**Economics 250 Mid-Term Test 2**
21 March 2014

Instructions: You may use an approved hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

**1.** Suppose that you believe there is a 55% chance of a political party winning any individual seat in an election.

(a) If 6 seats are contested, what is the chance that the party wins at least half? What is the chance the party wins all the seats?

(b) If 50 seats are contested, what is the chance that the party wins at least half?

**2.** Suppose that house prices in Vancouver are normally distributed with mean $\mu$ and standard deviation 20. Suppose that you collect a sample of 36 prices with an average value of 400.

(a) Find a 90% confidence interval for the population mean.

(b) Suppose that you would like the margin of error to be no larger than 4. How large must the sample size be?

(c) If you cannot increase the sample size to the value you found in part (b), what would the confidence level be to give a margin of error equal to 4 with the original sample size?

**3.** An economist is studying the properties of an asset price $x$. Suppose the economist knows that $x \sim N(\mu, 3)$, so that the standard deviation is 3. She studies a random sample of 16 observations and finds an average price 2. She wishes to test the null hypothesis that $\mu = 0$ against the alternative hypothesis that $\mu > 0$.

(a) Suppose that the significance level is pre-set at $\alpha = 2.5\%$. Find the critical value for the test, labelled $\overline{x}_c$. Does the economist reject the null at this level of significance?

(b) Unbeknownst to the economist, the true value of $\mu$ is 1. Find the probability of type II error and the power of the test that was adopted in part (a).

**4.** Researchers study a sample of 16 firms. Exports by these firms have an average value of 10 before a currency depreciation occurs. These exports have an average value of 12 after a depreciation occurs. When the researchers track the change in exports for each firm, they find that that change has a standard deviation of 2.

(a) Find a 90% confidence interval for the mean change in exports.

(b) Test the null hypothesis that the mean change in exports is 3.301 against the alternative that it is not equal to 3.301, by reporting the $P$-value.

**Economics 250 Midterm Test 2: Answer Guide**

**1.** (a) Three or more successes is the same thing as three or fewer failures. The probability of failing win a seat is 0.45. From table C, the probability of 3 or fewer losses is 0.7448 or 74.48%. So that is the probabi9lity of 3 or more wins.

And the probability of winning 6 is the probability of losing 0 which is 0.0277 or 2.77%.

(b) Using the normal approximation means that the number of successes,

$$X \sim N(27.5, \sqrt{12.375}).$$

Thus our $z$-statistic is

$$z = \frac{25 - 27.5}{3.5178} = -0.71067.$$

From table A there is probability 0.2389 below -0.71 or 76.11% above that point. So that is the probability of at least 25 wins.

**2.** (a) The confidence interval is

$$400 \pm 1.645 \frac{20}{6} = 400 \pm 5.48 = (394.52, 405.48).$$

(b) The margin of error is:

$$4 = 1.645 \frac{20}{\sqrt{n}},$$

so $\sqrt{n} = 8.225$ and so $n = 68$ is the smallest integer that gives a margin of error this small.

(b) Now

$$4 = z \frac{20}{6},$$

so $z = 1.2$ which has a cumulative probability of 0.8849. That leaves 1-0.8849 = 0.1151 in each tail so this is a 76.98% confidence interval.

**3.** (a) The one-sided critical value for $\alpha = 0.025$ is:

$$1.96\frac{3}{\sqrt{16}} = 1.47.$$

Because $2 > 1.47$, she rejects the null at this level of significance.

(b) Under this alternative the distribution is centred at 1 with standard deviation $\sigma/\sqrt{n} = 3/4 = 0.75$. The critical value is 1.47. Standardizing implies that critical value gives a $z$-value of 0.62666. From Table A I'll use the closest value which is 0.63 (you may be more precise with your calculator perhaps) giving a cumulative probability of 0.7357. That means there is a probability of type II error of 73.57% and a test power of 26.43%. (The alternative is quite close to the null, relative to the standard error, so it is difficult to distinguish them.)

**4.** (a) This is a paired difference with a mean of 2 and standard deviation of 2, as well as $n = 16$. With $df = 15$ from Table D the relevant $t$-statistic is 1.753. Thus the confidence interval is

$$2 \pm 1.753\frac{2}{4} = 2 \pm 0.8765 = (1.1235, 2.8765).$$

(b) The relevant $t$-statistic is:

$$t = \frac{2 - 3.301}{2/4} = -2.602.$$

From Table D, with $df = 15$ that leaves 1% probability in the lower tail. The $P$-value for a two-tailed test is the probability of finding a statistic this far from zero in absolute value, so the $P$-value is 0.02 or 2%.