Economics 250 Mid-Term Test 2
22 March 2012

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that a political party that you support has a 40% chance of winning any seat in an election.

(a) If 10 seats are being contested, what is the probability that your party wins 4 or more seats? What is the probability that they win 5 or more seats?

(b) If 100 seats are being contested, what is the probability that your party wins 40 or more seats? What is the probability that they win 50 or more seats?

2. Suppose that a blood test for athletes gives a numerical reading x and that $x > 4$ is considered a positive test for a banned drug.

(a) For those not taking the drug, x is continuously and uniformly distributed between 0 and 5. What is the probability of a positive test for them?

(b) For those who are taking the drug, x is continuously and uniformly distributed between 3 and 9. What is the probability of a positive test for them?

(c) Suppose that you believe 10% of athletes are taking the drug. If you test an athlete and find $x > 4$ what is the probability she or he is actually taking the drug?

3. A stress test of 100 European banks shows that half of them (*i.e.* a proportion 0.5) have enough capital.

(a) Find a 95% confidence interval for the population proportion.

(b) Test the null hypothesis that the population proportion is $P = 0.6$ against the alternative hypothesis that $P < 0.6$, with $\alpha = 0.10$. What is the prob-value (also known as the p-value) for your test statistic?

4. A researcher is studying historical data on human heights. She finds a sample of 20 people with an average height of 154 cm and a sample standard deviation of 8 cm. Assume the population distribution is normal.

(a) Find a 90% confidence interval for the population's average height.

(b) Perform a two-sided test (at the 10% level of significance) of the null hypothesis that the average height in the population is 158 cm.

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1. (a) This is a binomial problem with $n = 10$ and $P = 0.4$. From table 3 the probability of 3 or fewer seats is 0.382 so the probability of 4 or more is 61.8%. Similarly, the probability of 5 or more seats is 36.7%.

(b) With $n = 100$ we use the normal approximation to the binomial density: $x \sim N(40, 24)$. You can see that the mean is 40 so the probability of 40 or more seats is 0.5. As for 50 or more seats, standardizing gives:

$$z = \frac{50 - 40}{4.8989} = 2.04.$$

From table 1, there is 2.07% probability of 50 or more seats.

2. (a) The probability of a positive test is 1/5 or 0.20.

(b) The probability of a positive test is 5/6 or 0.833.

(c) The joint probability of taking the drug and testing positive is the product of the conditional and marginal probabilities: $0.10(0.833) = 0.0833$. The marginal probability of testing positive is $0.0833 + 0.2(0.9) = 0.2633$. So the probability of taking the drug conditional on a positive test is $0.0833/0.2633 = 0.316 = 31.6\%$.

3. (a) The 95% confidence interval is:

$$0.5 \pm 1.96\sqrt{0.5)(0.5)/100} = 0.5 \pm 0.098 = (0.402, 0.598)$$

(b) Our test statistic is:

$$z = \frac{0.5 - 0.6}{0.0489} = -2.0449.$$

(Note importantly that we use the value of P under the null to find the standard error.) Table 1 shows the critical value is -1.28 so we reject the null hypothesis. The p-value (from table 1) is 0.02 or 2%. There is only a 2% chance of finding a sample value this low when the null hypothesis is true.

4. (a) The sample mean is $\overline{x} = 154$. The sample standard deviation is $s = 8$. The cutoff points for the t-distribution with 19 df are ± 1.729; that leaves 5% in each tail. Thus the confidence interval is:

$$154 \pm 1.729(8/4.472) = 154 \pm 3.09 = (150.91, 157.09)$$

(b) Our t-statistic is:

$$t = \frac{154 - 158}{1.789} = -2.23.$$

With $df = 19$ the critical values are ± 1.729 with $\alpha = 0.10$. The test statistic is outside these values, so we can reject the null hypothesis.