Economics 250 Mid-Term Test 2
21 March 2013

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that taking a course before taking an aptitude test raises any student's score on the test (relative to their score on an earlier attempt) with probability $p = 0.6$.

(a) If 10 people take the course, then what is the probability that at least 5 of them experience raised scores?

(b) If 100 people take the test then what is the probability that at least 50 of them experience raised scores?

2. A financial economist believes that the exchange rate between the Canadian and US dollars is normally distributed with mean μ and variance $\sigma^2 = 0.0016$ (and so standard deviation $\sigma = 0.04$). She constructs the average exchange rate over a period of $n = 25$ trading days. From this one sample she finds an average exchange rate of 0.98.

(a) Find a 90% confidence interval for the population mean μ.

(b) Suppose that she wants the confidence interval to be no wider than 0.01 (*i.e.* with a total width of 0.01 or a margin of error half that). What confidence level is required to give this interval?

3. A demographer wishes to test the null hypothesis that average life expectancy in Japan is 80 years against the alternative hypothesis that it is greater than 80 years. Suppose that he knows that the standard deviation of life expectancy in the population is $\sigma = 3$ years. He studies a sample of $n = 9$ people.

(a) State the null and alternative hpotheses. Is this a two-tailed, upper one-tailed, or lower one-tailed test?

(b) If he chooses a significance level of $\alpha = 0.04$ then what is the critical value of the sample average life expectancy that divides outcomes into a reject region and a do-not-reject region?

(c) What is the power of this test against the specific alternative that the population average life expectancy is 82 years?

4. A researcher grades individuals' happiness X on a scale from 0 to 5. She interviews 6 people and finds an average grade of $\overline{X} = 3$ with a sample standard deviation of $s = 1$.

(a) Find a 95% confidence interval for the unknown, population average grade on this scale.

(b) The researcher wishes to test the null hypothesis that the average grade is 2 against the alternative hypothesis that it is greater than 2. Given the sample information, find a range within which the P-value must lie.

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1. (a) Five or more successes is the same thing as five or fewer failures. The probability of failing to raise one's score is 0.4. From table C, this is 0.8337.

(b) Using the normal approximation means that the number of successes, $X \sim N(60, 24)$. Thus our z-statistic is

$$z = \frac{50 - 60}{4.898} = -2.04.$$

From table A there is probability 0.0207 below that point or 97.93% above that point. So that is the probability of at least 50 improvements.

2. (a) The 90% CI is

$$\overline{x} \pm z^* \sigma/\sqrt{n} = 0.98 \pm 1.645(0.04/5) = 0.98 \pm 0.001316 = (0.96684, 0.99316)$$

(a bit more rounding is appropriate too).

(b) The margin of error is:

$$m = 0.005 = z^*(0.008),$$

so $z^* = 0.625$. From table A that is mid-way between two cumulative probability values (0.7324 and 0.7357) that average to 0.73405. Subtracting this from 1 gives the area in the right tail which is 0.26595. That means the area in two tails is 0.5319 so the area in the middle is 0.4681 and this is a 46.81% confidence interval.

3. (a)

$$H_0 : \mu = 80$$

$$H_A : \mu > 80$$

This is an upper one-tailed test.

(b) This is an upper, one-tailed test with σ known, so to leave 4% in the upper tail means $z = 1.75$. The standard deviation of the sample mean is 3/3 =1, so moving 1.75 standard deviations up from the null value gives a critical value of 81.75 years.

(c) If the true value is 82, then the power is the area above 81.75 assuming this alternative hypothesis is true. The z value that divides this new distribution is:

$$z = \frac{81.75 - 82}{1} = -0.25.$$

From table A that leaves 0.4013 to the left or .5987 to the right. Thus the power of the test is 59.87%.

4. (a) With $df = n - 1 = 5$ the critical value for t from table D is 2.571. Thus the CI is

$$3 \pm 2.571 \frac{1}{\sqrt{6}} = 3 \pm 1.0496 = (1.9504, 4.0496),$$

though you do not need all this precision.

(b) The t-statistic is simply:

$$t = \frac{3 - 2}{1/2.449} = 2.449,$$

with 5 degrees of freedom. From table D, then, the P-value is less than 5% but greater than 2.5%.