## Econ 435/835: Development Economics Assignment 1

Due Date: Wednesday, 2<sup>nd</sup> October 2013.

**1.** Consider the basic Solow growth model with a production technology in which aggregate output (income)  $Y_t$ , is given by:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

where  $K_t$  represents the physical capital stock and  $L_t$  represents labour (the population of adults), and  $\alpha \in (0, 1)$ .

Suppose in this model, we now introduce an unproductive government which taxes income at the rate  $\tau$  i.e. government income is  $G_t = \tau Y_t$ . It consumes this amount entirely without any benefit for the country. As in the standard model, a fraction  $s_K$  of disposable income gets invested to enhance the capital stock, which depreciates at the rate  $\delta$ .

 $L_t$  grows at the exogenously given rate of n.

(i) Write this model in terms of output per labor i.e. in terms of  $y_t = Y_t/L_t$ , and derive the differential equation characterizing the dynamic path for the evolution of  $k_t$ . Obtain the steady state values of k and y. How are these affected by the level of the tax rate  $\tau$ ?

(ii) Suppose the government was instead productive i.e. uses its income  $G_t$  to provide infrastructure, law and order etc. Now the aggregate output (income)  $Y_t$ , is given by:

$$Y_t = G_t^\beta K_t^\alpha L_t^{1-\alpha-\beta}$$

with  $\alpha + \beta < 1$ . Again derive the differential equation characterizing the dynamic path for the evolution of  $k_t$ . Obtain the steady state values of k and y. How are these affected by the level of the tax rate  $\tau$ ? Is there an optimal tax-rate that will maximize the steady state level of output in this model? What is it?

(iii) [For MA students only] In (ii), can you derive an equation characterizing the dynamics around the steady state. In particular, what is the rate of convergence? 2. [The Penn World Tables are a very useful source of data for development economics (and nearly all of the papers we have studied so far make use of it). This question is designed to get you to start using them too.] Go to the website for the Penn World Tables:

http://pwt.econ.upenn.edu/php\_site/pwt\_index.php

This contains data on 188 countries for the period 1950-2010, with 2005 as the base year. The website also contains instructions on how to download data into an Excel format (which can also be read by Stata), and also an Appendix containing explanations about the variables.

The notion of convergence suggests that poorer countries should grow faster and thus in the long run all countries should catch up. Barro (1991) looks at this issue by plotting the log of per capita GDP in 1960 (X-axis) against the growth rate of per capita GDP over the period 1960-1985 (Y-axis) and finds little evidence of convergence for the world as a whole.

(i) Let us try to do this same exercise for a different period. From the Penn World Tables, obtain data on real per capita GDP (Chain series) for all possible countries in 1970 and 2010, and then do the same plot as Barro. Do you see evidence for convergence? [Note that some countries maybe missing data for their GDP in 1970 or 2010. Eliminate those countries from your sample]. Also run a regression and report the coefficient on log of per capita GDP in 1970. Is it significant?

(ii) What about if you restricted the plot to OECD countries (presumably, a group of "similar" countries)? (As part of the exercise, check up which countries are in the OECD). What about if you looked only at countries in Africa (again presumably, a group of "similar" countries)? Do you see evidence for convergence in either of these sub-samples? Show both the plots and the regression coefficients.

(iii) Has the world become more unequal? A first pass at this question would be to compare the variance in GDP per capita across countries in 1970 and in 2010. What do you find?

(iv) Economists have conditioned on many different variables in an attempt to investigate the convergence question and analyze the importance of these variables in the growth regressions. In this spirit, let us consider the following variables from the Penn World Tables: countries' population growth over the period 1970-2010,  $n_i$ , average degree of openness  $(openk_i)$  over this period, and government's consumption share of PPP converted GDP per capita  $(kg_i)$  over this period. Include these variables in your regression of growth rate of per capita GDP over the period 1970-2010 on 1970 log GDP per capita. What do you find?

(v) [For MA students only] On the lines of Mankiw, Romer and Weil (1992), attempt to estimate the basic Solow model (not augmented for growth in human capital) with the above data for the period 1970-2010. Are the coefficients as expected? What is the estimate for  $\alpha$ ? Do the exercise also for the OECD countries only.