } x_h^N = x_\ell^N = x_u^N = -R, \tag{16}$$

(ii) intermediate R and $\overline{\alpha}$:

$$\infty > \alpha_H^{N*} > \alpha_L^{N*} = 0 \text{ and } x_h^N > x_\ell^N = x_u^N, \tag{17}$$

(iii) R high enough and $\overline{\alpha}$ low enough:

$$\infty > \alpha_H^{N*} > \alpha_L^{N*} > 0 \text{ and } x_h^N > x_\ell^N > x_u^N. \tag{18}$$

(c) Conditional Equality planner:
(i) R sufficiently negative and $\widetilde{\alpha}$ high enough:

$$\infty > \alpha_H^{C*} \geq \alpha_L^{C*} = \widetilde{\alpha} \text{ and } x_h^C \geq x_\ell^C > x_u^C, \tag{19}$$

(ii) R high enough and $\widetilde{\alpha}$ low enough: Non-welfarist and

$$\infty > \alpha_H^{C*} \geq \alpha_L^{C*} > \widetilde{\alpha} \text{ and } x_h^C \geq x_\ell^C > x_u^C.$$

(d) Egalitarian Equivalent, Roemer and Van de gaer planner:

$$\infty > \alpha_H^{X*} \geq \alpha_L^{X*} > 0 \text{ and } x_h^X \geq x_\ell^X > x_u^X \text{ with } X = E, R, V.$$

Theorem 3 makes clear that there is a big difference between the social planners that take into account (possibly weighted) individual utilities and those that are manifestly Non-welfarist. The former always want to have both high-skilled and low-skilled at work. For the individuals with the lowest disutility of work (which is zero), their productivity and hence the extra utility generated by consumption, is always larger than zero, hence $\alpha_L^{X*} > 0$ and $\alpha_H^{X*} > 0$. Moreover, $\alpha_H^{X*} > \alpha_L^{X*}$ from $w_H > w_L$. Who works under Non-welfarism depends on the social disutility of work and the amount of external resources the economy has. As the economy has fewer external resources at its disposal and/or $\overline{\alpha}$ decreases, we move from (16) over (17) to (18). We will assume in what follows that we are in a case where R is not too negative and $\overline{\alpha}$ is low enough, i.e. case (18), such that the situation with the Non-welfarist social planner resembles that of the other planners, (15). Therefore, for these types of planners, the utilities as a function of α, for low- and high-skilled agents, look as in the following graph.

The bold line is the utility of a high-skilled individual. He works if his disutility of work $\alpha \leq 0.75 = \alpha_H^*$, and he is inactive otherwise. Similarly the other line is the utility of a low-skilled individual. The latter works for $\alpha \leq 0.25 = \alpha_L^*$ and is inactive otherwise. Different planners choose different values for $(x_u, x_\ell, x_h, \alpha_L^*, \alpha_H^*)$, but the qualitative shape of the utilities as a function of α, for high- and low-skilled individuals, is always as indicated in the graph for our Utilitarian, Boadway *et al* , welfarist, Non-welfarist, Roemer and Van de gaer planner. For the CE and EE planners, however, we cannot exclude that the utility line for the high- skilled and low- skilled coincide.

The second best framework clearly leads to characteristics of consumption bundles and assignment (in Theorem 3) which are drastically distinct from the first best ones (Theorem 1). Unobservability of tastes makes it for the Welfarist, Boadway *et al.*, Roemer and Van de gaer planner impossible to offer different consumptions for people with different tastes in any other way than by assigning them to different tasks and award them the consumption associated with this task. The self-selection constraints prevent the Utilitarian and Non-welfarist social planners from offering all individuals the same consumption bundle and prevents the Welfarist, Boadway *et al.* and Roemer planner from offering the same consumption function to high- and low- skilled workers (equal to the consumption function of the inactive for the Boadway *et al.* planner). The self-selection constraint makes it impossible for the Boadway *et al.* planner with elasticity of $W(\alpha)$ smaller than -1 to assign those with a high disutility of labor to work. For the Boadway *et al.* planner, all corner solutions disappear.

Before we can characterize the optimal tax rates, we need to introduce more definitions. Let $T_\ell = w_L - x_\ell$, $T_h = w_H - x_h$, and $T_u = -x_u$, be the tax paid by the low-skilled workers, the high-skilled workers and the inactive, respectively. Define the elasticity of participation of the low-skilled with respect to x_ℓ as

$$\eta(x_\ell, \alpha_L^*) = \frac{x_\ell}{\gamma F(\alpha_L^*)} \frac{\partial(\gamma F(\alpha_L^*))}{\partial x_\ell}.$$

Since $\alpha_L^* = v(x_\ell) - v(x_u)$, we get $\frac{\partial \alpha_L^*}{\partial x_\ell} = v'(x_\ell)$ and so

$$\eta(x_\ell, \alpha_L^*) = \frac{x_\ell}{F(\alpha_L^*)} f(\alpha_L^*) v'(x_\ell). \tag{20}$$

Similarly it is easy to show that the elasticity of participation of the high-skilled with respect to x_h equals

$$\eta(x_h, \alpha_H^*) = \frac{x_h}{F(\alpha_H^*)} f(\alpha_H^*) v'(x_h). \tag{21}$$

Next, observe that the average of the inverse of the private marginal utility of consumption, is given by

$$g^P(x_u, x_\ell, x_h, \alpha_L^*, \alpha_H^*) \stackrel{\text{def}}{\equiv} \frac{\gamma F(\alpha_L^*)}{v'(x_\ell)} + \frac{\gamma(1-F(\alpha_L^*)) + (1-\gamma)(1-F(\alpha_H^*))}{v'(x_u)} + \frac{(1-\gamma)F(\alpha_H^*)}{v'(x_h)}.$$

Finally we define the average social marginal utility of consumption g_Y^X under objective function X $(= U, W, B, N, R, V, C$ or $E)$ and for working agents of skill level Y $(= L$ or $H)$ in the following table.

Low-skilled	High-skilled
$g_L^U = \frac{v'(x_\ell^U)}{\lambda^U}$	$g_H^U = \frac{v'(x_h^U)}{\lambda^U}$
$g_L^W = \frac{v'(x_\ell^W)}{\lambda^W} \frac{\int_0^{\alpha_L^{W*}} \Psi'(v(x_\ell^W)-\alpha)dF(\alpha)}{F(\alpha_L^{W*})}$	$g_H^W = \frac{v'(x_h^W)}{\lambda^W} \frac{\int_0^{\alpha_H^{W*}} \Psi'(v(x_h)-\alpha)dF(\alpha)}{F(\alpha_H^{W*})}$
$g_L^B = \frac{v'(x_\ell^B)}{\lambda^B} \frac{\int_0^{\alpha_L^{B*}} W(\alpha)dF(\alpha)}{F(\alpha_L^{B*})}$	$g_H^B = \frac{v'(x_h^B)}{\lambda^B} \frac{\int_0^{\alpha_H^{B*}} W(\alpha)dF(\alpha)}{F(\alpha_H^{B*})}$
$g_L^N = \frac{v'(x_\ell^N)}{\lambda^N}$	$g_H^N = \frac{v'(x_h^N)}{\lambda^N}$
$g_L^R = g_L^V = \frac{v'(x_\ell^R)}{\lambda^R \gamma}$	$g_H^R = g_H^V = 0$
$g_L^C = \frac{v'(x_\ell^C)}{\lambda^C \gamma F(\alpha_L^{C*})}$	$g_H^C = 0$
$g_L^E = \frac{1}{\lambda \gamma F(\alpha_L^*)}$	$g_H^E = 0$

The following theorem is the basis for a more detailed comparison of the tax rates.

Theorem 4: Under asymmetric information, the optimal consumption levels have to satisfy the budget constraint and the following equations:

(a) Utilitarian planner:

$$\frac{T_\ell^U - T_u^U}{x_\ell^U} = \frac{1}{\eta(x_\ell^U, \alpha_L^{U*})}\left[1 - g_L^U\right], \tag{22}$$

$$\frac{T_h^U - T_u^U}{x_h} = \frac{1}{\eta(x_h^U, \alpha_H^{U*})}\left[1 - g_H^U\right], \tag{23}$$

$$\left(\lambda^U\right)^{-1} = g^P\left(x_u^U, x_\ell^U, x_h^U, \alpha_L^{U*}, \alpha_H^{U*}\right). \tag{24}$$

(b) Welfarist planner:

$$\frac{T_\ell^W - T_u^W}{x_\ell^W} = \frac{1}{\eta(x_\ell^W, \alpha_L^{W*})}\left[1 - g_L^W\right], \tag{25}$$

$$\frac{T_h^W - T_u^W}{x_h^W} = \frac{1}{\eta(x_h^W, \alpha_H^{W*})}\left[1 - g_H^W\right], \tag{26}$$

$$\left(\lambda^W\right)^{-1} = \frac{g^P\left(x_u^W, x_\ell^W, x_h^W, \alpha_L^{W*}, \alpha_H^{W*}\right)}{D}, \tag{27}$$

where

$$D = \gamma \left[\int_0^{\alpha_L^{W*}} \Psi' \left(v\left(x_\ell^W\right) - \alpha \right) dF(\alpha) + \int_{\alpha_L^{W*}}^{\infty} \Psi' \left(v\left(x_u^W\right) \right) dF(\alpha) \right]$$
$$+ (1-\gamma) \left[\int_0^{\alpha_H^{W*}} \Psi' \left(v\left(x_h^W\right) - \alpha \right) dF(\alpha) + \int_{\alpha_H^{W*}}^{\infty} \Psi' \left(v\left(x_u^W\right) \right) dF(\alpha) \right].$$

(c) Boadway *et al.* planner:

$$\frac{T_\ell^B - T_u^B}{x_\ell^B} = \frac{1}{\eta\left(x_\ell^B, \alpha_L^{B*}\right)} \left[1 - g_L^B\right], \tag{28}$$

$$\frac{T_h^B - T_u^B}{x_h^B} = \frac{1}{\eta\left(x_h^B, \alpha_H^{B*}\right)} \left[1 - g_H^B\right], \tag{29}$$

$$\left(\lambda^B\right)^{-1} = \frac{g^P\left(x_u^B, x_\ell^B, x_h^B, \alpha_L^{B*}, \alpha_H^{B*}\right)}{\int_0^\infty W(\alpha)\, dF(\alpha)}. \tag{30}$$

(d) Non-welfarist planner:

$$\frac{T_\ell^N - T_u^N}{x_\ell^N} = \frac{1}{\eta\left(x_\ell^N, \alpha_L^{N*}\right)} \left[1 - g_L^N\right] - \frac{\alpha_L^{N*} - \overline{\alpha}}{\lambda x_\ell^N} \tag{31}$$

$$\frac{T_h^N - T_u^N}{x_h^N} = \frac{1}{\eta\left(x_h^N, \alpha_H^{N*}\right)} \left[1 - g_H^N\right] - \frac{\alpha_H^{N*} - \overline{\alpha}}{\lambda x_h^N} \tag{32}$$

$$\left(\lambda^N\right)^{-1} = g^P\left(x_u^N, x_\ell^N, x_h^N, \alpha_L^{N*}, \alpha_H^{N*}\right) \tag{33}$$

(e) Roemer and Van de gaer planner:[7]

$$\frac{T_\ell^R - T_u^R}{x_\ell^R} = \frac{1}{\eta\left(x_\ell^R, \alpha_L^{R*}\right)} \left[1 - g_L^R\right] \tag{34}$$

$$\frac{T_h^R - T_u^R}{x_h^R} = \frac{1}{\eta\left(x_h^R, \alpha_H^{R*}\right)} \tag{35}$$

$$\left(\lambda^R\right)^{-1} = g^P\left(x_u^R, x_\ell^R, x_h^R, \alpha_L^{R*}, \alpha_H^{R*}\right) \tag{36}$$

(f) Conditional Equality planner:

$$\frac{T_\ell^C - T_u^C}{x_\ell^C} = \frac{1}{\eta\left(x_\ell^C, \alpha_L^{C*}\right)} \left[1 - (1-\xi)\, g_L^C\right], \tag{37}$$

with ξ the Lagrangean multiplier of the constraint $\widetilde{\alpha} \geq \alpha_L^*$

$$\frac{T_h^C - T_u^C}{x_h^C} = \frac{1}{\eta\left(x_h^C, \alpha_H^{C*}\right)}, \tag{38}$$

$$\left(\lambda^C\right)^{-1} = g^P\left(x_u^C, x_\ell^C, x_h^C, \alpha_L^{C*}, \alpha_H^{C*}\right). \tag{39}$$

(g) Egalitarian Equivalent planner:

$$\frac{T_\ell^E - T_u^E}{x_\ell^E} = \frac{1}{\eta\left(x_\ell^E, \alpha_L^{E*}\right)} \left(1 - g_L^E\right) \tag{40}$$

$$\frac{T_h^E - T_u^E}{x_h^E} = \frac{1}{\eta\left(x_h^E, \alpha_H^{E*}\right)} \tag{41}$$

$$\left(\lambda^E\right)^{-1} = v'\left(x_\ell\right).g^P\left(x_u^E, x_\ell^E, x_h^E, \alpha_L^{E*}, \alpha_H^{E*}\right) \tag{42}$$

[7] As previously shown, Roemer and Van de gaer constrained optimization problems, hence solutions, are identical in second-best. We then use the same superscript, R, to denote the Roemer and Van de gaer solution.

We first interpret the λ^{-1} equations before moving to the tax rates, following closely Choné and Laroque (2008) in this interpretation. Equation (24), (27), (30), (33), (36) and (39) are similar to Diamond and Sheshinsky (1995)'s equation (6), p.6. and are associated with an equal marginal change of the consumption of everyone in the economy. All the equations for λ^{-1} have the same shape: they equate the inverse of the marginal cost of public funds to the ratio between the average of the inverse of the private utilities and the marginal social utility of a uniform increase in all individual utilities (the latter can be easily derived from corollary 3). Consider a uniform increase in all private utilities of one unit. This does not change the occupational choice decisions. To accomplish this uniform increase, we need per low-skilled worker $1/v'(x_\ell)$ extra units of consumption, per high-skilled worker we need $1/v'(x_h)$ extra units of consumption and per inactive person we need $1/v'(x_u)$ extra units of consumption. Weighting this by the frequencies of these groups in the population, we find that we need an additional $g^P(x_u, x_\ell, x_h, \alpha_L^*, \alpha_H^*)$ units of public means to finance this operation. In terms of social welfare, this is worth $\lambda g^P(x_u, x_\ell, x_h, \alpha_L^*, \alpha_H^*)$. This has to be equal to the increase in the social objective function caused by the uniform increase in utilities, which depends on the type of objective function. These can easily be derived from corollary 3 and is given by the denominators in expressions (24), (27), (30), (33), (36) and (39); they equal one for the Utilitarian, Non-welfarist, Roemer and Van de gaer and Conditional Equality planner.

Next, we give a simple heuristic interpretation of the optimal tax formula in the spirit of Saez (2002). Consider a small increase of the consumption x_ℓ (i.e. a small reduction of the income tax in low-skilled jobs), around the optimal tax schedule. This has a mechanical effect and a behavioral (or labor supply response) effect.

Mechanical effect

There is a mechanical decrease in tax revenue equal to $-\gamma F(\alpha_L^*)dx_\ell$ because low skilled workers have dx_ℓ additional consumption. This decrease in tax revenue is valued

$$-\left(1 - g_L^X\right)\gamma F(\alpha_L^*)dx_\ell$$

by the government because each euro lost increases the net income of low-skilled workers and this income gain is valued g_L^X by the government.

Behavioral effect

Behavioral responses imply a gain in tax revenue and, for the Non-welfarist criterion only, a direct change in welfare.

The change $dx_\ell > 0$ induces $\gamma f(\alpha_L^*)$ (pivotal) low skilled workers to enter the labor force. Each low skilled leaving non participation induces a gain in government revenue equal to $T_\ell - T_u$. By definition of the elasticity of participation (20), we have $d(\gamma F(\alpha_L^*)) = \gamma f(\alpha_L^*) = \eta(x_\ell, \alpha_L^*)(\gamma F(\alpha_L^*)/x_\ell)$ $(\gamma F(\alpha_L^*)/x_\ell)\, dx_\ell$. Therefore, the loss in tax revenue can be written as

$$(T_\ell - T_u)\,\eta(x_\ell, \alpha_L^*)\,\frac{\gamma F(\alpha_L^*)}{x_\ell}dx_\ell\,.$$

With the Utilitarian, Welfarist, Boadway *et al*, Roemer and Van de gaer criteria, there is no change in welfare due to the behavioral response because workers entering the labor force on the margin are indifferent between becoming active and remaining unemployed and are evaluated by the same in the normative criterion. At the optimum, the sum of the mechanical and labor supply response effects has to be nil, i.e.

$$-\left(1-g_L^X\right)\gamma F(\alpha_L^*)dx_\ell+(T_\ell-T_u)\,\eta\left(x_\ell,\alpha_L^*\right)\frac{\gamma F\left(\alpha_L^*\right)}{x_\ell}dx_\ell=0$$

which can be rewritten as (22), (25), (28) or (34).

However, with the Non-welfarist criterion, an extra term appears. The change dx_ℓ also induces a welfare gain since there are $\partial\left(\gamma F\left(\alpha_L^{N*}\right)\right)/\partial x_\ell^N$ new pivotal workers whose aversion to work α_L^{N*} is valued $\overline{\alpha}$ rather than α_L^{N*} into the welfare function. Valued in terms of public funds, this welfare gain is $\left(\left(\alpha_L^{N*}-\overline{\alpha}\right)/\lambda^N\right)\left(\partial\left(\gamma F\left(\alpha_L^{N*}\right)\right)/\partial x_\ell^N\right)dx_\ell^N$. By definition of the elasticity of participation (20), we have $\partial\left(\gamma F\left(\alpha_L^{N*}\right)\right)/\partial x_\ell^N=\eta\left(x_\ell^N,\alpha_L^{N*}\right)\gamma F\left(\alpha_L^{N*}\right)/x_\ell^N$. This gain is then equal to $\left(\left(\alpha_L^{N*}-\overline{\alpha}\right)/\lambda^N\right)\eta\left(x_\ell^N,\alpha_L^{N*}\right)\left(\gamma F\left(\alpha_L^{N*}\right)/x_\ell^N\right)dx_\ell^N$. The extra term captures the social value of divergence between private and social preferences. At the optimum, the sum of the mechanical and labor supply response effects has to be nil, i.e.

$$\begin{aligned}&-\left(1-g_L^N\right)\gamma F(\alpha_L^{N*})dx_\ell^N+\left(T_\ell^N-T_u^N\right)\eta\left(x_\ell^N,\alpha_L^*\right)\frac{\gamma F\left(\alpha_L^{N*}\right)}{x_\ell^N}dx_\ell^N\\&-\left(\left(\alpha_L^{N*}-\overline{\alpha}\right)/\lambda^N\right)\eta\left(x_\ell^N,\alpha_L^{N*}\right)\frac{\gamma F\left(\alpha_L^{N*}\right)}{x_\ell^N}dx_\ell^N=0\end{aligned}$$

which can be rewritten as (31).

Finally, looking at the Conditional Equality planner's optimal policy, note that the multiplier associated with the constraint $\widetilde{\alpha}\geq\alpha_L^*$ enters in (37). If the constraint is binding, the planner needs to bring α_L^* down, for which it has to decrease x_ℓ or increase x_u. The former increases T_l, the latter decreases T_u, and so $\left(T_l-T_u\right)/x_\ell$ must increase. This explains why an increase in ξ increases the right hand side of (37).

Equations (22)-(23), (25)-(26), (28)-(29), (34)-(35), (37)-(38) and (**??**)-(41) highlight that, under the Utilitarian, the Welfarist, the Boadway *et al.*, the Roemer, the Conditional Equality and Egalitarian Equivalent criteria, the optimal tax schedules depend on the average marginal social utilities of workers and on their elasticities of participation.[8] By contrast, the optimal tax schedule also depends on an extra term under the Non-welfarist criterion (see equations (31)-(32)). Let us focus on (31); the explanation for (32) is similar. The extra term appears since the effect of an infinitesimal change in the consumption bundles of low-skilled workers induces the marginal inactive low-skilled to start working, which has a first order effect on the Non-welfarist evaluation of their well being equal to $v\left(x_\ell^N\right)-\overline{\alpha}-v\left(x_u^N\right)$, which by virtue of (11) reduces to $\alpha_L^{N*}-\overline{\alpha}$. The denominator in (31) converts this effect in money terms and makes it relative to x_ℓ^N.

Since Diamond (1980), it is well known that subsidizing the low-skilled workers more than inactive people (i.e. $T_\ell<T_u$) can be optimal when the labor supply is modeled on the extensive margin. Using the definition of Saez (2002), an Earned Income Tax Credit (EITC) is then optimal. Alternatively, since $T_\ell<(>)T_u$ can be rewritten as $w_L<(>)x_\ell-x_u$, i.e. the income gain when a

[8] Note that the optimal tax formulas for the high-skilled workers, equations (35), (38) and (41), depend only on the elasticity of participation, since the utility of these workers is not valued in the Roemer and Van de gaer, Conditional Egalitarian and Egalitarian Equivalent criteria: $g_H^R=g_H^C=g_H^E=\mathbf{0}$.

low-skilled agent enters the labor force ($x_\ell - x_u$) is larger (lower) than the gross labor income (w_L). In other words, the labor supply of the low-skilled is distorted upwards (downwards), compared to laissez faire.

Theorem 5: optimality of EITC or NIT.

(a) For a Utilitarian, Welfarist, Boadway *et al.,* Roemer and Van de gaer and Egalitarian Equivalent planner, if the average social weight of the low-skilled workers is larger (lower) than one, an EITC (a NIT) is optimal.

(b) For a Non-welfarist planner, a sufficient condition for the EITC (NIT) to be optimal is that both the average social weight of the low-skilled workers is larger (lower) than one and $\alpha_L^{N*} > (<) \, \overline{\alpha}$.

(c) For a Conditional Equality planner, if the constraint $\widetilde{\alpha} \geq \alpha_L^*$ is not binding, then if the average social weight of the low-skilled workers is larger (lower) than one, an EITC (a NIT) is optimal. However, if this constraint binds, then for the EITC to be optimal, the average social weight of the low-skilled workers has to be sufficiently larger than one.

Under the Utilitarian, the Welfarist, the Boadway *et al.,* the Roemer and Van de gaer, the Egalitarian Equivalent and the Conditional Equality objective with non binding constraint, the average social weight of the low-skilled workers larger than one is a necessary condition for the EITC to be optimal. The necessary conditions to obtain unambiguous results under the Non-welfarist criterion are clearly more stringent. The EITC (NIT) encourages (discourages) participation of the marginal worker, which results in an increased (decreased) utility of consumption equal to α_L^{N*}, which is desirable if this is larger (smaller) than $\overline{\alpha}$, the utility cost of work in the eyes of the Non-welfarist planner. The extra term $-\left(\alpha_L^{N*} - \overline{\alpha}\right) / \left(\lambda^N x_\ell^N\right)$ (in (31)) which appears under the Non-welfarist criterion is used as a device to correct undesirable social outcomes. It corrects individual labor supply to correspond to social preferences. Hence, if social preferences are characterized by $\alpha_L^{N*} > \overline{\alpha}$, the government encourages participation and the EITC then becomes more attractive for the Non-welfarist planner. This term is sometimes called the paternalistic or first best motive for taxation since it arises from differences between social and private preferences (Kanbur *et al.,* 2006).

Under the Non-welfarist criterion, the sign of $T_\ell^N - T_u^N$ depends on the interaction between a second best motive for taxation which is to raises revenue in the least distortionary manner:

$$\frac{1}{\eta\left(x_\ell^N, \alpha_L^{N*}\right)} \left[1 - g_L^N\right] \quad \text{with } X = U, W, B \text{ or } N$$

and a first-best motive for taxation which arises when there is a difference between private and social preferences:

$$-\frac{\alpha_L^{N*} - \overline{\alpha}}{\lambda^N x_\ell^N}$$

The Utilitarian, Welfarist, Boadway *et al.,* Roemer and Van de gaer, Conditional Equality and Egalitarian Equivalent criteria all imply that $\alpha_L^{N*} = \overline{\alpha}$ hence the last term cancels out. Under the Non-welfarist criterion, the sign of $T_\ell^N - T_u^N$ will depend on the interaction between these terms. When the Non-welfarist government's views on working becomes more “Calvinistic”, i.e. when $\overline{\alpha}$ decreases, the term of first-best motive for taxation becomes negative hence plays in favor of an

EITC to promote participation of more people. An EITC can then prevail when the second best motive for taxation $\left[1-g_L^N\right]/\eta\left(x_\ell^N,\alpha_L^{N*}\right)$ is positive, under a Non-welfarist criterion.

Empirical studies suggest that participation decisions are more elastic at the bottom of the skill distribution (see the empirical evidence surveyed by Immervoll *et al.*, 2007, and Meghir and Phillips, 2008) which motivates the following assumption:

Assumption 1: $\eta\left(x_\ell,\alpha_L^*\right)>\eta\left(x_h,\alpha_H^*\right)$.

Corollary 4: Under assumption 1, for the Utilitarian, Welfarist, Roemer and Van de gaer, Conditional Equality, Egalitarian Equivalent and Boadway *et al.* when $W(\alpha)$ is a decreasing function:

$$\left(T_\ell-T_u\right)/x_\ell<\left(T_h-T_u\right)/x_h$$

Our model is an extensive model of labor supply. We have that the degree to which labor supply is distorted downwards depends on the difference between taxes paid when working and taxes paid when inactive (the latter is $-x_u$). The larger is this difference, the more labor supply is distorted downwards; if the difference is negative, labor supply is distorted upwards. We now have the following theorem.

Theorem 6: Under assumption 1, the Utilitarian, Welfarist, Roemer and Van de gaer, Conditional Equality, Egalitarian Equivalent and Boadway *et al.* criteria, the labor supply of the high-skilled is more distorted downwards than the labor supply of the low-skilled.

The statement that labor supply of the high-skilled is more downwardly distorted, also allows for the possibility that it is less upwardly distorted than the labor supply of the low-skilled. Whichever happens, depends crucially on the amount of external resources the economy has at its disposal.

6 Conclusion

This paper has studied optimal tax policies when agents differ in terms of skills and tastes for labor. We assumed quasilinear utility and that labor supply decision is at the extensive margin. The optimal tax policies under distinct objective functions have been derived, in full and asymmetric information.

The determination of appealing social criteria is important if one looks for social preferences applicable in public economics, in particular when dealing with redistribution. When agents differ in terms of skills and tastes for labor, the equality of opportunity approach is inspiring (Fleurbaey, 1995a) and broadly accepted (Alesina and Angeletos, 2005).

This paper has shown that many criteria in the optimal tax literature (Utilitarianism, Welfarism, Boadway *et al.*, Van de gaer and Non-welfarist criteria) fail the requirements of equality of opportunity. In full information, it has been shown that criteria respecting one of the equality of opportunity principles are Roemer's, the Conditional Equality and the Egalitarian Equivalent criterion, the latter two advocated by Fleurbaey (1995c). Given that these criteria were designed so as to meet one of the criteria in the first best, this should not come as a surprise. In asymmetric information, the self-selection constraints force even the policies following from these criteria to perform poorly. This says a lot about the force of the self selection constraints, and need not discourage us from using those criteria which have a clear normative foundation for the problem under study. Indeed, it has been shown that is possible to motivate some of the orderings considered in this paper using axioms that are tailored to deal with situations of double heterogeneity. This remains probably the most promising way to proceed.

Finally, we showed that the usual sufficient condition for advocating an Earned Income Tax Credit is valid under Utilitarianism, Welfarist, the criteria of Boadway et al., Roemer and Van de gaer and the Egalitarian Equivalent criterion. On the contrary, this sufficient condition is challenged under the paternalistic criterion of Schokkaert *et al.* (2004) and, for low values of the reference taste parameter, also by the Conditional Equality criterion.

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Appendix A: proof of theorem 1

We prove the results for each of the social objective functions.

(a) Utilitarian planner

The Lagrangean for the Utilitarian social welfare function is:

$$\begin{aligned}
\pounds &= \gamma \left[\int_0^\infty \left[\delta_L(\alpha) \left(v(x_\ell(\alpha, w_L)) - \alpha \right) + (1 - \delta_L(\alpha))\, v(x_u(\alpha, w_L)) \right] dF(\alpha) \right] \\
&+ (1-\gamma) \left[\int_0^\infty \left[\delta_H(\alpha) \left(v(x_h(\alpha, w_H)) - \alpha \right) + (1 - \delta_H(\alpha))\, v(x_u(\alpha, w_H)) \right] dF(\alpha) \right] \\
&+ \lambda \left\{ \gamma \int_0^\infty \left[\delta_L(\alpha) \left(w_L - x_\ell(\alpha, w_L) \right) - (1 - \delta_L(\alpha))\, x_u(\alpha, w_L) \right] dF(\alpha) \right. \\
&\left. + (1-\gamma) \int_0^\infty \left[\delta_H(\alpha) \left(w_H - x_h(\alpha, w_H) \right) - (1 - \delta_H(\alpha)) x_u(\alpha, w_H) \right] dF(\alpha) - R \right\},
\end{aligned}$$

where λ is the (non-negative) Lagrangean multiplier associated to the budget constraint.

The first-order conditions with respect to the four consumption functions are:

$$\begin{aligned}
\int_0^\infty \delta_L(\alpha) \left[v'(x_\ell(\alpha, w_L)) - \lambda \right] dF(\alpha) &= 0, \\
\int_0^\infty (1 - \delta_L(\alpha)) \left[v'(x_u(\alpha, w_L)) - \lambda \right] dF(\alpha) &= 0, \\
\int_0^\infty \delta_H(\alpha) \left[v'(x_h(\alpha, w_H)) - \lambda \right] dF(\alpha) &= 0, \\
\int_0^\infty (1 - \delta_H(\alpha)) \left[v'(x_u(\alpha, w_H)) - \lambda \right] dF(\alpha) &= 0.
\end{aligned}$$

Hence we get the following four conditions:

$$\begin{aligned}
\delta_L(\alpha)\left[v'(x_\ell(\alpha,w_L)-\lambda\right] &= 0,\\
(1-\delta_L(\alpha))\left[v'(x_u(\alpha,w_L))-\lambda\right] &= 0,\\
\delta_H(\alpha)\left[v'(x_h(\alpha,w_H))-\lambda\right] &= 0,\\
(1-\delta_H(\alpha))\left[v'(x_u(\alpha,w_H))-\lambda\right] &= 0,
\end{aligned}$$

but since $\delta_L(\alpha)$ and $\delta_H(\alpha)$ are equal to 1 or 0, for each value of α, only two of them matter; for those that matter the corresponding marginal utilities of consumption have to be equal, for the other two the consumption function does not matter (as nobody with this value for α is receiving it), and so we have that the first-order conditions with respect to consumption reduce to $(\forall\alpha)$ (since λ is a constant):

$$\begin{aligned}
& v'(x_\ell(\alpha,w_L)) = v'(x_u(\alpha,w_L)) = v'(x_h(\alpha,w_H)) = v'(x_u(\alpha,w_H)) = \lambda\\
\Longleftrightarrow\ & \overline{x} = x_\ell(\alpha,w_L) = x_u(\alpha,w_L) = x_h(\alpha,w_H) = x_u(\alpha,w_H)\,.
\end{aligned}\tag{43}$$

To study how the assignment functions $\delta_L(\alpha)$ and $\delta_H(\alpha)$ look like, use result (43) in the government budget constraint (1), from which we get

$$\overline{x} = \gamma w_L \int_0^\infty \delta_L(\alpha)\,dF(\alpha) + (1-\gamma)\,w_H \int_0^\infty \delta_H(\alpha)\,dF(\alpha) - R\,. \tag{44}$$

Evidently, $\overline{x}$ is independent of the taste parameter of the individuals that become employed: it only depends on the number of low types that are employed and the number of high types that are employed. Substituting (44) in (2), gives us the value of S^U as a function of the $\delta_L(\alpha)$ and $\delta_H(\alpha)$ functions:

$$S^U = v(\overline{x}) - \gamma \int_0^\infty \delta_L(\alpha)\,\alpha dF(\alpha) - (1-\gamma)\int_0^\infty \delta_H(\alpha)\,\alpha dF(\alpha)\,. \tag{45}$$

Keeping the number of employed of both types fixed, it is only though the last two terms that the shape of the $\delta_L(\alpha)$ and $\delta_H(\alpha)$ functions matter for a Utilitarian planner. Note that as α rises from zero to ∞ the function $1\cdot\alpha$ goes from zero to ∞. Hence, it will for a Utilitarian social planner always be optimal to have those in work with the lowest disutility of labor. Consequently, the functions $\delta_L(\alpha)$ and $\delta_H(\alpha)$ will have the following shape:

$$\delta_L(\alpha) = 1 \text{ for all } \alpha \le \alpha_L^* \text{ and } \delta_H(\alpha) = 1 \text{ for all } \alpha \le \alpha_H^*,$$

such that

$$\overline{x} = \gamma w_L \int_0^{\alpha_L^*} \delta_L(\alpha)\,dF(\alpha) + (1-\gamma)\,w_H \int_0^{\alpha_H^*} \delta_H(\alpha)\,dF(\alpha) - R\,, \tag{46}$$

$$S^U = v(\overline{x}) - \gamma \int_0^{\alpha_L^*} \alpha dF(\alpha) - (1-\gamma)\int_0^{\alpha_H^*} \alpha dF(\alpha)\,. \tag{47}$$

To determine the optimal value for α_L^*, note that from (46), a marginal increase in α_L^* increases all consumptions by $\gamma w_L f(\alpha_L^*)$, which, by (47), increases the value of the objective function by $v'(\overline{x})\,\gamma w_L f(\alpha_L^*)$. At the other hand, putting these people in work has a cost in terms of social welfare equal to $\gamma\alpha_L^* f(\alpha_L^*)$ -see (47). The optimal value of α_L^* balances these costs and benefits. A similar reasoning holds for α_H^*. Hence, the critical values are determined by

$$\alpha_i^* = v'(\overline{x})w_i > 0 \qquad i = L, H.$$

Since $w_H > w_L$, $\alpha_H > \alpha_L$.

(b) Welfarist planner

The Lagrangean for the Welfarist social objective function is:

$$\begin{aligned}
\pounds &= \gamma \left[\int_0^\infty [\delta_L(\alpha)\Psi(v(x_\ell(\alpha, w_L)) - \alpha) + \Psi(v(x_u(\alpha, w_L))] dF(\alpha) \right] \\
&+ (1-\gamma) \left[\int_0^\infty [\delta_H(\alpha)\Psi(v(x_h(\alpha, w_h)) - \alpha) + \int_{\alpha_h}^\infty \Psi(v(x_u(\alpha, w_H)))] dF(\alpha) \right] \\
&+ \lambda \left\{ \gamma \int_0^\infty [\delta_L(\alpha)(w_L - x_\ell(\alpha, w_L)) dF(\alpha) - (1 - \delta_L(\alpha)) x_u(\alpha, w_L)] dF(\alpha) \right. \\
&+ \left. (1-\gamma) \int_0^\infty [\delta_H(\alpha)(w_H - x_h(\alpha, w_H)) - (1 - \delta_H(\alpha)) x_u(\alpha, w_H)] dF(\alpha) - R \right\}.
\end{aligned}$$

The first order conditions with respect to consumption functions yield

$$\begin{aligned}
&\int_0^\infty \delta_L(\alpha)[\Psi'(v(x_\ell(\alpha, w_L)) - \alpha)v'(x_\ell(\alpha, w_L)) - \lambda] dF(\alpha) = 0, \\
&\int_0^\infty (1 - \delta_L(\alpha))[\Psi'(v(x_u(\alpha, w_L)))v'(x_u(\alpha, w_L)) - \lambda] dF(\alpha) = 0, \\
&\int_0^\infty \delta_H(\alpha)[\Psi'(v(x_h(\alpha, w_H)) - \alpha)v'(x_h(\alpha, w_H)) - \lambda] dF(\alpha) = 0, \\
&\int_0^\infty (1 - \delta_H(\alpha))[\Psi'(v(x_u(\alpha, w_H)))v'(x_u(\alpha, w_H)) - \lambda] dF(\alpha) = 0.
\end{aligned}$$

We get the following 4 first order conditions:

$$\begin{aligned}
\delta_L(\alpha)[\Psi'(v(x_\ell(\alpha, w_L)) - \alpha)v'(x_\ell(\alpha, w_L)) - \lambda] &= 0, \\
(1 - \delta_L(\alpha))[\Psi'(v(x_u(\alpha, w_L))v'(x_u(\alpha, w_L)) - \lambda] &= 0, \\
\delta_H(\alpha)[\Psi'(v(x_h(\alpha, w_H)) - \alpha)v'(x_h(\alpha, w_H)) - \lambda] &= 0, \\
(1 - \delta_H(\alpha))[\Psi'(v(x_u(\alpha, w_H))v'(x_u(\alpha, w_H)) - \lambda] &= 0,
\end{aligned}$$

of which again only two will matter. By the same reasoning as in the Utilitarian case, we get for all those that do not work:

$$\Psi'(v(x_u(\alpha, w_L)))v'(x_u(\alpha, w_L)) = \lambda = \Psi'(v(x_u(\alpha, w_H)))v'(x_u(\alpha, w_H)), \tag{48}$$

which means that their social marginal utility of consumption has to be the same. Due to the strict concavity of $\Psi'(\cdot)$ and $v'(\cdot)$, this can only hold true if

$$\overline{x}_u = x_u(\alpha, w_L) = x_u(\alpha, w_H).$$

For those that work, we get

$$\Psi'(v(x_\ell(\alpha, w_L)) - \alpha)v'(x_\ell(\alpha, w_L)) = \lambda = \Psi'(v(x_h(\alpha, w_H)) - \alpha)v'(x_h(\alpha, w_H)). \tag{49}$$

For a given value for α, the requirement is exactly the same for highly and low-skilled workers. Hence, for a given value of α, both get the same consumption bundle and so, for all α :

$$x_\ell(\alpha, w_L) = x_h(\alpha, w_H).$$

Note that λ does not depend on α, while (49) has to hold true for all values of α of those that work. As a result, worker's consumption bundles will depend on α. Moreover, from the implicit function theorem:

$$\frac{\partial x_\ell(\alpha, w_L)}{\partial \alpha} = \frac{\Psi''(v(x_\ell(.,.) - \alpha))v'(x_\ell(.,.))}{\Psi''(v(x_\ell(.,.) - \alpha))\left[v'(x_\ell(.,.))\right]^2 + \Psi'(v(x_\ell(.,.) - \alpha))v''(x_\ell(.,.))} > 0,$$

and so low-skilled workers with a higher disutility of work get a higher consumption bundle, such that for $\alpha_1 < \alpha_2$, $x_\ell(\alpha_1, w_L) < x_\ell(\alpha_2, w_L)$ and due to the concavity of $v(.)$,

$$v'(x_\ell(\alpha_1, w_L)) > v'(x_\ell(\alpha_2, w_L)).$$

From (49), we also need to have that

$$\Psi'(v(x_\ell(\alpha_1, w_L)) - \alpha_1)v'(x_\ell(\alpha_1, w_L)) = \Psi'(v(x_\ell(\alpha_2, w_L)) - \alpha_2)v'(x_\ell(\alpha_2, w_L)),$$

which combined with the last inequality requires that $\Psi'(v(x_\ell(\alpha_1, w_L) - \alpha_1) < \Psi'(v_\ell(\alpha_2, w_L) - \alpha_2)$. Since Ψ is strictly concave, this requires that

$$v(x_\ell(\alpha_1, w_L) - \alpha_1 > v(x_\ell(\alpha_2, w_L)) - \alpha_2,$$

and so workers with a high disutility of labor are not fully compensated for this high disutility of labor. Repeating the reasoning starting after (49) for high-skilled workers shows that also here workers with a high disutility of work are not fully compensated for a higher disutility of work.

Note that from (49) with $\alpha = 0$ and (48) we get that

$$x_\ell(0, w_L) = x_h(0, w_H) = \overline{x}_u.$$

The government budget constraint only depends on the number of high- and low-skilled that work, not on which low- and high-skilled. From (3), the Welfarist objective function can be written as

$$\begin{aligned}
S^W &= \gamma \int_0^\infty \delta_L(\alpha)\Psi(v(x_\ell(\alpha, w_L)) - \alpha)\,dF(\alpha) \\
&+\gamma \int_0^\infty (1 - \delta_L(\alpha))\Psi(v(\overline{x}_u))\,dF(\alpha) \\
&+(1-\gamma) \int_0^\infty \delta_H(\alpha)\Psi(v(x_h(\alpha, w_H)) - \alpha)\,dF(\alpha) \\
&+(1-\gamma) \int_0^\infty (1 - \delta_H(\alpha))\Psi(v(\overline{x}_u))\,dF(\alpha)
\end{aligned}$$

where only the expressions on the first and the third line depend on which low- and high-skilled are working. Since it is less costly to have those at work with the lowest disutility of labor, we will have, just like in the Utilitarian case that the functions $\delta_L(\alpha)$ and $\delta_H(\alpha)$ will have the following shape:

$$\delta_L(\alpha) = 1 \text{ for all } \alpha \leq \alpha_L^* \text{ and } \delta_H(\alpha) = 1 \text{ for all } \alpha \leq \alpha_H^*.$$

Again, since the high-skilled produce more goods that can be distributed, it will be the case that more highly than low-skilled individuals will be put at work:

$$\alpha_L^* < \alpha_H^*.$$

(c) Boadway *et al.* planner

The Lagrangean function is now

$$\begin{aligned}
\pounds &= \gamma \left[\int_0^\infty W(\alpha) \left[\delta_L(\alpha) \left(v(x_\ell(\alpha, w_L)) - \alpha \right) + (1 - \delta_L(\alpha)) v(x_u(\alpha, w_L)) \right] dF(\alpha) \right] \\
&+ (1-\gamma) \left[\int_0^\infty W(\alpha) \left[\delta_H(\alpha) \left(v(x_h(\alpha, w_H)) - \alpha \right) + (1 - \delta_H(\alpha)) v(x_u(\alpha, w_H)) \right] dF(\alpha) \right] \\
&+ \lambda \{ \gamma \int_0^\infty \left[\delta_L(\alpha) (w_L - x_\ell(\alpha, w_L)) - (1 - \delta_L(\alpha)) x_u(\alpha, w_L) \right] dF(\alpha) \\
&+ (1-\gamma) \int_0^\infty \left[\delta_H(\alpha) (w_H - x_h(\alpha, w_H)) - (1 - \delta_H(\alpha)) x_u(\alpha, w_H) \right] dF(\alpha) - R \Big\} .
\end{aligned}$$

The first-order conditions with respect to consumption functions (assuming an interior solution) yield:

$$\begin{aligned}
\int_0^\infty \delta_L(\alpha) \left[W(\alpha) v'(x_\ell(\alpha, w_L)) - \lambda \right] dF(\alpha) &= 0, \\
\int_0^\infty (1 - \delta_L(\alpha)) \left[W(\alpha) v'(x_u(\alpha, w_L)) - \lambda \right] dF(\alpha) &= 0, \\
\int_0^\infty \delta_H(\alpha) \left[W(\alpha) v'(x_h(\alpha, w_H)) - \lambda \right] dF(\alpha) &= 0, \\
\int_0^\infty (1 - \delta_H(\alpha)) \left[W(\alpha) v'(x_u(\alpha, w_H)) - \lambda \right] dF(\alpha) &= 0.
\end{aligned}$$

Consequently, we get

$$\begin{aligned}
& v'(x_\ell(\alpha, w_L)) = v'(x_u(\alpha, w_L)) = v'(x_h(\alpha, w_H)) = v'(x_u(\alpha, w_H)) = \frac{\lambda}{W(\alpha)} \\
& \Longleftrightarrow x(\alpha) = x_\ell(\alpha, w_L) = x_u(\alpha, w_L) = x_h(\alpha, w_H) = x_u(\alpha, w_H) . \qquad (50)
\end{aligned}$$

Consumption now depends on taste for leisure. Application of the implicit function theorem to the equation $v'(x(\alpha)) = \frac{\lambda}{W(\alpha)}$ yields the following result:

$$\frac{\partial x(\alpha)}{\partial \alpha} = -\frac{\lambda}{[W(\alpha)]^2} \frac{W'(\alpha)}{v''(x(\alpha))} \geq (\leq) 0 \text{ if } W'(.) \geq (\leq) 0.$$

Using (50) in the government budget constraint (1) yields that the function $x(\alpha)$ must be such that

$$\int_0^\infty x(\alpha) \, dF(\alpha) = \gamma w_L \int_0^\infty \delta_L(\alpha) \, dF(\alpha) + (1-\gamma) w_H \int_0^\infty \delta_H(\alpha) \, dF(\alpha) - R.$$

For the government budget constraint it only matters how many high- and low-skilled people work, it does not matter which low and high skilled people work. Hence, differential treatment in job assignment between equally skilled people must be based on the objective function. Using (4), the value of the objective function is given by:

$$S^B = \int_0^\infty W(\alpha)\, v(x(\alpha))\, dF(\alpha)$$
$$-\gamma \int_0^\infty W(\alpha)\, \delta_L(\alpha)\, \alpha dF(\alpha) - (1-\gamma) \int_0^\infty W(\alpha)\, \delta_H(\alpha)\, \alpha dF(\alpha).$$

Whether people with high or low disutility of effort should be working depends on the last two terms of this expression. If $W(\alpha)\, a$ is increasing, having people with a high disutility working is not a good idea. From this it follows that, if the elasticity of the weight function $(\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)})$ is larger than -1, then it is optimal for the government not to employ people that have a high disutility of work. If this elasticity is smaller than -1, it will be optimal to employ people with a high disutility of work. Consequently, the functions $\delta_L(\alpha)$ and $\delta_H(\alpha)$ can have different shapes:

Case 1: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} > -1$:
$\delta_L(\alpha) = 1$ for all $\alpha \leq \alpha_L^*$ and $\delta_H(\alpha) = 1$ for all $\alpha \leq \alpha_H^*$,

Case 2: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} = -1$ (i.e. $W(\alpha)\,\alpha$ is a constant):
see discussion below.

Case 3: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} < -1$:
$\delta_L(\alpha) = 1$ for all $\alpha \geq \alpha_L^{**}$ and $\delta_H(\alpha) = 1$ for all $\alpha \geq \alpha_H^{**}$.

Analyzing case 2 in more detail, the problem facing the planner with $W(\alpha)\,\alpha$ constant has the following Lagrangean:

$$\begin{aligned} \pounds &= \int_0^\infty W(\alpha)\, v(x(\alpha))\, dF(\alpha) - \gamma W(\alpha)\, \alpha n_L - (1-\gamma)\, W(\alpha)\, \alpha n_H \\ &+\lambda \left[\gamma w_L n_L + (1-\gamma)\, w_H n_H - \int_0^\infty x(\alpha)\, dF(\alpha) - R\right], \end{aligned}$$

which leads to the following:

$$\begin{aligned} \frac{\partial \pounds}{\partial x(\alpha)} &= 0 \Leftrightarrow \int_0^\infty \left[W(\alpha)\, v'(x(\alpha)) - \lambda\right] dF(\alpha) = 0 \\ \frac{\partial \pounds}{\partial n_L} &= -\gamma W(\alpha)\, \alpha + \lambda \gamma w_L \\ \frac{\partial \pounds}{\partial n_H} &= (1-\gamma)\, W(\alpha)\, \alpha + \lambda (1-\gamma)\, w_H \end{aligned}$$

From the first condition, $\lambda = \int_0^\infty W(\alpha)\, v'(x(\alpha))\, dF(\alpha)$. Note that the second and third condition cannot hold simultaneously with equality:

$$\begin{aligned} \frac{\partial \pounds}{\partial n_L} &\geq (\leq)\, 0 \Leftrightarrow \left[\lambda w_L - W(\alpha)\, \alpha\right] \geq (\leq)\, 0 \\ \frac{\partial \pounds}{\partial n_H} &\geq (\leq)\, 0 \Leftrightarrow \left[\lambda w_H - W(\alpha)\, \alpha\right] \geq (\leq)\, 0 \end{aligned}$$

Hence, since $w_H > w_L$, we always have that $\frac{\partial \pounds}{\partial n_L} \geq 0 \Rightarrow \frac{\partial \pounds}{\partial n_H} > 0$ and $\frac{\partial \pounds}{\partial n_H} \leq 0 \Rightarrow \frac{\partial \pounds}{\partial n_L} < 0$. We then get the possibilities listed in the theorem.

(d) Non-welfarist social planner

The corresponding Lagrangean becomes

$$\begin{aligned}
\pounds &= \gamma \left[\int_0^\infty \left[\delta_L(\alpha) \left(v(x_\ell(\alpha, w_L)) - \overline{\alpha} \right) + (1 - \delta_L(\alpha))\, v(x_u(\alpha, w_L)) \right] dF(\alpha) \right] \\
&+(1-\gamma) \left[\int_0^\infty \left[\delta_H(\alpha) \left(v(x_h(\alpha, w_H)) - \overline{\alpha} \right) + (1 - \delta_H(\alpha))\, v(x_u(\alpha, w_H)) \right] dF(\alpha) \right] \\
&+\lambda \left\{ \gamma \int_0^\infty \left[\delta_L(\alpha) (w_L - x_\ell(\alpha, w_L)) - (1 - \delta_L(\alpha))\, x_u(\alpha, w_L) \right] dF(\alpha) \right. \\
&\left. + (1-\gamma) \int_0^\infty \left[\delta_H(\alpha) (w_H - x_h(\alpha, w_H)) - (1 - \delta_H(\alpha)) x_u(\alpha, w_H) \right] dF(\alpha) - R \right\}.
\end{aligned}$$

It is easy to see that we obtain the same first order conditions as with the Utilitarian objective, and so the consumption functions are similar to (43). The budget constraint gives us (44). Consequently, using (5), the value of our Non-welfaristic objective function becomes

$$\begin{aligned}
S^N &= v(\overline{x}) - \gamma\overline{\alpha} \int_0^\infty \delta_L(\alpha)\, dF(\alpha) - (1-\gamma)\,\overline{\alpha} \int_0^\infty \delta_H(\alpha)\, dF(\alpha) \\
&= v(\overline{x}) - \gamma\overline{\alpha} n_L - (1-\gamma)\,\overline{\alpha} n_H.
\end{aligned}$$

This entire expression only depends on the number of low- and high-skilled that are employed, just like the budget constraint. Hence, within the group of low-skilled and within the group of high-skilled the social planner is indifferent what the taste parameter is of those that are employed. The planner determines η_L and η_H so as to maximize

$$v(\gamma w_L n_L + (1-\gamma)\, w_H n_H) - \gamma\overline{\alpha} n_L - (1-\gamma)\,\overline{\alpha} n_H.$$

The derivatives of this expression with respect to n_H and n_L are, respectively

$$(1-\gamma)\left[w_H v'(\overline{x}) - \overline{\alpha} \right], \tag{51}$$

$$\gamma \left[w_L v'(\overline{x}) - \overline{\alpha} \right]. \tag{52}$$

Since $w_H > w_L$, we can distinguish the cases listed in the theorem.

(e) Roemer planner

There is no point in allowing the two elements in the min operator of Roemer's objective function to be different in the first best. Hence there are in principle four possibilities:

(i) $\delta_L(\alpha) = \delta_H(\alpha) = 1 \Rightarrow x_\ell(\alpha, w_L) = x_h(\alpha, w_H)$,

(ii) $\delta_L(\alpha) = 0, \delta_H(\alpha) = 1 \Rightarrow v(x_u(\alpha, w_L)) = v(x_h(\alpha, w_H)) - \alpha$
$\Rightarrow x_u(\alpha, w_L) < x_h(\alpha, w_H)$,

(iii) $\delta_L(\alpha) = \delta_H(\alpha) = 0 \Rightarrow x_u(\alpha, w_L) = x_u(\alpha, w_H)$,

(iv) $\delta_L(\alpha) = 1, \delta_H(\alpha) = 0 \Rightarrow v(x_\ell(\alpha, w_L)) - \alpha = v(x_u(\alpha, w_H))$
$\Rightarrow x_\ell(\alpha, w_L) > x_u(\alpha, w_H)$.

There is an equivalence between the maximin approach and the revenue-maximizing approach. Maximizing tax revenue subject to a minimal utility level is equivalent to maximizing the minimum of utility subject to the revenue constraint. Here, the objective function maximizes the sum of the minimal utility levels but the logic is similar. The government maximizes the tax revenue subject to minimal utility levels. The tax revenue will be maximized the more people are working, in particular productive people. The minimal utility levels avoid that people with large α work. Therefore, if anyone, we would like the ones with low values for α to work, and since highly skilled have a higher productivity, we might like more highly skilled to work ($\alpha_H^* \geq \alpha_L^*$); for α increasing, we move from (i) over (ii) to (iii). If we plug this in, we get the following objective function:

$$\int_0^{\alpha_L^*} \min\left\{v\left(x_\ell\left(\alpha,w_L\right)\right)-\alpha, v\left(x_h\left(\alpha,w_H\right)\right)-\alpha\right\} dF\left(\alpha\right)$$
$$+\int_{\alpha_L^*}^{\alpha_H^*} \min\left\{v\left(x_u\left(\alpha,w_L\right)\right), v\left(x_h\left(\alpha,w_H\right)\right)-\alpha\right\} dF\left(\alpha\right)$$
$$+\int_{\alpha_H^*}^{\infty} \min\left\{v\left(x_u\left(\alpha,w_L\right)\right), v(x_u\left(\alpha,w_H\right))\right\} dF\left(\alpha\right).$$

Maximizing this objective function implies

$$x_\ell\left(\alpha,w_L\right) = x_h\left(\alpha,w_H\right) \quad \forall\alpha\in[0,\alpha_L^*), \tag{53}$$
$$x_u\left(\alpha,w_L\right) = v^{-1}\left(v\left(x_h\left(\alpha,w_H\right)\right)-\alpha\right) \quad \forall\alpha\in[\alpha_L^*,\alpha_H^*), \tag{54}$$
$$x_u\left(\alpha,w_L\right) = x_u\left(\alpha,w_H\right) \quad \forall\alpha\in[\alpha_H^*,\infty). \tag{55}$$

Therefore, the objective function can be rewritten as

$$\int_0^{\alpha_L^*}\left(v\left(x_\ell\left(\alpha,w_L\right)\right)-\alpha\right) dF\left(\alpha\right)+\int_{\alpha_L^*}^{\infty} v\left(x_u\left(\alpha,w_L\right)\right) dF\left(\alpha\right).$$

The government budget constraint can be formulated as follows:

$$\gamma\left[\int_0^{\alpha_L^*}\left(w_L-x_\ell(\alpha,w_L)\right) dF(\alpha)-\int_{\alpha_L^*}^{\infty} x_u\left(\alpha,w_L\right) dF(\alpha)\right]$$
$$+(1-\gamma)\left[\int_0^{\alpha_L^*}\left(w_H-x_\ell(\alpha,w_L)\right) dF(\alpha)+\int_{\alpha_L^*}^{\alpha_H^*}\left(w_H-x_h\left(\alpha,w_H\right)+\alpha\right) dF(\alpha)\right]$$
$$-\int_{\alpha_H^*}^{\infty} x_u(\alpha,w_H) dF(\alpha) \geq R.$$

The corresponding Lagrangean function is:

$$\pounds = \int_0^{\alpha_L^*}\left(v\left(x_\ell\left(\alpha,w_L\right)\right)-\alpha\right) dF\left(\alpha\right)+\int_{\alpha_L^*}^{\infty} v\left(x_u\left(\alpha,w_L\right)\right) dF\left(\alpha\right)$$
$$+\lambda\left\{\gamma\left[\int_0^{\alpha_L^*}\left(w_L-x_\ell(\alpha,w_L)\right) dF(\alpha)-\int_{\alpha_L^*}^{\infty} x_u\left(\alpha,w_L\right) dF(\alpha)\right]+\right.$$
$$+(1-\gamma)\left[\int_0^{\alpha_L^*}\left(w_H-x_\ell(\alpha,w_L)\right) dF(\alpha)+\int_{\alpha_L^*}^{\alpha_H^*}\left(w_H-v^{-1}\left(v\left(x_u\left(\alpha,w_L\right)\right)+\alpha\right)\right) dF(\alpha)\right]$$
$$\left.-\int_{\alpha_H^*}^{\infty} x_u(\alpha,w_H) dF(\alpha)-R\right\}.$$

The first-order conditions with respect to $x_\ell\left(\alpha,w_L\right)$ and $x_u(\alpha,w_L)$ are:

$$\alpha\leq\alpha_L^*: v'\left(x_\ell\left(\alpha,w_L\right)\right)=\lambda,$$
$$\alpha_H^*<\alpha: v'(x_u(\alpha,w_L))=\lambda$$
$$\alpha_L^*<\alpha\leq\alpha_H^*: v'(x_u(\alpha,w_L))=\lambda\left[\gamma+(1-\gamma)\frac{v'\left(x_u\left(\alpha,w_L\right)\right)}{v'\left(x_h\left(\alpha,w_H\right)\right)}\right]$$

with λ as the Lagrangean multiplier associated to the government budget constraint.

From the first and second first-order conditions, we have (since λ is constant):

$$\forall\alpha \in [0,\alpha_L^*) \cup [\alpha_H^*,\infty) : x_\ell(\alpha,w_L) = x_u(\alpha,w_L) = \overline{x}. \tag{56}$$

For $\alpha_L^* < \alpha \leq \alpha_H^*$, from (54), it follows that $x_u(\alpha,w_L) < x_h(\alpha,w_H)$ and so $v'(x_u(\alpha,w_L)) > v'(x_h(\alpha,w_H))$, such that $v'(x_u(\alpha,w_L)) > \lambda$ and

$$\forall\alpha \in [\alpha_L^*,\alpha_H^*) : x_u(\alpha,w_L) < \overline{x}. \tag{57}$$

From (53), (54) and (55) it is clear that the first best allocation satisfies EWEP, but from (56) and (57) it follows that it does not satisfy ETES.

(f) Van de gaer planner:

In the first best, there is no reason for having different opportunity sets for different types. For the same reasons as usual, if anybody works, it will be those with a low disutility of work. Hence the objective function reduces to:

$$\int_0^{\alpha_L^*}[v(x_l(\alpha,w_L))-\alpha]\,dF(\alpha)+\int_{\alpha_L^*}^{\infty}v(x_u(\alpha,w_L))\,dF(\alpha).$$

This objective function must be maximized subject to two constraints. The first is that both opportunity sets must be equal:

$$\begin{aligned}&\int_0^{\alpha_L^*}[v(x_l(\alpha,w_L))-\alpha]\,dF(\alpha)+\int_{\alpha_L^*}^{\infty}v(x_u(\alpha,w_L))\,dF(\alpha)\\ =\;&\int_0^{\alpha_H^*}[v(x_h(\alpha,w_H))-\alpha]\,dF(\alpha)+\int_{\alpha_H^*}^{\infty}v(x_u(\alpha,w_H))\,dF(\alpha).\end{aligned} \tag{58}$$

The second is the budget constraint:

$$\begin{aligned}&\gamma\left[\int_0^{\alpha_L^*}(w_L-x_\ell(\alpha,w_L))\,dF(\alpha)-\int_{\alpha_L^*}^{\infty}x_u(\alpha,w_L)dF(\alpha)\right]\\ &+(1-\gamma)\left[\int_0^{\alpha_H^*}(w_H-x_h(\alpha,w_H))\,dF(\alpha)-\int_{\alpha_H^*}^{\infty}x_u(\alpha,w_H)dF(\alpha)\right]=R.\end{aligned}$$

The corresponding Lagrangean function is:

$$\begin{aligned}\pounds =\;&\int_0^{\alpha_L^*}[v(x_l(\alpha,w_L))-\alpha]\,dF(\alpha)+\int_{\alpha_L^*}^{\infty}v(x_u(\alpha,w_L))\,dF(\alpha)\\ &+\lambda\left\{\gamma\left[\int_0^{\alpha_L^*}(w_L-x_\ell(\alpha,w_L))\,dF(\alpha)-\int_{\alpha_L^*}^{\infty}x_u(\alpha,w_L)\,dF(\alpha)\right]\right.\\ &\left.(1-\gamma)\int_0^{\alpha_H^*}(w_H-x_h(\alpha,w_H))\,dF(\alpha)-\int_{\alpha_H^*}^{\infty}x_u(\alpha,w_H)\,dF(\alpha)-R\right\}\\ &+\mu\left\{\int_0^{\alpha_L^*}[v(x_l(\alpha,w_L))-\alpha]\,dF(\alpha)+\int_{\alpha_L^*}^{\infty}v(x_u(\alpha,w_L))\,dF(\alpha)\right.\\ &\left.-\int_0^{\alpha_H^*}[v(x_h(\alpha,w_H))-\alpha]\,dF(\alpha)-\int_{\alpha_H^*}^{\infty}v(x_u(\alpha,w_H))\,dF(\alpha)\right\}\end{aligned}$$

where μ is the multiplier associated to the equality of opportunity sets constraint (58).

The first-order conditions with respect to $x_\ell(\alpha, w_L)$, $x_u(\alpha, w_L)$, $x_h(\alpha, w_H)$ and $x_u(\alpha, w_H)$ are:

$$v'(x_\ell(\alpha, w_L))(1+\mu) = \lambda\gamma, \tag{59}$$

$$v'(x_u(\alpha, w_L))(1+\mu) = \lambda\gamma, \tag{60}$$

$$-\mu v'(x_h(\alpha, w_H)) = \lambda(1-\gamma), \tag{61}$$

$$-\mu v'(x_u(\alpha, w_H)) = \lambda(1-\gamma). \tag{62}$$

From (**??**) and (60) we have:

$$x_\ell(\alpha, w_L) = x_u(\alpha, w_L) = \overline{x},$$

and from (61) and (62):

$$x_h(\alpha, w_H) = x_u(\alpha, w_H) = \overline{\overline{x}}.$$

It is clear that the ETES axiom is violated.

Substituting these two equations into the equality of opportunity sets constraint (58) gives:

$$v(\overline{x}) - \int_0^{\alpha_L^*} \alpha dF(\alpha) = v(\overline{\overline{x}}) - \int_0^{\alpha_H^*} \alpha dF(\alpha).$$

Therefore,

$$\alpha_L^* < (\geq)\alpha_H^* \Leftrightarrow \overline{x} > (\leq)\overline{\overline{x}}.$$

Since high skilled people have a higher productivity, we want at least as many high skilled people to work as low skilled: $\alpha_H^* \geq \alpha_L^*$ hence $\overline{\overline{x}} \geq \overline{x}$.

Appendix B: proof of theorem 2

Lemma 1: for an allocation that satisfies EWEP and ETES, there cannot exist a measurable set $A \subset [0, \infty]$ such that for all α in $A : \delta_L(\alpha) \neq \delta_H(\alpha)$.

Proof of Lemma 1. If such a measurable set existed, we would have by EWEP that for all elements in this set either $v(x_u(\alpha, w_L)) = v(x_h(\alpha, w_H)) - \alpha$ or $v(x_\ell(\alpha, w_L)) - \alpha = v(x_u(\alpha, w_H))$, both of which are impossible since by ETES the consumption bundles cannot depend on α.

Proof of Theorem 2.

(a) FEO allocation: in view of Lemma 1, we have that for all $\alpha : \delta_L(\alpha) = \delta_H(\alpha)$. Suppose there exists an allocation satisfying EWEP and ETES in which a measurable set of people work and others do not work. From ETES we know that all low-skilled in work have to get the same consumption bundle, which with some abuse of notation we denote as $x_\ell(w_L)$. Similarly, all high-skilled in work get the same consumption bundle, denoted as $x_h(w_H)$. In addition, we need

(i) $x_\ell(w_L) - w_L = x_u(w_L)$

(ii) $x_h(w_H) - w_H = x_u(w_H)$.

EWEP requires that $x_u(w_L) = x_u(w_H)$. Combining this with (i) and (ii) we get that

$x_\ell(w_L) = w_L - w_H + x_h(w_H)$,

which contradicts with the other requirement from EWEP that $x_\ell(w_L) = x_h(w_H)$. Hence an allocation that satisfies EWEP and ETES cannot have a measurable set of people working and not working.

It is easy to verify that the allocations (1) and (2) satisfy both axioms. Their consumption bundles follow from the government budget constraint (1).

(b) CE allocation: a first thing to note is that for the allocation to equalize $u(x(\alpha, w_Y), \delta_Y(\alpha), \widetilde{\alpha})$ for all α and $Y = L, H$ requires that utility is independent of w_Y. This has the following implications:

i) for all α such that $\delta(\alpha, w_L) = \delta(\alpha, w_H) = 1 \Rightarrow x_\ell(\alpha, w_L) = x_h(\alpha, w_H)$. In addition, all those assigned in a job have to get the same level of utility, which implies that their consumption bundle cannot depend on α, and thus $x_\ell = x_\ell(\alpha, w_L) = x_h(\alpha, w_H) = x_h$.

ii) for all α such that $\delta(\alpha, w_L) = \delta(\alpha, w_H) = 0 \Rightarrow x_u(\alpha, w_L) = x_u(\alpha, w_H)$. In addition, all those that are unemployed have to get the same level of utility, implying that their consumption bundle cannot depend on α, such that $x_u(\alpha, w_L) = x_u(\alpha, w_H) = x_u$.

iii) for all α such that $\delta(\alpha, w_L) = 1$ and $\delta(\alpha, w_H) = 0 \Rightarrow x_\ell(\alpha, w_H) = v^{-1}(v(x_u(\alpha, w_H)) + \widetilde{\alpha})$, which combined with case 1 and 2 gives $x_\ell = v^{-1}(v(x_u) + \widetilde{\alpha})$

iv) for all α such that $\delta(\alpha, w_L) = 0$ and $\delta(\alpha, w_H) = 1 \Rightarrow x_h(\alpha, w_H) = v^{-1}(v(x_u(\alpha, w_H)) + \widetilde{\alpha})$, which combined with case 1 and 2 gives $x_h = v^{-1}(v(x_u) + \widetilde{\alpha})$

Combining these results, we get

$$x_\ell = x_h = v^{-1}(v(x_u) + \widetilde{\alpha}).$$

Everybody gets the same level of utility $v(x_u)$ in the optimum, and so the problem of the first best allocation then amounts to maximize the equal utility level

$$v(x_u)$$

with respect to x_u, n_L and n_H subject to the budget constraint

$$\begin{aligned} R &\leq \gamma\left(w_L - v^{-1}(v(x_u) + \widetilde{\alpha})\right) n_L - \gamma x_u [1 - n_L] \\ &+ (1-\gamma)\left(w_H - v^{-1}(v(x_u) + \widetilde{\alpha})\right) n_H - (1-\gamma) x_u [1 - n_H]. \end{aligned}$$

The Lagrangean for this problem is

$L = v(x_u) + \lambda[\gamma\left(w_L - v^{-1}(v(x_u) + \widetilde{\alpha})\right) n_L - \gamma x_u [1 - n_L]$
$+ (1-\gamma)\left(w_H - v^{-1}(v(x_u) + \widetilde{\alpha})\right) n_H - (1-\gamma) x_u [1 - n_H] - R]$.

Taking derivatives, we get :

$\frac{\partial L}{\partial x_u} = v'(x_u) + \lambda\gamma \frac{\partial v^{-1}(v(x_u)+\widetilde{\alpha})}{\partial x_u} n_L + \lambda(1-\gamma) \frac{\partial v^{-1}(v(x_u)+\widetilde{\alpha})}{\partial x_u} n_H$
$\quad -\lambda\gamma[1 - n_L] - \lambda(1-\gamma)[1 - n_H] = 0,$

$\frac{\partial L}{\partial n_L} = \lambda\gamma\left[w_L - v^{-1}(v(x_u) + \widetilde{\alpha})\right] + \lambda\gamma x_u$
$\quad = \lambda\gamma\left[x_u + \left[w_L - v^{-1}(v(x_u) + \widetilde{\alpha})\right]\right],$

$\frac{\partial L}{\partial n_H} = \lambda(1-\gamma)\left[w_H - v^{-1}(v(x_u) + \widetilde{\alpha})\right] n_H + \lambda(1-\gamma) x_u n_H$
$\quad = \lambda(1-\gamma)\left[x_u + \left[w_H - v^{-1}(v(x_u) + \widetilde{\alpha})\right]\right].$

The two last first order derivatives cannot possibly both be equal to zero at the same time:

$w_H > w_L \Rightarrow w_H - v^{-1}(v(x_u) + \widetilde{\alpha}) > w_L - v^{-1}(v(x_u) + \widetilde{\alpha})$

$\Rightarrow x_u + \left[w_H - v^{-1}\left(v\left(x_u\right)+\widetilde{\alpha}\right)\right] > x_u + \left[w_L - v^{-1}\left(v\left(x_u\right)+\widetilde{\alpha}\right)\right].$

Hence we either have that

(i) $\frac{\partial L}{\partial n_L} > 0 \Rightarrow \frac{\partial L}{\partial n_H} > 0$, implying that $n_H = 1 = n_L$,

(ii) $-x_u = \left[w_L - v^{-1}\left(v\left(x_u\right)+\widetilde{\alpha}\right)\right]$ and $\frac{\partial L}{\partial n_H} > 0$, implying $n_H = 1$ and n_L follows from the budget constraint,

(iii) $\frac{\partial L}{\partial n_H} > 0$ and $\frac{\partial L}{\partial n_L} > 0$, implying that $n_H = 1$ and $n_L = 0$,

(iv)$-x_u = \left[w_H - v^{-1}\left(v\left(x_u\right)+\widetilde{\alpha}\right)\right]$ and $\frac{\partial L}{\partial n_L} < 0$, implying $n_L = 0$ and n_H follows from the budget constraint or

(v) $\frac{\partial L}{\partial n_H} < 0 \Rightarrow \frac{\partial L}{\partial n_L} < 0$, implying that $n_H = 0 = n_L$.

Which of these allocations yields the highest value for $v\left(x_u\right)$ depends on the parameters of the model. If $\widetilde{\alpha}$ is sufficiently low, the optimum will be case (i),as $\widetilde{\alpha}$ rises, we move from (i) to (ii), as it increases further we move to (iii) and (iv) and for values of $\widetilde{\alpha}$ sufficiently high, the optimum will be case (v).

(c) EE allocation: we want everybody to be indifferent between his actual resources (consumption and activity) and a reference resource bundle where he works and gets consumption $\widetilde{x}$. The best thing to do is to give all employed exactly this reference consumption bundle: $x_\ell = x_h = \widetilde{x}$. Clearly, to bring the equivalent wage of the inactive with a very high α down can lead to negative consumption levels. To prevent this, we impose that $x_u\left(\alpha, w_Y\right) \geq 0$. If this constraint is binding, these individuals get an equivalent wage larger than $\widetilde{x}$; we have to give up the ideal of equalizing equivalent incomes. The logical alternative then becomes Fleurbaey and Maniquet's maximin solution.

To get an equivalent wage of exactly $\widetilde{x}$, a person with taste parameter α needs an inactivity transfer equal to $v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right)$, which is independent of his skill level. Since we maximin the equivalent wages, the transfer for the inactive is $x_u\left(\alpha\right) = \min\left\{v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right), 0\right\}$. There exists a value for α, say $\widehat{\alpha}$, such that, if $\alpha \leq \widehat{\alpha}$ we have $x_u\left(\alpha\right) = v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right) \geq 0$, and if $\alpha > \widehat{\alpha}$, $x_u\left(\alpha\right) = 0$. In both cases, $x_u\left(\alpha\right) \leq \widetilde{x}$ such that it is cheaper to have people inactive than to have them working.

However, working people produce w_L or w_H, while inactive people produce nothing. As a consequence, it can never be optimal to have people inactive for which $\alpha \leq \widehat{\alpha}$: they cost $v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right) \geq 0$, but produce nothing. The best policy that maximizes (9) is therefore $x_\ell = x_h = \gamma w_L + (1-\gamma) w_H - R$, $x_u = 0$ and $\alpha_L^* = \alpha_H^* = v\left(\gamma w_L + (1-\gamma) w_H\right) - v(0)$.

Appendix C: proof of Corollary 1

Parts (a), (b), (c) and (d) are straightforward to prove.

To see part (e), observe that (12) (due to incentive constraints) implies that for all α, $v(x_\ell) - \alpha \leq v(x_h) - \alpha$. Therefore, Roemer's objective function

$$\int_0^\infty \min\{\underset{\delta_L(\alpha)}{\operatorname{oper}}\left\{v\left(x_\ell\right)-\alpha, v\left(x_u\right)\right\},$$
$$\underset{\delta_H(\alpha)}{\operatorname{oper}}\left\{v\left(x_h\right)-\alpha, v\left(x_u\right)\right\}\}dF\left(\alpha\right)$$

becomes

$$\int_0^{\alpha_L^*} (v(x_\ell) - \alpha)\, dF(\alpha) + \int_{\alpha_L^*}^{\infty} v(x_u)\, dF(\alpha).$$

To see part (f), note that, in second-best, Van de gaer's objective function is

$$\min\{\int_0^{\infty} \underset{\delta_L(\alpha)}{\text{oper}}\{v(x_\ell) - \alpha, v(x_u)\}dF(\alpha),$$
$$\int_0^{\infty} \underset{\delta_H(\alpha)}{\text{oper}}\{v(x_h) - \alpha, v(x_u)\}dF(\alpha)\}.$$

As noted before with Roemer's criterion, due to the incentive constraints, the objective function reduces to:

$$\int_0^{\alpha_L^*} [v(x_l) - \alpha]\, dF(\alpha) + \int_{\alpha_L^*}^{\infty} v(x_u)\, dF(\alpha).$$

The latter is the same as in Roemer's case, hence both problems yield the same solution. The reason being that the opportunity sets cannot cross in the second best.

To see part (g), observe that, since the policy can no longer depend on α, (8) reduces to

$$\widetilde{S}^C = \min\{u(x(w_L), \delta_L, \widetilde{\alpha}), u(x(w_H), \delta_H, \widetilde{\alpha})\},$$

where, for $Y = L$ or H, $\delta_Y = 1$ or 0 and $x(w_Y) = x_y$ if $\delta_Y = 1$ and $x(w_Y) = x_u$ if $\delta_Y = 0$. However, since (12) holds true, the first element in the set behind the min sign is always the smallest; the low skilled will always be the worst off and

$$\widetilde{S}^C = \min\{v(x_\ell) - \widetilde{\alpha}, v(x_u)\}. \tag{63}$$

If maximization of $v(x_\ell) - \widetilde{\alpha}$ yields a value $\alpha_L^* > \widetilde{\alpha}$, then $v(x_\ell) - \widetilde{\alpha} > v(x_\ell) - \alpha_L^* = v(x_u)$, and so objective function (63) was not maximized. To prevent this from occurring, we maximize $v(x_\ell) - \widetilde{\alpha}$ subject to the constraint that $\widetilde{\alpha} \geq \alpha_L^*$. The multiplier associated to this constraint is denoted by ξ.

To see part (h), note that the equivalent wages for the employed are equal to x_y ($y = h$ or l) and for the inactive $v^{-1}(v(x_u) + \alpha)$. The objective is to maximize the lowest equivalent wage. Consider the inactive. Since $v^{-1}(.)$ is an increasing function, the equivalent wage is lowest for those inactive having the lowest value for α; this are those with $\alpha = \alpha_L^*$. Hence the lowest value for the equivalent wage is $v^{-1}(v(x_u) + \alpha_L^*) = v^{-1}(v(x_\ell)) = x_\ell$.

Appendix D: proof of theorems 3 and 4

STEP 1: derivation of first order conditions

(a) Utilitarian planner:

The Lagrangean function is

$$
\begin{aligned}
\pounds &= \gamma \left[\int_0^{\alpha_L^*} \left(v\left(x_\ell\right) - \alpha \right) dF(\alpha) + v(x_u) \left(1 - F\left(\alpha_L^*\right) \right) \right] + \\
&+ (1-\gamma) \left[\int_0^{\alpha_H^*} (v(x_h) - \alpha) dF(\alpha) + v(x_u) \left(1 - F\left(\alpha_H^*\right) \right) \right] \\
&+ \lambda \left\{ \gamma \left[\left(w_L - x_\ell\right) F\left(\alpha_L^*\right) - x_u \left(1 - F\left(\alpha_L^*\right) \right) \right] + \right. \\
&+ (1-\gamma) \left[\left(w_H - x_h\right) F(\alpha_H^*) - x_u \left(1 - F\left(\alpha_H^*\right) \right) \right] - R \left. \right\} \\
&+ \mu_L \left[v(x_\ell) - \alpha_L^* - v(x_u) \right] + \mu_H \left[v(x_h) - \alpha_H^* - v(x_u) \right].
\end{aligned}
$$

The first order conditions are:

$$\gamma F(\alpha_L^*) \left[v'(x_\ell) - \lambda \right] = -\mu_L v'(x_\ell), \tag{64}$$

$$\left[\gamma(1 - F(\alpha_L^*)) + (1-\gamma)(1 - F(\alpha_H^*)) \right] \left[v'(x_u) - \lambda \right] = (\mu_L + \mu_H) v'(x_u), \tag{65}$$

$$(1-\gamma) F(\alpha_H^*)(v'(x_h) - \lambda) = -\mu_H v'(x_h), \tag{66}$$

$$\lambda \gamma f(\alpha_L^*)(w_L - x_\ell + x_u) - \mu_L = 0, \tag{67}$$

$$\lambda (1-\gamma) f(\alpha_H^*)(w_H - x_h + x_u) - \mu_H = 0. \tag{68}$$

(b) Boadway *et al.* planner:

The Lagrangean function is:

$$
\begin{aligned}
\pounds &= \gamma \left[\int_0^{\alpha_L^*} W\left(\alpha\right) \left[\left(v(x_\ell) - \alpha \right) \right] dF(\alpha) + \int_{\alpha_L^*}^{\infty} W\left(\alpha\right) v(x_u) dF(\alpha) \right] \\
&+ (1-\gamma) \left[\int_0^{\alpha_H^*} W\left(\alpha\right) \left[v(x_h) - \alpha \right] dF(\alpha) + \int_{\alpha_H^*}^{\infty} W(\alpha) v(x_u) dF(\alpha) \right] \\
&+ \lambda \left\{ \gamma \left[\left(w_L - x_\ell\right) F(\alpha_L^*) - x_u \left(1 - F\left(\alpha_L^*\right) \right) \right] + \right. \\
&+ (1-\gamma) \left[\left(w_H - x_h\right) F(\alpha_H^*) - x_u \left(1 - F\left(\alpha_H^*\right) \right) \right] - R \left. \right\} \\
&+ \mu_L \left[v(x_\ell) - \alpha_L^* - v(x_u) \right] + \mu_H \left[v(x_h) - \alpha_H^* - v(x_u) \right].
\end{aligned}
$$

The first order conditions are:

$$\gamma v'(x_\ell) \int_0^{\alpha_L^*} W\left(\alpha\right) dF(\alpha) - \lambda \gamma F(\alpha_L^*) = -\mu_L v'(x_\ell), \tag{69}$$

$$
\begin{aligned}
& v'\left(x_u\right) \left[\gamma \int_{\alpha_L^*}^{\infty} W\left(\alpha\right) dF\left(\alpha\right) + (1-\gamma) \int_{\alpha_H^*}^{\infty} W\left(\alpha\right) dF\left(\alpha\right) \right] \\
& -\lambda \left[\gamma \left(1 - F\left(\alpha_L^*\right) \right) + (1-\gamma) \left(1 - F\left(\alpha_H^*\right) \right) \right] = \left(\mu_L + \mu_H \right) v'\left(x_u\right),
\end{aligned} \tag{70}
$$

$$(1-\gamma) v'(x_h) \int_0^{\alpha_H^*} W\left(\alpha\right) dF(\alpha) - \lambda(1-\gamma) F(\alpha_H^*) = -\mu_H v'(x_h), \tag{71}$$

$$\lambda \gamma f(\alpha_L^*)(w_L - x_\ell + x_u) - \mu_L = 0, \tag{72}$$

$$\lambda (1-\gamma) f(\alpha_H^*)(w_H - x_h + x_u) - \mu_H = 0. \tag{73}$$

(c) Welfarist planner:

The Lagrangean function is:

$$\begin{aligned}
\pounds &= \gamma\left[\int_0^{\alpha_L^*}\Psi\left(v(x_\ell)-\alpha\right)dF(\alpha)+\Psi\left(v(x_u)\right)\left(1-F\left(\alpha_L^*\right)\right)\right] \\
&+(1-\gamma)\left[\int_0^{\alpha_H^*}\Psi\left(v(x_h)-\alpha\right)dF(\alpha)+\Psi\left(v(x_u)\right)\left(1-F\left(\alpha_H^*\right)\right)\right] \\
&+\lambda\left\{\gamma\left[\left(w_L-x_\ell\right)F(\alpha_L^*)-x_u\left(1-F\left(\alpha_L^*\right)\right)\right]+\right. \\
&\left.+(1-\gamma)\left[\left(w_H-x_h\right)F(\alpha_H^*)-x_u\left(1-F\left(\alpha_H^*\right)\right)\right]-R\right\} \\
&+\mu_L\left[v(x_\ell)-\alpha_L^*-v(x_u)\right]+\mu_H\left[v(x_h)-\alpha_H^*-v(x_u)\right].
\end{aligned}$$

The first order conditions are

$$\gamma v'\left(x_\ell\right)\int_0^{\alpha_L^*}\Psi'\left(v(x_\ell)-\alpha\right)dF\left(\alpha\right)-\lambda\gamma F\left(\alpha_L^*\right)=-\mu_L v'\left(x_\ell\right), \tag{74}$$

$$\begin{aligned}
&v'\left(x_u\right)\left[\gamma\int_{\alpha_L^*}^{\infty}\Psi'\left(v\left(x_u\right)\right)dF\left(\alpha\right)+\left(1-\gamma\right)\int_{\alpha_H^*}^{\infty}\Psi'\left(v\left(x_u\right)\right)dF\left(\alpha\right)\right] \\
&-\lambda\left[\gamma\left(1-F\left(\alpha_L^*\right)\right)+\left(1-\gamma\right)\left(1-F\left(\alpha_H^*\right)\right)\right]=\left(\mu_L+\mu_H\right)v'\left(x_u\right),
\end{aligned} \tag{75}$$

$$\begin{aligned}
\left(1-\gamma\right)v'(x_h)\textstyle\int_0^{\alpha_H^*}\Psi'\left(v\left(x_h\right)-\alpha\right)dF(\alpha)-\lambda(1-\gamma)F(\alpha_H^*) \\
=-\mu_H v'(x_h),
\end{aligned} \tag{76}$$

$$\lambda\gamma f(\alpha_L^*)(w_L-x_\ell+x_u)-\mu_L=0, \tag{77}$$

$$\lambda(1-\gamma)f(\alpha_H^*)(w_H-x_h+x_u)-\mu_H=0. \tag{78}$$

(d) Non-welfarist planner:

The Lagrangean function is:

$$\begin{aligned}
\pounds &= \gamma\left[\left(v\left(x_\ell\right)-\overline{\alpha}\right)F(\alpha_L^*)+v(x_u)\left(1-F(\alpha_L^*)\right)\right]+ \\
&+(1-\gamma)\left[(v(x_h)-\overline{\alpha})F(\alpha_H^*)+v(x_u)\left(1-F(\alpha_H^*)\right)\right] \\
&+\lambda\left\{\gamma\left[\left(w_L-x_\ell\right)F(\alpha_L^*)-x_u\left(1-F\left(\alpha_L^*\right)\right)\right]+\right. \\
&\left.+(1-\gamma)\left[\left(w_H-x_h\right)F(\alpha_H^*)-x_u\left(1-F\left(\alpha_H^*\right)\right)\right]-R\right\} \\
&+\mu_L\left[v(x_\ell)-\alpha_L^*-v(x_u)\right]+\mu_H\left[v(x_h)-\alpha_H^*-v(x_u)\right].
\end{aligned}$$

The first order conditions are:

$$\gamma F(\alpha_L^*)\left[v'(x_\ell)-\lambda\right]=-\mu_L v'(x_\ell), \tag{79}$$

$$\left[\gamma(1-F(\alpha_L^*))+(1-\gamma)(1-F(\alpha_H^*))\right]\left[v'(x_u)-\lambda\right]=(\mu_L+\mu_H)v'(x_u), \tag{80}$$

$$(1-\gamma)F(\alpha_H^*)(v'(x_h)-\lambda)=-\mu_H v'(x_h), \tag{81}$$

$$\gamma f\left(\alpha_L^*\right)\left[\alpha_L^*-\overline{\alpha}\right]+\lambda\gamma f(\alpha_L^*)(w_L-x_\ell+x_u)-\mu_L=0, \tag{82}$$

$$(1-\gamma) f(\alpha_H^*)[\alpha_H^* - \overline{\alpha}] + \lambda(1-\gamma) f(\alpha_H^*)(w_H - x_h + x_u) - \mu_H = 0. \tag{83}$$

(f) Roemer and Van de gaer planners

The Lagrangean function is

$$\begin{aligned}
&\pounds = \int_0^{\alpha_L^*} (v(x_\ell) - \alpha)\, dF(\alpha) + \int_{\alpha_L^*}^{\infty} v(x_u)\, dF(\alpha) + \\
&+\lambda \{\gamma [(w_L - x_\ell) F(\alpha_L^*) - x_u (1 - F(\alpha_L^*))] + \\
&+(1-\gamma) [(w_H - x_h) F(\alpha_H^*) - x_u (1 - F(\alpha_H^*))] - R\} \\
&+\mu_L [v(x_\ell) - \alpha_L^* - v(x_u)] + \mu_H [v(x_h) - \alpha_H^* - v(x_u)].
\end{aligned}$$

The first order conditions are:

$$F(\alpha_L^*)[v'(x_\ell) - \lambda\gamma] = -\mu_L v'(x_\ell), \tag{84}$$

$$(1 - F(\alpha_L^*))v'(x_u) - \lambda[\gamma(1 - F(\alpha_L^*)) + (1-\gamma)(1 - F(\alpha_H^*))] = (\mu_L + \mu_H) v'(x_u), \tag{85}$$

$$\lambda(1-\gamma) F(\alpha_H^*) = \mu_H v'(x_h), \tag{86}$$

$$\lambda\gamma f(\alpha_L^*)(w_L - x_\ell + x_u) - \mu_L = 0, \tag{87}$$

$$\lambda(1-\gamma) f(\alpha_H^*)(w_H - x_h + x_u) - \mu_H = 0. \tag{88}$$

(g) Conditional Equality planner:

The Lagrangean is:

$$\begin{aligned}
&\pounds = v(x_\ell) - \widetilde{\alpha} \\
&+\lambda \{\gamma [(w_L - x_\ell) F(\alpha_L^*) - x_u (1 - F(\alpha_L^*))] + \\
&+(1-\gamma) [(w_H - x_h) F(\alpha_H^*) - x_u (1 - F(\alpha_H^*))] - R\} \\
&+\mu_L [v(x_\ell) - \alpha_L^* - v(x_u)] + \mu_H [v(x_h) - \alpha_H^* - v(x_u)]. \\
&+\xi [\widetilde{\alpha} - \alpha_L^*]
\end{aligned}$$

The first order conditions are:

$$v'(x_\ell) - \lambda\gamma F(\alpha_L^*) = -\mu_L v'(x_\ell), \tag{89}$$

$$-[\gamma(1 - F(\alpha_L^*)) + (1-\gamma)(1 - F(\alpha_H^*))]\lambda = (\mu_L + \mu_H) v'(x_u), \tag{90}$$

$$-(1-\gamma) F(\alpha_H^*)\lambda = -\mu_H v'(x_h), \tag{91}$$

$$\lambda\gamma f(\alpha_L^*)(w_L - x_\ell + x_u) - \xi - \mu_L = 0, \tag{92}$$

$$\lambda(1-\gamma) f(\alpha_H^*)(w_H - x_h + x_u) - \mu_H = 0. \tag{93}$$

(h) Egalitarian Equivalent planner

The Lagrangean is:

$$\begin{aligned}
&\pounds = x_\ell \\
&+\lambda\left\{\gamma\left[(w_L - x_\ell)\,F(\alpha_L^*) - x_u\,(1 - F\,(\alpha_L^*))\right] + \right. \\
&\left.+(1-\gamma)\left[(w_H - x_h)\,F(\alpha_H^*) - x_u\,(1 - F\,(\alpha_H^*))\right] - R\right\} \\
&+\mu_L\left[v(x_\ell) - \alpha_L^* - v(x_u)\right] + \mu_H\left[v(x_h) - \alpha_H^* - v(x_u)\right].
\end{aligned}$$

The first order conditions are:

$$1 - \lambda\gamma F(\alpha_L^*) = -\mu_L v'(x_\ell), \tag{94}$$

$$-\left[\gamma(1 - F(\alpha_L^*)) + (1-\gamma)(1 - F(\alpha_H^*))\right]\lambda = (\mu_L + \mu_H) v'(x_u), \tag{95}$$

$$-(1-\gamma)F(\alpha_H^*)\lambda = -\mu_H v'(x_h), \tag{96}$$

$$\lambda\gamma f(\alpha_L^*)(w_L - x_\ell + x_u) - \mu_L = 0, \tag{97}$$

$$\lambda(1-\gamma) f(\alpha_H^*)(w_H - x_h + x_u) - \mu_H = 0. \tag{98}$$

STEP 2: proof of theorem 3

PART 1: both α_H^* and α_L^* are smaller than ∞.

As $\forall\alpha : f(\alpha) > 0$, all low-ability (resp. high-ability) people work means $\alpha_L^* \to \infty$ (resp. $\alpha_H^* \to \infty$) at the optimum. Since consumption levels are finite, from (11) and (resp. (13)), α_L^* and α_H^* cannot tend to ∞.

PART 2: for the Utilitarian, Welfarist, Boadway *et al.,* Roemer and Van de gaer planners we always have that $\alpha_L^* > 0$ and $\alpha_H^* > 0$. For a Non-welfarist planner it is possible that $\alpha_L^* = 0$ or $\alpha_H^* = \alpha_L^* = 0$. For a Conditional Equality planner, $\alpha_H^* \geq \alpha_L^* = \widetilde{\alpha}$ or $\alpha_H^* \geq \alpha_L^* > \widetilde{\alpha}$.

(a) Utilitarian planner:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (64) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $\mu_L \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, and so $0 \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, from which, since λ, γ and $f(0) > 0$, we have that $w_L - x_\ell + x_u \leq 0$. However, from (11) with $\alpha_L^* = 0$, we get $x_u = x_\ell$, and so the previous inequality becomes $w_L \leq 0$, which was excluded by assumption.

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (66) evaluated at $\alpha_H^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $\mu_H \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, and so $0 \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, from which, since $\lambda, 1-\gamma$ and $f(0) > 0$, we have that $w_H - x_h + x_u \leq 0$. However, from (13) with $\alpha_H^* = 0$, we get $x_u = x_h$, and so the previous inequality becomes $w_H \leq 0$, which was excluded by assumption.

(b) Welfarist planner:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (74) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $\mu_L \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, and so $0 \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, from which, since λ, γ and $f(0) > 0$, we have that $w_L - x_\ell + x_u \leq 0$. However, from (11) with $\alpha_L^* = 0$, we get $x_u = x_\ell$, and so the previous inequality becomes $w_L \leq 0$, which was excluded by assumption.

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (76) evaluated at $\alpha_H^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $\mu_H \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, and so $0 \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, from which, since $\lambda, 1-\gamma$ and $f(0) > 0$, we have that $w_H - x_h + x_u \leq 0$. However, from (13) with $\alpha_H^* = 0$, we get $x_u = x_h$, and so the previous inequality becomes $w_H \leq 0$, which was excluded by assumption.

(c) Boadway *et al.* planner:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (69) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $\mu_L \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, and so $0 \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, from which, since λ, γ and $f(0) > 0$, we have that $w_L - x_\ell + x_u \leq 0$. However, from (11) with $\alpha_L^* = 0$, we get $x_u = x_\ell$, and so the previous inequality becomes $w_L \leq 0$, which was excluded by assumption.

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (71) evaluated at $\alpha_H^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $\mu_H \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, and so $0 \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, from which, since $\lambda, 1-\gamma$ and $f(0) > 0$, we have that $w_H - x_h + x_u \leq 0$. However, from (13) with $\alpha_H^* = 0$, we get $x_u = x_h$, and so the previous inequality becomes $w_H \leq 0$, which was excluded by assumption.

(d) Non-welfarist planner:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (79) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $-\gamma f(0)\overline{\alpha} + \lambda\gamma f(0)(w_L - x_\ell + x_u) \leq 0$, and so a condition for $\alpha_L^* = 0$ to be optimal is that

$$\overline{\alpha} \geq \lambda(w_L - x_\ell + x_u).$$

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (81) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $-(1-\gamma) f(0)\overline{\alpha} + \lambda(1-\gamma) f(0)(w_H - x_h + x_u) \leq 0$, and so a condition for $\alpha_H^* = 0$ to be optimal is that

$$\overline{\alpha} \geq \lambda(w_H - x_h + x_u).$$

Whether these conditions hold true or not crucially depends on $\overline{\alpha}$ and the magnitude of λ, the shadow price of public funds. This shadow price crucially depends on the amount of external resources R that the economy has at its disposal. For an economy richly endowed with R, λ will be small and the inequalities can hold for moderate values of $\overline{\alpha}$.

If $\alpha_L^* = 0$, because of (11), we get $x_u = x_\ell$, and if then $\alpha_H^* > 0$, by (13), $x_h > x_u$. Note that, if $\alpha_H^* = 0$, then by (13), it must be the case that $x_u = x_h$. At the same time, (12) requires that $x_h \geq x_\ell$, such that $x_u \geq x_\ell$, but then none of the low-skilled wants to work, and so $\alpha_L^* = 0$ and nobody works. Hence we get $x_u = x_h = x_\ell$, which by virtue of the government budget constraint must be equal to $-R$.

(e) Roemer and Van de gaer planners:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (84) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $\mu_L \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, and so $0 \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, from which, since λ, γ and $f(0) > 0$, we have that $w_L - x_\ell + x_u \leq 0$. However, from (11) with $\alpha_L^* = 0$, we get $x_u = x_\ell$, and so the previous inequality becomes $w_L \leq 0$, which was excluded by assumption.

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (86) evaluated at $\alpha_H^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $\mu_H \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, and so $0 \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, from which, since $\lambda, 1-\gamma$ and $f(0) > 0$, we have that $w_H - x_h + x_u \leq 0$. However, from (13) with $\alpha_H^* = 0$, we get $x_u = x_h$, and so the previous inequality becomes $w_H \leq 0$, which was excluded by assumption.

(f) Conditional Equality planner:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (89) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $\mu_L + \xi \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, and so $\xi \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, from which, since λ, γ, ξ and $f(0) > 0$, we have that $w_L - x_\ell + x_u \leq 0$. However, from (11) with $\alpha_L^* = 0$, we get $x_u = x_\ell$, and so the previous inequality becomes $w_L \leq 0$, which was excluded by assumption.

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (91) evaluated at $\alpha_H^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $\mu_H \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, and so $0 \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, from which, since $\lambda, 1-\gamma$ and $f(0) > 0$, we have that $w_H - x_h + x_u \leq 0$. However, from (13) with $\alpha_H^* = 0$, we get $x_u = x_h$, and so the previous inequality becomes $w_H \leq 0$, which was excluded by assumption.

(g) Egalitarian Equivalent planner:

Suppose that $\alpha_L^* = 0$ is an optimum. Equality (94) evaluated at $\alpha_L^* = 0$ still holds, from which since $F(0) = 0$, $\mu_L = 1/v'(x_\ell)$. In addition, $\frac{\partial \pounds}{\partial \alpha_L}$ evaluated at $\alpha_L^* = 0$ must be non positive, from which $\mu_L \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, and so $1/v'(x_\ell) \geq \lambda\gamma f(0)(w_L - x_\ell + x_u)$, from which, since $\lambda, \gamma, f(0)$ and $v'(x_\ell) > 0$, we have that $w_L - x_\ell + x_u \leq 0$. However, from (11) with $\alpha_L^* = 0$, we get $x_u = x_\ell$, and so the previous inequality becomes $w_L \leq 0$, which was excluded by assumption.

Suppose that $\alpha_H^* = 0$ is an optimum. Equality (96) evaluated at $\alpha_H^* = 0$ still holds, from which since $F(0) = 0$, $\mu_H = 0$. In addition, $\frac{\partial \pounds}{\partial \alpha_H}$ evaluated at $\alpha_H^* = 0$ must be non positive, from which $\mu_H \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, and so $0 \geq \lambda(1-\gamma) f(0)(w_H - x_h + x_u)$, from which, since $\lambda, 1-\gamma$ and $f(0) > 0$, we have that $w_H - x_h + x_u \leq 0$. However, from (13) with $\alpha_H^* = 0$, we get $x_u = x_h$, and so the previous inequality becomes $w_H \leq 0$, which was excluded by assumption.

PART 3: at interior solutions, with Utilitarian, Welfarist, Boadway *et al.* and Non-welfarist planners: $\alpha_H^* > \alpha_L^*$.

We know from (14) that $\alpha_H^* \geq \alpha_L^*$. We prove that $\alpha_H^* > \alpha_L^*$ by contradiction for the criteria listed above.

(a) Utilitarian planner:

Suppose $\alpha_H^* = \alpha_L^* = \alpha^*$ and $x_\ell = x_h = x$ is optimal. Solving (64) and (67) for μ_L and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - 1\right] = \lambda f(\alpha^*)\left[w_L - x + x_u\right]. \tag{99}$$

Similarly solving (66) and (68) for μ_H and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - 1\right] = \lambda f(\alpha^*)\left[w_H - x + x_u\right]. \tag{100}$$

However, both (99) and (100) can only hold true if $w_H = w_L$, which was excluded by assumption.

(b) Boadway *et al.* planner:

Suppose $\alpha_H^* = \alpha_L^* = \alpha^*$ and $x_\ell = x_h = x$ is optimal. Solving (69) and (72) for μ_L and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - \frac{\int_0^{\alpha^*} W(\alpha)\, dF(\alpha)}{F(\alpha^*)}\right] = \lambda f(\alpha^*)\left[w_L - x + x_u\right]. \tag{101}$$

Similarly solving (71) and (73) for μ_H and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - \frac{\int_0^{\alpha^*} W(\alpha)\, dF(\alpha)}{F(\alpha^*)}\right] = \lambda f(\alpha^*)\left[w_H - x + x_u\right]. \tag{102}$$

However, both (101) and (102) can only hold true if $w_H = w_L$, which was excluded by assumption.

(c) Welfarist planner:

Suppose $\alpha_H^* = \alpha_L^* = \alpha^*$ and $x_\ell = x_h = x$ is optimal. Solving (74) and (77) for μ_L and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - \frac{\int_0^{\alpha^*} \Psi'(v(x) - \alpha)\, dF(\alpha)}{F(\alpha^*)}\right] = \lambda f(\alpha^*)\left[w_L - x + x_u\right]. \tag{103}$$

Similarly solving (76) and (78) for μ_H and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - \frac{\int_0^{\alpha^*} \Psi'(v(x) - \alpha)\, dF(\alpha)}{F(\alpha^*)}\right] = \lambda f(\alpha^*)\left[w_H - x + x_u\right]. \tag{104}$$

However, both (103) and (104) can only hold true if $w_H = w_L$, which was excluded by assumption.

(d) Non-welfarist planner:

Suppose $\alpha_H^* = \alpha_L^* = \alpha^*$ and $x_\ell = x_h = x$ is optimal. Solving (79) and (82) for μ_L and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - 1\right] = \lambda f(\alpha^*)\left[w_L - x + x_u\right] + f(\alpha^*)\left[\alpha^* - \overline{\alpha}\right]. \tag{105}$$

Similarly solving (81) and (83) for μ_H and equating the resulting expressions yields that

$$F(\alpha^*)\left[\frac{\lambda}{v'(x)} - 1\right] = \lambda f(\alpha^*)\left[w_H - x + x_u\right] + f(\alpha^*)\left[\alpha^* - \overline{\alpha}\right]. \tag{106}$$

However, both (105) and (106) can only hold true if $w_H = w_L$, which was excluded by assumption.

STEP 3: proof of theorem 4

(a) Utilitarian planner:

Combining (64) and (67), we get

$$\gamma F(\alpha_L^*)\left[v'(x_\ell) - \lambda\right] = -\lambda\gamma f(\alpha_L^*)\left[w_L - x_\ell + x_u\right] v'(x_\ell). \tag{107}$$

Using (20), this reduces to

$$\frac{1}{\eta(x_\ell, \alpha_L^*)}\left[1 - \frac{v'(x_\ell)}{\lambda}\right] = \frac{w_L - x_\ell + x_u}{x_\ell}. \tag{108}$$

Combining (66) and (68), we get

$$(1-\gamma)F(\alpha_H^*)(v'(x_h) - \lambda) = -\lambda(1-\gamma)f(\alpha_H^*)\left[w_H - x_h + x_u\right] v'(x_h). \tag{109}$$

Using (21), this reduces to

$$\frac{1}{\eta(x_h, \alpha_H^*)}\left[1 - \frac{v'(x_h)}{\lambda}\right] = \frac{w_H - x_h + x_u}{x_h}. \tag{110}$$

Dividing equations (64)-(66) by the marginal utilities on the right-hand side and adding, we see that the inverse of the shadow price of the public funds equals the average of the inverse of the marginal utilities of consumption:

$$\lambda^{-1} = \frac{\gamma F(\alpha_L^*)}{v'(x_\ell)} + \frac{\gamma(1 - F(\alpha_L^*)) + (1-\gamma)(1 - F(\alpha_H^*))}{v'(x_u)} + \frac{(1-\gamma)F(\alpha_H^*)}{v'(x_h)} \tag{111}$$

This expression is similar to Diamond and Sheshinsky (1995)'s equation (6), p.6. $x_h > x_\ell > x_u$ (Theorem 3) and the fact that a weighted average with positive weights is bounded by its least and greatest elements ensure that

$$v'(x_u) > \lambda > v'(x_h) \tag{112}$$

(b) Boadway *et al.* planner:

Combining (69) and (72), we get

$$\begin{aligned} &\gamma v'(x_\ell)\textstyle\int_0^{\alpha_L^*} W(\alpha)\, dF(\alpha) - \lambda\gamma F(\alpha_L^*) \\ &= -\lambda\gamma f(\alpha_L^*)\left[w_L - x_\ell + x_u\right] v'(x_\ell). \end{aligned} \tag{113}$$

Using (20), this reduces to

$$\frac{1}{\eta\left(x_\ell,\alpha_L^*\right)}\left[1-\frac{v'\left(x_\ell\right)}{\lambda}\frac{\int_0^{\alpha_L^*} W\left(\alpha\right)dF(\alpha)}{F(\alpha_L^*)}\right]=\frac{w_L-x_\ell+x_u}{x_\ell}. \tag{114}$$

Combining (71) and (73), we get

$$\begin{array}{c}(1-\gamma)v'(x_h)\int_0^{\alpha_H^*} W\left(\alpha\right)dF(\alpha)-\lambda(1-\gamma)F(\alpha_H^*)\\ =-\lambda(1-\gamma)f(\alpha_H^*)\left[w_H-x_h+x_u\right]v'(x_h).\end{array} \tag{115}$$

Using (21), this reduces to

$$\frac{1}{\eta\left(x_h,\alpha_H^*\right)}\left[1-\frac{v'\left(x_h\right)}{\lambda}\frac{\int_0^{\alpha_H^*} W\left(\alpha\right)dF(\alpha)}{F(\alpha_H^*)}\right]=\frac{w_H-x_h+x_u}{x_h}. \tag{116}$$

Dividing equations (69)-(71) by the marginal utilities on the right-hand side and adding, we obtain that the inverse of the shadow price of the public funds

$$\lambda^{-1}=\frac{\frac{\gamma F(\alpha_L^*)}{v'(x_\ell)}+\frac{\gamma(1-F(\alpha_L^*))+(1-\gamma)(1-F(\alpha_H^*))}{v'(x_u)}+\frac{(1-\gamma)F(\alpha_H^*)}{v'(x_h)}}{\int_0^\infty W\left(\alpha\right)dF\left(\alpha\right)}. \tag{117}$$

(c) Welfarist planner:

Combining (74) and (77), we get

$$\begin{array}{c}\gamma v'(x_\ell)\int_0^{\alpha_L^*}\Psi'\left(v\left(x_\ell\right)-\alpha\right)dF(\alpha)-\lambda\gamma F(\alpha_L^*)\\ =-\lambda\gamma f\left(\alpha_L^*\right)\left[w_L-x_\ell+x_u\right]v'(x_\ell).\end{array} \tag{118}$$

Using (20), this reduces to

$$\frac{1}{\eta\left(x_\ell,\alpha_L^*\right)}\left[1-\frac{v'\left(x_\ell\right)}{\lambda}\frac{\int_0^{\alpha_L^*}\Psi'\left(v\left(x_\ell\right)-\alpha\right)dF(\alpha)}{F(\alpha_L^*)}\right]=\frac{w_L-x_\ell+x_u}{x_\ell}. \tag{119}$$

Combining (76) and (78), we get

$$\begin{array}{c}(1-\gamma)v'(x_h)\int_0^{\alpha_H^*}\Psi'\left(v\left(x_h\right)-\alpha\right)dF(\alpha)-\lambda(1-\gamma)F(\alpha_H^*)\\ =-\lambda(1-\gamma)f(\alpha_H^*)\left[w_H-x_h+x_u\right]v'(x_h).\end{array} \tag{120}$$

Using (21), this reduces to

$$\frac{1}{\eta\left(x_h,\alpha_H^*\right)}\left[1-\frac{v'\left(x_h\right)}{\lambda}\frac{\int_0^{\alpha_H^*}\Psi'\left(v\left(x_h\right)-\alpha\right)dF(\alpha)}{F(\alpha_H^*)}\right]=\frac{w_H-x_h+x_u}{x_h}. \tag{121}$$

Dividing equations (74)-(76) by the marginal utilities on the right-hand side and adding, we obtain that the inverse of the shadow price of the public funds

$$\lambda^{-1}=\frac{\frac{\gamma F(\alpha_L^*)}{v'(x_\ell)}+\frac{\gamma(1-F(\alpha_L^*))+(1-\gamma)(1-F(\alpha_H^*))}{v'(x_u)}+\frac{(1-\gamma)F(\alpha_H^*)}{v'(x_h)}}{D}, \tag{122}$$

where

$$\begin{aligned}D &= \gamma\left[\int_0^{\alpha_L^*}\Psi'\left(v\left(x_\ell\right)-\alpha\right)dF(\alpha)+\int_{\alpha_L^*}^\infty\Psi'\left(v\left(x_u\right)\right)dF\left(\alpha\right)\right]\\ &\quad+(1-\gamma)\left[\int_0^{\alpha_H^*}\Psi'\left(v\left(x_h\right)-\alpha\right)dF(\alpha)+\int_{\alpha_H^*}^\infty\Psi'\left(v\left(x_u\right)\right)dF\left(\alpha\right)\right].\end{aligned}$$

(d) Non-welfarist planner:

Combining (79) and (82), we get

$$\gamma F(\alpha_L^*)\left[v'(x_\ell)-\lambda\right]=-\lambda\gamma f\left(\alpha_L^*\right)\left[\frac{\alpha_L^*-\overline{\alpha}}{\lambda}+w_L-x_\ell+x_u\right]v'(x_\ell). \tag{123}$$

Using (20), this reduces to

$$\frac{1}{\eta\left(x_\ell,\alpha_L^*\right)}\left[1-\frac{v'\left(x_\ell\right)}{\lambda}\right]-\frac{\alpha_L^*-\overline{\alpha}}{\lambda x_\ell}=\frac{w_L-x_\ell+x_u}{x_\ell}. \tag{124}$$

Combining (81) and (83), we get

$$\begin{aligned}(1-\gamma)F(\alpha_H^*)(v'(x_h)-\lambda)\\=-\lambda(1-\gamma)f(\alpha_H^*)\left[\tfrac{\alpha_H^*-\overline{\alpha}}{\lambda}+w_H-x_h+x_u\right]v'(x_h).\end{aligned} \tag{125}$$

Using (21), this reduces to

$$\frac{1}{\eta\left(x_h,\alpha_H^*\right)}\left[1-\frac{v'\left(x_h\right)}{\lambda}\right]-\frac{\alpha_H^*-\overline{\alpha}}{\lambda x_h}=\frac{w_H-x_h+x_u}{x_h}. \tag{126}$$

Note that the first three first order conditions (79)-(81) are the same as in the Utilitarian case, (64)-(66). Hence we obtain the same expression for the inverse of the Lagrangean multiplier, i.e. (111).

(e) Roemer and Van de gaer planners:

Combining (84) and (87), we get

$$F(\alpha_L^*)\left[v'(x_\ell)-\lambda\gamma\right]=-\lambda\gamma f\left(\alpha_L^*\right)\left[w_L-x_\ell+x_u\right]v'(x_\ell)$$

Using (20), this reduces to

$$\frac{1}{\eta\left(x_\ell,\alpha_L^*\right)}\left[1-g_L^R\right]=\frac{w_L-x_\ell+x_u}{x_\ell}.$$

Combining (86) and (88), we get

$$F(\alpha_H^*)=f(\alpha_H^*)\left[w_H-x_h+x_u\right]v'(x_h).$$

Using (21), this reduces to

$$\frac{1}{\eta\left(x_h,\alpha_H^*\right)}=\frac{w_H-x_h+x_u}{x_h}.$$

Dividing equations (84)-(86) by the marginal utilities on the right-hand side and adding, we see that the inverse of the shadow price of the public funds equals the average of the inverse of the marginal utilities of consumption. Hence we obtain the same expression for the inverse of the Lagrangean multiplier as under the utilitarian and the non-welfarist criteria, i.e. expression (111).

(f) Conditional Equality planner:

Combining (89) and (92), we get

$$v'(x_\ell) - \lambda\gamma F(\alpha_L^*) = -\left[\lambda\gamma f(\alpha_L^*)(w_L - x_\ell + x_u) - \xi\right] v'(x_\ell),$$

Using (20), this reduces to

$$\frac{1}{\eta\left(x_\ell, \alpha_L^*\right)}\left(1 - \frac{(1-\xi)\, v'(x_\ell)}{\lambda\gamma F(\alpha_L^*)}\right) = \frac{w_L - x_\ell + x_u}{x_\ell}$$

Combining (91) and (93), we get

$$F(\alpha_H^*) = f(\alpha_H^*)(w_H - x_h + x_u)v'(x_h),$$

Using (21), this reduces to

$$\frac{1}{\eta\left(x_h, \alpha_H^*\right)} = \frac{w_H - x_h + x_u}{x_h}$$

Dividing equations (89)-(91) by the marginal utilities on the right-hand side and adding, we see that the inverse of the shadow price of the public funds equals the average of the inverse of the marginal utilities of consumption. Hence we obtain the same expression for the inverse of the Lagrangean multiplier as under the utilitarian, Roemer and Van de gaer and the non-welfarist criteria, i.e. expression (111).

(g) Egalitarian Equivalent planner

Combining (94) and (97), we get

$$1 - \lambda\gamma F(\alpha_L^*) = -\left[\lambda\gamma f(\alpha_L^*)(w_L - x_\ell + x_u)\right] v'(x_\ell),$$

Using (20), this reduces to

$$\frac{1}{\eta\left(x_\ell, \alpha_L^*\right)}\left(1 - \frac{1}{\lambda\gamma F(\alpha_L^*)}\right) = \frac{w_L - x_\ell + x_u}{x_\ell}$$

Combining (96) and (98), we get

$$F(\alpha_H^*) = f(\alpha_H^*)(w_H - x_h + x_u)v'(x_h),$$

Using (21), this reduces to

$$\frac{1}{\eta\left(x_h, \alpha_H^*\right)} = \frac{w_H - x_h + x_u}{x_h}$$

Dividing equations (94)-(96) by the marginal utilities on the right-hand side and adding, we obtain again expression (111) for the inverse of the shadow price of the public funds.

Appendix E: interpretation of second best

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Note that

$$\frac{\int_0^{\alpha_L^*} W(\alpha)\, dF(\alpha)}{F(\alpha_L^*)} \geq (\leq) \frac{\int_0^{\alpha_H^*} W(\alpha)\, dF(\alpha)}{F(\alpha_H^*)} \Leftrightarrow$$
$$\frac{\int_0^{\alpha_L^*} W(\alpha)\, dF(\alpha)}{F(\alpha_L^*)} \geq (\leq) \frac{\int_0^{\alpha_L^*} W(\alpha)\, dF(\alpha)}{F(\alpha_L^*)} \frac{F(\alpha_L^*)}{F(\alpha_H^*)} + \frac{\int_{\alpha_L^*}^{\alpha_H^*} W(\alpha)\, dF(\alpha)}{F(\alpha_H^*)} \Leftrightarrow$$
$$\frac{\int_0^{\alpha_L^*} W(\alpha)\, dF(\alpha)}{F(\alpha_L^*)} \left[1 - \frac{F(\alpha_L^*)}{F(\alpha_H^*)}\right] \geq (\leq) \frac{\int_{\alpha_L^*}^{\alpha_H^*} W(\alpha)\, dF(\alpha)}{F(\alpha_H^*)} \Leftrightarrow$$
$$\frac{\int_0^{\alpha_L^*} W(\alpha)\, dF(\alpha)}{F(\alpha_L^*)} \geq (\leq) \frac{\int_{\alpha_L^*}^{\alpha_H^*} W(\alpha)\, dF(\alpha)}{F(\alpha_H^*) - F(\alpha_L^*)},$$

which holds as $\geq$ automatically if $W(\alpha)$ is a decreasing function, and as $\leq$ if $W(\alpha)$ is an increasing function.

Appendix F: proof of theorem 5

From (22), (25), (28), (34) and (40) it follows immediately that when the average marginal social utility of the low-skilled workers is $\geq (\leq) 1$, then $T_\ell < (>) T_u$. Moreover, $w_L = x_\ell - x_u$ implies no distortion of the labor supply, compared to laissez faire.

Under a Non-welfarist criterion the result follows from observing from (31) that

$$T_\ell < (>) T_u \Leftrightarrow \frac{1}{\eta(x_\ell, \alpha_L^*)} \left[1 - \frac{v'(x_\ell)}{\lambda}\right] - \frac{\alpha_L^* - \overline{\alpha}}{\lambda x_\ell} < (>) 0$$

Appendix G: proof of corollary 2

(i) Utilitarian optimum.

We know from Theorem 3 that $x_h > x_\ell$, such that $v'(x_h) < v'(x_\ell)$ and $g_H^U < g_L^U$. Combined with $\eta(x_\ell, \alpha_L^*) > \eta(x_h, \alpha_H^*)$, the right hand side (22) is smaller than the right hand side of (23) such that $(T_\ell - T_u)/x_\ell < (T_h - T_u)/x_h$ at the Utilitarian optimum.

(ii) Welfarist optimum.

Since $x_h > x_\ell$, $v(x_\ell) - \alpha < v(x_h) - \alpha$ and since $\Psi'' < 0$, $\Psi'(v(x_\ell) - \alpha_1) > \Psi'(v(x_h) - \alpha_1) > \Psi'(v(x_h) - \alpha_2)$ when $\alpha_2 > \alpha_1$, such that $g_H^W < g_L^W$. Combined with $\eta(x_\ell, \alpha_L^*) > \eta(x_h, \alpha_H^*)$, it follows that the right hand side (25) is smaller than the right hand side of (26). Therefore the left hand sides of these equations can be ranked as follows $(T_\ell - T_u)/x_\ell < (T_h - T_u)/x_h$.

(iii) Boadway *et al.* optimum.

Assume that $W(\alpha)$ is a decreasing function, $g_L^B > g_H^B$. Following the same proof as in the Utilitarian case, it can easily be shown that the right hand side (28) is smaller than the right hand side of (29). Therefore the left hand sides of these equations can be ranked as follows $(T_\ell - T_u)/x_\ell < (T_h - T_u)/x_h$.

(iv) Roemer and Van de gaer optimum.

The result follows immediately from $g_L^R > 0$, $\eta(x_\ell, \alpha_L^*) > \eta(x_h, \alpha_H^*)$, (34) and (35).

(v) Conditional Equality optimum.

The result follows immediately from $g_L^C > 0$, $\eta(x_\ell, \alpha_L^*) > \eta(x_h, \alpha_H^*)$, (37) and (38).

(vi) Egalitarian Equivalent optimum:

The result follows immediately from $g_L^E > 0$, $\eta(x_\ell, \alpha_L^*) > \eta(x_h, \alpha_H^*)$, (40) and (41).

Appendix H: proof of theorem 6

By definition, $\frac{T_\ell - T_u}{x_\ell} < \frac{T_h - T_u}{x_h} \Leftrightarrow \frac{w_L - x_\ell + x_u}{x_\ell} < \frac{w_H - x + x_u}{x_h}$. Therefore under assumption A1, from corollary 2, we have that for the planners considered in the corollary, $x_h(w_L - x_\ell + x_u) < x_\ell(w_H - x_h + x_u)$. Since $x_h \geq x_\ell$ (from Theorem 3), we have: $w_L - x_\ell + x_u < w_H - x_h + x_u$.